Towards Next-Generation Nuclear Matrix Elements for Double-Beta Decay

Lotta Jokiniemi (she/her)
TRIUMF, Theory Department
NME2025 Workshop, RCNP, Osaka, Japan
20/01/2025

Discovery, accelerated

Collaborators

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Universitat de Barcelona — D. Castillo, P. Soriano, J. Menéndez

Lawrence Livermore National Laboratory — K. Kravvaris

The University of North Carolina at Chapel Hill — B. Romeo

Jyväskylän Yliopisto / University of Jyväskylä — J. Kotila

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Outline

Introduction

Corrections to $0\nu\beta\beta$-Decay Nuclear Matrix Elements

Correlations with Other Observables to Constrain the Matrix Elements

Muon Capture as a Probe of $0\nu\beta\beta$ Decay

Summary

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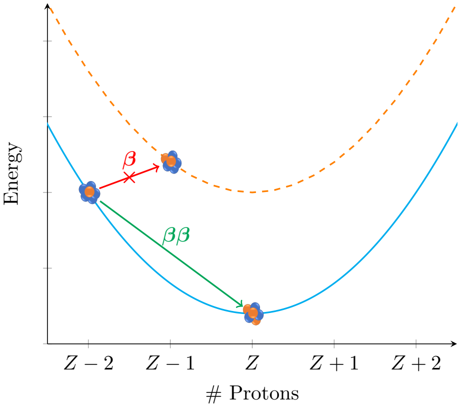
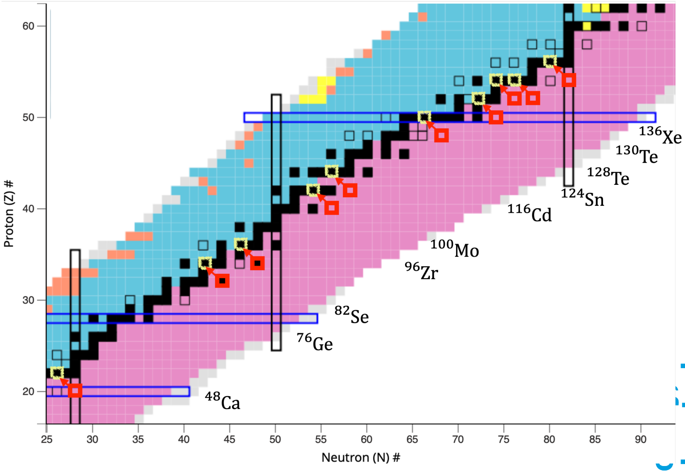
Double-Beta Decay





nndc.bnl.gov

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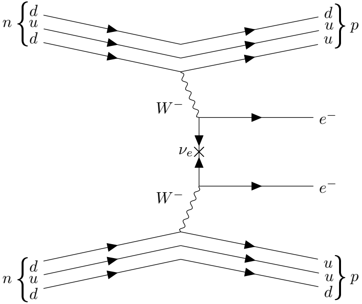
Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- **Violates lepton-number conservation**

$$(A, Z) \rightarrow (A, Z+2) + 2e^{-} \cancel{+2\nu_e}$$



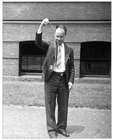
Maria Goeppert-Mayer

$2\nu\beta\beta$

Ettore Majorana

Majorana particles

Wendell H. Furry



$0\nu\beta\beta$

1935 → 1937 → 1939 → …

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- Requires that **neutrinos are Majorana particles**

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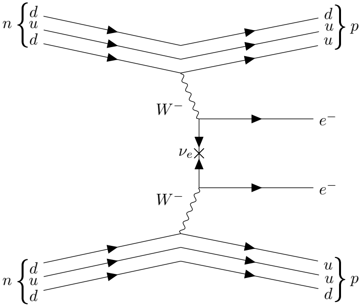

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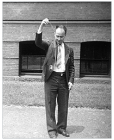

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- If observed, $t_{1/2}^{0\nu} \gtrsim 10^{25}$ **years**

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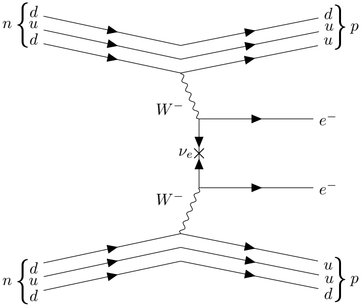

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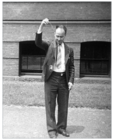

$0\nu\beta\beta$

1935 → 1937 → 1939 → …

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Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

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- Requires that **neutrinos are Majorana particles**
- If observed, $t_{1/2}^{0\nu} \gtrsim 10^{25}$ **years**
  ($t_{1/2}^{2\nu} \approx 10^{20}$ years,
  age of the Universe $\approx 10^{10}$ years)

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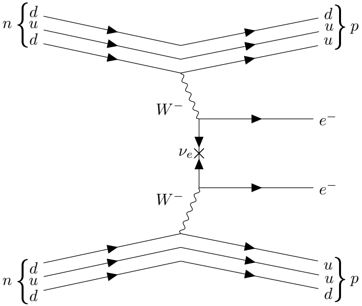
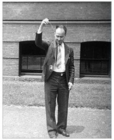


Maria Goeppert-Mayer — *Ettore Majorana* — *Wendell H. Furry*





$2\nu\beta\beta$ — Majorana particles — $0\nu\beta\beta$

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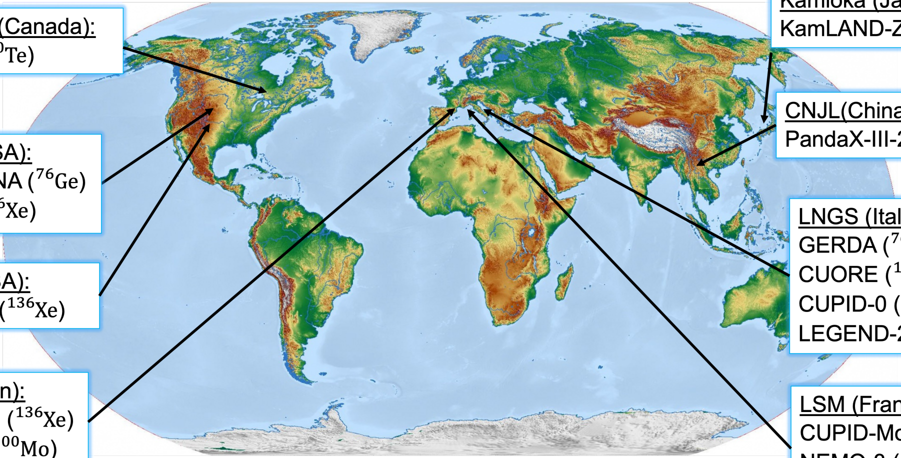
$0\nu\beta\beta$-Decay Experiments



SNOLAB (Canada):
SNO+ (^{130}Te)

SURF (USA):
MAJORANA (^{76}Ge)
LZ-nat (^{136}Xe)

WIPP (USA):
EXO-200 (^{136}Xe)

LSC (Spain):
NEXT-100 (^{136}Xe)
CROSS (^{100}Mo)

Kamioka (Japan):
KamLAND-Zen (^{136}Xe)

CNJL(China):
PandaX-III-200 (^{136}Xe)

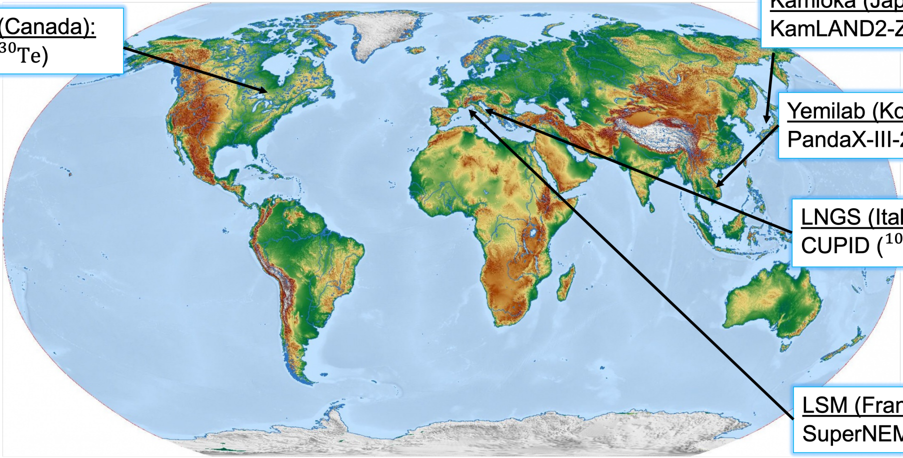
LNGS (Italy):
GERDA (^{76}Ge)
CUORE (^{130}Te)
CUPID-0 (^{82}Se)
LEGEND-200 (^{76}Ge)

LSM (France):
CUPID-Mo (^{100}Mo)
NEMO-3 (^{100}Mo)
SuperNEMO-D (^{82}Se)

Current record: $t_{1/2}^{0\nu\beta\beta}(^{136}\mathrm{Xe}) > 3.8\times10^{26}$ years

KamLAND-Zen, arXiv:2407:11438

Next-Generation $0\nu\beta\beta$-Decay Experiments



+nEXO (^{136}Xe), LEGEND-1000 (^{76}Ge), NEXT-HD (^{136}Xe), Darwin (^{136}Xe), …

M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

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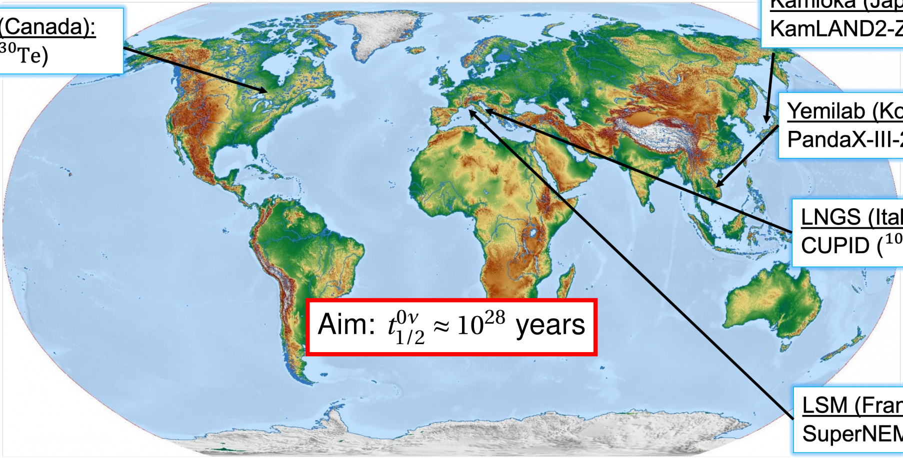

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Aim: $t_{1/2}^{0\nu} \approx 10^{28}$ years

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$0\nu\beta\beta$-Decay Half-Life

What would be measured

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

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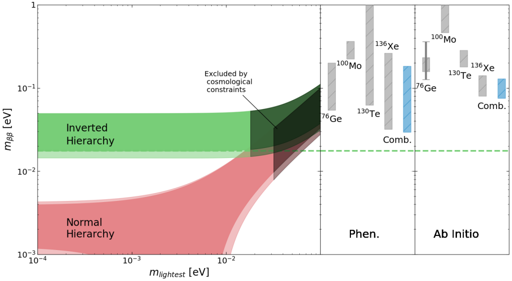
Majorana mass

$m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$



T. Shickele, LJ, A. Belley, J. D. Holt, in preparation

Discovery, accelerated

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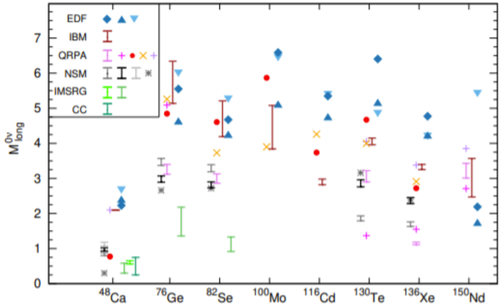
Nuclear matrix element



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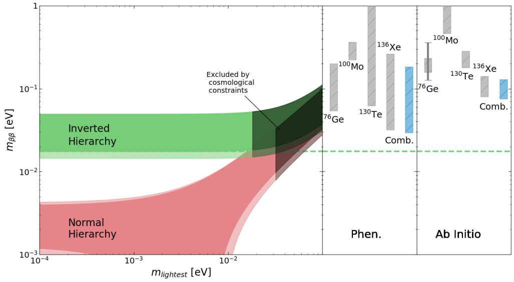

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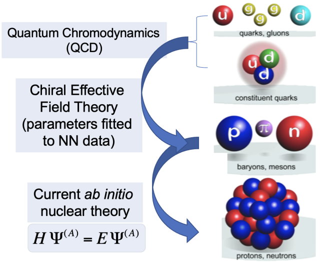
Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

- *Ab initio methods* (IMSRG, NCSM,...)



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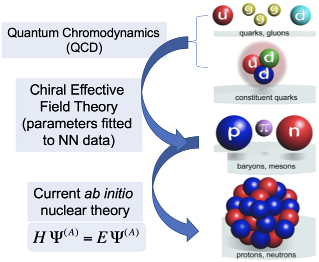

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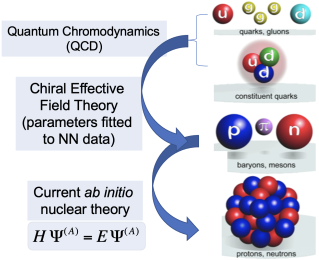

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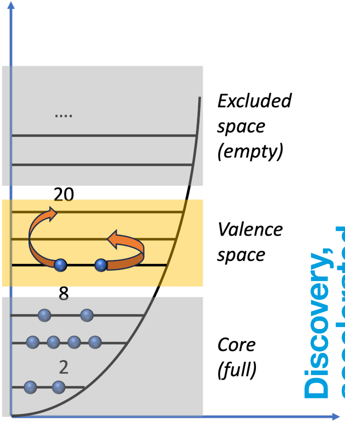

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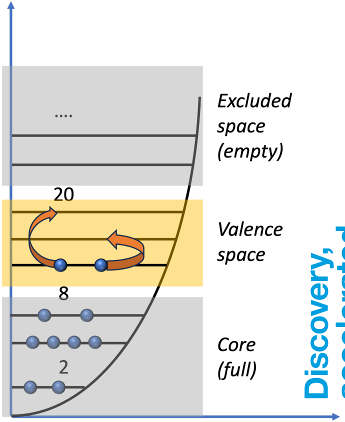

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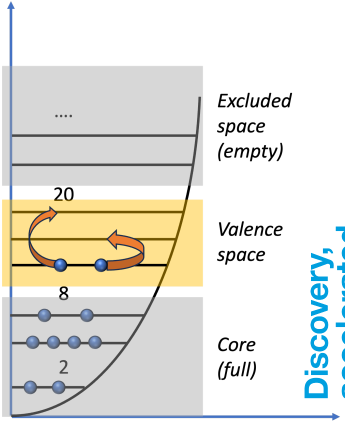

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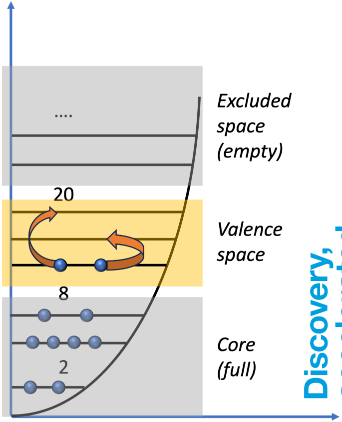

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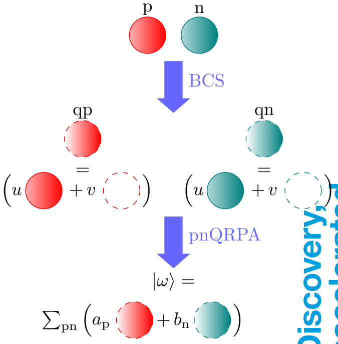

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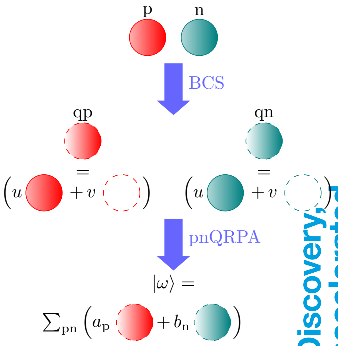

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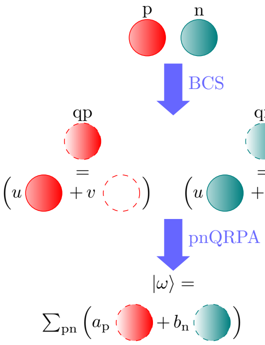

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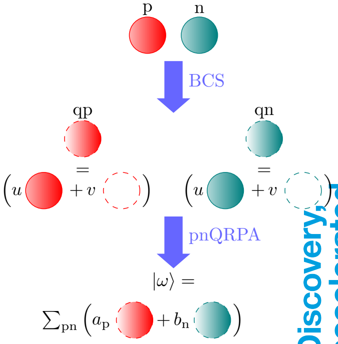

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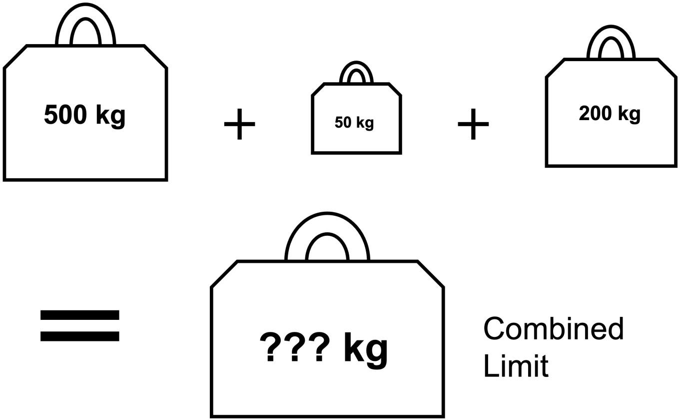
Combining the Experimental Efforts

Individual Limits



Combined Limit

Courtesy of T. Shickele

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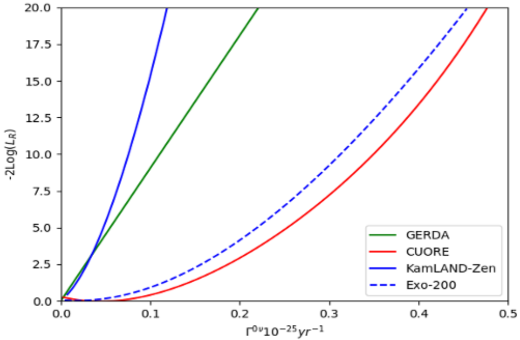
Combined Limits for Effective Majorana Mass

Current Experiments

- Idea from:

  S. D. Biller, Phys. Rev. D 104, 012002 (2021)



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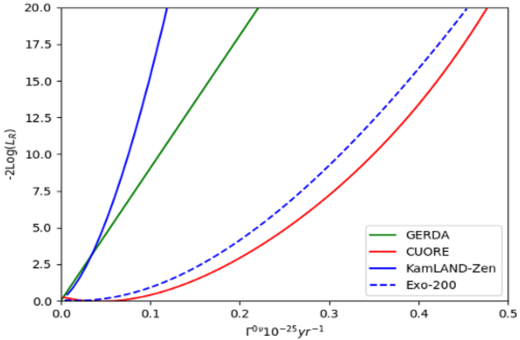

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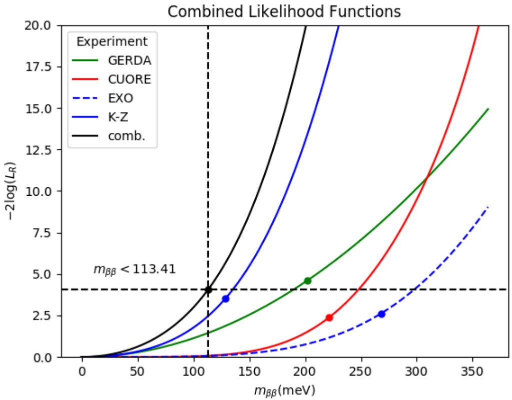

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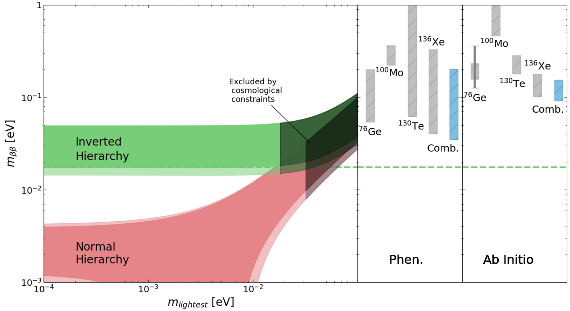

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Next-Generation Experiments

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LEGEND-1000 Collaboration, arXiv:2107.11462(2021)

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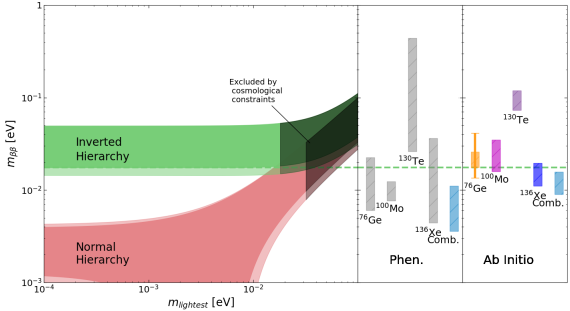

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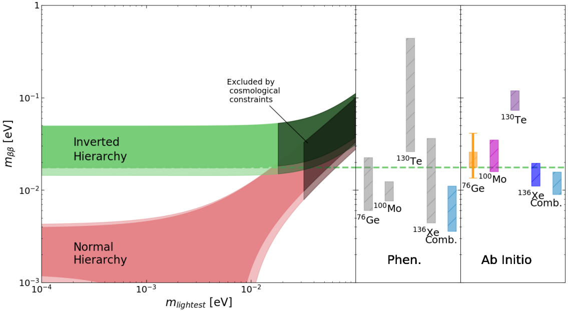


T. Shickele, LJ, A. Belley, J. D. Holt, in preparation

Discovery, accelerated

Outline

Introduction

Corrections to $0\nu\beta\beta$-Decay Nuclear Matrix Elements

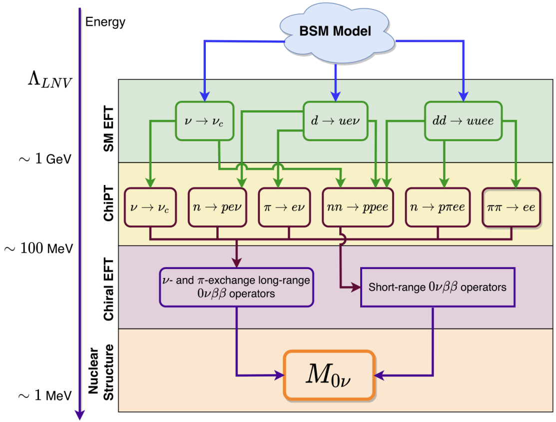
Correlations with Other Observables to Constrain the Matrix Elements

Muon Capture as a Probe of $0\nu\beta\beta$ Decay

Summary

Discovery, accelerated

Effective Field Theory For $0\nu\beta\beta$ Decay

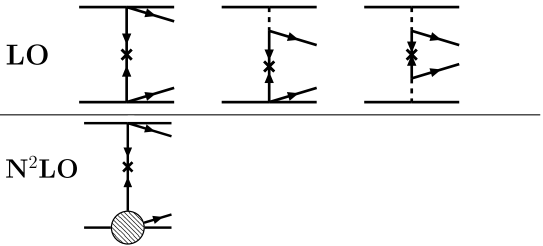


V. Cirigliano et al., J. Phys. G: Nucl. Part. Phys. 49, 120502 (2022)

Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

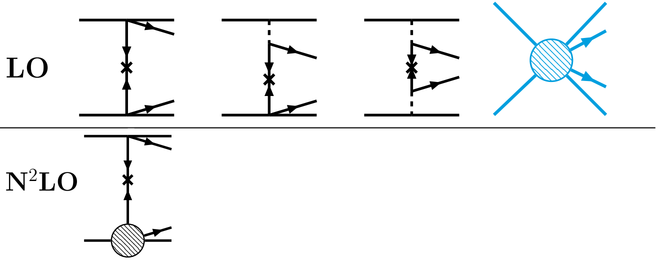
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

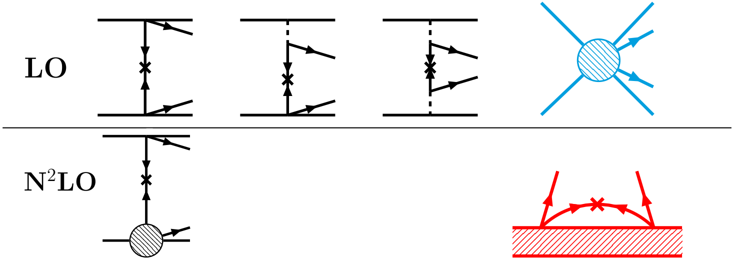
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_L^{0\nu} + \mathbf{M_S^{0\nu}} + \mathbf{M_{usoft}^{0\nu}} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

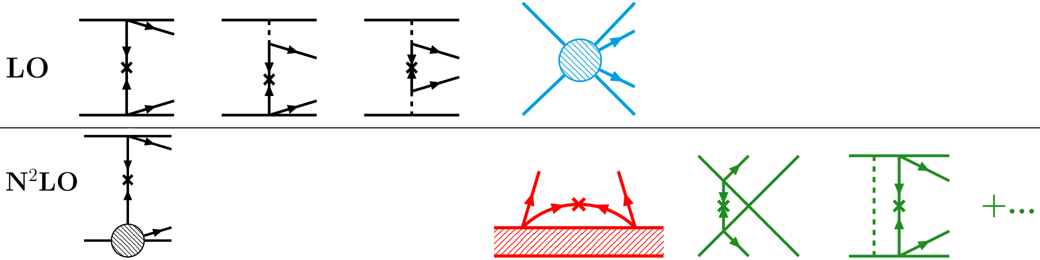
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loop}}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Traditional $0\nu\beta\beta$-Decay Operators

$$M^{0\nu} = \frac{R}{g_{\mathbf{A}}^2} \int \frac{\mathbf{dk}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

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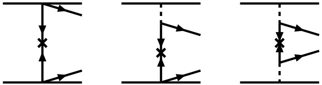
$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n\rangle \langle n | J^\mu(\mathbf{y}) | i\rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Traditionally, the nuclear current includes the leading-order (LO) transition operators

$$\mathscr{J}^0 = \tau[g_V(0)]$$
$$\boldsymbol{J} = \tau[g_A(0)\boldsymbol{\sigma} - g_P(0)\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{\sigma})]$$

LO



Traditional $0\nu\beta\beta$-Decay Operators

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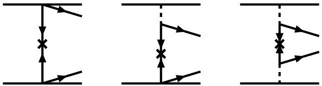
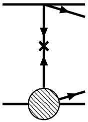


- and next-to-next-to-leading-order (N^2LO) corrections absorbed into **form factors** and **induced weak-magnetism terms**

$$\mathcal{J}^0 = \tau [g_V(p^2)]$$

$$\boldsymbol{J} = \tau \left[g_A(p^2)\boldsymbol{\sigma} - g_P(p^2)\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{\sigma}) + i g_M(p^2) \frac{\boldsymbol{\sigma}\times\boldsymbol{p}}{2m_N} \right]$$



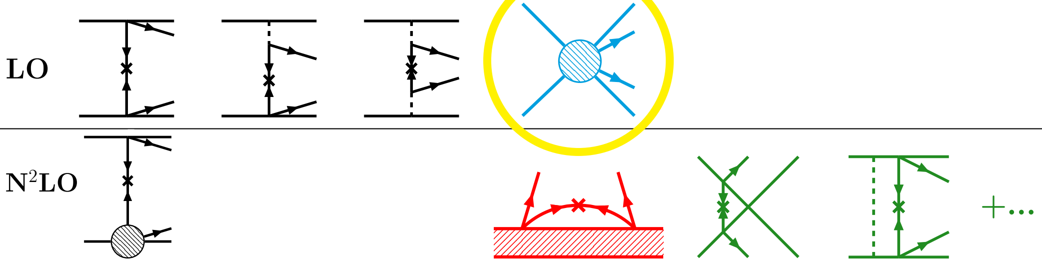
Discovery, accelerated

TRIUMF

Leading-order short-range contribution to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{N^2LO}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Discovery, accelerated

TRIUMF

Contact Term in pnQRPA and NSM

$$\int C_{\mathrm{L/S}}(r)\mathrm{d}r = M^{0\nu}_{\mathrm{L/S}}$$

- The contact term reads

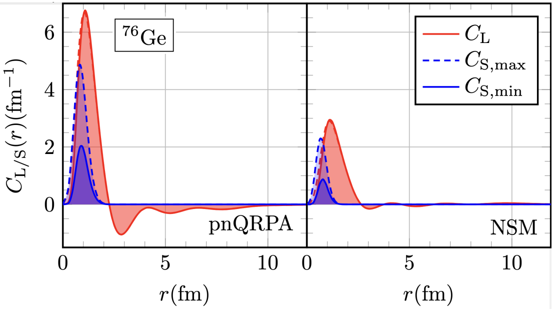
$$M^{0\nu}_{\mathrm{S}} = \frac{2R}{\pi g_{\mathrm{A}}^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr)\, \mathbf{h_S(q^2)}\, q^2 \ldots$$

with

$$\mathbf{h_S(q^2)} = 2\mathbf{g}_\nu^{\mathbf{NN}}\, e^{-q^2/(2\Lambda^2)}\ .$$

Relative effect:

$M_{\mathrm{S}}/M_{\mathrm{L}} \approx 15\% - 80\%$



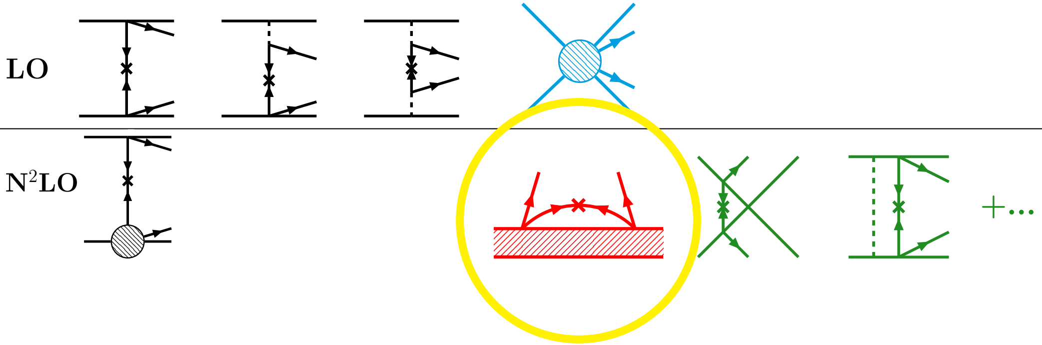
LJ, P. Soriano and J. Menéndez, Phys. Lett. B **823**, 136720 (2021)

Discovery, accelerated

Ultrasoft-neutrino contribution to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_{\mathrm{L}}^{0\nu} + M_{\mathrm{S}}^{0\nu} + M_{\mathrm{usoft}}^{0\nu} + M_{\mathrm{N^2LO}}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



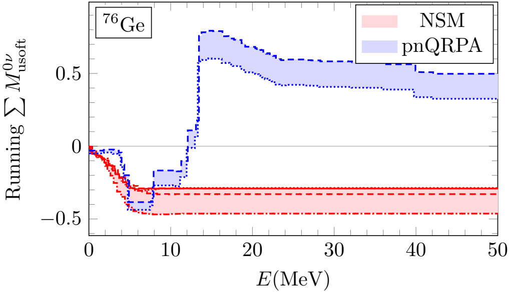
TRIUMF

Ultrasoft Neutrinos in pnQRPA and NSM

- Contribution of ultrasoft neutrinos ($|\mathbf{k}| << k_F \approx 100$ **MeV**) to $0\nu\beta\beta$ decay:

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018)

$$M^{0\nu}_{\text{usoft}} = -\frac{2R}{\pi}\sum_n \langle f||\sum_a \boldsymbol{\sigma}_a\tau_a^+||n\rangle\langle n||\sum_b \boldsymbol{\sigma}_b\tau_b^+||i\rangle$$
$$\times (E_e + E_n - E_i)\left(\ln\frac{\mu_{\text{us}}}{2\,(E_e+E_n-E_i)} + 1\right)$$



Relative effect

$|M^{0\nu}_{\text{usoft}}/M^{0\nu}_{\text{LO}}| \leq 10\%$

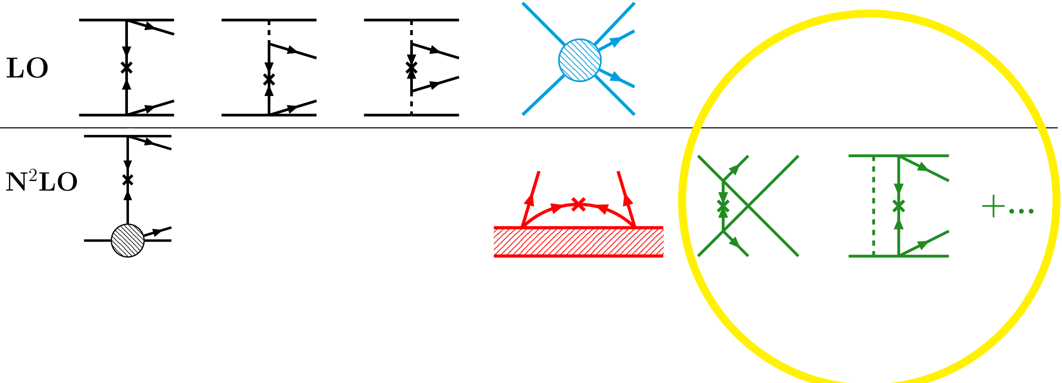
D. Castillo, LJ, P. Soriano, J. Menéndez, Phys. Lett. B 860, 139181 (2025)

Discovery, accelerated

$\mathrm{N^2LO}$ Loop Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_L^{0\nu} + M_S^{0\nu} + M_{usoft}^{0\nu} + M_{N^2LO}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



N^2LO Loop Corrections in pnQRPA and NSM

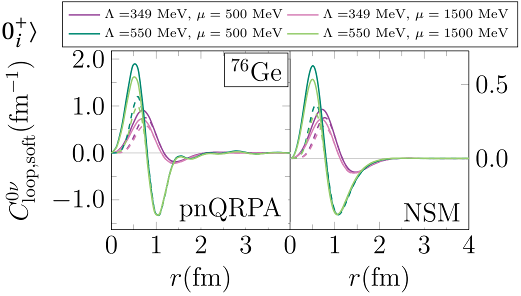
- The N^2LO loop corrections read as

$$M^{0\nu}_{\text{loops}} = \frac{4R}{\pi g_A^2}\langle 0_f^+|\sum_{a,b}\tau_a^-\tau_b^-\int e^{-\frac{q^2}{2\Lambda^2}} j_u(qr)\, V^{(a,b)}_{\nu,2}\, q^2 \mathrm{d}q|0_i^+\rangle$$

with

$$V^{(a,b)}_{\nu,2} = V^{(a,b)}_{\text{VV}} + V^{(a,b)}_{\text{AA}} + \ln\frac{m_\pi^2}{\mu_{\text{us}}^2} V^{(a,b)}_{\text{us}} + V^{(a,b)}_{\text{CT}}$$

$$\int C^{0\nu}_{\text{N}^2\text{LO}}(r)\mathrm{d}r = M^{0\nu}_{\text{loops}}$$



Relative effect:

$|M_{\text{loops}}/M_{\text{LO}}| \leq 10\%$

D. Castillo, LJ, P. Soriano, J Menéndez, Phys. Lett. B 860, 139181 (2025)

Outline

Discovery, accelerated

$0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

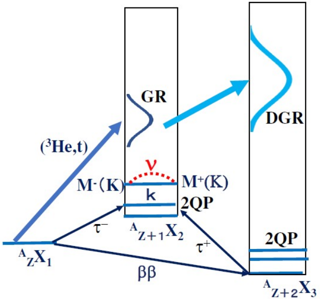
$$M^{0\nu} = M^{0\nu}_{\rm GT} - \left(\frac{g_{\rm V}}{g_{\rm A}}\right)^2 M^{0\nu}_{\rm F} + M^{0\nu}_{\rm T} + M^{0\nu}_{\rm S} + M^{0\nu}_{\rm N^2LO}$$

Leading contribution

$$M^{0\nu}_{\rm GT} = \langle f || \sum_{jk} \tau^-_j \tau^-_k \sigma^-_j \sigma^-_k V_{\rm GT}(r_{jk}) || i \rangle$$

- Double-Gamow-Teller (DGT) strength function

$$B({\rm DGT};\lambda) = \frac{1}{2J_i+1} |\langle f || [\sum_{jk} \sigma_j \tau^-_j \times \sigma_k \tau^-_k]^{(\lambda)} || i \rangle|^2$$



H. Ejiri, LJ, J. Suhonen, Phys. Rev. C 105, L022501 (2022)

$0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

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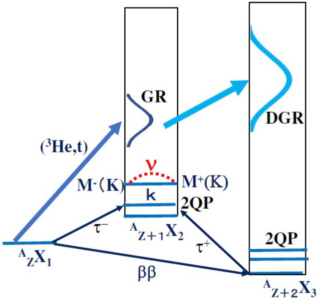
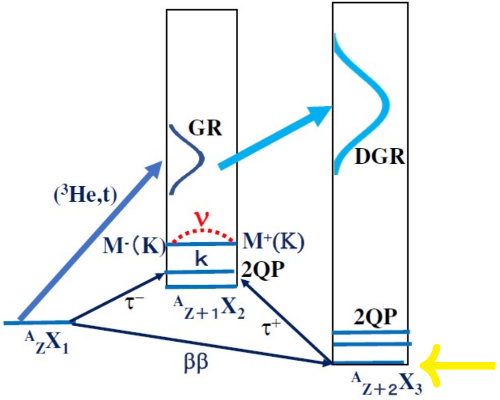
  - **Could we probe $0\nu\beta\beta$ decay by DGT reactions?**



H. Ejiri, LJ, J. Suhonen, Phys. Rev. C 105, L022501 (2022)

Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\mathrm{DGT}} = -\langle 0^+_{\mathrm{gs,f}} || [\sum_{jk} \boldsymbol{\sigma}_j \tau^-_j \times \boldsymbol{\sigma}_k \tau^-_k]^{(0)} || 0^+_{\mathrm{gs,i}} \rangle$$



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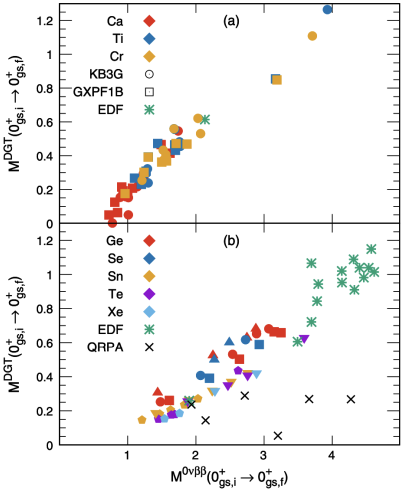
- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**



N. Shimizu, J. Menéndez, K. Yako, Phys. Rev. Lett. 120, 142502 (2018)

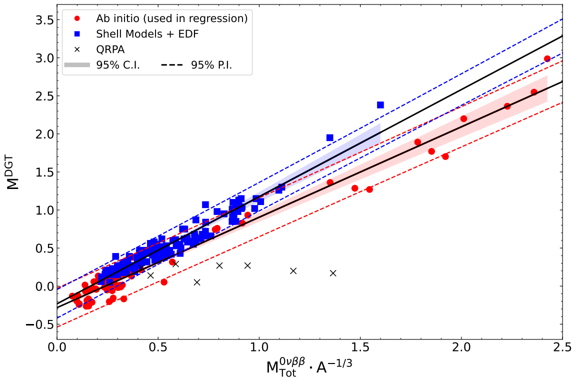
Discovery, accelerated

TRIUMF

Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\rm DGT} = -\langle 0^+_{\rm gs,f} || [\sum_{jk} \boldsymbol{\sigma}_j \tau^-_j \times \boldsymbol{\sigma}_k \tau^-_k]^{(0)} || 0^+_{\rm gs,i} \rangle$$

- Correlation between $M^{0\nu}$ and $M_{\rm DGT}$ found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**



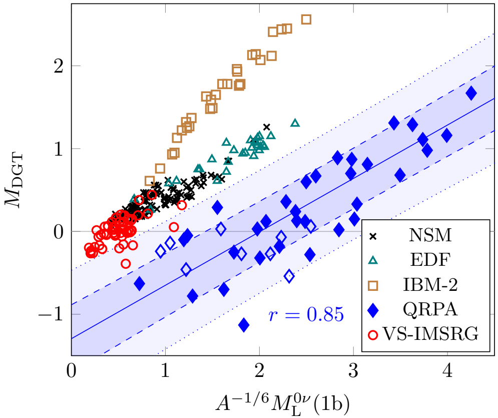
J. M. Yao, I. Ginnett, A. Belley et al., Phys. Rev. C 106, 014315 (2022)

Discover accelerated

Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\mathrm{DGT}} = -\langle 0^+_{\mathrm{gs,f}} || [\sum_{jk} \boldsymbol{\sigma}_j \tau^-_j \times \boldsymbol{\sigma}_k \tau^-_k]^{(0)} || 0^+_{\mathrm{gs,i}} \rangle$$

- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**
- ...and **QRPA**, when proton-neutron pairing varied
  - **Observation of M_{DGT} → constraints for $M^{0\nu}$**



LJ, J. Menéndez, Phys. Rev. C 107, 044316 (2023)

Discovery, accelerated

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

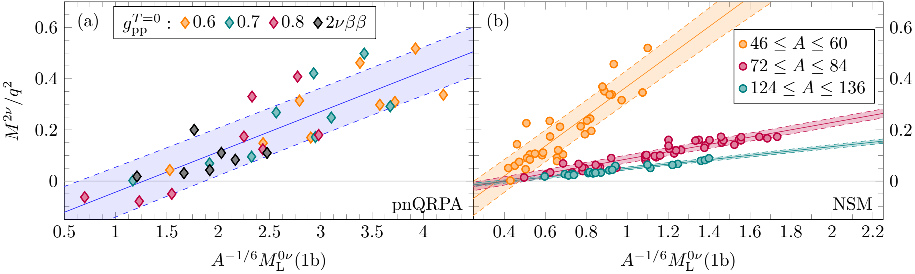
- ***How about $2\nu\beta\beta$ decay?***

Discovery, accelerated

TRIUMF

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- ***How about $2\nu\beta\beta$ decay?***
- $2\nu\beta\beta$-decay also correlated with $0\nu\beta\beta$-decay!



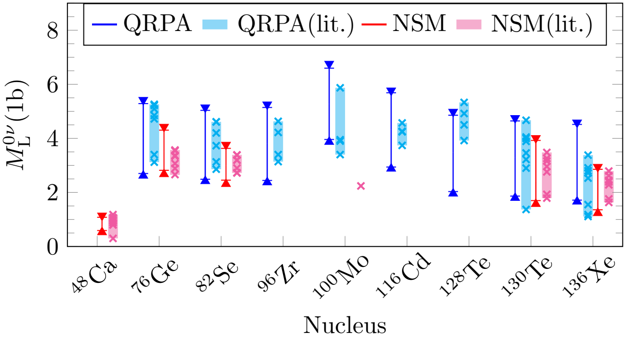
LJ, B. Romeo, P. Soriano and J. Menéndez, Phys. Rev. C **107**, 044305 (2023)

Discovery, accelerated

TRIUMF

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- ***How about $2\nu\beta\beta$ decay?***
- $2\nu\beta\beta$-decay also correlated with $0\nu\beta\beta$-decay!
- **NMEs with uncertainties** based on the correlations and experimental data



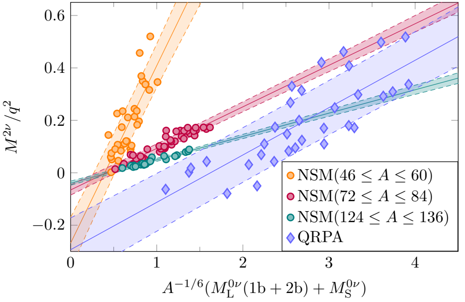
*LJ, B. Romeo, P. Soriano and J. Menéndez, Phys. Rev. C **107**, 044305 (2023)*

Discovery, accelerated

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

Two-Body Currents & Contact Term

- Correlations survive when adding approximate two-body currents (2BCs) and the contact term



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Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

Two-Body Currents & Contact Term

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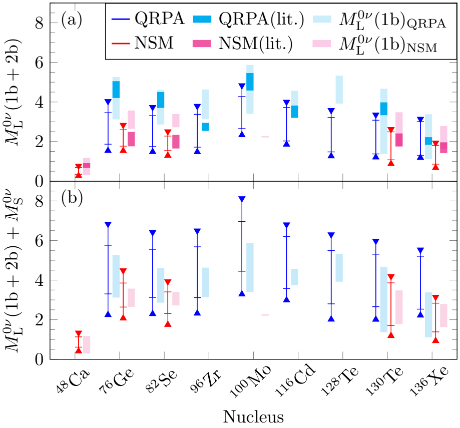
- Effect of 2BCs larger than in previous studies

J. Menéndez, D. Gazit, A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011)

J. Engel, F. Šimkovic, P. Vogel, Phys. Rev. C 89, 064308 (2014)



LJ, B. Romeo, P. Soriano and J. Menéndez, Phys. Rev. C ***107****, 044305 (2023)*

TRIUMF

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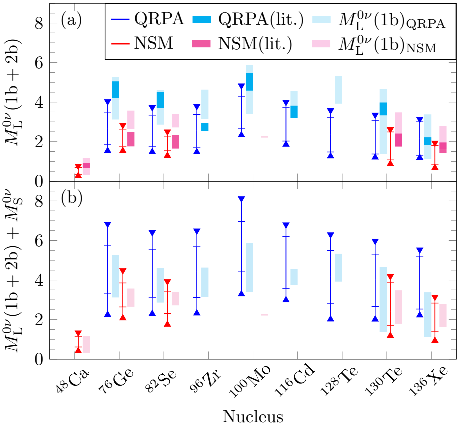
J. Engel, F. Šimkovic, P. Vogel, Phys. Rev. C 89, 064308 (2014)

- 2BCs and the contact term largely cancel each other



*LJ, B. Romeo, P. Soriano and J. Menéndez, Phys. Rev. C **107**, 044305 (2023)*

Discovery, accelerated

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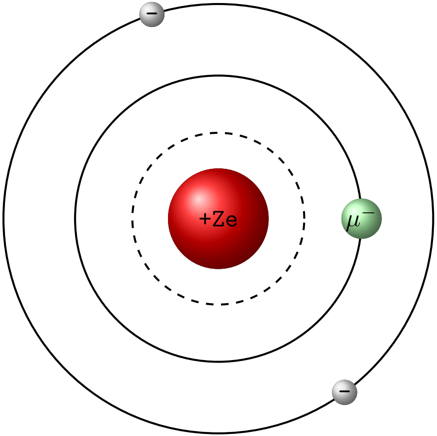
Muon Capture as a Probe of $0\nu\beta\beta$ Decay

Summary

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*



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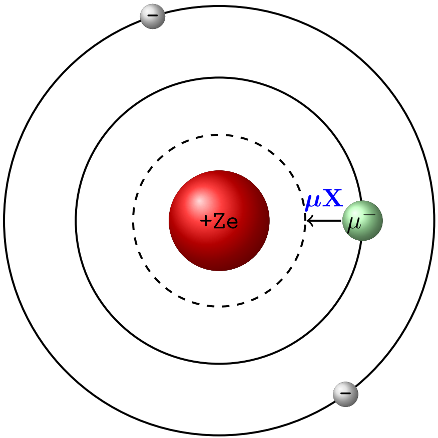
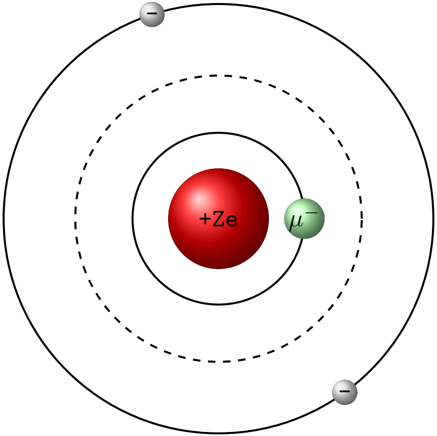


Discovery, accelerated

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*
  - Eventually bound on **the $1s_{1/2}$ orbit**

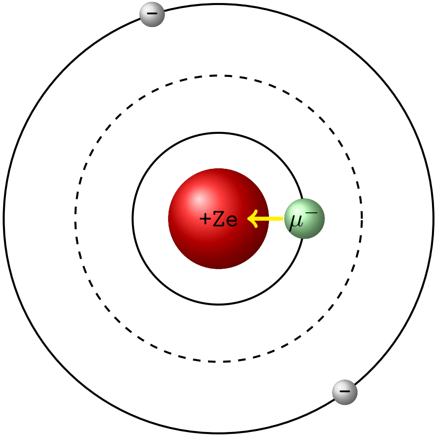


TRIUMF

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*
  - Eventually bound on **the $1s_{1/2}$ orbit**
- The *muon can then be captured* by the nucleus

$$\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})$$



Discovery, accelerated

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*
  - Eventually bound on **the $1s_{1/2}$ orbit**
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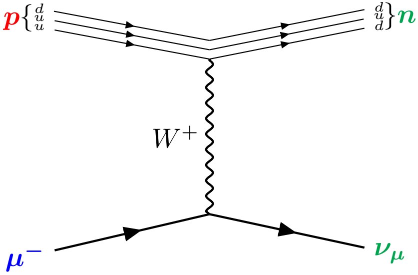
$$\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})$$



Discovery, accelerated

TRIUMF

Ordinary Muon Capture (OMC)

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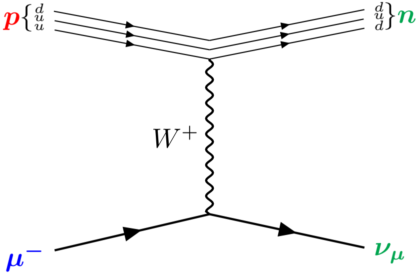
$$\boldsymbol{\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})}$$



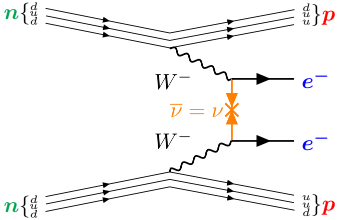
Ordinary = non-radiative

(Radiative muon capture (RMC):

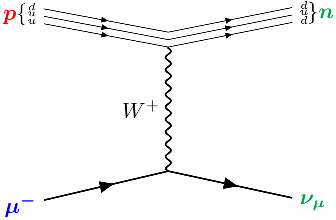
$$\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f}) + \boldsymbol{\gamma}$$)

Discovery, accelerated

$0\nu\beta\beta$ Decay vs. Muon Capture



$$ {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \to {}^{A}_{Z+2}\mathrm{X}'(J_f^{\pi_f}) + 2e^- $$



$$ \mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \to \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f}) $$

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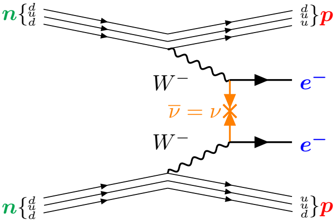

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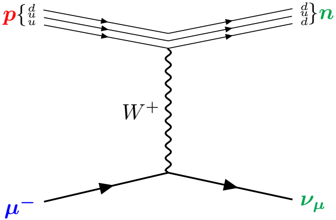

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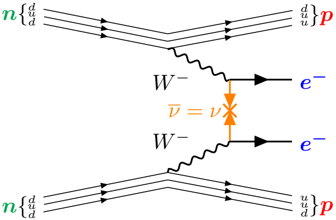
Both involve hadronic current:

$$ \langle \boldsymbol{p} | j^{\alpha\dagger} | \boldsymbol{p} \rangle = \bar{\Psi} \left[g_{\mathrm{V}}(q^2)\gamma^\alpha - g_{\mathrm{A}}(q^2)\gamma^\alpha\gamma_5 - g_{\mathrm{P}}(q^2) q^\alpha \gamma_5 + i g_{\mathrm{M}}(q^2) \frac{\sigma^{\alpha\beta}}{2m_p} q_\beta \right] \tau^{\pm} \Psi $$

Discovery, accelerated

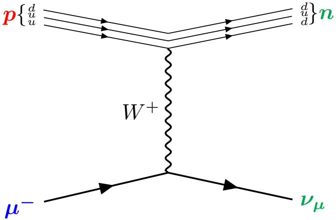
TRIUMF

$0\nu\beta\beta$ Decay vs. Muon Capture



$$ {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow {}_{Z+2}^{\ \ A}\mathrm{X}'(J_f^{\pi_f}) + 2e^- $$

- $q \approx 1/|\mathbf{r}_1 - \mathbf{r}_2| \approx 100 - 200$ **MeV**



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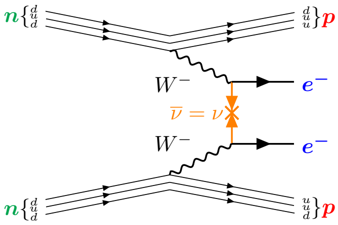
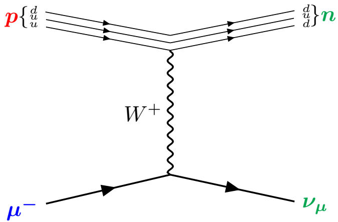
Discovery, accelerated

TRIUMF

$0\nu\beta\beta$ Decay vs. Muon Capture





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- $q \approx 1/|\mathbf{r}_1 - \mathbf{r}_2| \approx 100 - 200$ **MeV**

- $q \approx m_\mu + M_i - M_f - m_e - E_X \approx 100$ **MeV**

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Discovery, accelerated

TRIUMF

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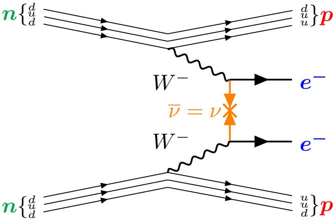

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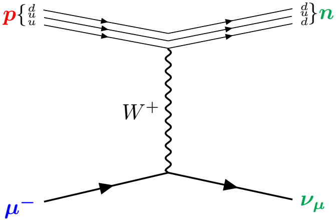
- **Yet hypothetical**



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Discovery, accelerated

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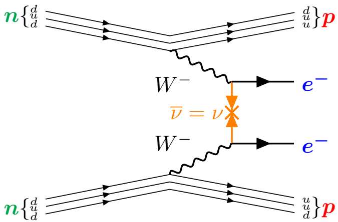

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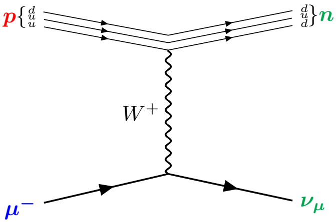
- **Yet hypothetical**



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- **Has been measured!**

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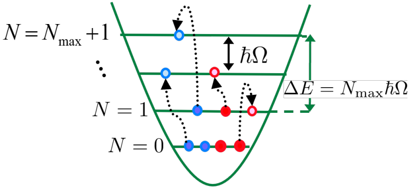
Discovery, accelerated

Ab initio No-Core Shell Model (NCSM)

- Solve nuclear many-body problem

$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$



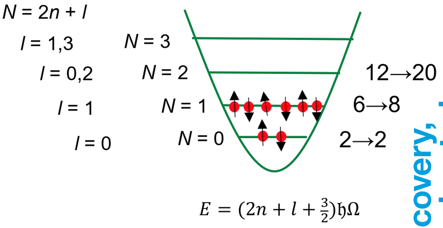


Figure courtesy of P. Navrátil

Discovery, accelerated

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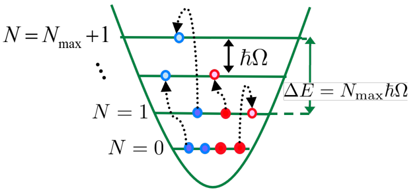
$$H^{(A)} = \sum_{i=1}^{A}\frac{p_i^2}{2m} + \sum_{i<j=1}^{A} V^{NN}(\mathbf{r}_i-\mathbf{r}_j) + \sum_{i<j<k=1}^{A} V^{3N}_{ijk}$$



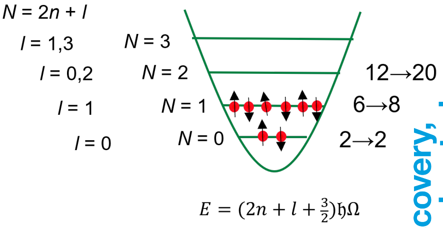


Figure courtesy of P. Navrátil

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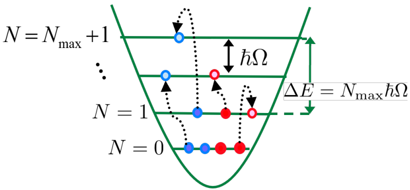
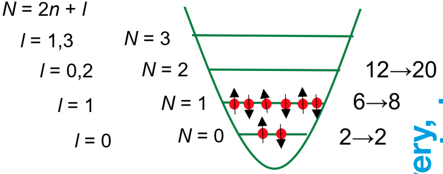
- Expansion in harmonic oscillator (HO) basis

$$\Psi^{(A)} = \sum_{N=0}^{N_{\max}}\sum_{j} c_{Nj}\Phi_{Nj}^{\mathrm{HO}}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$





$$E = (2n + l + \tfrac{3}{2})\hbar\Omega$$

Figure courtesy of P. Navrátil

TRIUMF

Dependency on the Harmonic-Oscillator Frequency

$$\Psi^{(A)} = \sum_{N=0}^{N_{\max}} \sum_{j} c_{Nj} \Phi_{Nj}^{\text{HO}}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$$

- The expansion depends on the HO frequency because of the $N_{\max}$ truncation

Discovery, accelerated

Dependency on the Harmonic-Oscillator Frequency

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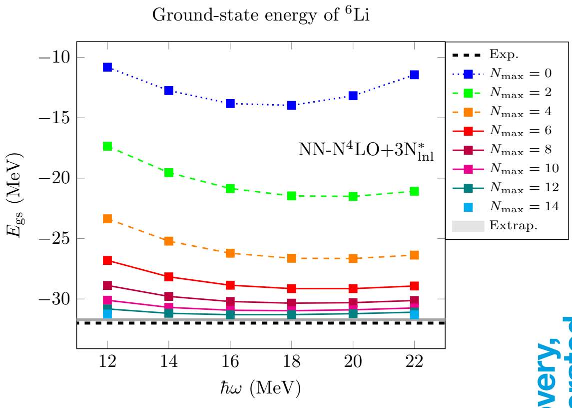
- The expansion depends on the HO frequency because of the $N_{\max}$ truncation
  - **Increasing $N_{\max}$ leads towards convergenced results**

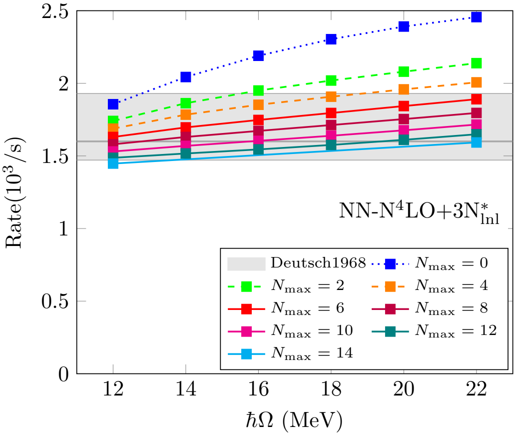


LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

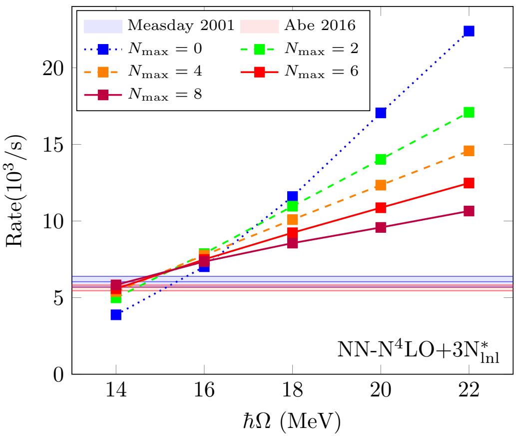
Discovery, accelerated

Harmonic-Oscillator Frequency Dependence of Muon Capture

$^{6}\mathrm{Li}(1^+_{\mathrm{gs}}) + \mu^- \rightarrow {}^{6}\mathrm{He}(0^+_{\mathrm{gs}}) + \nu_\mu$



$^{12}\mathrm{C}(0^+_{\mathrm{gs}}) + \mu^- \rightarrow {}^{12}\mathrm{B}(1^+_{\mathrm{gs}}) + \nu_\mu$



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Discovery, accelerated

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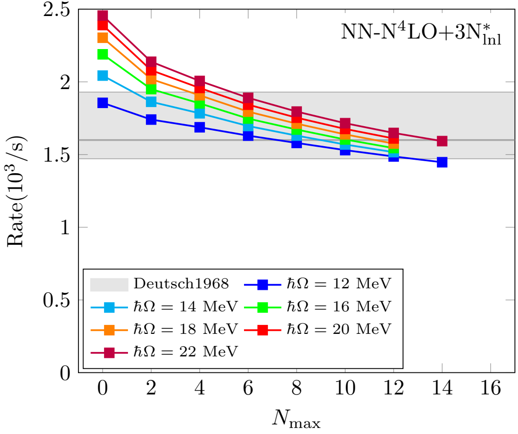

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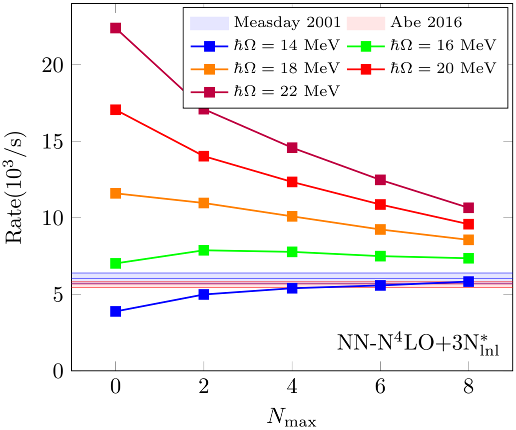

LJ, Navrátil, Kotila and Kravvaris, Phys. Rev. C 109, 065501 (2024)

Discovery, accelerated

Muon Capture on ^{6}Li

- NCSM slightly underestimating experiment

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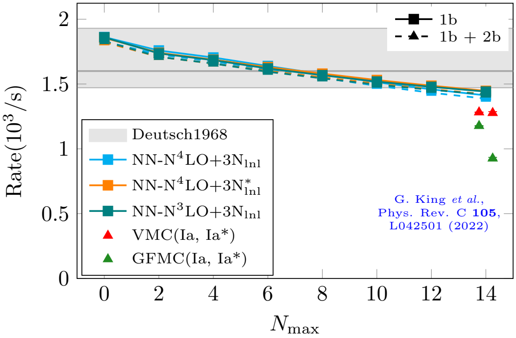

LJ, Navrátil, Kotila, Kravvaris,

Phys. Rev. C 109, 065501 (2024)

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King *et al.*, Phys. Rev. C **105**, L042501 (2022)

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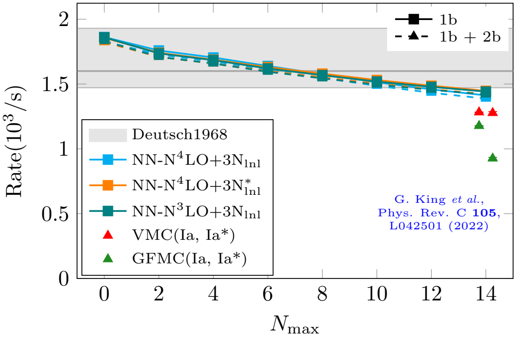

LJ, Navrátil, Kotila, Kravvaris,

Phys. Rev. C 109, 065501 (2024)

Discovery, accelerated

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- Slow convergence due to cluster-structure?

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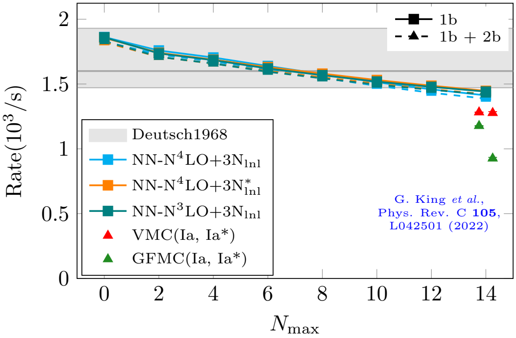

LJ, Navrátil, Kotila, Kravvaris,

Phys. Rev. C 109, 065501 (2024)

Discovery, accelerated

TRIUMF

Muon Capture on ^{6}Li

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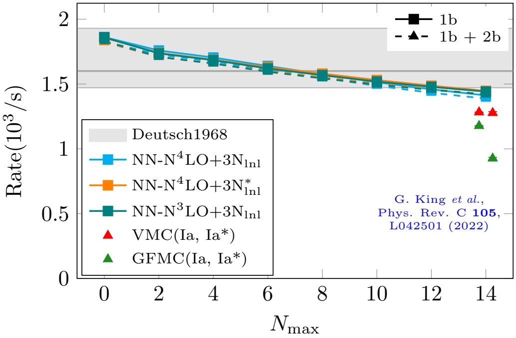
  King *et al.*, Phys. Rev. C **105**, L042501 (2022)
- Slow convergence due to cluster-structure?
  - **NCSM with continuum (NCSMC) might give better results?**

$$^6\mathrm{Li}(1^+_{\mathrm{gs}}) + \mu^- \rightarrow \ ^6\mathrm{He}(0^+_{\mathrm{gs}}) + \nu_\mu$$



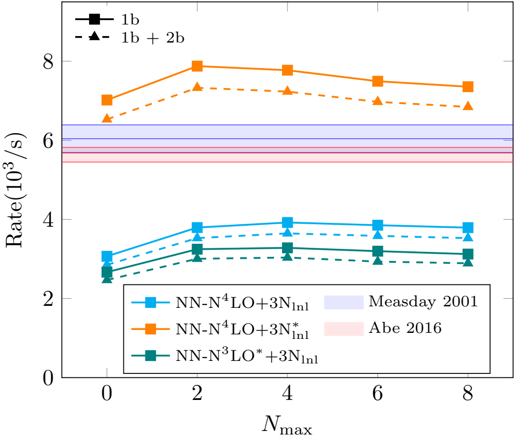
LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

Discovery, accelerated

Muon capture on ^{12}C

$$^{12}\mathrm{C}(0^+_{\mathrm{gs}}) + \mu^- \rightarrow \ ^{12}\mathrm{B}(1^+_{\mathrm{gs}}) + \nu_\mu$$

- The **NN-N^{4}LO+3N$^*_{\mathrm{lnl}}$** interaction with the additional spin-orbit 3N-force term most consistent with experiment



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

Discovery, accelerated

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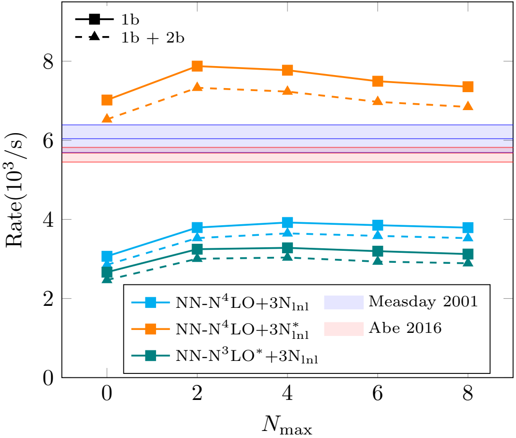
- Capture rates to excited states in ^{12}B also well reproduced



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

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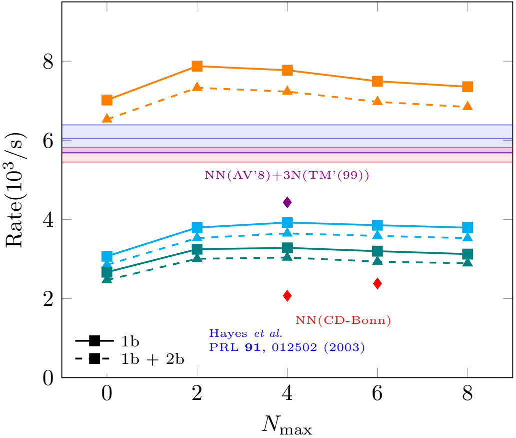
Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

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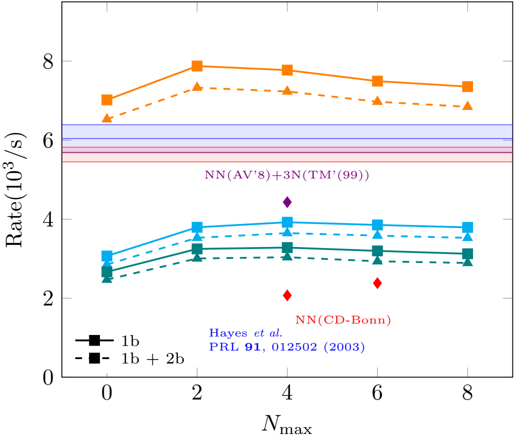
Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)

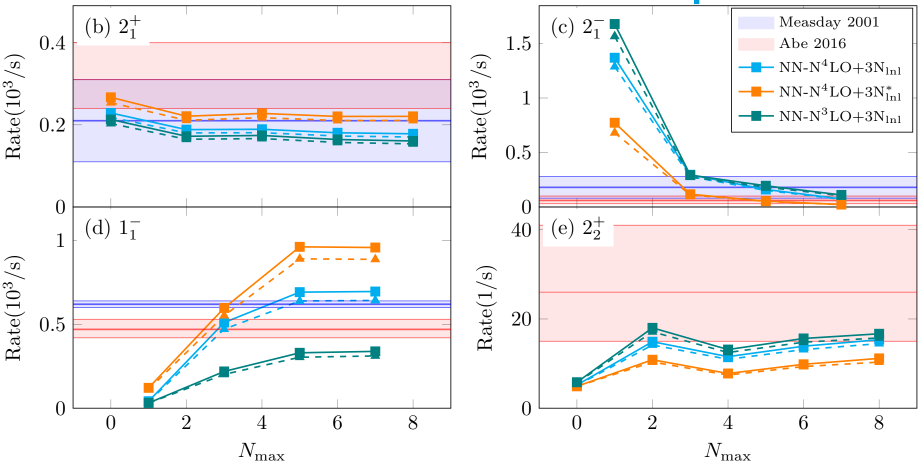
- 3N-forces essential to reproduce the measured rate



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

TRIUMF

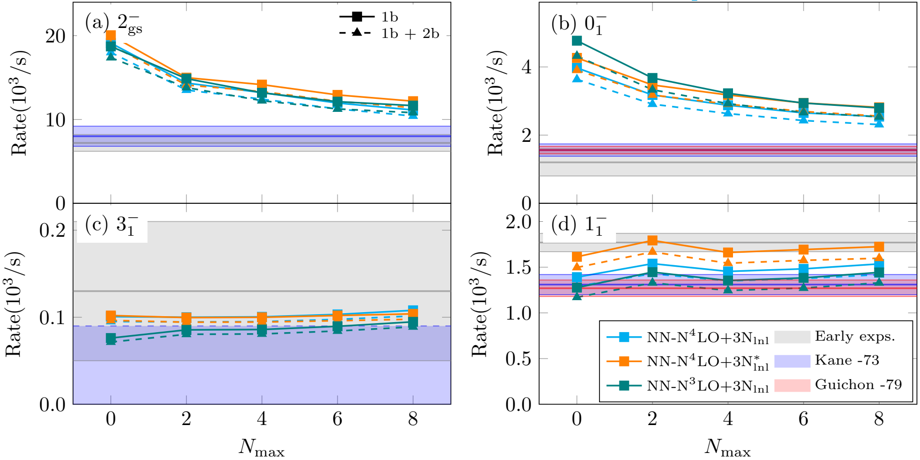
Muon capture on ^{12}C



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

Discovery, accelerated

Muon capture on ^{16}O



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

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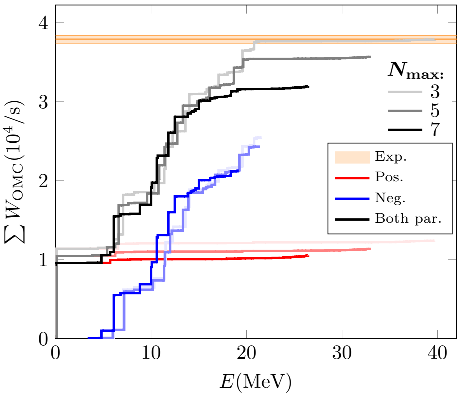
Total Muon-Capture Rates

- Rates obtained summing over ~ 50 final states of each parity

$$\mu^- + {}^{12}\mathrm{C}(0^+_{\mathrm{gs}}) \to \nu_\mu + {}^{12}\mathrm{B}(J^\pi_k)$$



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

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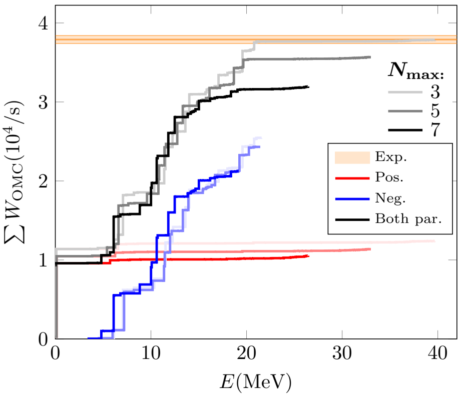
- Summing up the rates, we capture **$\sim$ 85% of the total rate** in both ^{12}B and ^{16}N

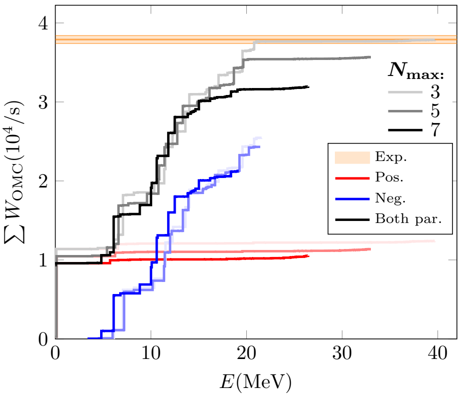


LJ, Navrátil, Kotila, Kravvaris,
Phys. Rev. C 109, 065501 (2024)

Total Muon-Capture Rates

- Rates obtained summing over ~ 50 final states of each parity
- Summing up the rates, we capture **$\sim$ 85% of the total rate** in both ^{12}B and ^{16}N
- Where is the rest?

$$\mu^- + {}^{12}\mathrm{C}(0^+_{\mathrm{gs}}) \rightarrow \nu_\mu + {}^{12}\mathrm{B}(J^\pi_k)$$



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

Total Muon-Capture Rates with the Lanczos Strength Function Method

$$W_{\text{tot.}} = \sum_f W_{\text{OMC}}(i \to f) \approx \sum_{f,J} g(q_f) |\langle \Psi_f | O_J(q_f) | \Psi_i \rangle|^2$$

- For each operator (in a q grid), compute pivot

$$|\Phi_1\rangle = \frac{O_J |\Psi_i\rangle}{\sqrt{\langle \Psi_i | O_J^\dagger O_J |\Psi_i\rangle}}$$

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- Construct Lanczos basis $\{|\Phi_i\rangle\}$ recursively:

$$\beta_{i+1} |\Phi_{i+1}\rangle = H|\Phi_i\rangle - \alpha_i |\Phi_i\rangle - \beta_{i+1} \Phi_{i+1} \,,$$

where $\alpha_i = \langle \Phi_i | H | \Phi_i \rangle$ and β_{i+1} s.t. $\langle \Phi_{i+1} | \Phi_{i+1} \rangle = 1$

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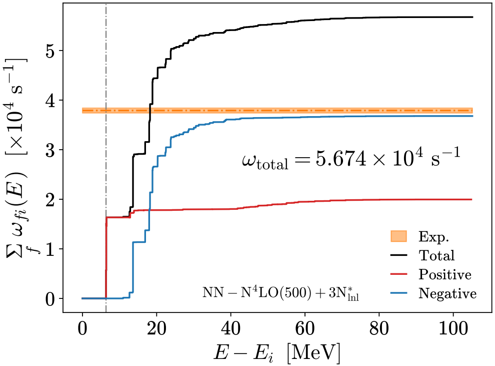
where $\alpha_i = \langle \Phi_i | H |\Phi_i\rangle$ and β_{i+1} s.t. $\langle \Phi_{i+1} | \Phi_{i+1}\rangle = 1$
- Extract strength from the orthonormality of the basis

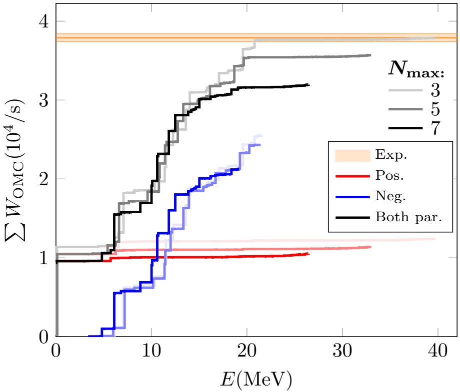
$$\langle \Psi_f | O_J(q_f) | \Psi_i \rangle = \sqrt{\langle \Psi_i | O_J^\dagger O_J |\Psi_i\rangle} \langle \Phi_0 \big| \Phi_f \rangle$$

VERY PRELIMINARY: Total Muon-Capture Rates with the Lanczos Strength Function



D. Araujo Najera, M. Gennari, LJ, M. Drissi, P. Navrátil, in preparation

$\mu^- + {}^{12}\mathrm{C}(0^+_{\mathrm{gs}}) \to \nu_\mu + {}^{12}\mathrm{B}(J^\pi_k)$



LJ, Navrátil, Kotila, Kravvaris, Phys. Rev. C 109, 065501 (2024)

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Outline

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Summary

- χEFT corrections to $0\nu\beta\beta$-decay seem to respect the power counting, but N^{2}LO corrections still significant
- Correlation between $0\nu\beta\beta$ and $2\nu\beta\beta$ decays helped us predict $0\nu\beta\beta$-decay NMEs with uncertainties
- Correlations with DGT and M1M1 transitions with future data can help us further constrain the NMEs
- Ab initio muon-capture studies could shed light on nuclear-weak current at finite momentum exchange regime relevant for $0\nu\beta\beta$ decay

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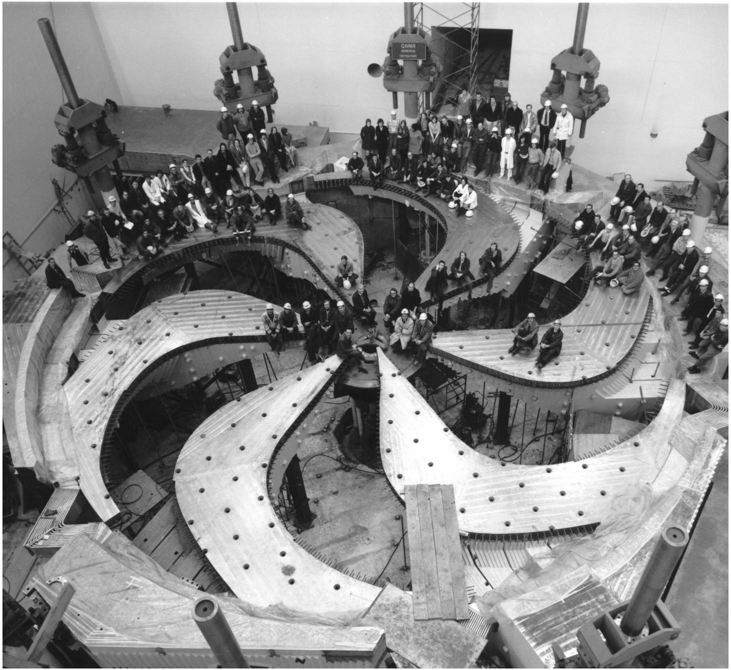
Thank you
Merci



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