Double β decay in the EDF-mapped IBM

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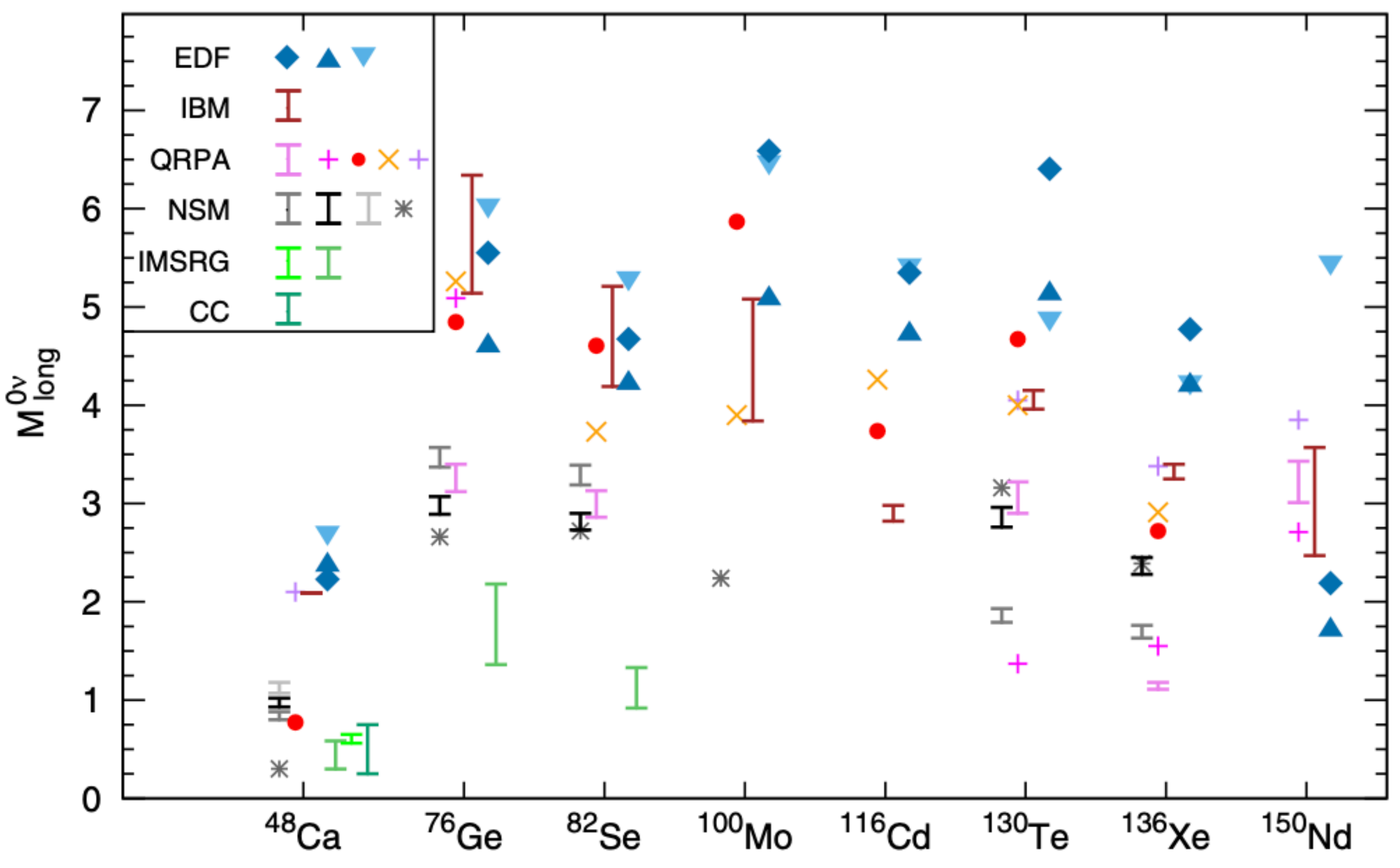
Contents

- Theoretical framework for $\beta\beta$ decay
- Low-lying structures of even-even and odd-odd nuclei
- $2\nu\beta\beta$ decay, source of uncertainty

K. Nomura, Phys. Rev. C 105 (2022) 044301
K. Nomura, Phys. Rev. C 110 (2024) 024304

Study of $\beta\beta$ decay

- Nature of neutrinos, BSM physics, …
- Experiments: GERDA, NEMO, KamLAND …
- Predicted NMEs differ by a factor of 2-3 among theories



Avignone et al., RMP (2008)
Agostini et al., RMP (2023)
Engel, Menendez, RPP (2017), etc.

Interacting Boson Model (IBM)

Arima, Iachello (1975)

- Collective J=0+ and 2+ pairs of valence nucleons $\mapsto$ s, d bosons

- Microscopic derivations from nucleonic degrees of freedom

  - Otsuka, Arima, Iachello (1979)
  - Mizusaki, Otsuka (1997)
  - Nomura, Shimizu, Otsuka (2008)

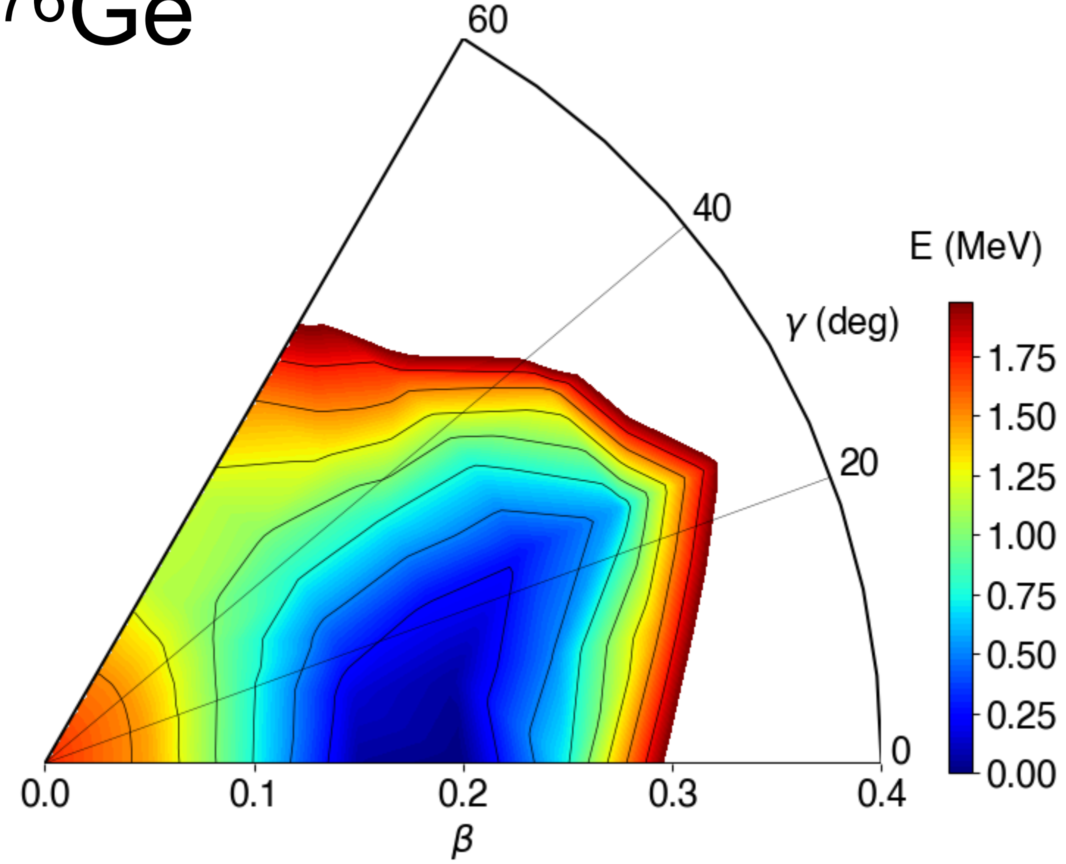
- Applications to $\beta\beta$ decays:

  - 0ν and 2ν in the closure approximation: Barea, Iachello (2009), Barea et al (2015), Deppisch et al. (2020), …
  - 2ν without the closure Yoshida, Iachello (2013)

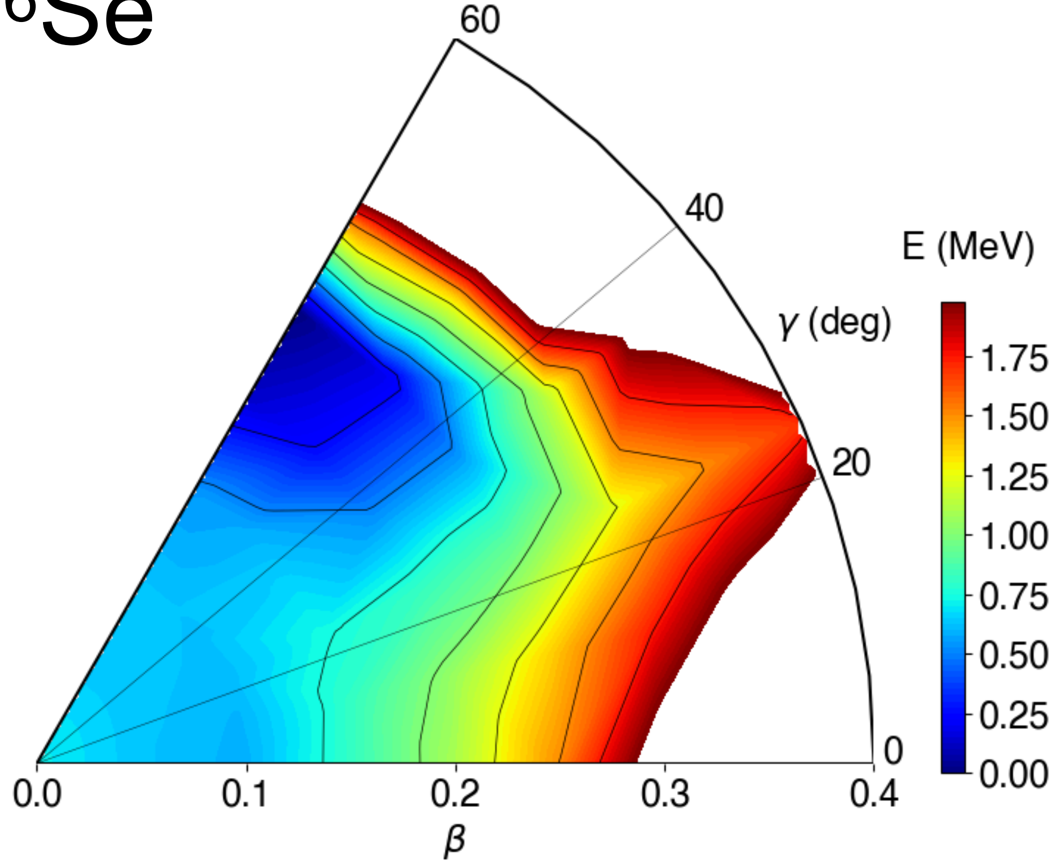
… based largely on the phenomenological fit

Self-consistent mean-field

^{76}Ge



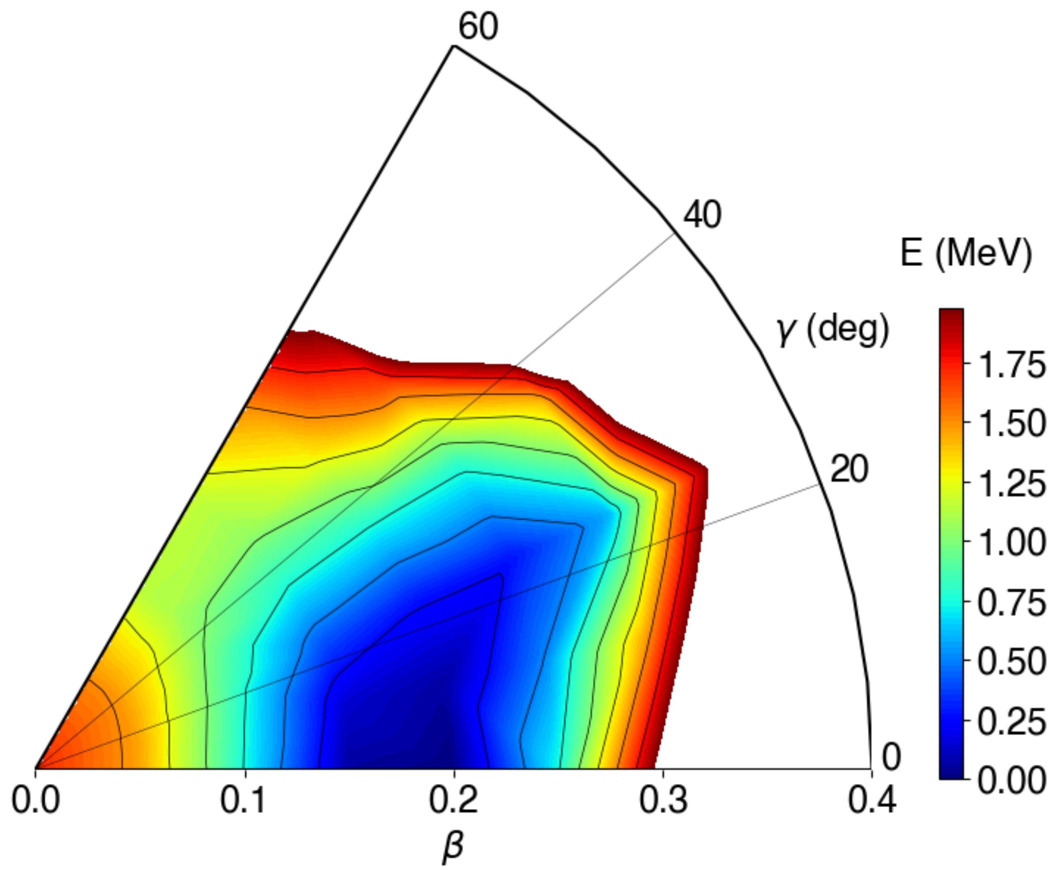
^{76}Se



… potential energy surfaces (PESs) from the relativistic Hartree-Bogoliubov method using DD-PC1 energy density functional (EDF) and separable pairing

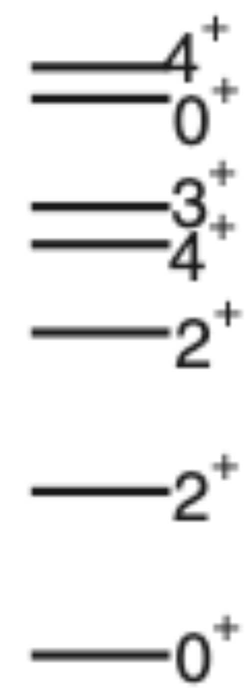
Computing energy spectra

Intrinsic frame



Beyond-mean-field treatments
- Symmetry projections, GCM
- Collective Hamiltonian
- Interacting Boson Model

lab. frame

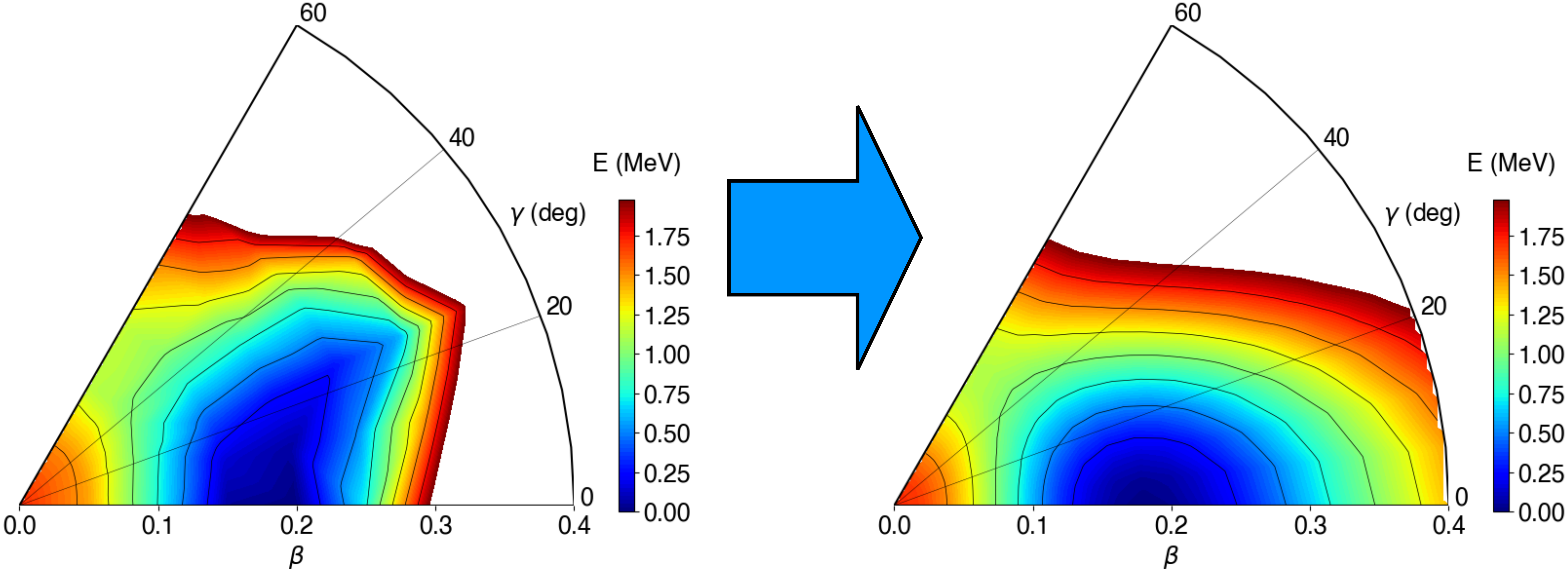


Observables:
Excitation spectra, EM properties, β, $\beta\beta$ decay?

Mean-field to IBM

Fermionic

Bosonic



- SCMF energy surface is mapped onto that of the IBM
- the mapped Hamiltonian yields nuclear wave functions

KN et al. PRL101 (2008) 142501; PRC81 (2010) 044307

IBM for even-even nuclei

- building blocks: (s_ν, s_π) and (d_ν, d_π) bosons

- Hamiltonian:

$$\hat{H}_{\mathrm{IBM}} = \epsilon_d (\hat{n}_{d_\nu} + \hat{n}_{d_\pi}) + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi$$

pairing-like (spherical driving) → $\epsilon_d (\hat{n}_{d_\nu} + \hat{n}_{d_\pi})$

quadrupole-quadrupole (deformation driving) → $\kappa \hat{Q}_\nu \cdot \hat{Q}_\pi$

$$\hat{n}_{d_\rho} = d_\rho^\dagger \cdot \tilde{d}_\rho$$

$$\hat{Q}_\rho = s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger s_\rho + \chi_\rho (d_\rho^\dagger \times \tilde{d}_\rho)^{(2)}$$

… with 4 parameters: $\epsilon_d, \kappa, \chi_\nu, \chi_\pi$

Geometry of the IBM

Energy surface: $$E_{\mathrm{IBM}}(\beta,\gamma) = \langle \phi | \hat{H}_{\mathrm{IBM}} | \phi \rangle$$

… with boson coherent state

$$|\phi\rangle \propto \Pi_{\rho=\nu,\pi} \left[s_\rho^\dagger + \beta \cos\gamma d_{\rho,0}^\dagger + \frac{1}{\sqrt{2}} \beta \sin\gamma \left(d_{\rho,+2}^\dagger + d_{\rho,-2}^\dagger \right) \right]^{N_\rho} |0\rangle$$

Ginocchio-Kirson (1980)

IBM Hamiltonian is determined by

$$E_{\mathrm{SCMF}}(\beta,\gamma) \approx E_{\mathrm{IBM}}(\beta,\gamma)$$

KN et al. PRL101 (2008) 142501

IBFFM for odd-odd nuclei

$$\hat{H}_{\mathrm{IBFFM}} = \hat{H}_{\mathrm{IBM}} + \hat{H}_{\mathrm{F}} + \hat{V}_{\mathrm{BF}} + \hat{V}_{\nu\pi}$$

Single-fermion Hamiltonian

$$\hat{H}_{\mathrm{F}} = \sum_{j_\rho} \epsilon_{j_\rho} \hat{n}_{j_\rho}$$

Boson-fermion interactions

$$\hat{V}_{\mathrm{BF}} = \Gamma_0 \hat{V}_{\mathrm{dyn}} + \Lambda_0 \hat{V}_{\mathrm{exc}} + A_0 \hat{V}_{\mathrm{mon}}$$

dynamical (direct) term

exchange term

monopole term

neutron-proton interaction

$$\hat{V}_{\nu\pi} = 4\pi [v_{\mathrm{d}} + v_{\mathrm{ssd}} \boldsymbol{\sigma}_\nu \cdot \boldsymbol{\sigma}_\pi] \delta(\boldsymbol{r}) \delta(\boldsymbol{r}_\nu - r_0) \delta(\boldsymbol{r}_\pi - r_0) - \frac{1}{\sqrt{3}} v_{\mathrm{ss}} \boldsymbol{\sigma}_\nu \cdot \boldsymbol{\sigma}_\pi + v_{\mathrm{t}} \left[\frac{3(\boldsymbol{\sigma}_\nu \cdot \mathbf{r})(\boldsymbol{\sigma}_\pi \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_\nu \cdot \boldsymbol{\sigma}_\pi \right]$$

Iachello, Van Isacker, “The interacting boson-fermion model” (1991)

Boson-fermion interactions

$$\hat{V}^{\rho}_{\rm dyn} = \sum_{j_\rho j'_\rho} \gamma_{j_\rho j'_\rho} (a^\dagger_{j_\rho} \times \tilde{a}_{j'_\rho})^{(2)} \cdot \hat{Q}_{\rho'},$$

$$\hat{V}^{\rho}_{\rm exc} = -\left(s^\dagger_{\rho'} \times \tilde{d}_{\rho'}\right)^{(2)} \cdot \sum_{j_\rho j'_\rho j''_\rho} \sqrt{\frac{10}{N_\rho(2j_\rho+1)}} \beta_{j_\rho j'_\rho} \beta_{j''_\rho j_\rho} : \left[(d^\dagger_\rho \times \tilde{a}_{j''_\rho})^{(j_\rho)} \times (a^\dagger_{j'_\rho} \times \tilde{s}_\rho)^{(j'_\rho)}\right]^{(2)} : +({\rm H.c.}) ,$$

$$\hat{V}^{\rho}_{\rm mon} = \hat{n}_{d_\rho} \hat{n}_{j_\rho} ,$$

with (u,v)-dependent factors:

$$\gamma_{j_\rho j'_\rho} = (u_{j_\rho} u_{j'_\rho} - v_{j_\rho} v_{j'_\rho}) Q_{j_\rho j'_\rho}$$

$$\beta_{j_\rho j'_\rho} = (u_{j_\rho} v_{j'_\rho} + v_{j_\rho} u_{j'_\rho}) Q_{j_\rho j'_\rho}$$

… derived within the generalized seniority

e.g., Schoten, PPNP (1985)

Building the IBFFM Hamiltonian

$$\hat{H}_{\mathrm{IBFFM}} = \hat{H}_{\mathrm{IBM}} + \hat{H}_{\mathrm{F}} + \hat{V}_{\mathrm{BF}} + \hat{V}_{\nu\pi}$$

mapping the PES

spherical s.p. e. (ϵ_j)

occupation probabilities v_j^2 at spherical configuration

Microscopic input from EDF

… 3 strength parameters for $\hat{V}_{\mathrm{BF}}$ fitted for each nucleus (odd-N, odd-Z, and parity)

… 4 strength parameters for $\hat{V}_{\nu\pi}$ fitted for each odd-odd nucleus

KN et al. PRC93 (2016) 054305; PRC99 (2019) 034308

Calculation of NME

$$M_{2\nu} = g_{\mathrm{A}}^2 \cdot m_e c^2 \left[M_{2\nu}^{\mathrm{GT}} - \left(\frac{g_{\mathrm{V}}}{g_{\mathrm{A}}} \right)^2 M_{2\nu}^{\mathrm{F}} \right]$$

Gamow-Teller:

$$M_{2\nu}^{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+ \| \tau^\dagger \sigma \| 1_n^+ \rangle \langle 1_n^+ \| \tau^\dagger \sigma \| 0_{1,i}^+ \rangle}{E_n - E_i + (Q_{\beta\beta} + 2 m_e c^2)/2}$$

Fermi:

$$M_{2\nu}^{\mathrm{F}} = \sum_n \frac{\langle 0_f^+ \| \tau^\dagger \| 0_n^+ \rangle \langle 0_n^+ \| \tau^\dagger \| 0_{1,i}^+ \rangle}{E_n - E_i + (Q_{\beta\beta} + 2 m_e c^2)/2}$$

- $\tau^\dagger \sigma$ and $\tau^\dagger$ are GT and F operators
- $|1_n^+\rangle$, $|0_n^+\rangle$ intermediate states of the odd-odd nucleus

One particle transfer operator

Mapping onto the boson image:

$$\tau^\dagger\sigma \longmapsto \hat{O}^{\mathrm{GT}} = \sum_{j_\nu j_\pi} \eta^{\mathrm{GT}}_{j_\nu j_\pi} \left(\hat{P}_{j_\nu} \times \hat{P}_{j_\pi}\right)^{(1)},$$

$$\tau^\dagger \longmapsto \hat{O}^{\mathrm{F}} = \sum_{j_\nu j_\pi} \eta^{\mathrm{F}}_{j_\nu j_\pi} \left(\hat{P}_{j_\nu} \times \hat{P}_{j_\pi}\right)^{(0)},$$

with

$$\eta^{\mathrm{GT}}_{j_\nu j_\pi} = -\frac{1}{\sqrt{3}} \langle \ell_\nu 1/2; j_\nu \| \sigma \| \ell_\pi 1/2; j_\pi \rangle \, \delta_{\ell_\nu \ell_\pi}$$

$$\eta^{\mathrm{F}}_{j_\nu j_\pi} = -\sqrt{2j_\nu + 1}\delta_{j_\nu j_\pi} .$$

One-particle transfer operator $\hat{P}_{j_\rho}$ corresponds to:

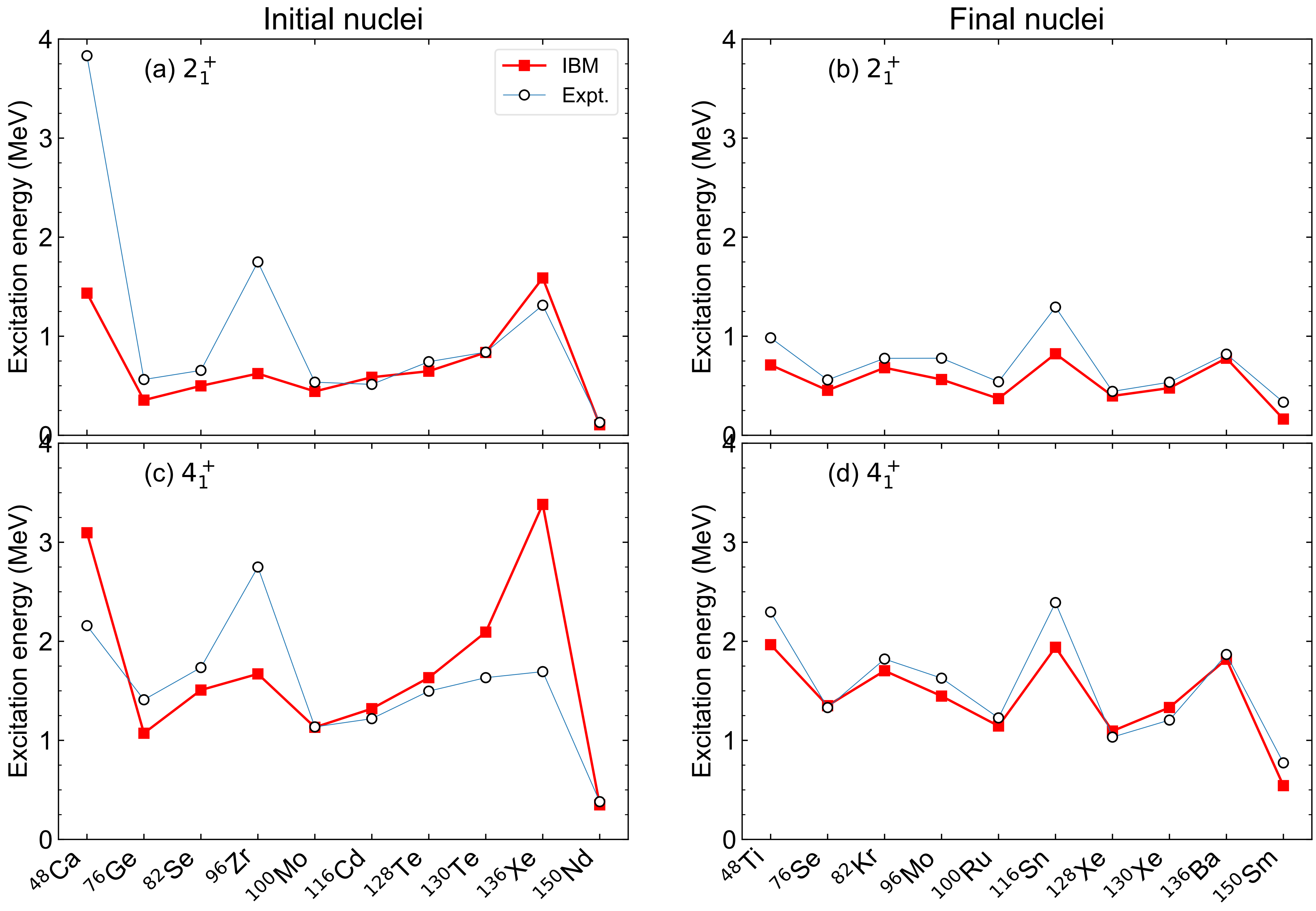
$$A^\dagger_{j_\rho m_\rho} = \zeta_{j_\rho} a^\dagger_{j_\rho m_\rho} + \sum_{j'_\rho} \zeta_{j_\rho j'_\rho} s^\dagger_\rho (\tilde{d}_\rho \times a^\dagger_{j'_\rho})^{(j_\rho)}_{m_\rho}$$

$$B^\dagger_{j_\rho m_\rho} = \theta_{j_\rho} s^\dagger_\rho \tilde{a}_{j_\rho m_\rho} + \sum_{j'_\rho} \theta_{j_\rho j'_\rho} (d^\dagger_\rho \times \tilde{a}_{j'_\rho})^{(j_\rho)}_{m_\rho},$$

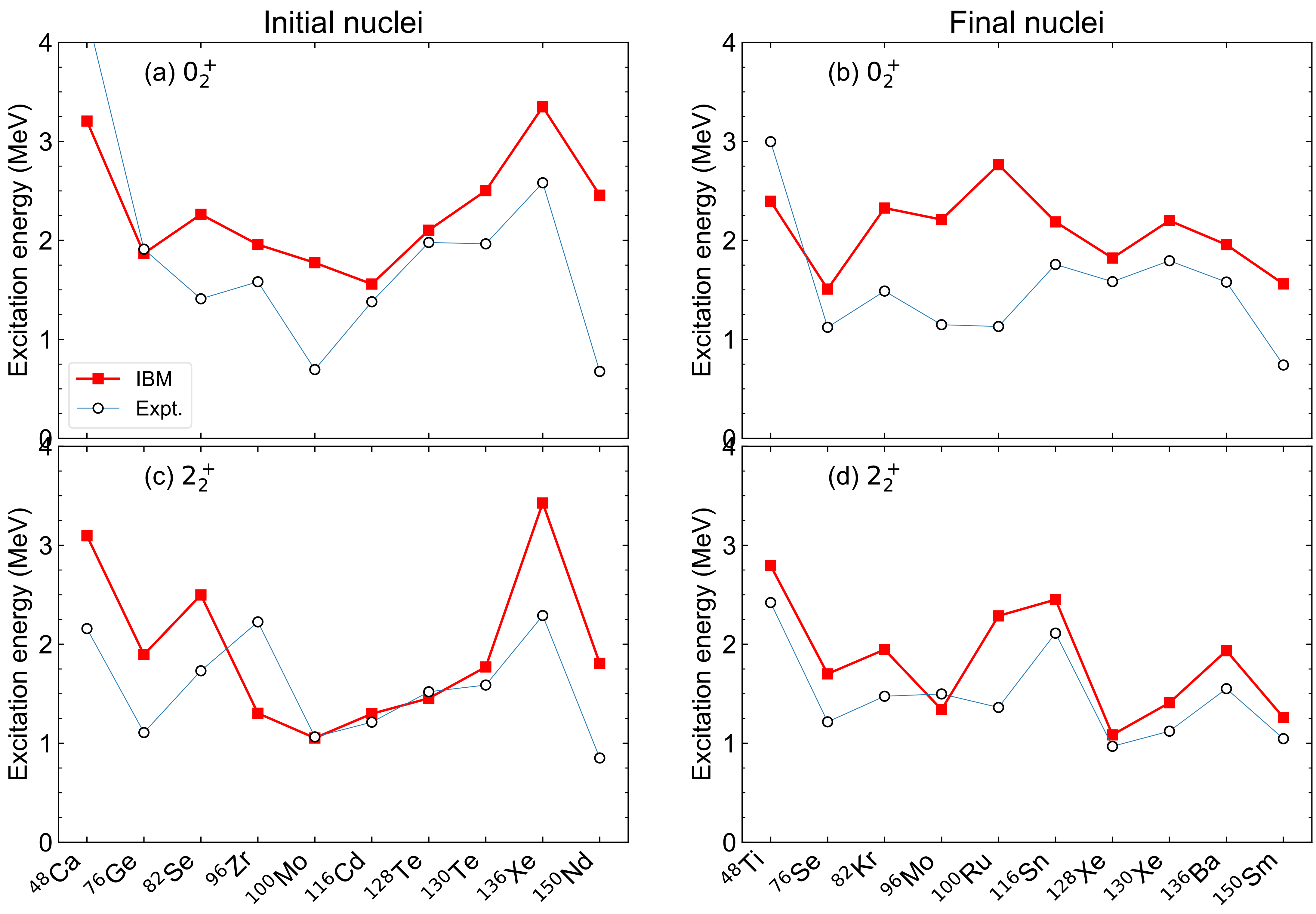
Coefficients ζ 's and θ 's are functions of (u_j , v_j) factors, which are computed by the SCMF

Low-lying structure

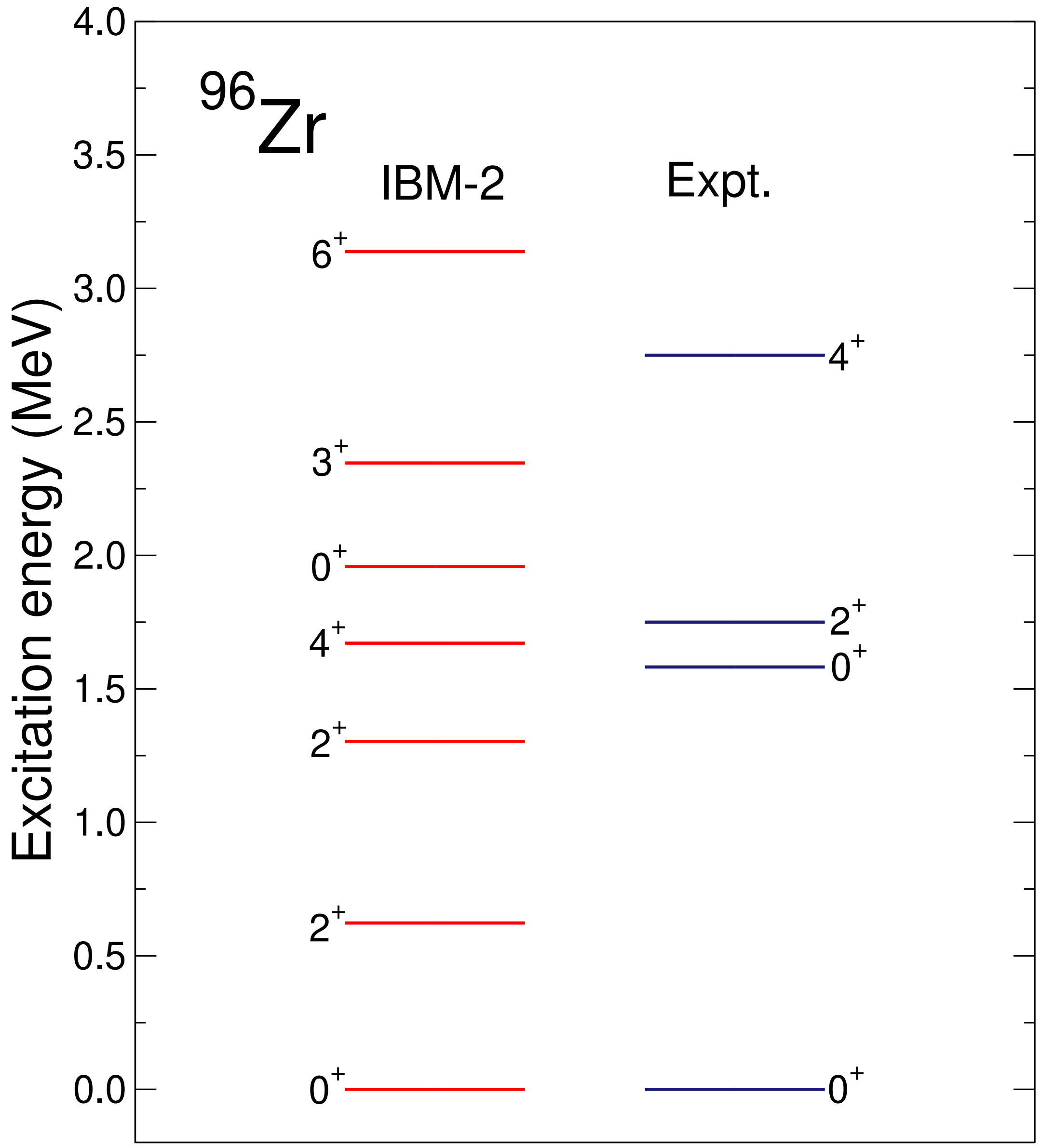
Energy spectra for even-even nuclei

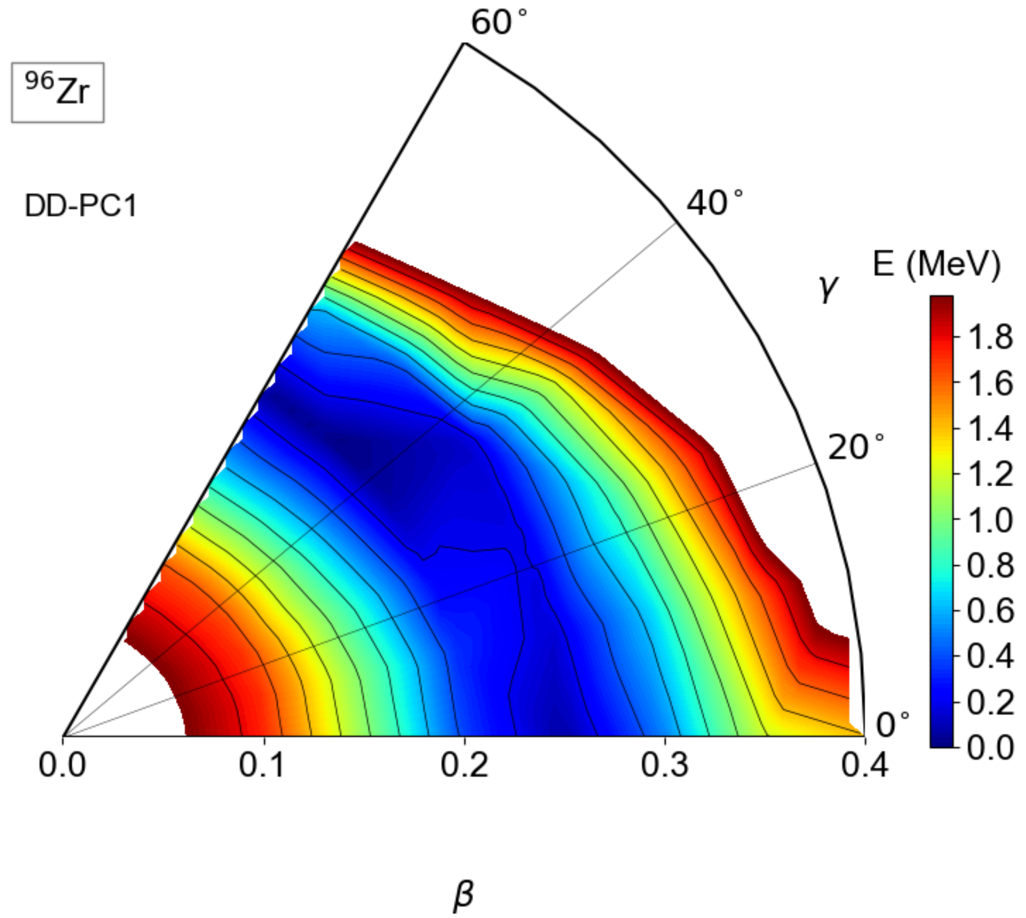


Energy spectra for even-even nuclei



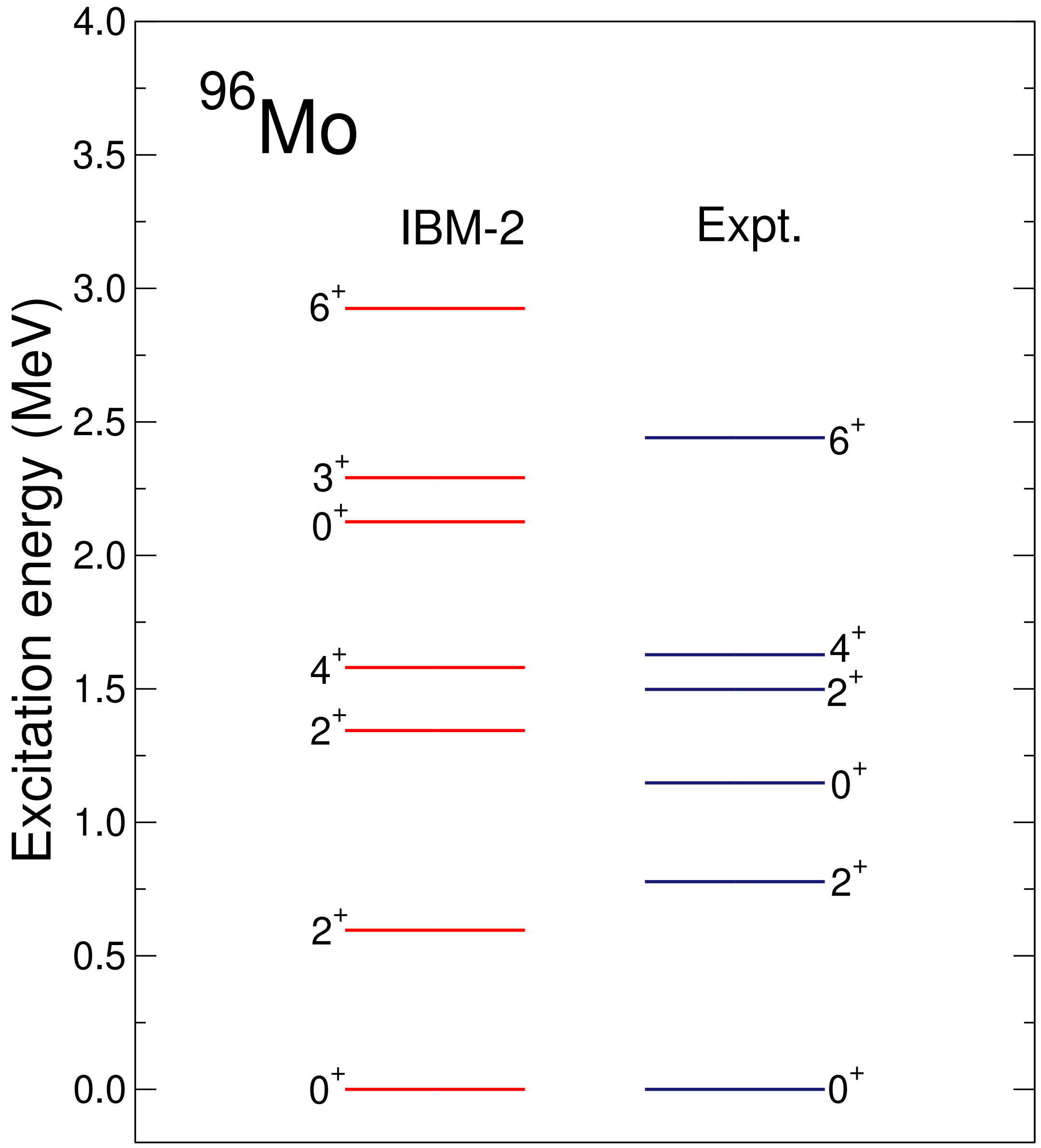
^{96}Zr spectrum

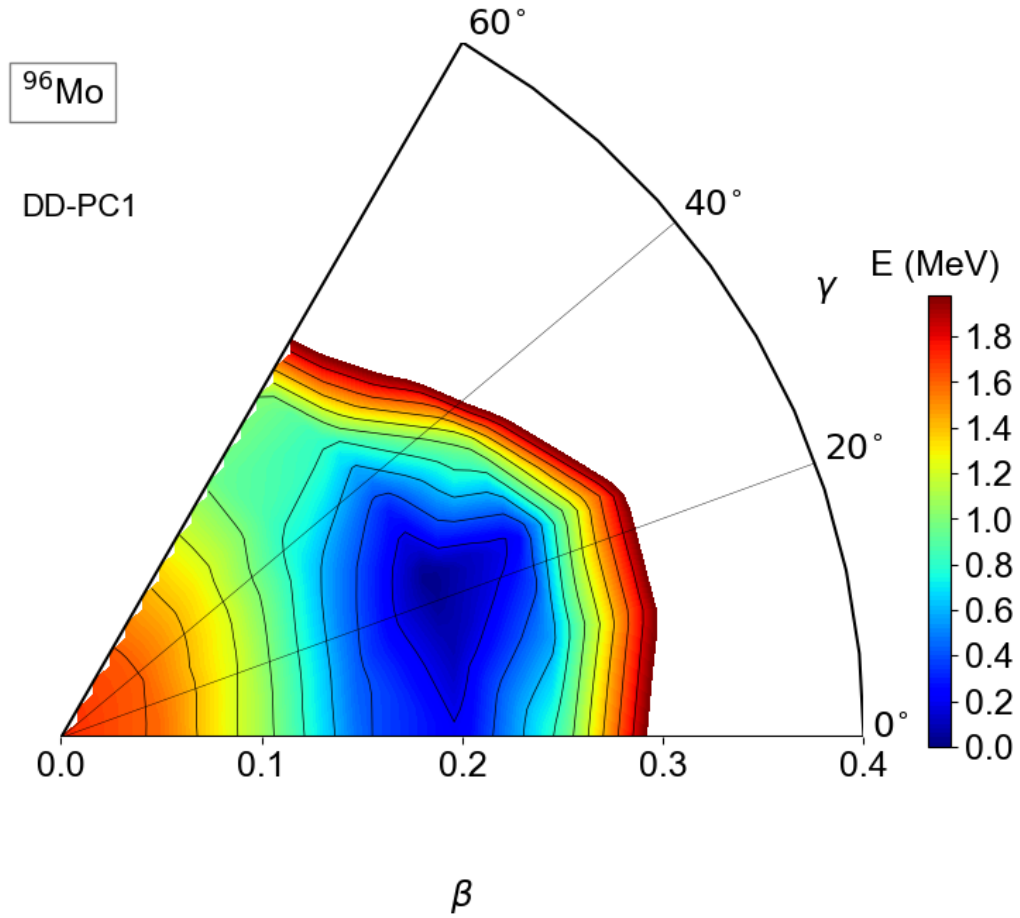




… the N=56 sub-shell effect not described due to the PES exhibiting a strong deformation

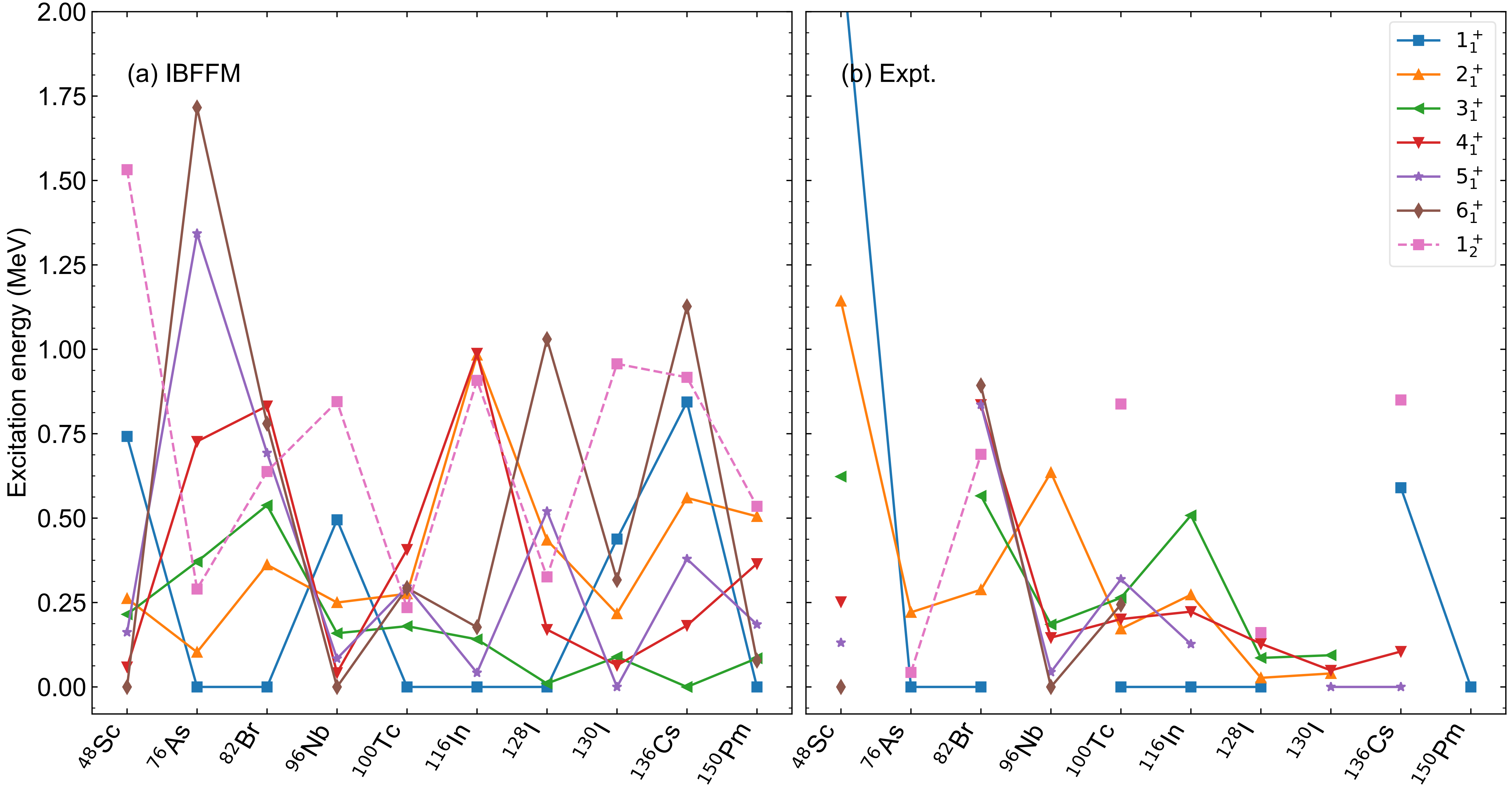
^{96}Mo spectrum





… low-energy 0_2^+ level not reproduced due to the lack of configuration mixing

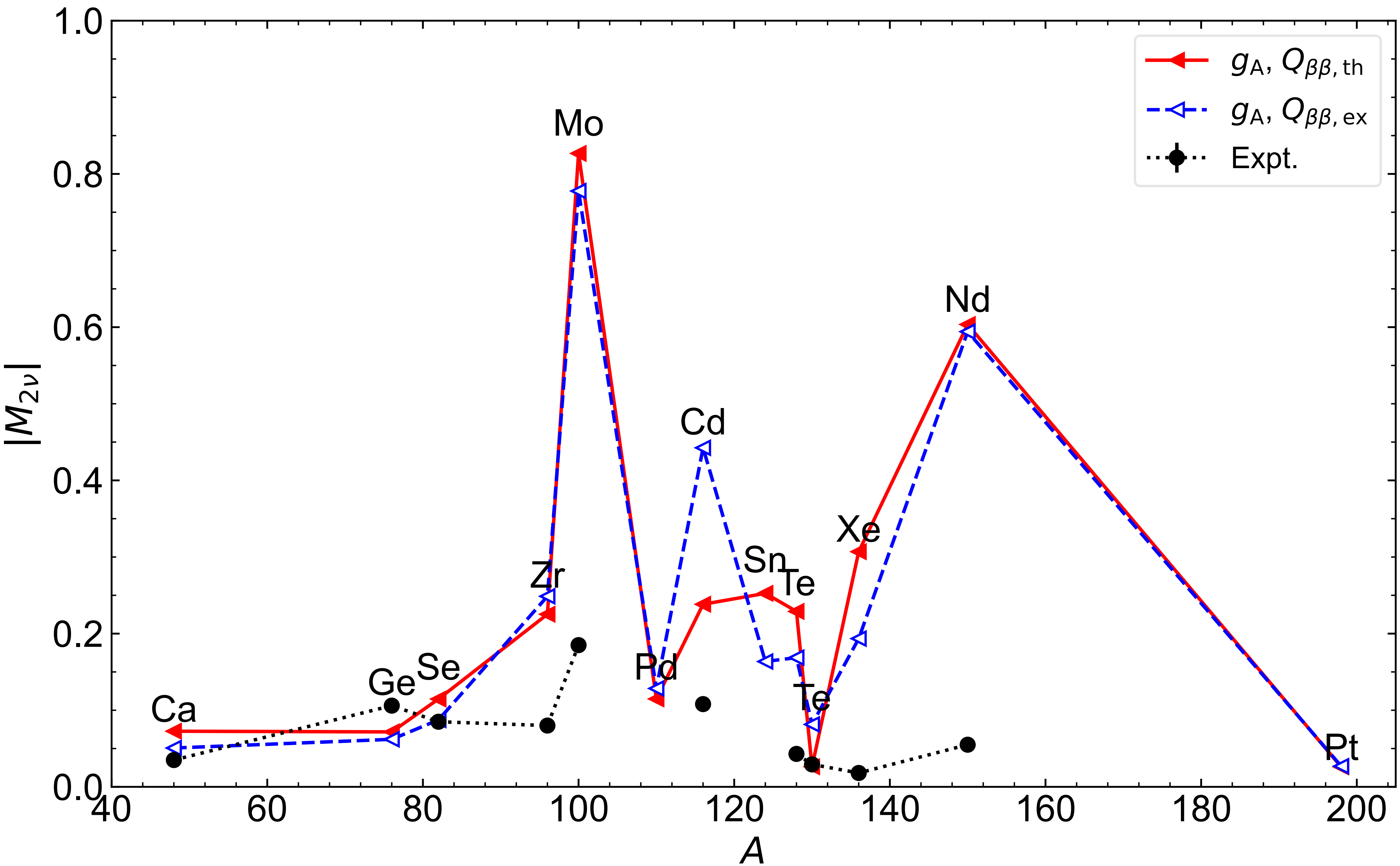
Energy spectra of odd-odd nuclei



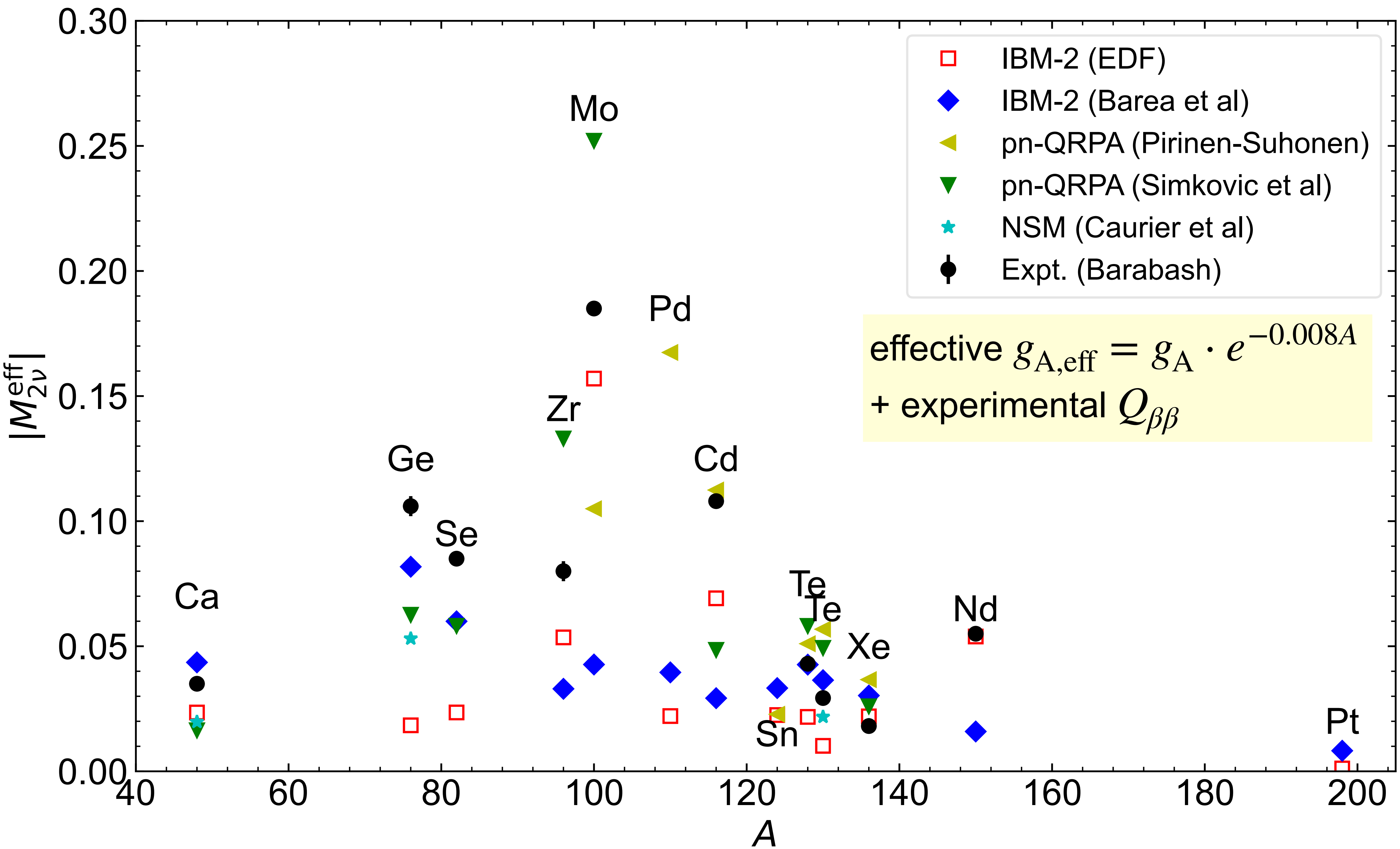
… ground state spin and locations of 1^+ levels reproduced to a reasonable accuracy

$2\nu\beta\beta$ decay

NMEs (GT unquenched)

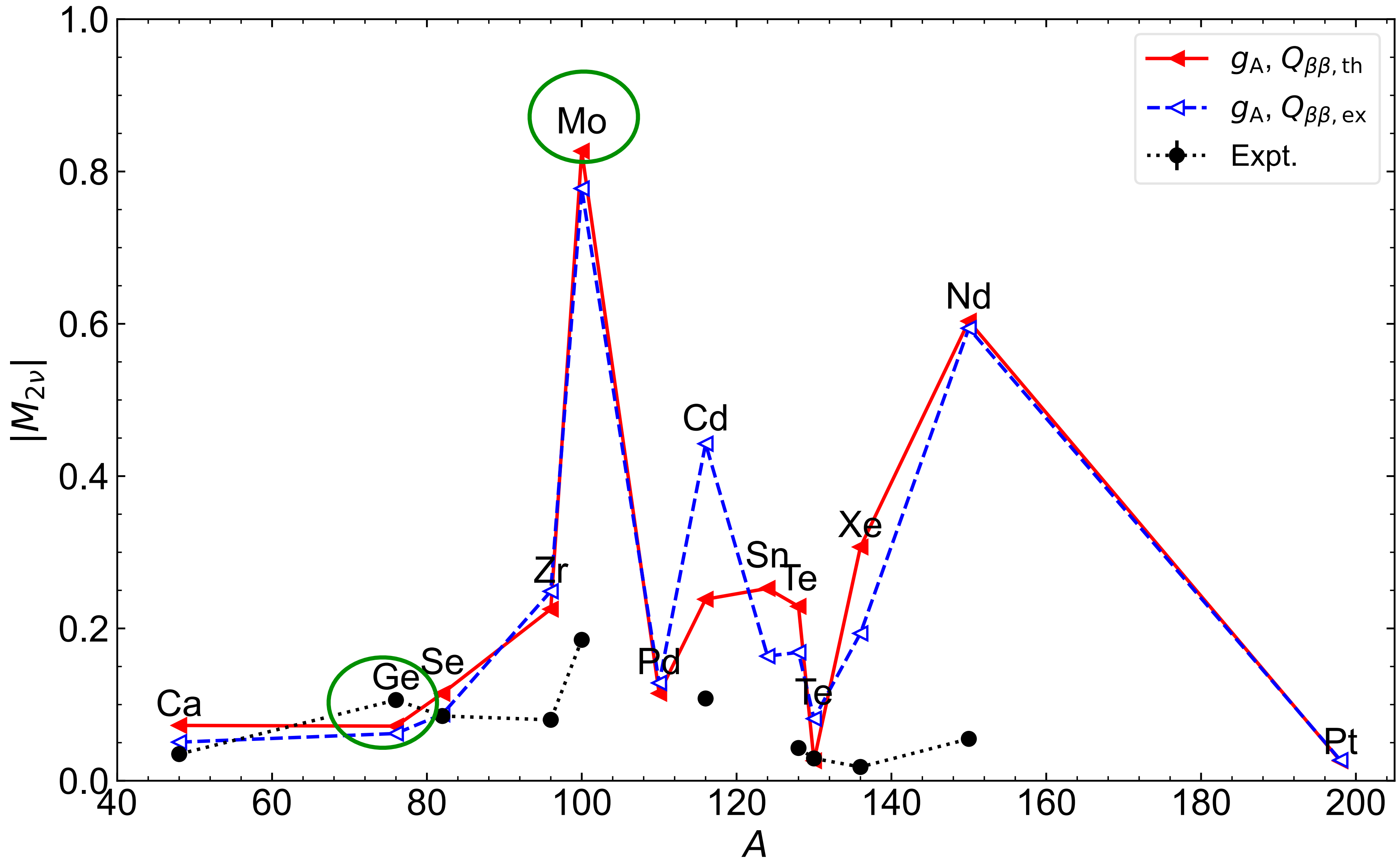


Effective NMEs



Source of uncertainties

- SCMF: choice of the EDF, pairing, single-particle energies, …
- IBM/IBFFM: Hamiltonian, configuration mixing, ….



Pairing interaction in the RHB

… separable pairing force of finite range

Tian, Ma, Ring (2009)

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_1', \mathbf{r}_2') = -V\delta(\mathbf{R}-\mathbf{R}')P(\mathbf{r})P(\mathbf{r}')\frac{1}{2}(1-P^\sigma),$$

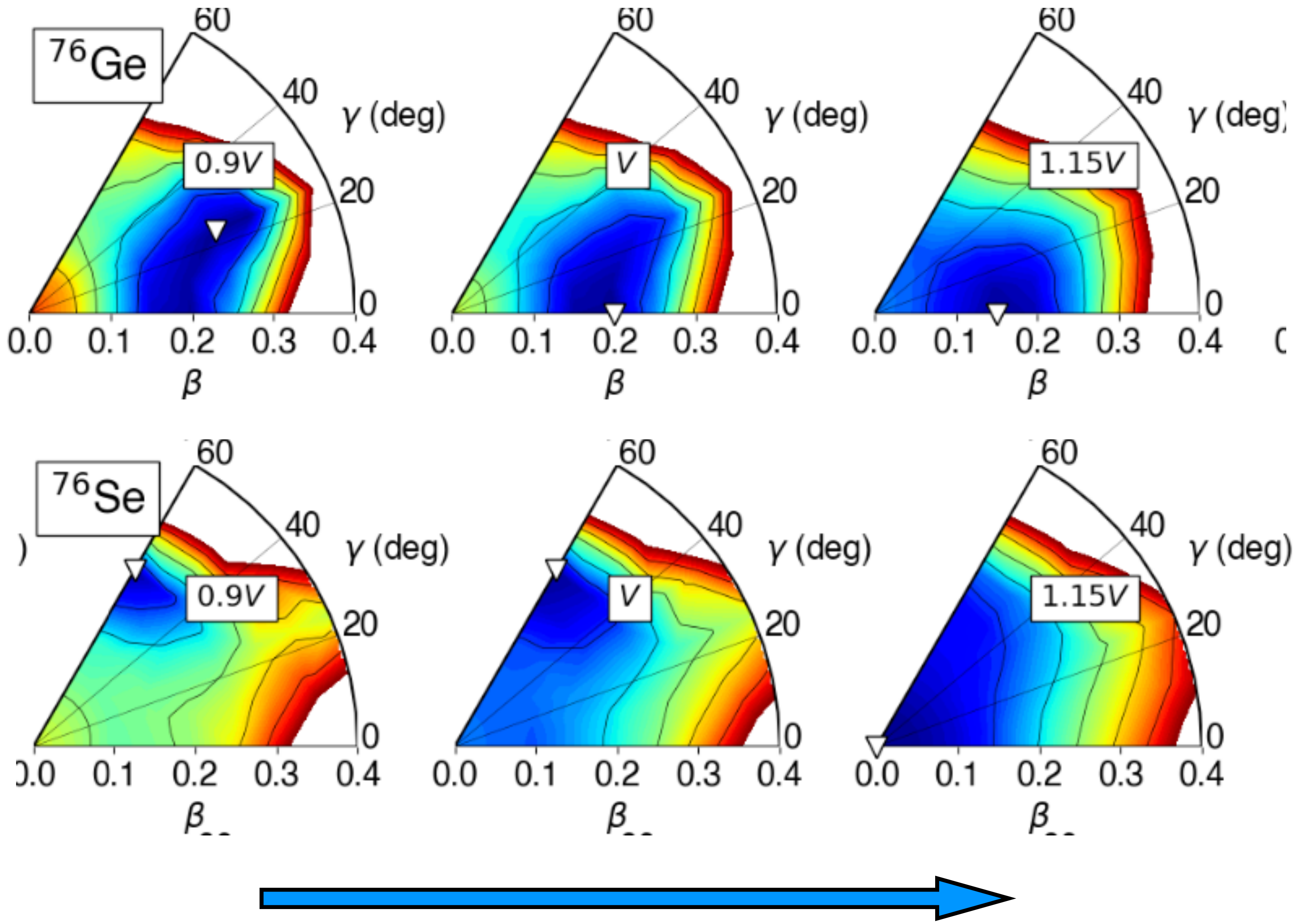
$$P(\mathbf{r}) = \frac{1}{(4\pi a^2)^{3/2}} e^{-\mathbf{r}^2/4a^2}$$

… strength V=728 MeV fm3 fit to Gogny D1S pairing gap

compare results with the pairing strength V

- 10 % reduced
- default
- 15 % enhanced

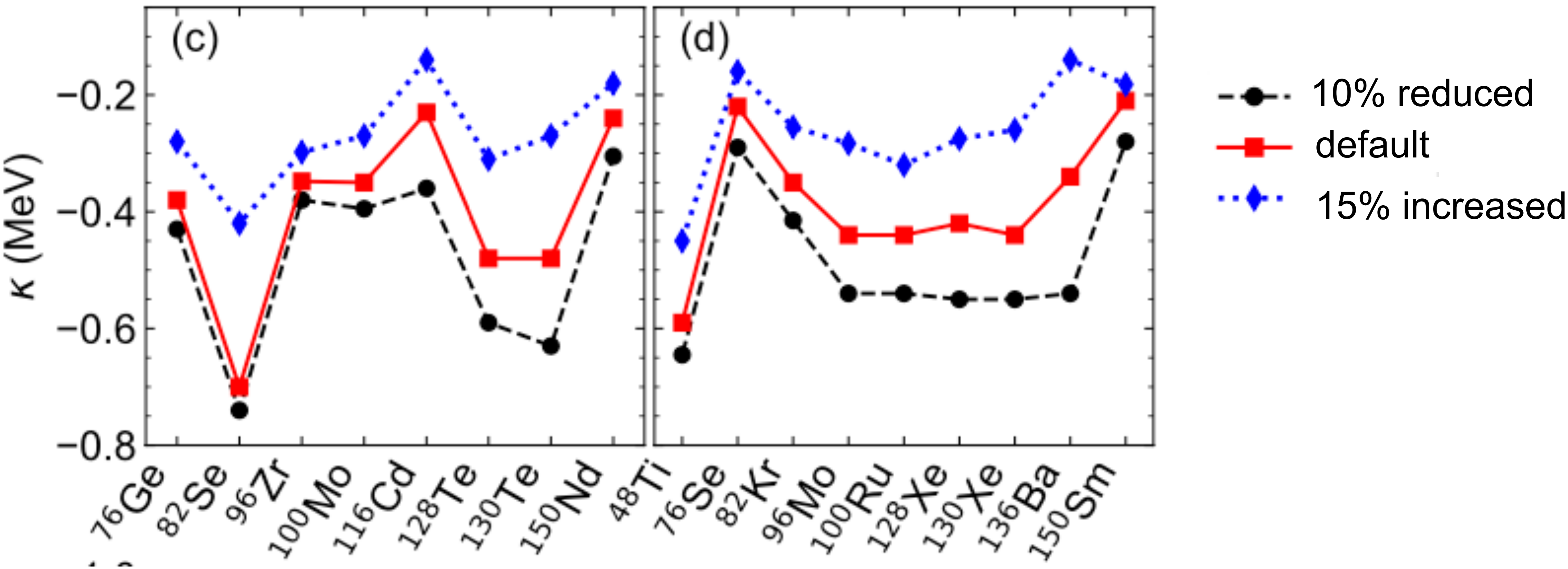
Sensitivity to the pairing strength



increased pairing strength favors less pronounced deformation

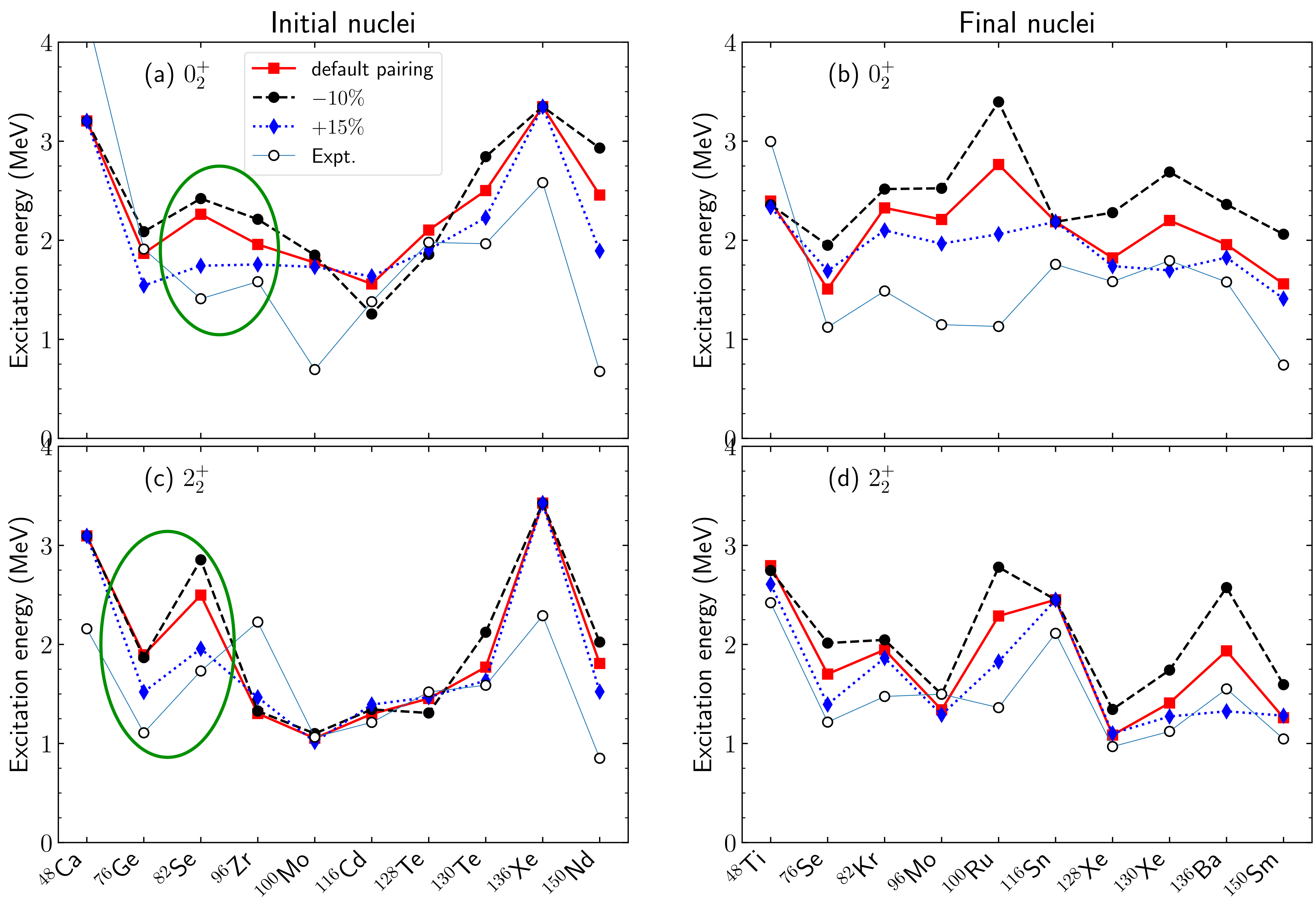
Influence on IBM parameters

$$\hat{H}_{\mathrm{IBM}} = \epsilon_d (\hat{n}_{d_\nu} + \hat{n}_{d_\pi}) + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi$$



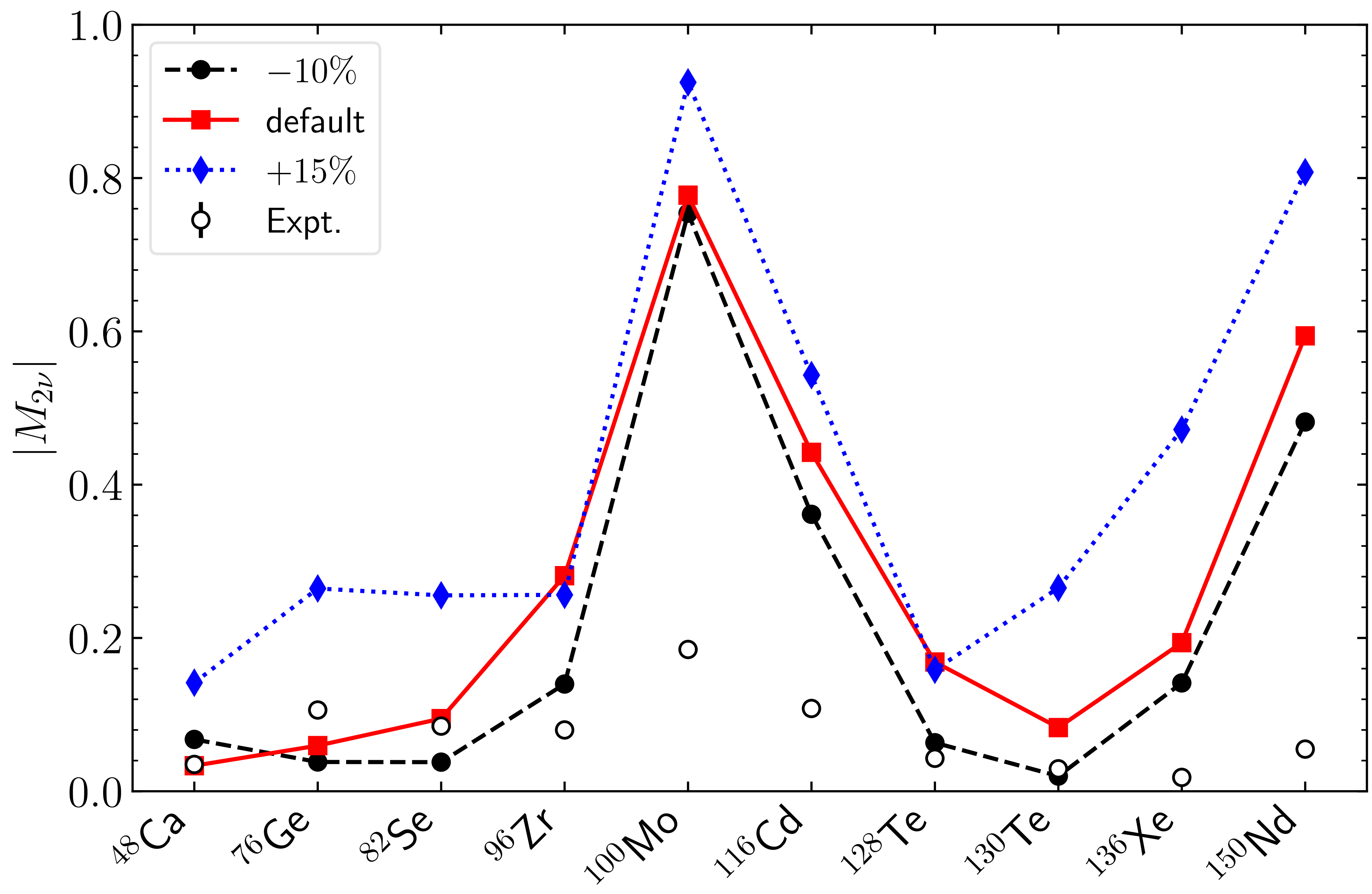
… quadrupole-quadrupole strength parameter (κ) is significantly reduced with the enhanced pairing

Impacts on energy spectra



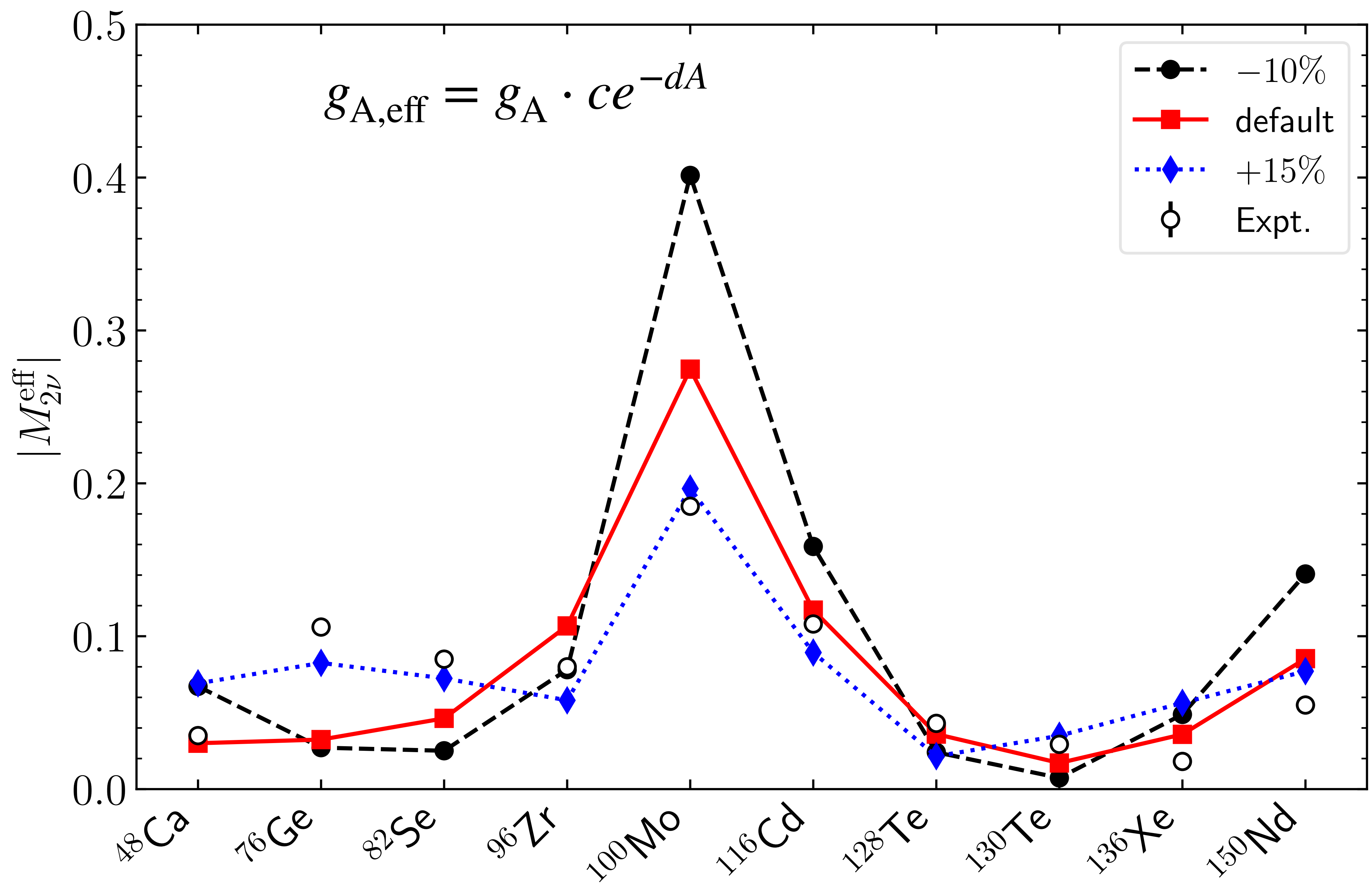
0_2^+ and 2_2^+ energy levels are lowered with the increased (+15%) pairing

Unquenched NMEs



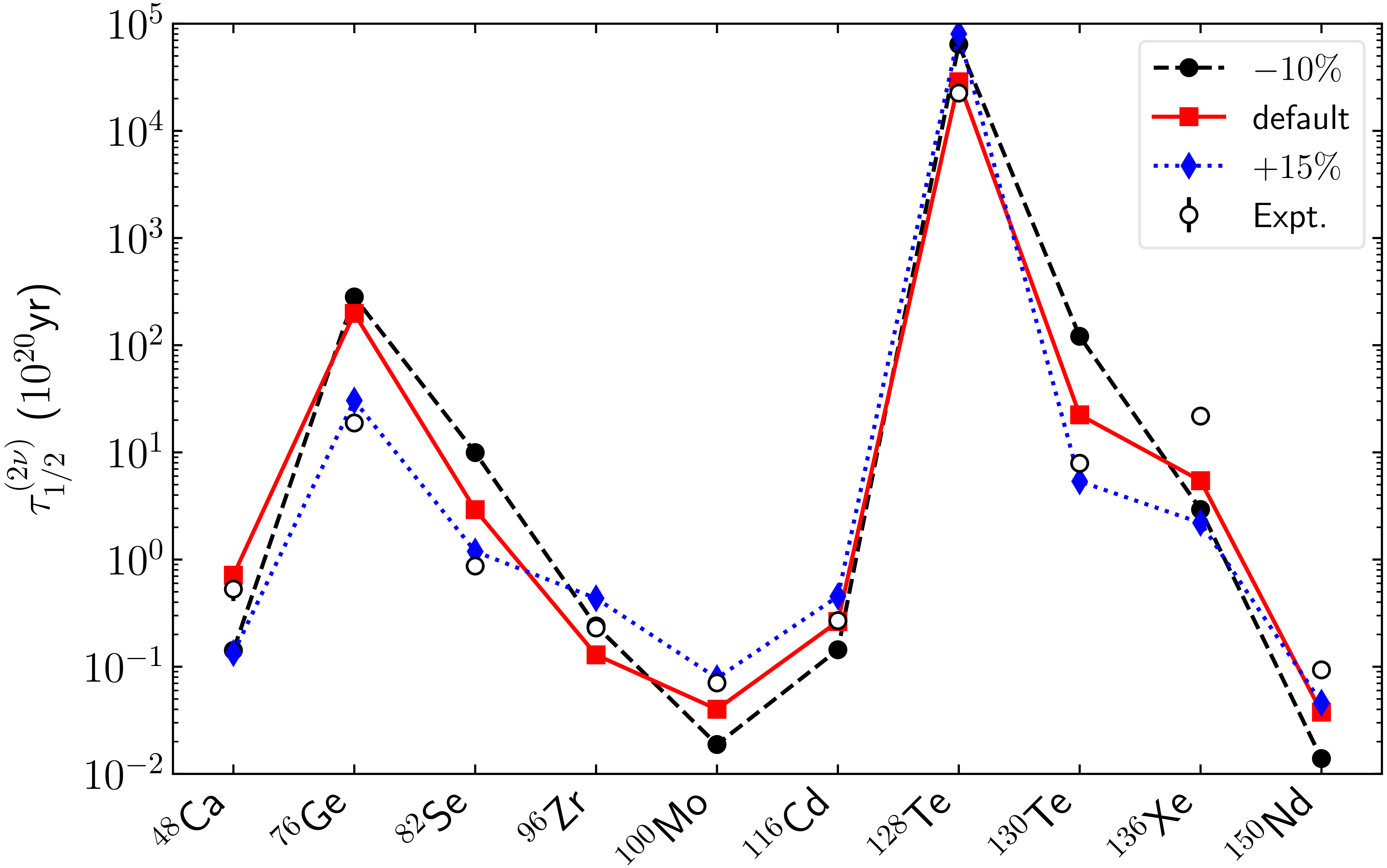
… NMEs increased by the increased pairing strength

Effective NMEs



… agree with the 76Ge, 82Se, and 100Mo decay NMEs with the 15% increased pairing strengths and $g_{A,\mathrm{eff}}$ ~ 0.9-0.4

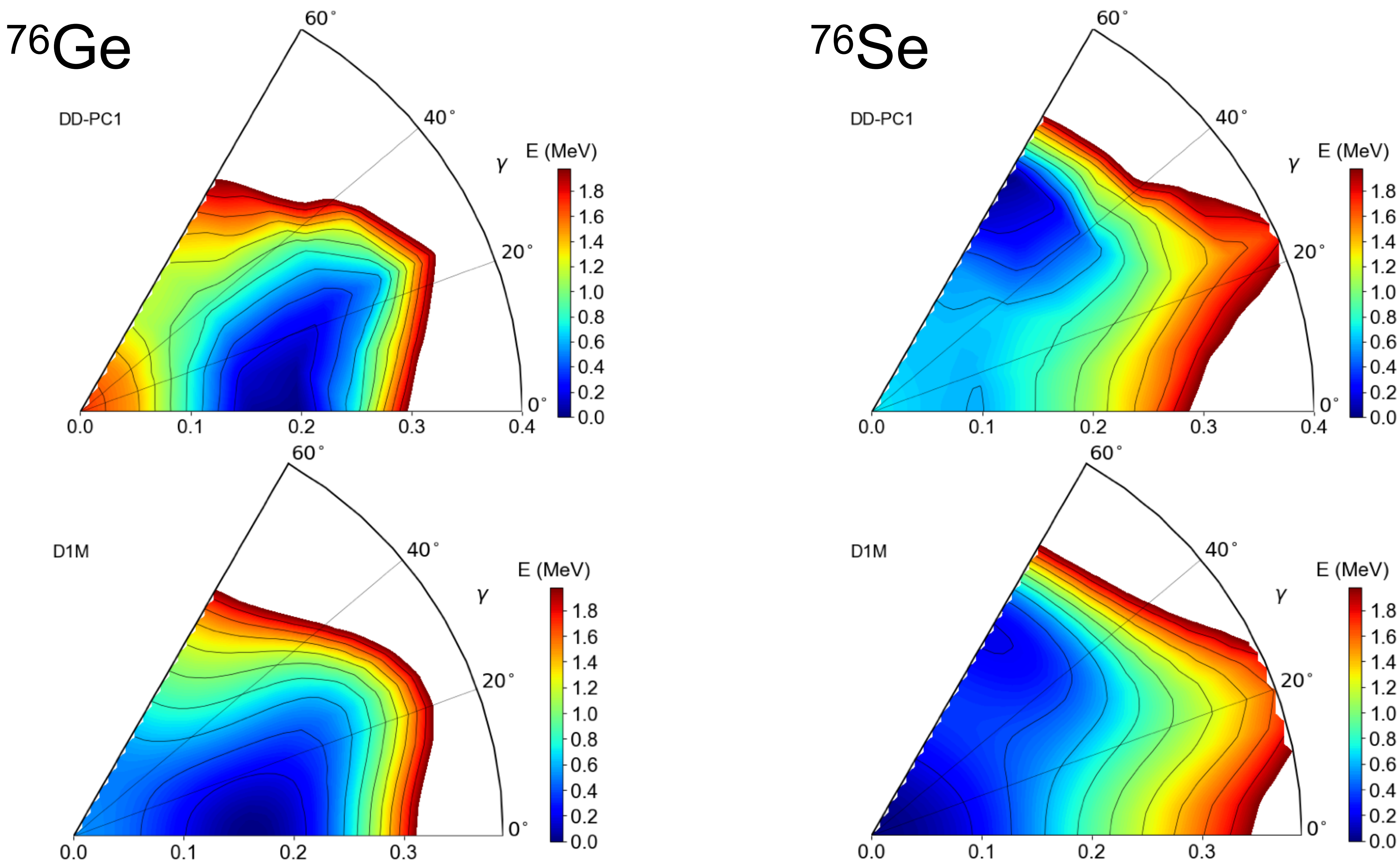
Half-lives



$$\left[\tau_{1/2}^{(2\nu)}\right]^{-1} = G_{2\nu}|M_{2\nu}|^2$$

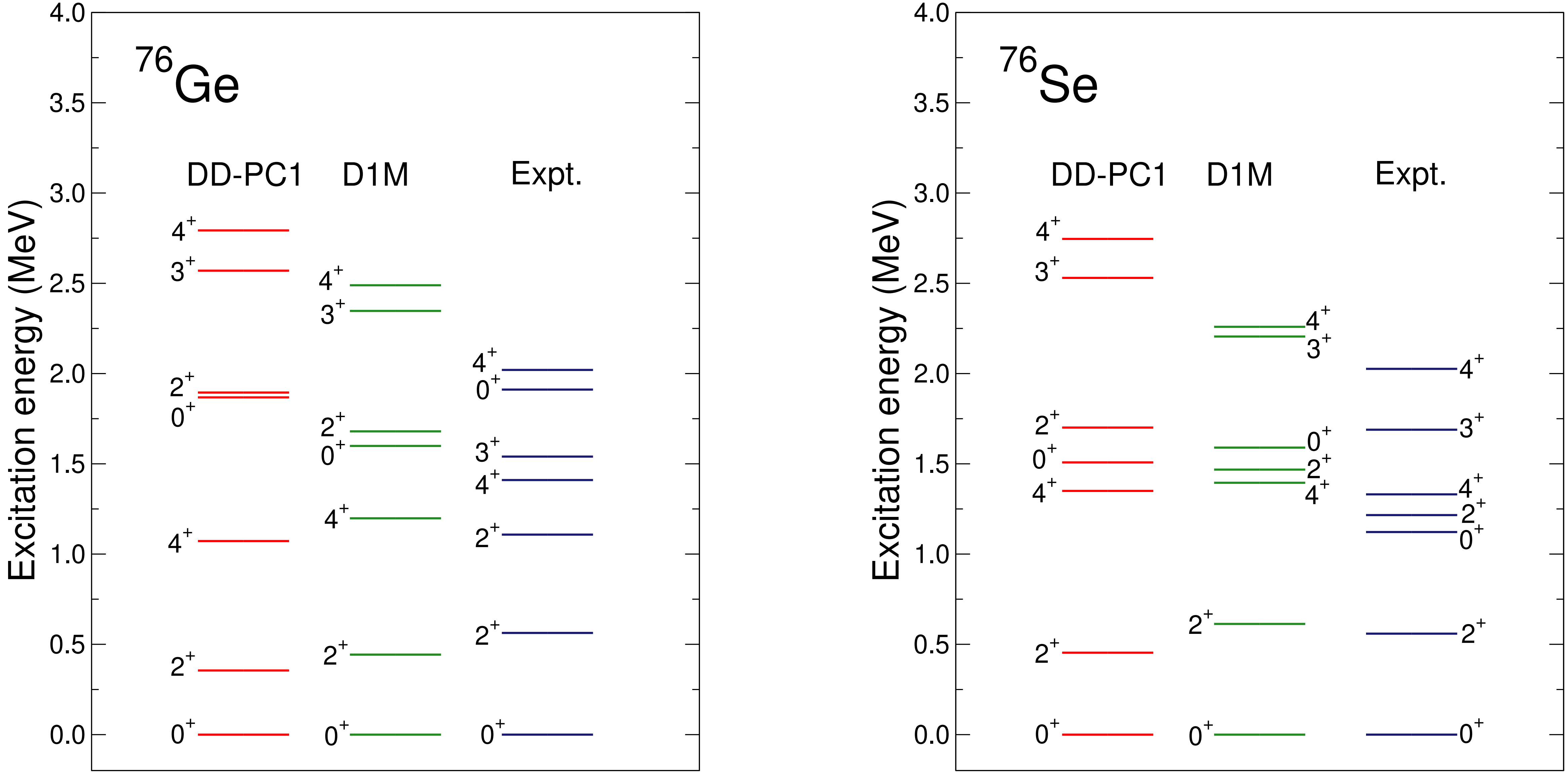
Phase-space factor: Kotila-Iachello (2012)
Experiment: Barabash, Universe (2020)

Sensitivity to the EDFs



… Gogny (D1M) EDF HFB yields less pronounced deformation, and spherical-oblate coexistence for 76Se.

Comparison of spectra



… slightly better description with Gogny

Comparison of 2ν-NMEs

76Ge -> 76Se decay

EDF	$\beta_{\min}$ (^{76}Se)	$M_{2\nu}^{\mathrm{F}}$	$M_{2\nu}^{GT}$	$\|M_{2\nu}\|$		$\|M_{2\nu}\|$ (Expt.)
				$g_{\mathrm{A}} = 1.27$	$g_{\mathrm{A}} = 1$	
DD-PC1 RMF	0.2 (Oblate)	-0.002	0.036	0.060	0.038	0.106 ± 0.004
D1M Gogny	0.2 (Oblate)	0.007	-0.053	0.092	0.060	
	0.0 (Spherical)	0.016	-0.068	0.136	0.085	

… larger $|M_{2\nu}|$ for Gogny

100Mo -> 100Ru decay

EDF	$M_{2\nu}^{\mathrm{F}}$	$M_{2\nu}^{GT}$	$\|M_{2\nu}\|$		$\|M_{2\nu}\|$ (Expt.)
			$g_{\mathrm{A}} = 1.27$	$g_{\mathrm{A}} = 1$	
DD-PC1 RMF	-0.000	0.483	0.778	0.483	0.185 ± 0.002
D1M Gogny	-0.000	0.336	0.542	0.336	

… reduced $|M_{2\nu}|$ with Gogny

Summary

- Consistent description of low-lying states and $2\nu\beta\beta$ NMEs beyond closure approximation
- Coupling to higher-order deformations, shape coexistence…?
- Derivation of the IBFFM parameters only from the EDF
- Extension to the $0\nu\beta\beta$ NME

Thank you