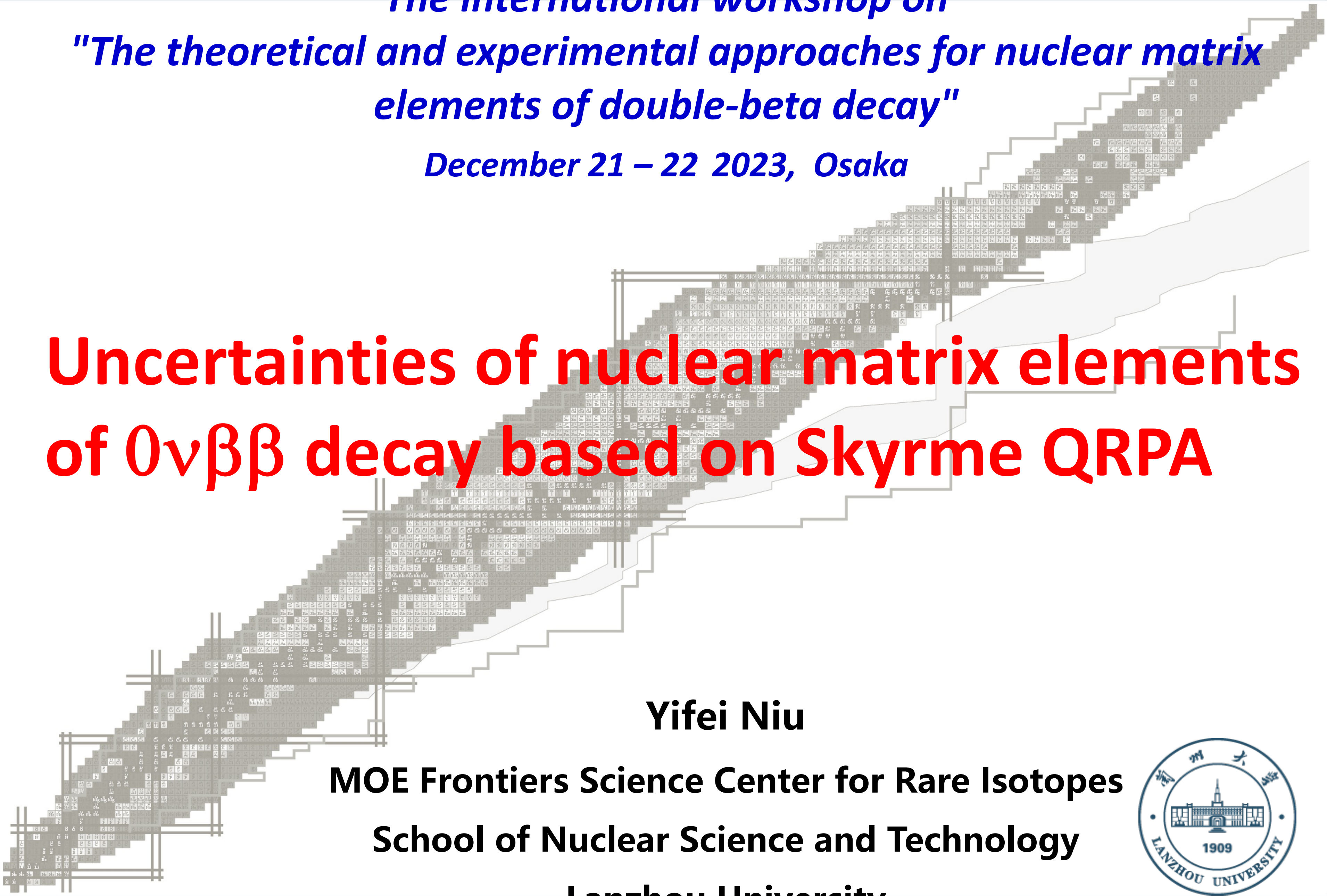
The international workshop on
"The theoretical and experimental approaches for nuclear matrix elements of double-beta decay"
December 21 – 22 2023, Osaka



Uncertainties of nuclear matrix elements of $0\nu\beta\beta$ decay based on Skyrme QRPA

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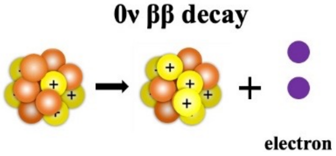
MOE Frontiers Science Center for Rare Isotopes
School of Nuclear Science and Technology
Lanzhou University

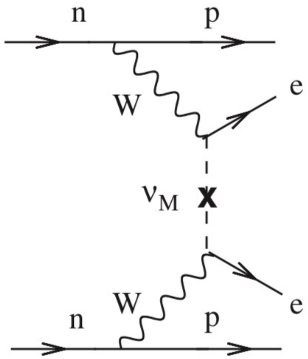
Outline

- **Introduction**
- **Theoretical Framework**
- **Uncertainties from pairing interactions**
- **Summary and Perspective**

Neutrinoless double beta decay

☐ 0νββ decay

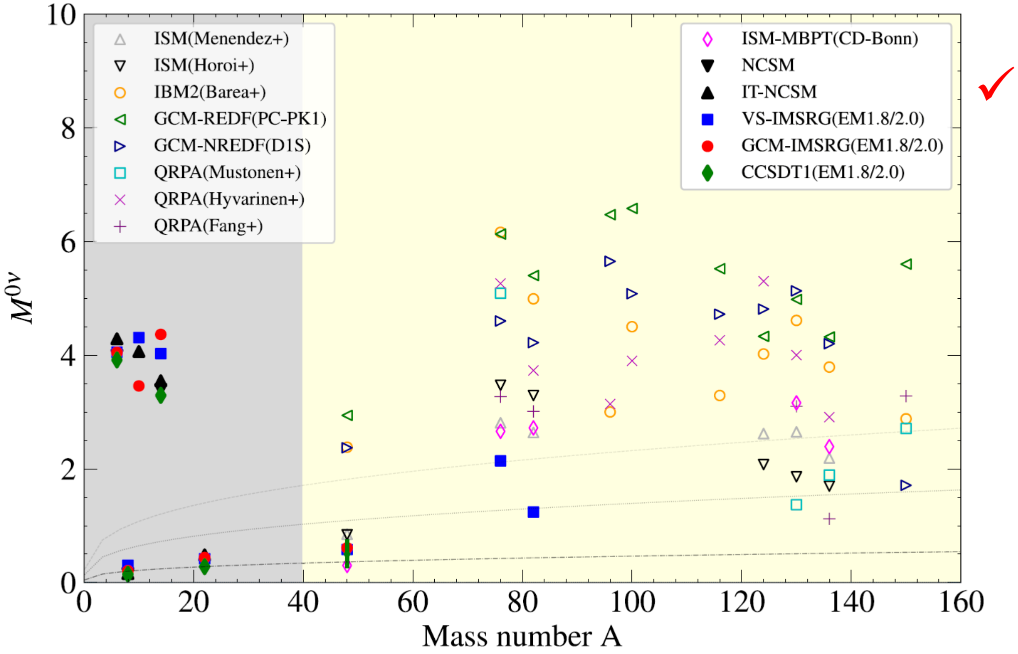




- Neutrino

  Majorana or Dirac nature?
- Neutrino Mass
- Lepton number conservation

F. T. Avignone, et al., Rev. Mod. Phys. 80, 481 (2008)



✓ Challenge to nuclear physicists:

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G_{0\nu}|M^{0\nu}|^2 m^2_{\beta\beta}$$

Discrepancies of nuclear matrix elements (NMEs) obtained by different nuclear models are large!

J. M. Yao, J. Meng, Y. F. Niu, and P. Ring, Prog. Phys. Nucl. Phys. 126, 103965 (2022)

Uncertainties of NMEs

- ☐ *To understand the discrepancies, great efforts have been made to analyze the uncertainties of NMEs*

  - **Uncertainty sources**
    - ✓ *The axial-vector coupling constant g_A* : g_A=1 or 1.25
    - ✓ *The two-nucleon short-range correlations (s.r.c.)* : *UCOM / Jastrow*
    - ✓ *The higher order terms of the nucleon current* : *weak- magnetism and pseudoscalar couplings*
    - ✓ *The finite size of the nucleon*: *nucleon form factors*
    - ✓ *The size of the model space*: 2/3/4 *oscillator shells (QRPA)*
    - ✓ *The closure approximation*

    …

  - **These uncertainties were studied within**
    - ✓ Quasiparticle Random Phase Approximation (QRPA)

      F. Šimkovic et al. Phys. Rev. C 60, 055502 (1999) *V. A. Rodin et al. Nucl. Phys. A 766, 107 (2006)*

      F. Šimkovic et al. Phys. Rev. C 77, 045503 (2008)

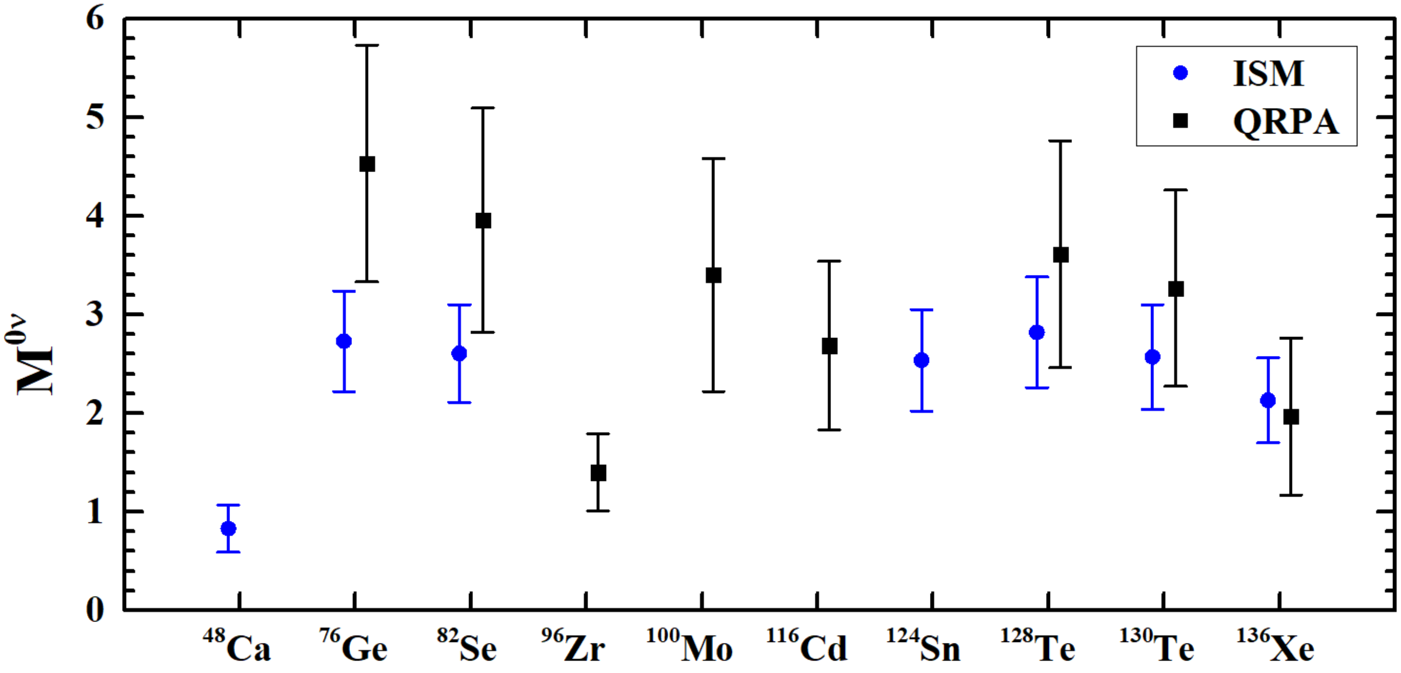
    - ✓ Interacting Shell Model (ISM)

      E. Caurier et al. Phys. Rev. Lett. 052503, 100 (2008) *J. Menéndez et al. Nucl. Phys. A 818, 139 (2009)*

Uncertainties of NMEs

- *To understand the discrepancies, great efforts have been made to analyze the uncertainties of NMEs*

  - The NMEs of QRPA and ISM with error bar evaluated from those uncertainties



data from *J. Menéndez et al. Nucl. Phys. A 818, 139 (2009)* *for shell model*

F. Šimkovic et al. Phys. Rev. C 77, 045503 (2008) *for QRPA*

Uncertainties of NMEs: Nuclear interactions (ph)

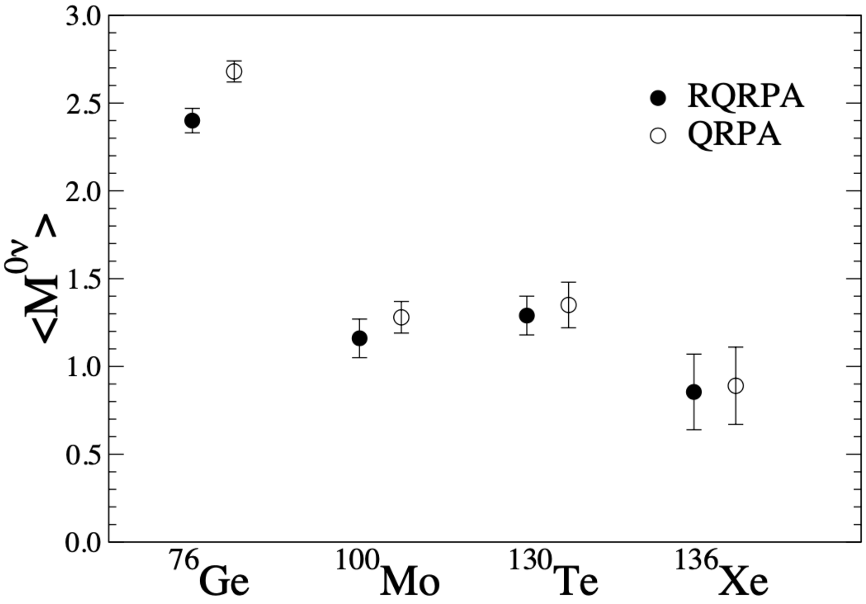
- **Uncertainty sources: nuclear interactions --- particle-hole channel**

  - *G*-matrix QRPA:

    Mean field: Coulomb corrected Woods-Saxon potential

    Residual interaction: Bonn, Argonne, Nijmegen renormalized by Brückner *G* matrix

  - Averaged over three potentials and three choices of the s.p. space



✓ The strength of the particle-particle interaction is adjusted so that the $2\nu\beta\beta$ decay rate is correctly reproduced

✓ $M^{0\nu}$ values are essentially independent of the form of different realistic *NN* potentials.

V. A. Rodin, F. Šimkovic , et al. Phys. Rev. C 68, 044302 (2003)

Uncertainties of NMEs: Nuclear interactions (ph)

- **Uncertainty sources: nuclear interactions --- particle-hole channel**

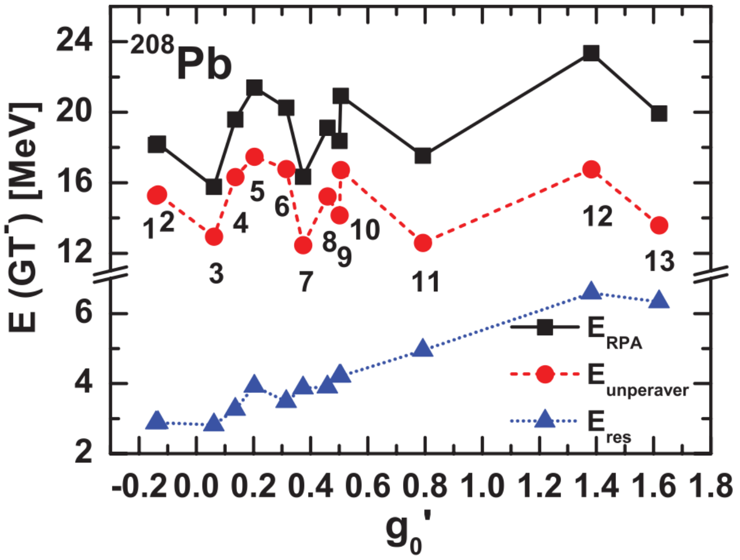
➢ Self-consistent QRPA:

Mean field, residual interaction: same interaction from energy density functionals

✓ Particle-hole (ph) channel

Hundreds of Skyrme interactions:

- Nucleon effective mass m^* : single-particle level density near Fermi level
- Landau parameter g_0' : the strength of spin-isospin part of nuclear interactions



Y. F. Niu et al. Phys. Rev. C 85, 034313 (2012)

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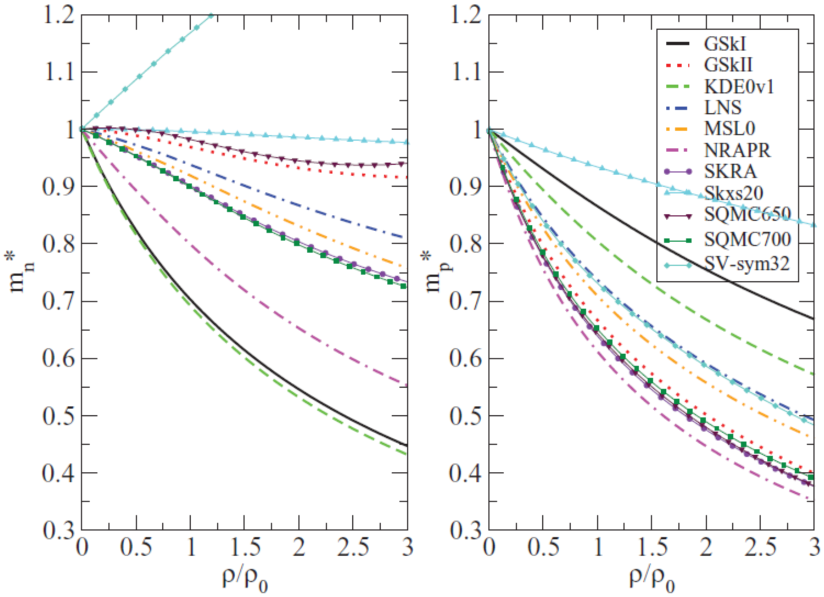

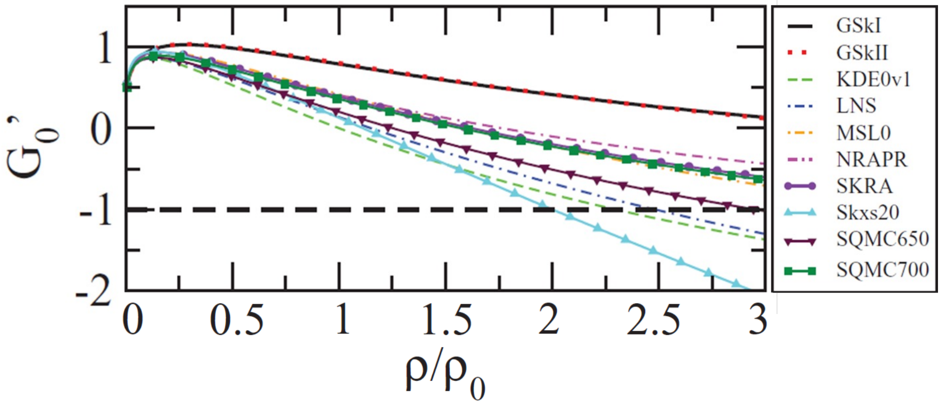

M. Dutra et al. Phys. Rev. C 85, 035201 (2012)

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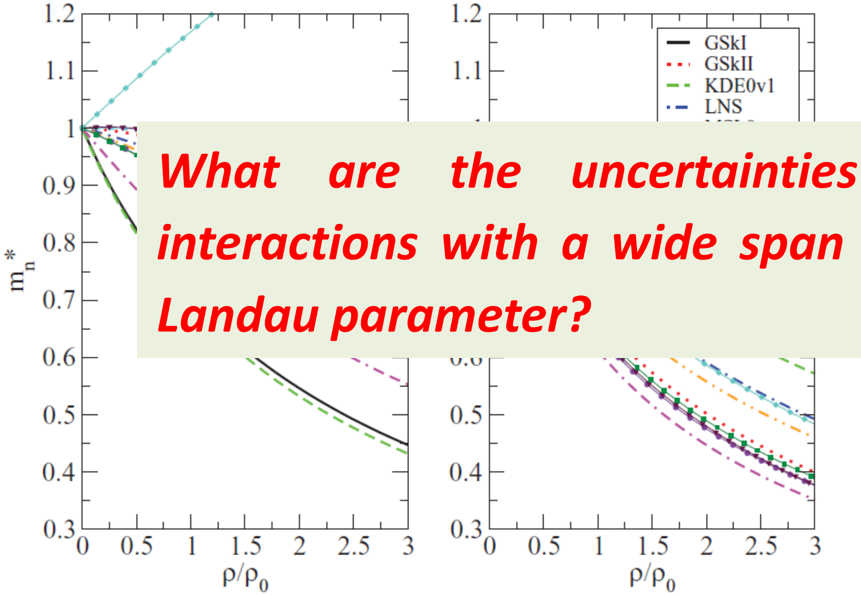
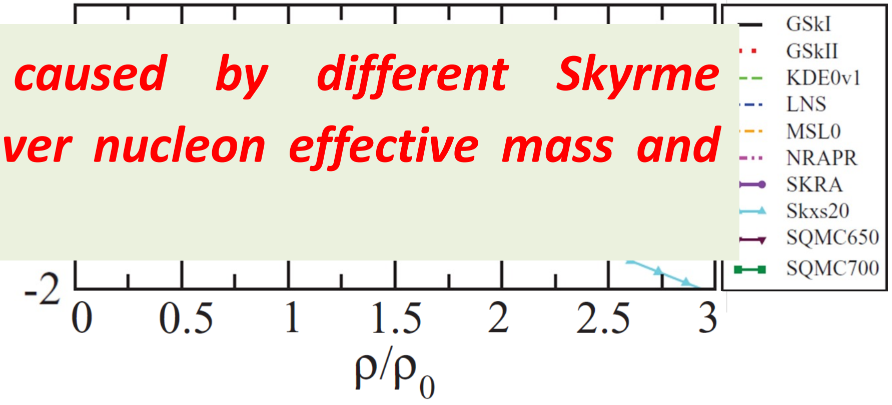


What are the uncertainties caused by different Skyrme interactions with a wide span over nucleon effective mass and Landau parameter?

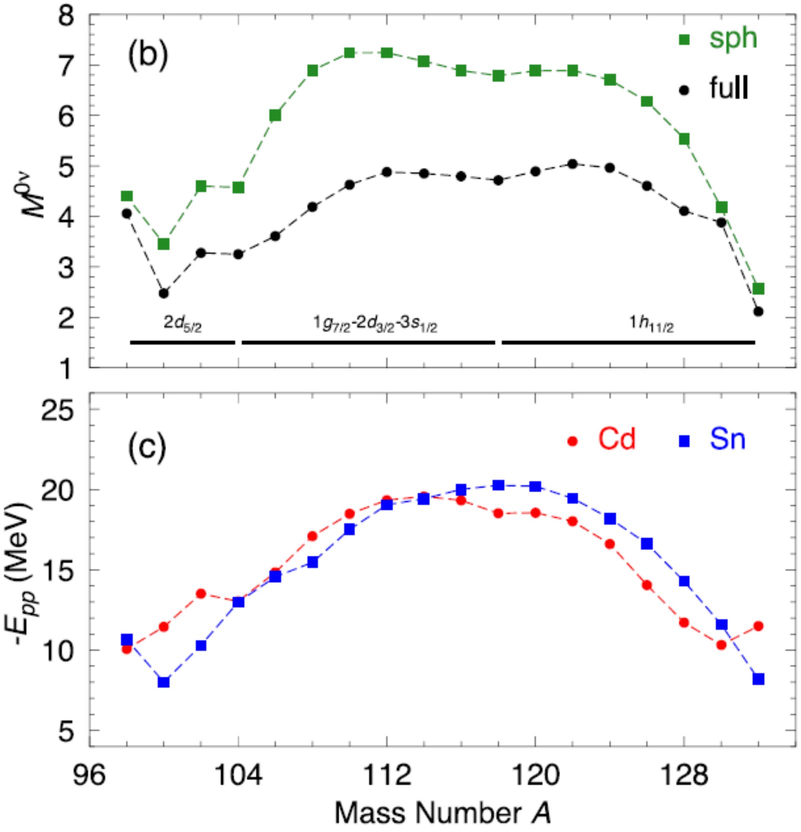


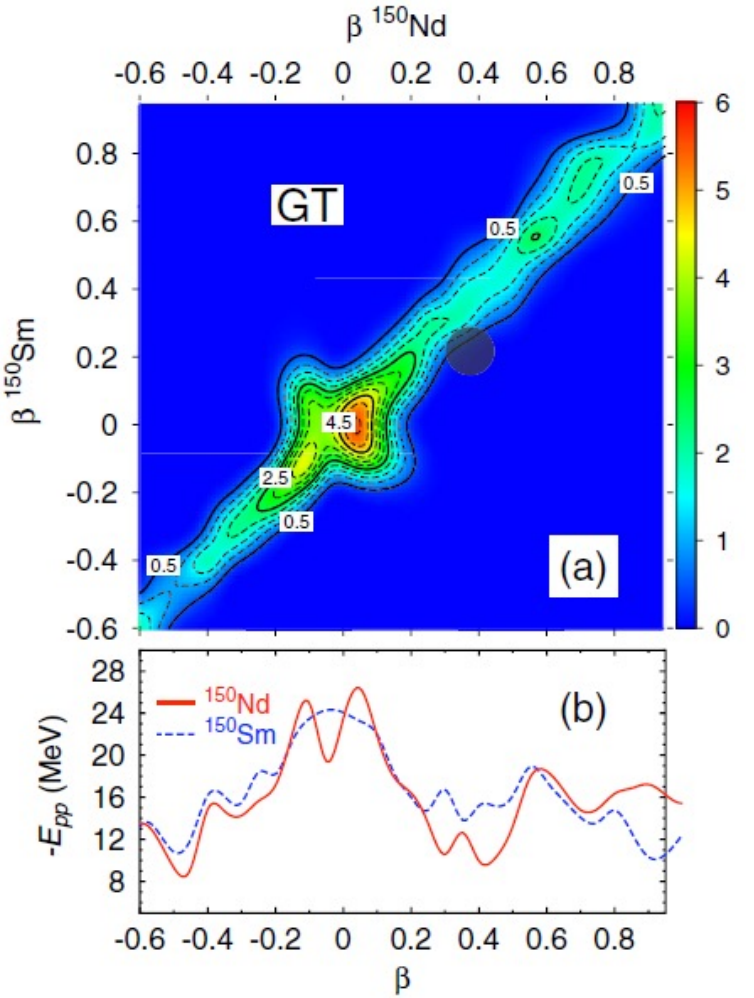
M. Dutra et al. Phys. Rev. C 85, 035201 (2012)

Uncertainties of NMEs: Nuclear interactions (pp)

- **Uncertainty sources: nuclear interactions --- particle-particle channel**

✓ Correlation between pairing energy and $M^{0\nu}$ studied by GCM with Gogny interaction





T. R. Rodríguez et al. Phys. Lett. B 719, 174 (2013); Phys. Rev. Lett. 105, 252503 (2010)

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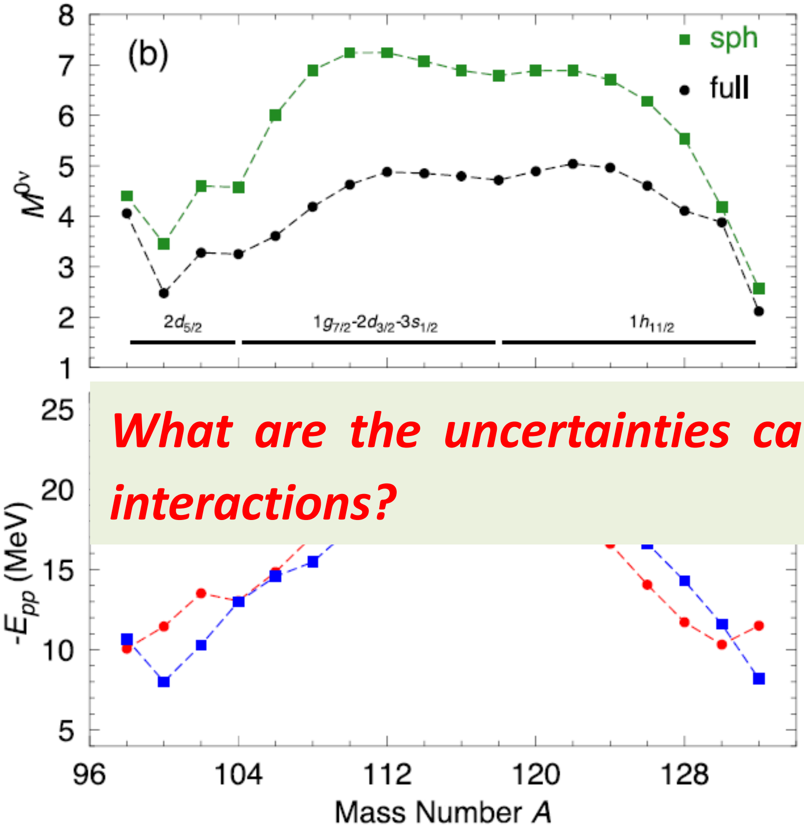

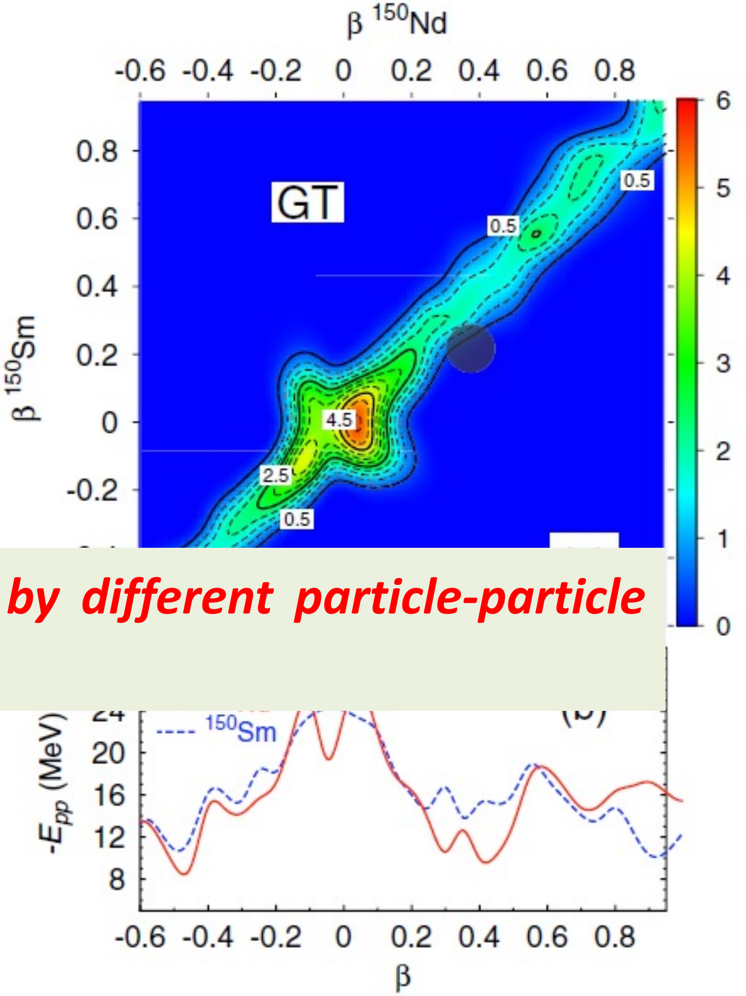

What are the uncertainties caused by different particle-particle interactions?

T. R. Rodríguez et al. Phys. Lett. B 719, 174 (2013); Phys. Rev. Lett. 105, 252503 (2010)

Motivation

❑ *In order to study the uncertainties caused by particle-hole channel and particle-particle channel of nuclear effective interaction, we need self-consistent QRPA models with large variety of different interactions*

- Self-consistent QRPA for $M^{0\nu}$ and $M^{2\nu}$
  - ✓ Spherical Skyrme QRPA *J. Terasaki, Phys. Rev. C 86, 021301(R) (2012); Phys. Rev. C 102, 044303 (2020)*
  - ✓ Axially deformed Skyrme QRPA (matrix diagonalization / finite amplitude method)
    M. T. Mustonen and J. Engel, Phys. Rev. C 87, 064302 (2013)
    N. Hinohara and J. Engel, Phys. Rev. C 105, 044314 (2022)
  - ✓ Spherical relativistic QRPA *N. Popara, A. Ravlić, and N. Paar, Phys. Rev. C 105, 064315 (2022)*

Uncertainties from nuclear effective interactions are not discussed so far

➢ In this work:

With self-consistent Skyrme QRPA, we study the NMEs for ^{76}Ge, ^{82}Se, ^{128}Te, ^{130}Te, and ^{136}Xe. The uncertainties from nuclear effective interaction will be emphasized.

- ph channel: 18 Skyrme interactions
- pp channel: 2 kinds of pairing forces

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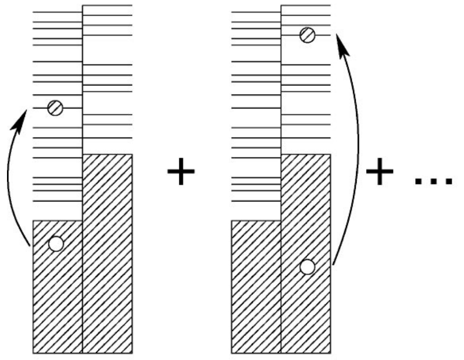
NME calculated by QRPA

- ❑ **QRPA:** widely used for the description of spin-isospin excitations
  - **The QRPA excited state is generated by**

$$Q_\nu^\dagger = \sum_{mi} X^\nu_{mi} \alpha^\dagger_m \alpha^\dagger_i - Y^\nu_{mi} \alpha_i \alpha_m$$

- ✓ **Full 2 quasiparticle configuration space ⇒ almost whole nuclear chart**

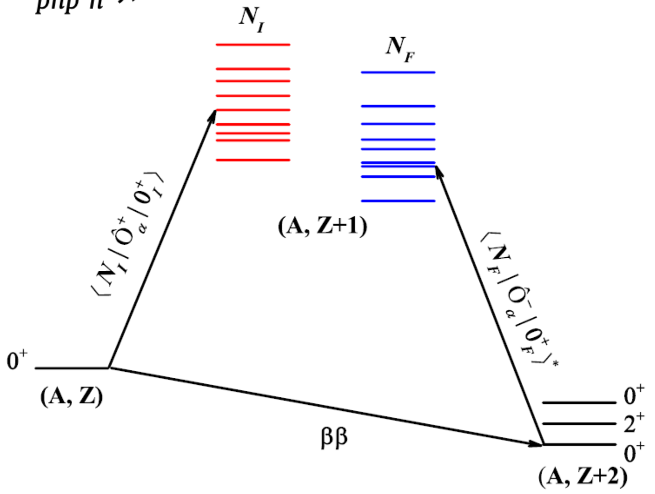


RPA

- ❑ **NME** $M^{0\nu} \equiv -M^{0\nu}_{\rm F} + M^{0\nu}_{\rm GT} + M^{0\nu}_{\rm T}$

$$M^{0\nu} = \frac{8R_0}{g_A^2(0)} \sum_{N_F N_I} \sum_{pnp'n'} \langle N_F | c_n^\dagger c_p | 0_F^+ \rangle \boxed{\langle N_F | N_I \rangle} \langle N_I | c_{p'}^\dagger c_{n'} | 0_I^+ \rangle (K^{\rm F}_{pnp'n'} + K^{\rm GT}_{pnp'n'}),$$

overlap factor



$$K^\alpha_{pnp'n'} = \int dqq \sum_{LM} \frac{h_\alpha(q^2)}{q + E_N - (E_I + E_F)/2} \langle n | \mathcal{O}^-_\alpha | p \rangle^* \langle p' | \mathcal{O}^+_\alpha | n' \rangle$$

$$\mathcal{O}^\pm_{\rm F} = j_L(qr) Y_{LM}(\hat{\mathbf{r}}) \tau^\pm,$$
$$\mathcal{O}^\pm_{\rm GT} = j_L(qr) Y_{LM}(\hat{\mathbf{r}}) \boldsymbol{\sigma} \tau^\pm.$$

induced current

$$h_F(\boldsymbol{q}^2) = -g_V^2 \qquad h_{GT}(\boldsymbol{q}^2) = g_A^2 - \boxed{g_A g_P \frac{\boldsymbol{q}^2}{3m_p} + g_P^2 \frac{\boldsymbol{q}^4}{12m_p^2} + g_M^2 \frac{\boldsymbol{q}^2}{6m_p^2}}$$

$$g_A = 1.27$$

Nuclear effective interaction

- ph channel: Skyrme interaction

$$\begin{aligned}V^{ph}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \; & t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}) + \frac{1}{2}t_1(1+x_1P_\sigma)\left[\boldsymbol{P}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{P}^2\right] \\ & + t_2(1+x_2P_\sigma)\boldsymbol{P}'\cdot\delta(\boldsymbol{r})\boldsymbol{P} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\boldsymbol{R})\delta(\boldsymbol{r}) \\ & + iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[\boldsymbol{P}'\times\delta(\boldsymbol{r})\boldsymbol{P}]\end{aligned}$$

- pp channel: δ interaction

$$V^{pp}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \left[t_0' + \frac{t_3'}{6}\rho(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2})\right]\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$

i) Volume pairing (the pairing field follows the shape of the density), $t_3' = 0$.

ii) Surface pairing (the pairing field is peaked at the surface and follows roughly the variations of the density), $t_3' = -37.5t_0'$.

The pairing strengths are determined by fitting the experimental pairing gap.

K. Bennaceur, J. Dobaczewski, Comput. Phys. Comm. 168, 96 (2005)

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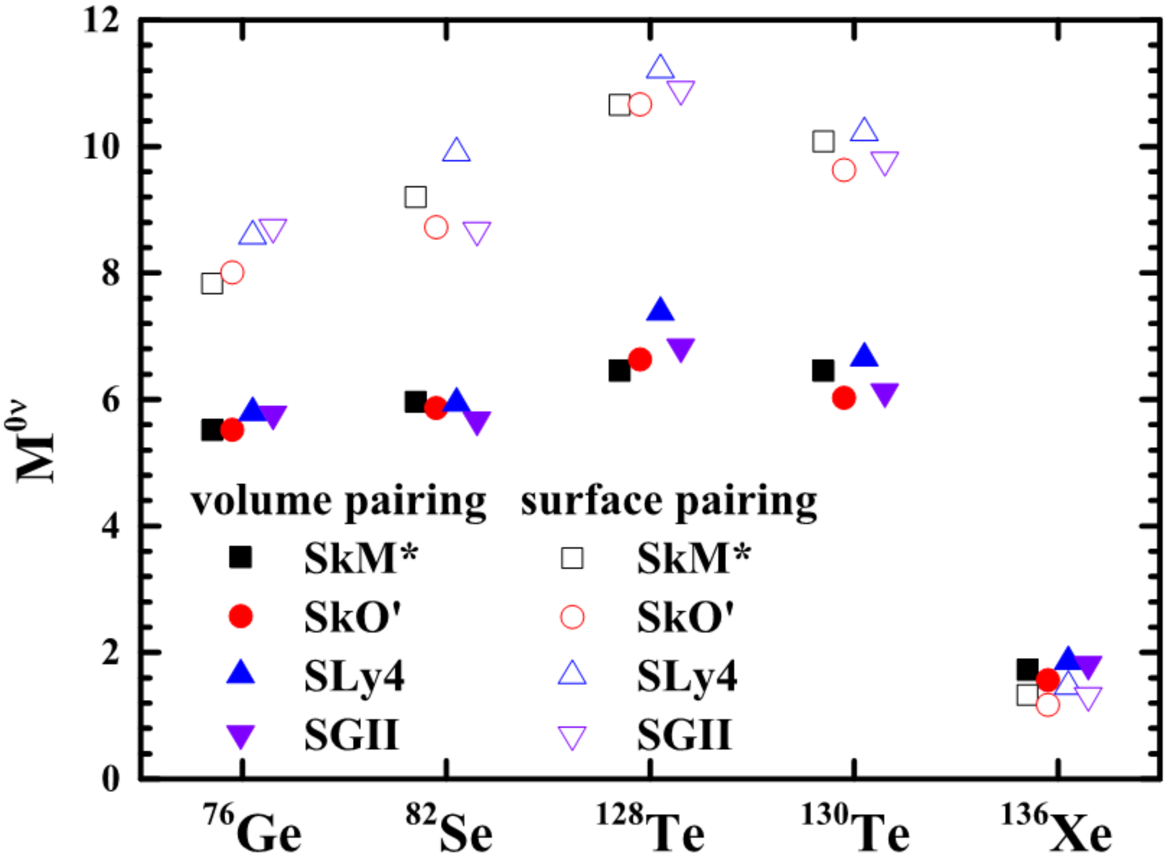
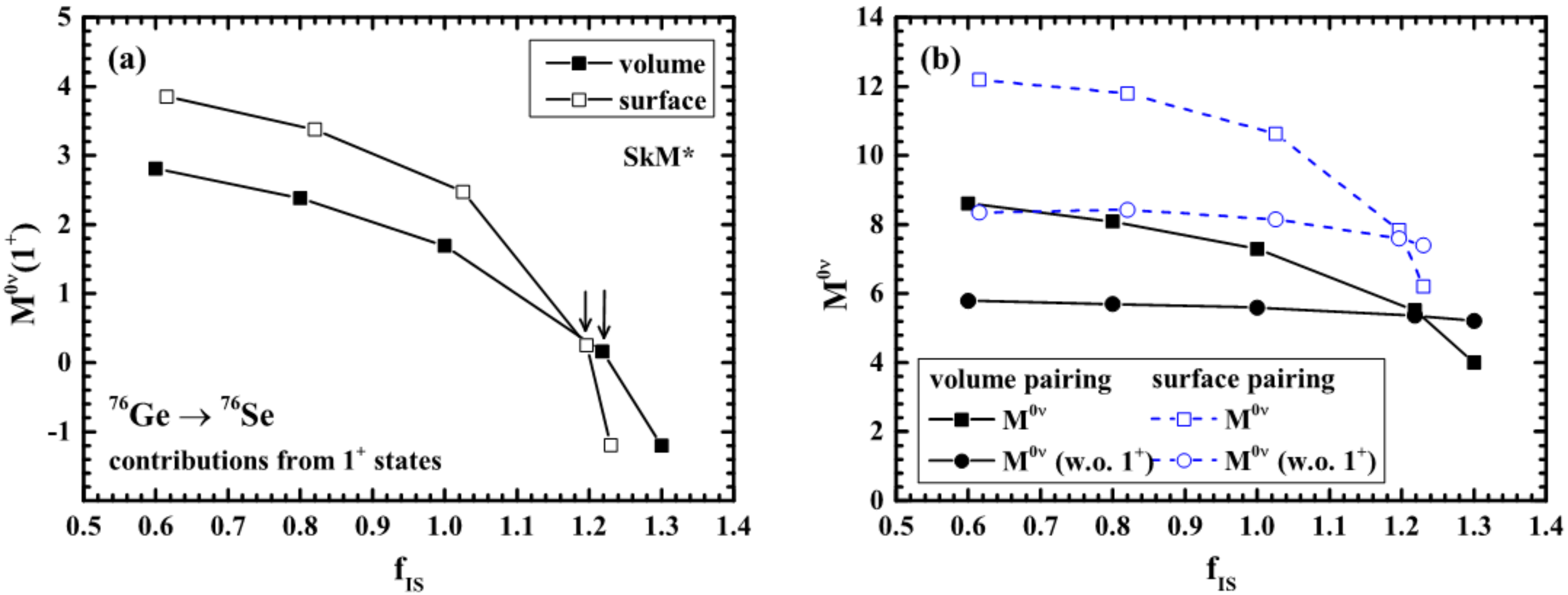
$M^{0\nu}$ by different ph and pp interactions



- ✓ For the same kind of pp interaction, $M^{0\nu}$ obtained by different ph interaction are close.
- ✓ Except for ^{136}Xe, $M^{0\nu}$ calculated by surface pairing are larger.

Isoscalar pairing dependence of $M^{0\nu}$



- ✓ By adjusting f_{IS} to reproduce the experimental $M_{\text{GT}}^{2\nu}$, $M^{0\nu}(1^+)$ by different pp interactions are close.
- ✓ The difference of $M^{0\nu}$ between volume pairing and surface pairing mainly comes from contributions of other multipoles rather than 1^+.
- ✓ Contributions from other multipoles are almost independent of f_{IS}.

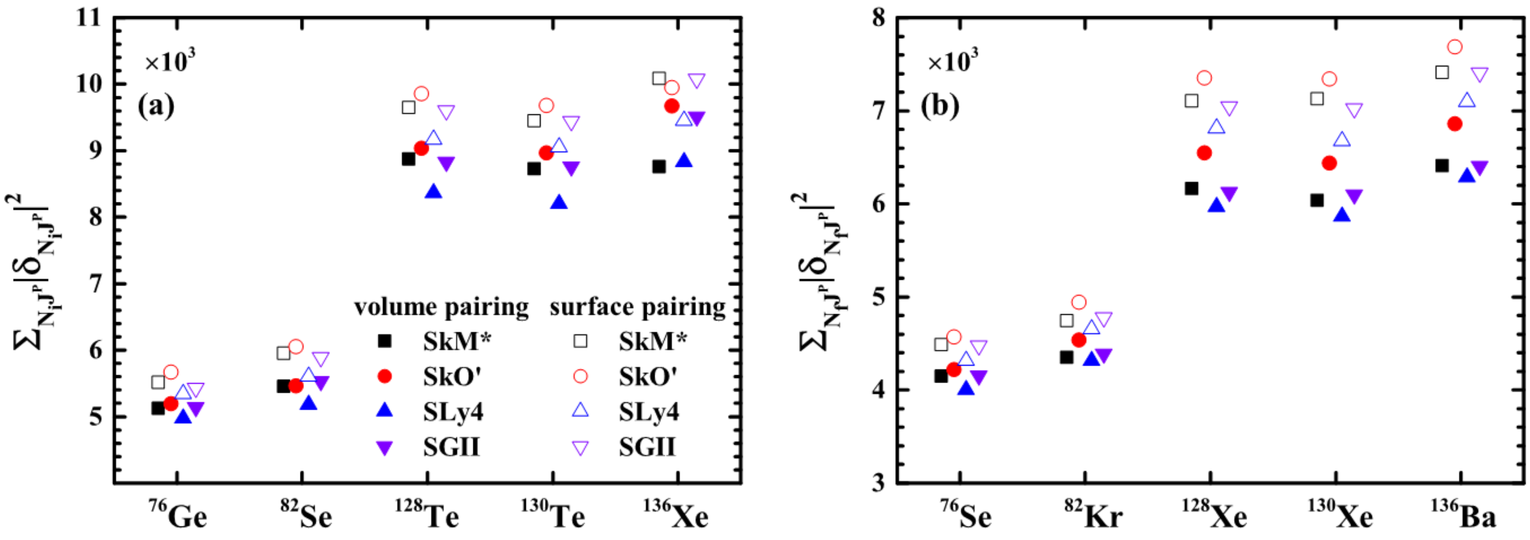
The difference in $M^{0\nu}$ from the different form of pairing interaction should be caused by the isovector pairing part.

Isovector pairing effects

❑ Isovector pairing plays its role on NME through the following factors

- ✓ the overlap of HFB wavefunctions $\langle \mathrm{HFB}_f | \mathrm{HFB}_i \rangle$.
- ✓ one-body transition densities
- ✓ the number of two quasiparticle (2qp) proton-neutron configurations

$$\delta_{N_i J^P} \equiv -\hat{J}^{-1} \sum_{\pi_i \nu_i} \langle N_i J^P \| [c^{\dagger}_{\pi_i} \tilde{c}_{\nu_i}]_J \| 0^{(i)+}_{\mathrm{g.s.}} \rangle = \boxed{\sum_{\pi_i \nu_i}} \boxed{\left(X^{N_i J^P *}_{\pi_i \nu_i} u_{\pi_i} v_{\nu_i} + Y^{N_i J^P *}_{\pi_i \nu_i} v_{\pi_i} u_{\nu_i} \right)}$$



✓ The distribution of occupation probability is more diffuse for the surface pairing than the volume pairing. → Larger configuration space

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- ✓ one-body transition densities
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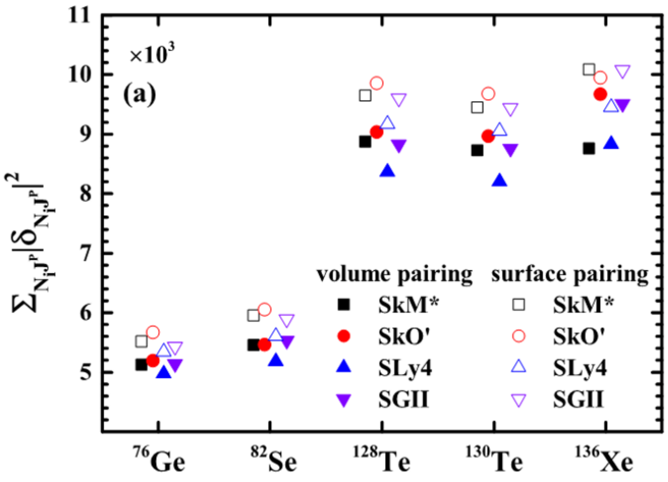
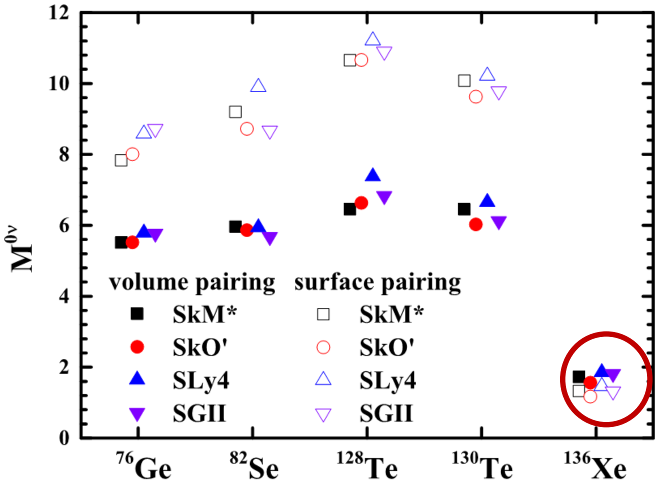
^{76}Ge, ^{82}Se, ^{128,130}Te: $\langle \mathrm{HFB}_f|\mathrm{HFB}_i\rangle \simeq 0.82$ for volume and surface pairing

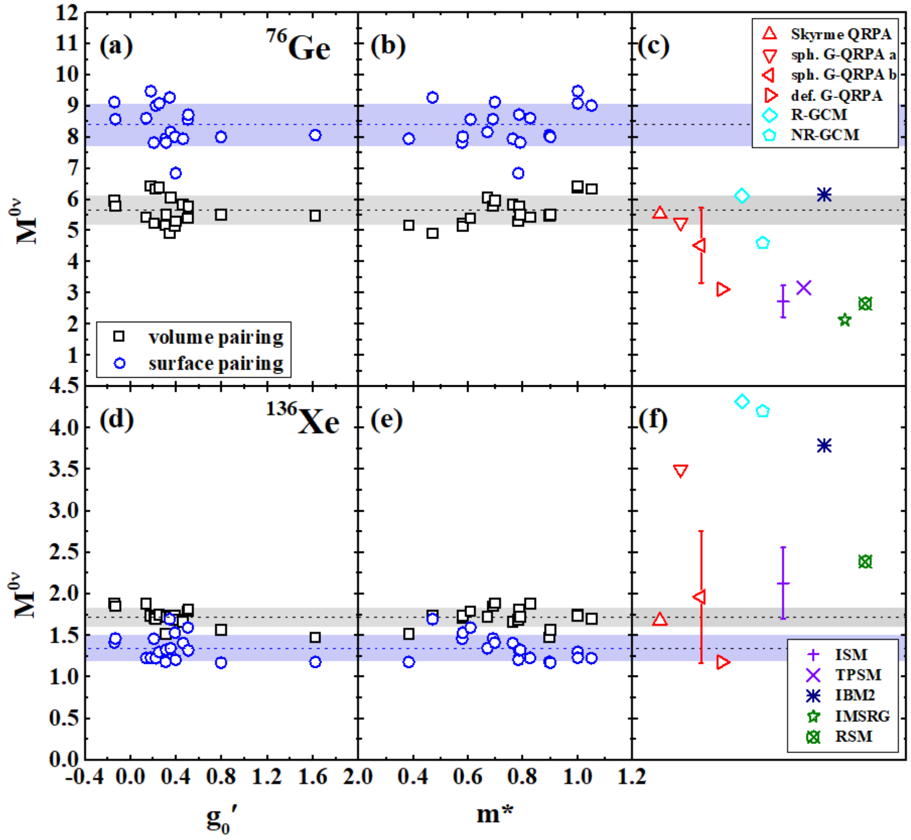
^{136}Xe: $\langle \mathrm{HFB}_f|\mathrm{HFB}_i\rangle = 0.45$ for volume pairing

$\langle \mathrm{HFB}_f|\mathrm{HFB}_i\rangle = 0.25$ for surface pairing





$M^{0\nu}$ by 18 Skyrme interactions



	Volume pairing		Surface pairing	
Nucleus	^{76}Ge	^{136}Xe	^{76}Ge	^{136}Xe
$\overline{M}^{0\nu}$	5.65	1.72	8.40	1.35
σ of $M^{0\nu}$	0.45	0.11	0.66	0.15

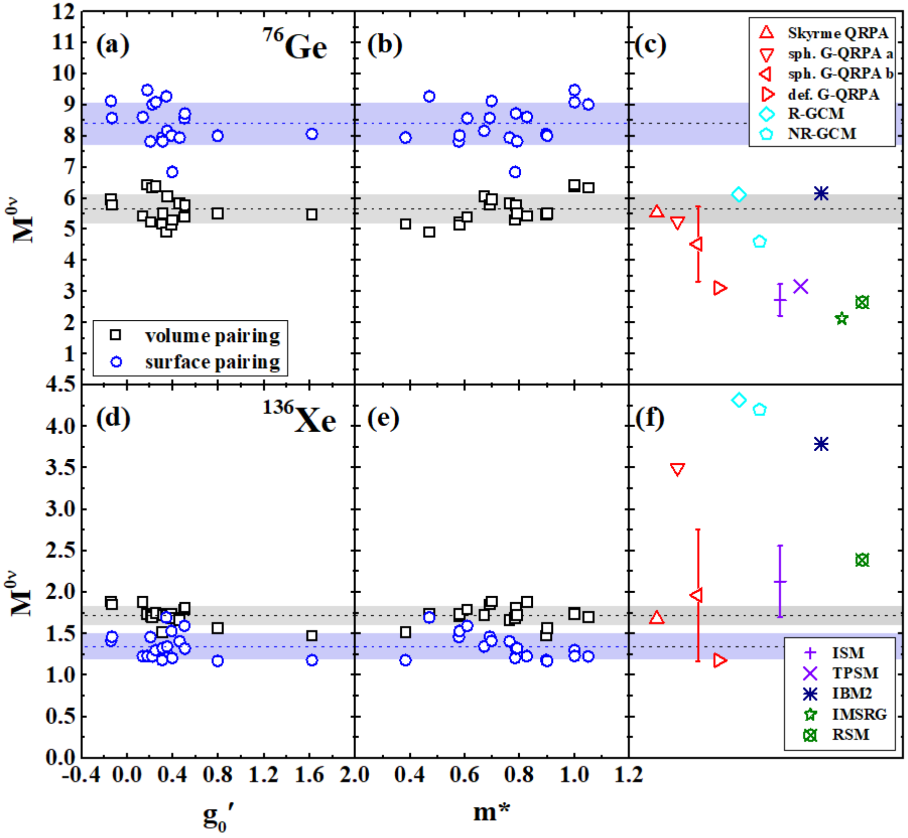
- Although the effective mass m^* and Landau parameter g_0' span a wide range, for each kind of pp interaction, σ is only around 10% of $\overline{M}^{0\nu}$.

- For $M^{0\nu}$(^{76}Ge)

✓ spherical *G*-QRPA, relativistic and non-relativistic GCM, and IBM2 results lie within 1.0～2.0 σ from our $\overline{M}^{0\nu}$ by volume pairing.

✓ deformed *G*-QRPA, ISM, triaxial projected SM, and ab initio approaches are much smaller, since they consider more many-body correlations.

$M^{0\nu}$ by 18 Skyrme interactions



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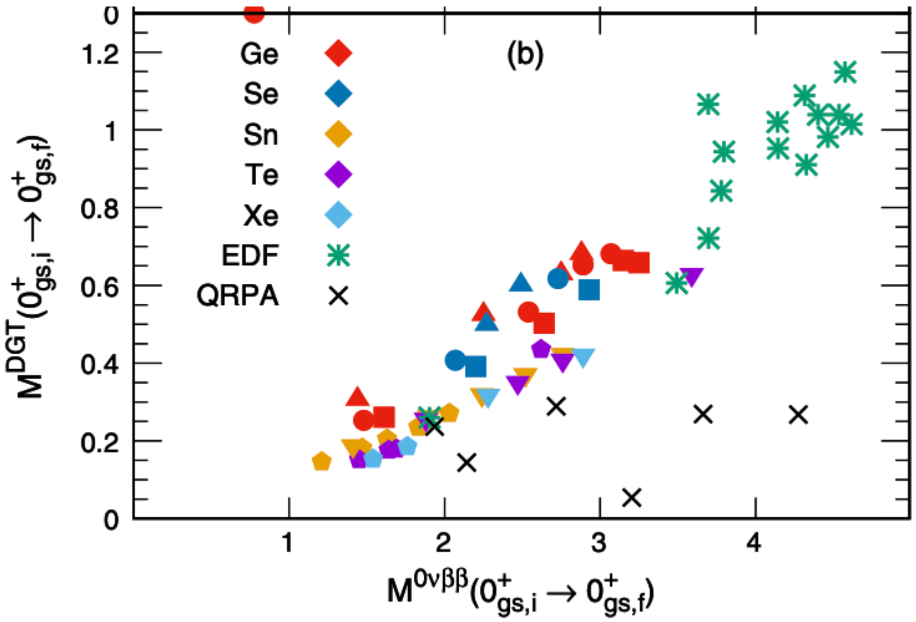
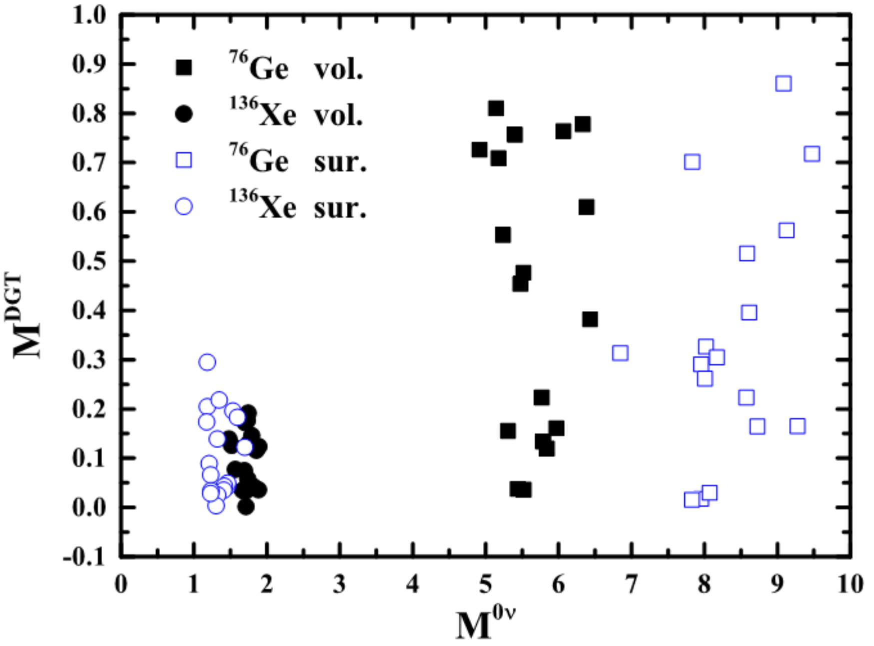
- Although the effective mass m^* and Landau parameter g_0' span a wide range, for each kind of pp interaction, σ is only around 10% of $\overline{M}^{0\nu}$.

- For $M^{0\nu}$(^{136}Xe), either by volume pairing or surface pairing, our results are smaller than many other models, which could be caused by the sharp neutron Fermi surface in ^{136}Xe that suppresses the NMEs through $\langle \mathrm{HFB}_f | \mathrm{HFB}_i \rangle$.

W. L. Lv, Y. F. Niu, D. L. Fang, J. M. Yao, C. L. Bai, and J. Meng, Phys. Rev. C 108, L051304 (2023).

Correlation between M^{DGT} and $M^{0\nu}$

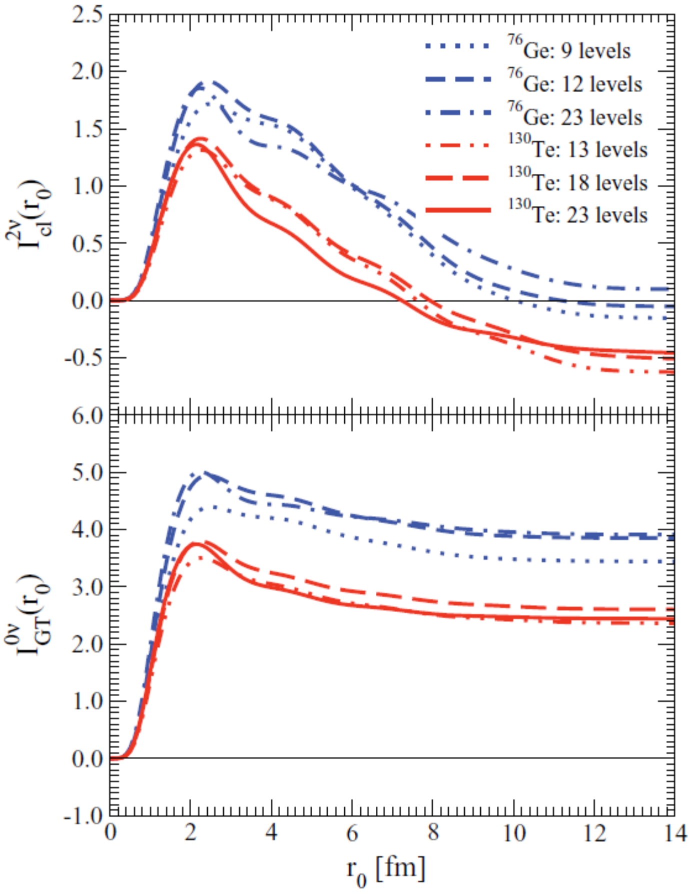




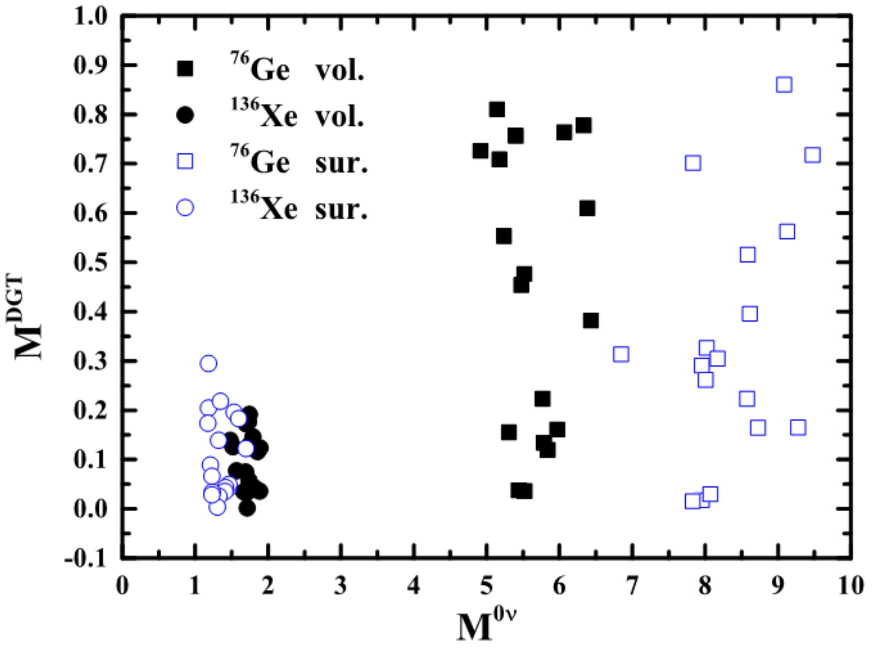
N. Shimizu et al. Phys. Rev. Lett. 120, 142502 (2018)

✓ M^{DGT} is strongly affected by the choice of ph interactions. There seems no correlation between M^{DGT} and $M^{0\nu}$ in QRPA model.

Correlation between M^{DGT} and $M^{0\nu}$



F. Šimkovic et al. Phys. Rev. C 83, 015502 (2011)



$$M^{2\nu}_{\mathrm{cl}} = \int_0^\infty C^{2\nu}_{\mathrm{cl}}(r)\,dr.$$

$$I^{2\nu}(r_0) = \int_0^{r_0} C^{2\nu}_{\mathrm{cl}}(r)\,dr,$$

$$I^{0\nu}_{\mathrm{GT}}(r_0) = \int_0^{r_0} C^{0\nu}_{\mathrm{GT-cl}}(r)\,dr.$$

- ✓ M^{DGT} : both short range and long range physics matter.
- ✓ $M^{0\nu}$: only short range physics matters.

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Summary and Perspectives

☐ Summary

- Uncertainties raising from nuclear effective interactions within Skyrme QRPA model are investigated
  - ✓ NME are not sensitive to ph interactions
  - ✓ NME are very sensitive to pp interactions: surface pairing with more diffused Fermi surface gives larger NMEs.

☐ Perspective

- Which pairing is more suitable for NME calculation?
  - ✓ Besides the mean pairing gaps, other constraints on the pairing interactions need to be considered.

- Effects of beyond QRPA model (QPVC) on NME

Acknowledgment

- Collaborators

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Bai Chunlin	Sichuan University
Fang Dongliang	Insitute of Modern Physics
Meng Jie	Peking University
Yao Jiangming	Sun Yat-sen University

Thank you!