Improved description of double beta-decay

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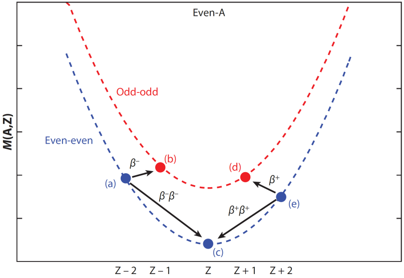
Outlook

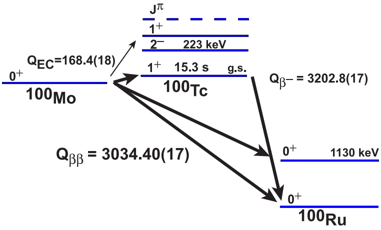
1. Introduction and motivation
2. Radiative and exchange corrections in $2\nu\beta\beta$-decay
3. Angular correlations in DBD
4. Systematic study of 2νECEC process
5. Semi-empirical formula for $2\nu\beta\beta$-decay (and 2νECEC)
6. Conclusions

Double beta-decay (DBD)

$2\nu\beta\beta$-decay is the rarest observed decay in nature, $T^{2\nu}_{1/2} > 10^{18}$ yrs.

$$2\nu\beta\beta : (A, Z) \to (A, Z+2) + 2e^- + 2\overline{\nu}_e$$





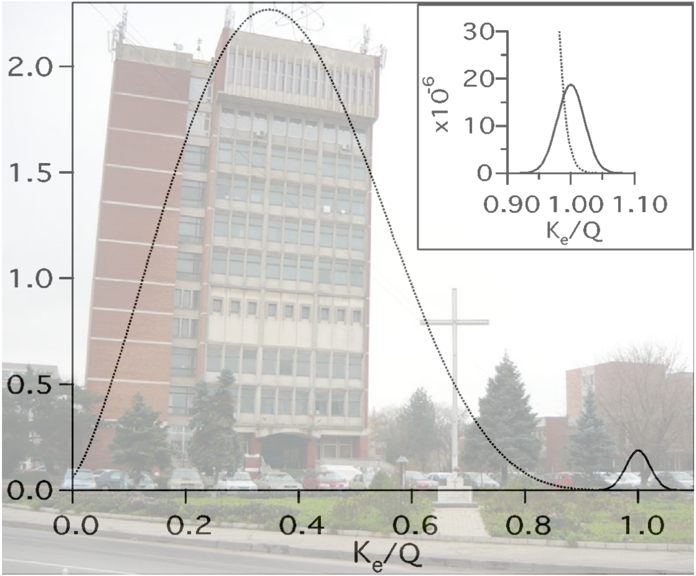
There is an increasing interest in the process due to the possible observation of $0\nu\beta\beta$ decay.

$$0\nu\beta\beta : (A, Z) \to (A, Z+2) + 2e^-$$

$\nu_e = \overline{\nu}_e$ Majorana particle

GERDA, CUPID-0, CUPID-Mo, NEMO, CUORE, EXO-200 , KamLAND-ZEN
nEXO, LEGEND-200($\times$5), CUPID, SuperNEMO, CROSS, AMoRE, SNO+, PandaX
COBRA, CROSS, ACCESS, MONUMENT, BINGO, ...

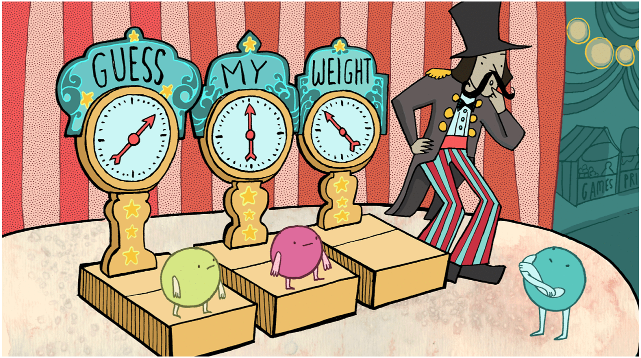
Finding the needle ... under the car ($0\nu\beta\beta$ vs $2\nu\beta\beta$)



Why precise $\beta\beta$ (and β)-decay theoretical predictions?

Neutrino mass scale determination:

- $0\nu\beta\beta$-decay NMEs and best half-life limits GERDA,KamLAND-Zen
- (ultra-)low Q value β and EC transitions
  KATRIN, Project-8, ECHo, HOLMES, PTOLEMY, NuMECS



Why precise $\beta\beta$ (and β)-decay theoretical predictions?

Underground experiments that use liquid Xenon as the detection medium to search for rare interactions:

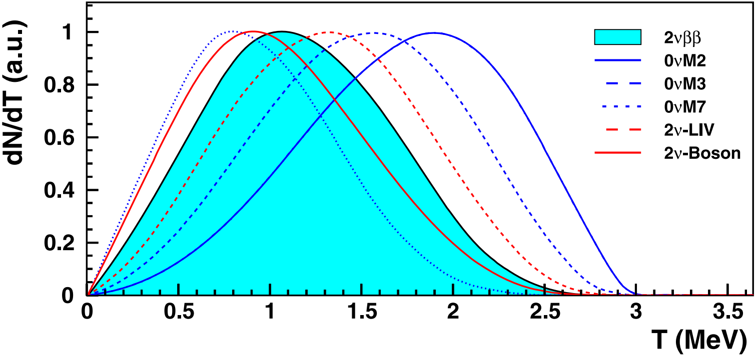
- WIMPs: XENON, LUX, PandaX, XMASS, DarkSide
- CEνNS: XENONnT, LUX-ZEPLIN, DARWIN
- 2νECEC process of ^{124}Xe
- $2\nu\beta\beta$ and $0\nu\beta\beta$ decay of ^{136}Xe

The infrastructure of XENONnT experiment

Why precise $\beta\beta$ (and β)-decay theoretical predictions?*

With the increasing statistics, the idea of searching for new physics scenarios in β and $2\nu\beta\beta$-decay becomes reliable:

- new physics parameters constrains from the shape of β and $\beta\beta$ spectra



NEMO-3 Collaboration, Eur. Phys. J. C 79, 440 (2019)

$2\nu\beta\beta$-decay rate via Taylor expansion formalism

We get a more precise expression of the $2\nu\beta\beta$-decay rate

$$\left[T^{2\nu}_{1/2}(\xi_{31},\xi_{51})\right]^{-1} = G^{2\nu}_0 \left(g_A^{\text{eff}}\right)^4 \left|M^{2\nu}_{GT}\right|^2 \times \left\{1 + \xi_{31}\frac{G^{2\nu}_2}{G^{2\nu}_0} + \frac{1}{3}\xi_{31}^2\frac{G^{2\nu}_{22}}{G^{2\nu}_0} + \left(\frac{1}{3}\xi^2_{31} + \xi_{51}\right)\frac{G^{2\nu}_4}{G^{2\nu}_0}\right\},$$

by performing in the matrix elements the following Taylor expansion

$$M^{K,L}_{F,GT} = m_e \sum_n M_{F,GT}(n)\frac{E_n - (E_i - E_f)/2}{[E_n - (E_i - E_f)/2]^2 - \epsilon^2_{K,L}}$$

$$= m_e \sum_n M_{F,GT}(n)\frac{1}{E_n - (E_i - E_f)/2} \times \left\{1 + \left(\frac{\epsilon_{K,L}}{E_n - (E_i - E_f)/2}\right)^2 + \left(\frac{\epsilon_{K,L}}{E_n - (E_i - E_f)/2}\right)^4 + \dots\right\}$$

*F.Šimkovic, R. Dvornický, D. Štefánik, and A. Faessler, Phys. Rev. C **97**, 034315 (2018)*

$2\nu\beta\beta$-decay rate via Taylor expansion

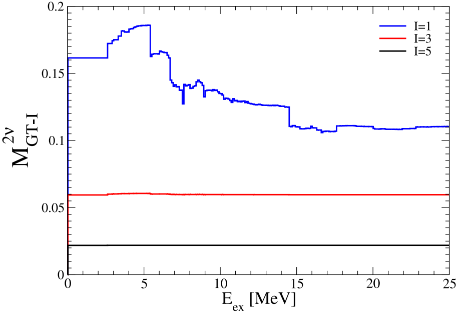
$$\xi_{31} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}},$$

$$\xi_{51} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}.$$

$$M^{2\nu}_{GT-1} \equiv M^{2\nu}_{GT} = \sum_n M_{GT}(n) \frac{m_e}{E_n(1^+) - (E_i + E_f)/2},$$

$$M^{2\nu}_{GT-3} = \sum_n M_{GT}(n) \frac{4\ m_e^3}{[E_n(1^+) - (E_i + E_f)/2]^3},$$

$$M^{2\nu}_{GT-5} = \sum_n M_{GT}(n) \frac{16\ m_e^5}{[E_n(1^+) - (E_i + E_f)/2]^5},$$



$$M_{GT}(n) = \langle 0^+_f \| \sum_m \tau^+_m \sigma_m \| 1^+_n \rangle \langle 1^+_n \| \sum_m \tau^+_m \sigma_m \| 0^+_i \rangle,$$

Adding exchange and radiative corrections to $2\nu\beta\beta$-decay

*O. Nițescu, S. Stoica, R. Dvornický and F.Šimkovic, Univers **7** (5), 147 (2021)*

$$G_N^{2\nu} = \frac{m_e (G_\beta m_e^2)^4}{8\pi^7 \ln 2} \frac{1}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} dE_{e_1} \int_{m_e}^{E_i - E_f - E_{e_1}} dE_{e_2}$$
$$\times p_{e_1} E_{e_1} \left[1 + \eta^T(E_{e_1})\right] R(E_{e_1}, E_i - E_f - m_e) p_{e_2} E_{e_2} \left[1 + \eta^T(E_{e_2})\right]$$
$$\times R(E_{e_2}, E_i - E_f - E_{e_1}) F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \mathscr{I}_N$$

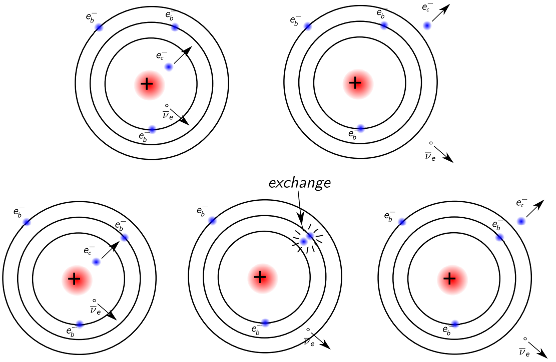
with $N = \{0, 2, 22, 4\}$.

$$R(E_e, E_e^{\text{max}}) = 1 + \frac{\alpha}{2\pi} g(E_e, E_e^{\text{max}})$$

$$g(E_e, E_e^{\text{max}}) = 3\ln(m_p) - \frac{3}{4} - \frac{4}{\beta}\text{Li}_2\left(\frac{2\beta}{1+\beta}\right)$$
$$+ \frac{\tanh^{-1}\beta}{\beta}\left[2\left(1+\beta^2\right) + \frac{(E_e^{\text{max}} - E_e)^2}{6E_e^2} - 4\tanh^{-1}\beta\right]$$
$$+4\left(\frac{\tanh^{-1}\beta}{\beta} - 1\right)\left\{\frac{E_e^{\text{max}} - E_e}{3E_e} - \frac{3}{2} + \ln\left[2\left(E_e^{\text{max}} - E_e\right)\right]\right\}$$

where $\beta = p_e/E_e$.

Atomic exchange correction in β-decay



$$\frac{d\Gamma}{dE_e} \Rightarrow \frac{d\Gamma}{dE_e} \times \left[1 + \eta^T(E_e)\right]$$

$$\eta^T(E_e) = f_s(2T_s + T_s^2) + (1 - f_s)(2T_{\bar{p}} + T_{\bar{p}}^2) = \eta_s(E_e) + \eta_{\bar{p}}(E_e)$$

$$T_s = \sum_{(ns)'} T_{ns} = -\sum_{(ns)'} \frac{\left\langle \psi'_{E_e s} \middle| \psi_{ns} \right\rangle}{\langle \psi'_{ns} | \psi_{ns} \rangle} \frac{g'_{n,-1}(R)}{g'_{-1}(E_e, R)}, \quad f_s = \frac{g'^2_{-1}(E_e, R)}{g'^2_{-1}(E_e, R) + f'^2_{+1}(E_e, R)}.$$

DHFS self-consistent method

The relativistic wave function of the bound electron

$$\psi_{n,\kappa,m}(\boldsymbol{r}) = \begin{pmatrix} g_{n,\kappa}(r)\Omega_{\kappa,m}(\hat{\boldsymbol{r}}) \\ if_{n,\kappa}(r)\Omega_{-\kappa,m}(\hat{\boldsymbol{r}}) \end{pmatrix},$$

where the large- and small-component radial functions obey the radial Dirac equation

$$\left(\frac{d}{dr} + \frac{\kappa+1}{r}\right) g_{n,\kappa}(r) - (E_{n\kappa} - V(r) + m_e) f_{n,\kappa}(r) = 0,$$

$$\left(\frac{d}{dr} - \frac{\kappa-1}{r}\right) f_{n,\kappa}(r) + (E_{n\kappa} - V(r) - m_e) g_{n,\kappa}(r) = 0.$$

Bound states ($E_e < m_e$): discrete energy levels (n, κ, $t_{n\kappa}$, $E_{n\kappa} = m_e - |t_{n\kappa}|$), $\langle \psi_{n\kappa} | \psi_{n'\kappa'} \rangle = \delta_{nn'}\delta_{\kappa\kappa'}$ and

$$V(r) \equiv V_{\text{DHFS}}(r) = V_{\text{nuc}}(r) + V_{\text{el}}(r) + V_{\text{ex}}(r)$$

The nuclear potential is obtained from a Fermi distribution for the proton density

$$\rho_p(r) = \frac{\rho_0}{1 + e^{(r - c_{\text{rms}})/a}}, \qquad V_{\text{nuc}}(r) = -\alpha \int \frac{\rho_p(r')}{|\boldsymbol{r} - \boldsymbol{r'}|} d\boldsymbol{r'}.$$

DHFS self-consistent method

The electronic potential is

$$V_{\text{el}}(r) = \alpha \int \frac{\rho(r')}{|\boldsymbol{r} - \boldsymbol{r'}|} d\boldsymbol{r'}.$$

The local exchange potential with the correct the asymptotic behavior is

$$V_{\text{ex}}(r) = \begin{cases} V_{\text{ex}}^{\text{Slater}}(r) = -\frac{3}{2}\alpha\left(\frac{3}{\pi}\right)^{1/3}[\rho(r)]^{1/3} & r < r_{\text{Latter}}, \\ -\frac{\alpha(Z-N+1)}{r} - V_{\text{nuc}}(r) - V_{\text{el}}(r) & r \geq r_{\text{Latter}}. \end{cases}$$

- start from an approximate electron density (Molière parametrization of the Thomas-Fermi potential)
- solve the Dirac equation
- update the electron density from the obtained wave functions

$$\rho(r) = \sum_{n\kappa} \psi_{n\kappa}^{\dagger}(r)\psi_{n\kappa}(r)$$

- repeat and stop when the atomic binding energy is not changing (ϵ tolerance)

Exchange correction calculation: current status

The overlap between final continuum and initial bound states,

$$\left\langle \psi'_{E_e\kappa} \middle| \psi_{n\kappa} \right\rangle = \int_0^\infty r^2 \left[g'_\kappa(E_e, r) g_{n,\kappa}(r) + f'_\kappa(E_e, r) f_{n,\kappa}(r) \right] dr$$

- good knowledge of the continuum w.f. over large distances
- integration method
- orthogonal continuum and bound w.f. for the same atomic system

*Phys. Rev. A **45**, 6282 (1992)*

$$\left\langle \psi'_{E_e\kappa} \middle| \psi'_{n\kappa} \right\rangle = 0$$

Standard calculations (true DHFS)

$|\psi_{n\kappa}\rangle$, $|\psi'_{n\kappa}\rangle$: $V_{\mathsf{nuc}}(r) + V_{\mathsf{el}}(r) + V_{\mathsf{ex}}(r)$

$\left|\psi'_{E_e s}\right\rangle$: $V_{\mathsf{nuc}}(r) + V_{\mathsf{el}}(r)$

*Phys. Rev. A **90**, 012501 (2014)*
*Rev. Mod. Phys. **90**, 015008 (2018)*
*Phys. Rev. C **102**, 065501 (2020)*
*Phys. Rev. D **102**, 072004 (2020)*
*Appl. Radiat. Isot. **185**, 110237 (2022)*

Our calculations (modified DHFS)

$|\psi_{n\kappa}\rangle$, $|\psi'_{n\kappa}\rangle$ and $\left|\psi'_{E_e s}\right\rangle$:

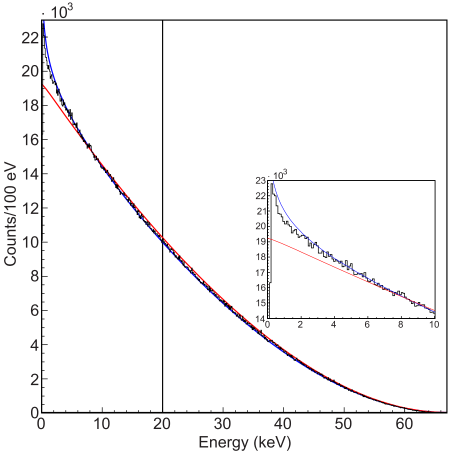
$V_{\mathsf{nuc}}(r) + V_{\mathsf{el}}(r) + V_{\mathsf{ex}}^{\mathrm{Slater}}(r)$

*our result: Phys. Rev. C **107**, 025501 (2023)*

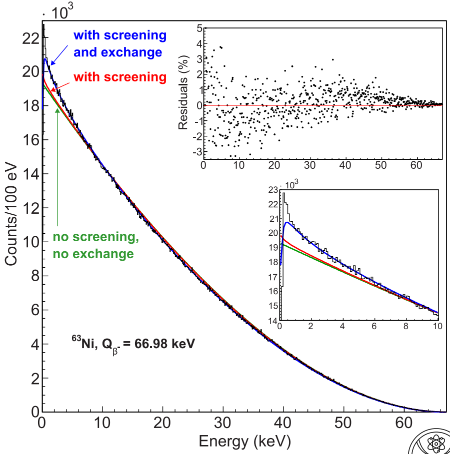
Experiment vs Theory

$$^{63}\text{Ni} \rightarrow {}^{63}\text{Cu}^{+} + e^{-} + \overline{\nu}_e \qquad Q = 66.945 \text{ keV}$$

*our result: Phys. Rev. C **107**, 025501 (2023)*

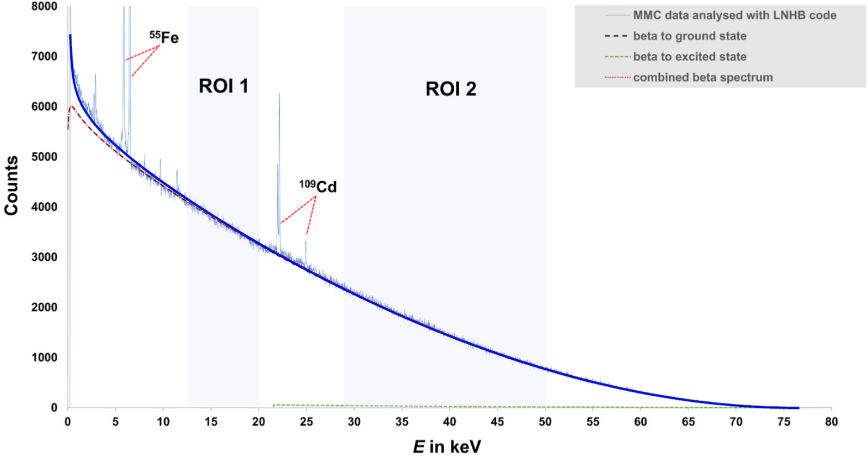


*Phys. Rev. A **90**, 012501 (2014)*



Experiment vs Theory

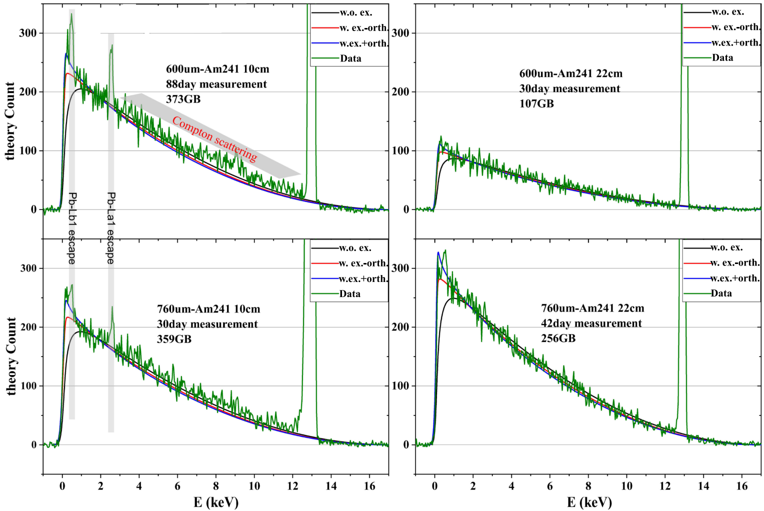
$$^{151}\text{Sm} \rightarrow {}^{151}\text{Eu}^{+} + e^{-} + \overline{\nu}_e \qquad Q = 76.6 \text{ keV}$$



Appl. Radiat. Isot. ***185***, *110237 (2022)*

Experiment vs Theory

$$^{210}\text{Pb} \to {}^{210}\text{Bi}^{*} + e^{-} + \overline{\nu}_e \qquad Q = 16.96 \text{ keV}$$

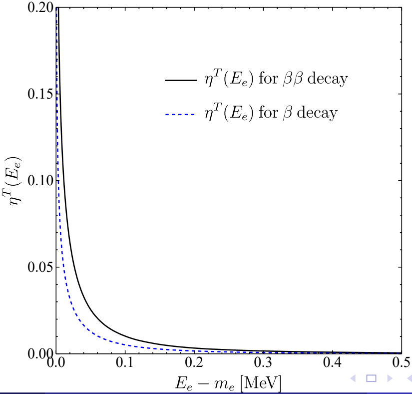


arXiv:2307.16276 (2023)

Exchange correction for $\beta\beta$-decay

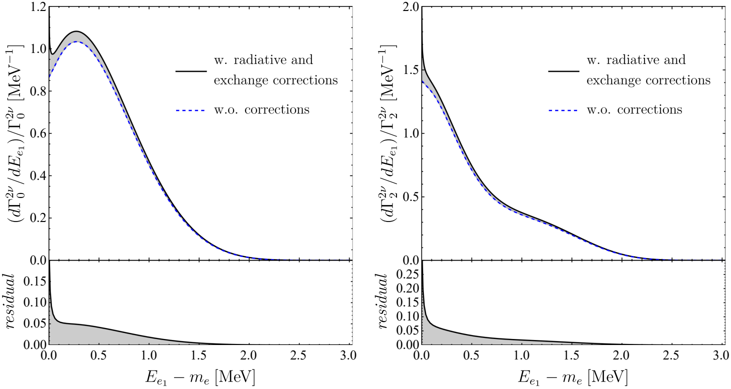
$$G_N^{2\nu} = \frac{(G_F\,|V_{ud}|)^4}{8\pi^7 m_e^2 \ln 2} \int_{m_e}^{E_i - E_f - m_e} \int_{m_e}^{E_i - E_f - E_{e_1}} F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \mathscr{I}_N$$
$$\times\, p_{e_1} E_{e_1} \left[1 + \eta^T(E_{e_1})\right] R(E_{e_1}, E_i - E_f - m_e)$$
$$\times\, p_{e_2} E_{e_2} \left[1 + \eta^T(E_{e_2})\right] R(E_{e_2}, E_i - E_f - E_{e_1}) dE_{e_2} dE_{e_1}$$

O. Nițescu and F. Šimkovic *arXiv:2411.05405 (2024)*



Single electron distributions and decay rates for ^{100}Mo

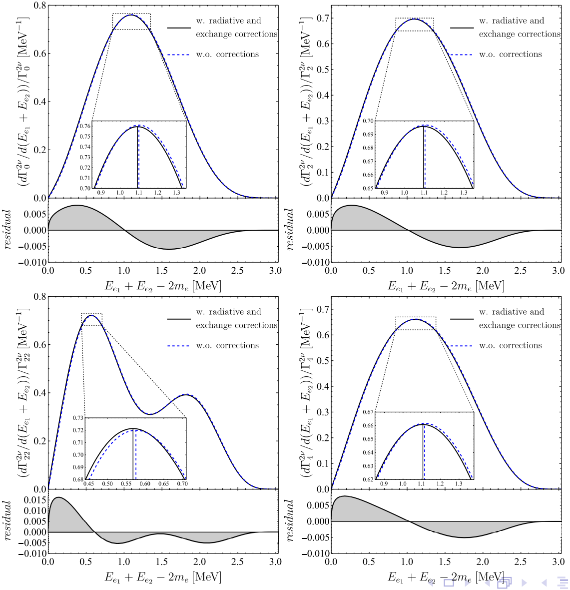
O. Nițescu and F. Šimkovic arXiv:2411.05405 (2024)



Nucleus	Correction(s)	$G_0^{2\nu}$	$G_2^{2\nu}$	$G_{22}^{2\nu}$	$G_4^{2\nu}$
^{100}Mo	DHFS	3.307×10^{-18}	1.511×10^{-18}	1.989×10^{-19}	8.652×10^{-19}
	Exchange	3.343×10^{-18}	1.536×10^{-18}	2.031×10^{-19}	8.835×10^{-19}
	Radiative	3.432×10^{-18}	1.568×10^{-18}	2.066×10^{-19}	8.974×10^{-19}
	Radiative and Exchange	3.470×10^{-18}	1.593×10^{-18}	2.109×10^{-19}	9.164×10^{-19}
	δ	4.91%	5.42%	5.97%	5.92%

Summed electron distributions*

O. Nițescu and F. Šimkovic arXiv:2411.05405 (2024)



The differential DBD rate for a $0^+ \rightarrow 0^+$ nuclear transition with respect to the angle $0 \leq \theta \leq \pi$ between the emitted electrons can be written as,

$$\frac{d\Gamma}{d\left(\cos\theta\right)} = \frac{\Gamma}{2}\left(1 + K\cos\theta\right), \quad \text{where} \quad K = -\frac{\Lambda}{\Gamma}.$$

*O. Nițescu, S. Ghinescu and F. Šimkovic, Universe **10** (12), 442, (2024)*

For $2\nu\beta\beta$-decay we have,

$$\left\{\begin{matrix} \Gamma^{2\nu} \\ \Lambda^{2\nu} \end{matrix}\right\} = (g_A^{\text{eff}})^4 \left|M_{GT}^{2\nu}\right|^2 \ln(2) \left\{\begin{matrix} G_0^{2\nu} + \xi_{31} G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51}\right) G_4^{2\nu} \\ H_0^{2\nu} + \xi_{31} H_2^{2\nu} + \frac{5}{9}\xi_{31}^2 H_{22}^{2\nu} + \left(\frac{2}{9}\xi_{31}^2 + \xi_{51}\right) H_4^{2\nu} \end{matrix}\right\},$$

$$\left\{\begin{matrix} G_N^{2\nu} \\ H_N^{2\nu} \end{matrix}\right\} = \frac{m_e (G_F \left|V_{ud}\right| m_e^2)^4}{8\pi^7 \ln(2)} \frac{1}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} p_{e_1} E_{e_1} \int_{m_e}^{E_i - E_f - E_{e_1}} p_{e_2} E_{e_2} \mathscr{I}_N \left\{\begin{matrix} F_{ss}(E_{e_1}) F_{ss}(E_{e} \\ E_{ss}(E_{e_1}) E_{ss}(E_{e} \end{matrix}\right.$$

For $0\nu\beta\beta$-decay we have,

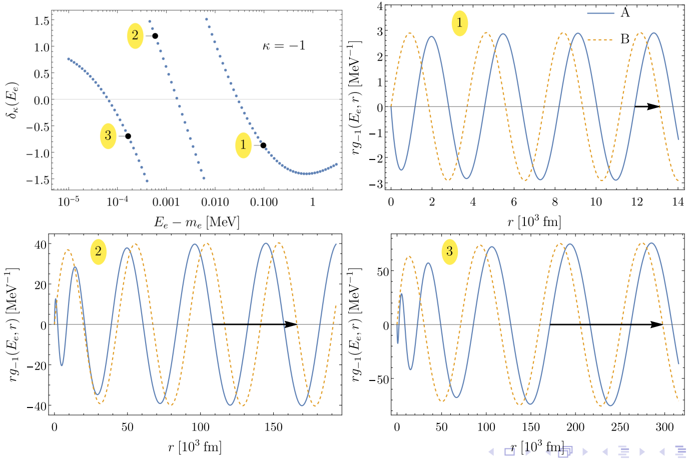
$$\left\{\begin{matrix} \Gamma^{0\nu} \\ \Lambda^{0\nu} \end{matrix}\right\} = \frac{(G_F \left|V_{ud}\right|)^4 m_e^2}{32\pi^5 R^2} (g_A)^4 \left|M^{0\nu}\right|^2 \frac{\left|m_{\beta\beta}\right|^2}{m_e^2} \int_{m_e}^{E_i - E_f - m_e} p_{e_1} E_{e_1} p_{e_2} E_{e_2} \left\{\begin{matrix} F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \\ E_{ss}(E_{e_1}) E_{ss}(E_{e_2}) \end{matrix}\right\} d$$

Electron phase shifts

$$\begin{Bmatrix} \tilde{g}_\kappa(E_e, r) \\ \tilde{f}_\kappa(E_e, r) \end{Bmatrix} \xrightarrow[r\to\infty]{} \frac{\exp\left(-i\bar{\Delta}_\kappa\right)}{p_e r} \begin{Bmatrix} \sqrt{\frac{E_e+m_e}{2E_e}} \sin\left(p_e r - \ell_\kappa \frac{\pi}{2} + \eta \ln(2p_e r) + \bar{\Delta}_\kappa\right) \\ \sqrt{\frac{E_e-m_e}{2E_e}} \cos\left(p_e r - \ell_\kappa \frac{\pi}{2} + \eta \ln(2p_e r) + \bar{\Delta}_\kappa\right) \end{Bmatrix},$$

$$\bar{\Delta}_\kappa = \Delta_\kappa + \delta_\kappa .$$

*O. Nițescu, S. Ghinescu and F. Šimkovic, Universe **10** (12), 442, (2024)*



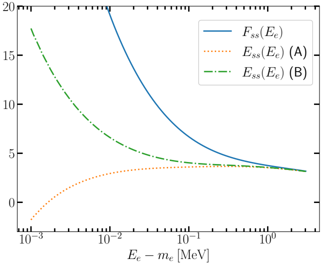
Angular distributions for $2\nu\beta\beta$-decay*

$$F_{ss}(E_e) = |\tilde{g}_{-1}(E_e, R)|^2 + \left|\tilde{f}_1(E_e, R)\right|^2$$

$$E_{ss}(E_e) = 2\,\mathrm{Re}\{\tilde{g}_{-1}(E_e, R)\tilde{f}_1^*(E_e, R)\}$$

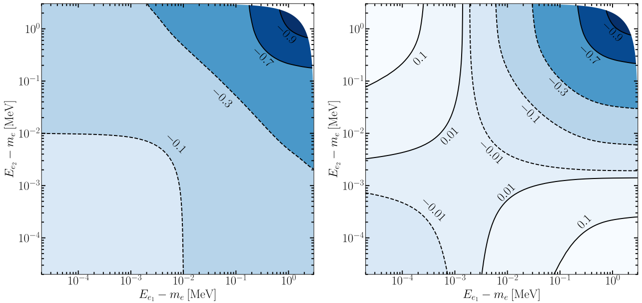
$$\simeq 2\,|\tilde{g}_{-1}(E_e, R)|\left|\tilde{f}_1(E_e, R)\right|.$$

7% increase in $K^{2\nu}$



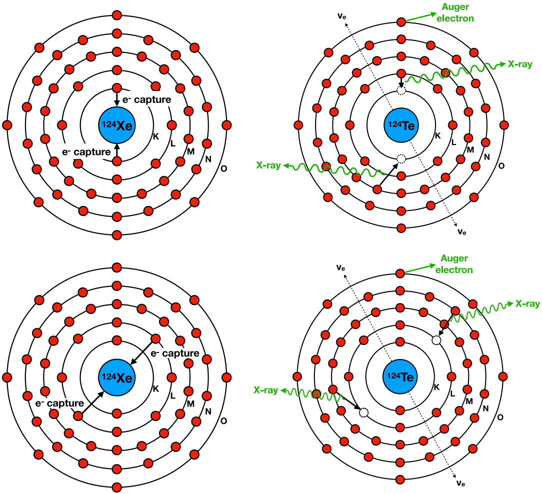
$$\kappa^{2\nu}(E_{e_1}, E_{e_2}) = -\frac{d^2\Lambda^{2\nu}/(dE_{e_1}dE_{e_2})}{d^2\Gamma^{2\nu}/(dE_{e_1}dE_{e_2})}$$

*O. Nițescu, S. Ghinescu and F. Šimkovic, Universe **10** (12), 442, (2024)*



Two-neutrino double electron capture (2νECEC)

*Figure reproduced from Journal of High Energy Physics **2021**, 1029 (2021)*

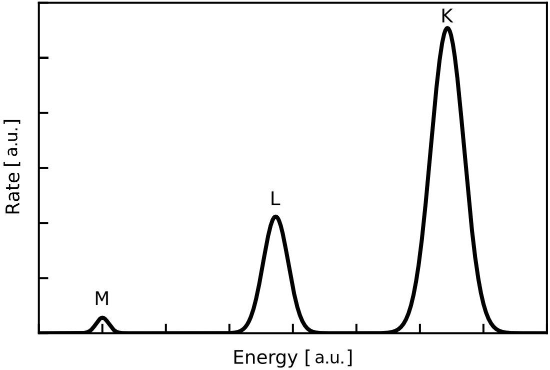


$$2e^{-} + (A, Z+2) \rightarrow (A, Z)^{*} + 2\nu_e$$

$$R_{\rm xy} = \mathscr{M}_{\rm xy}(A, Z) - \mathscr{M}_{gs}(A, Z) = B_{\rm gs}(Z) - B_{\rm xy}(Z)$$

Expected signals from electron captures

Figure reproduced from Physical Review C ***106****, 024328 (2022)*



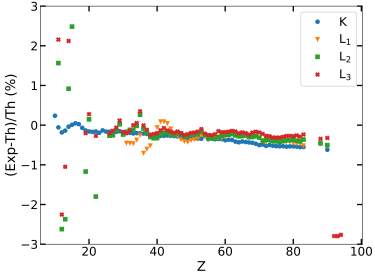
- width $\rightarrow$ energy resolution of the detector
- area $\rightarrow$ capture fractions (Γ_x/Γ)
- position $\rightarrow$ atomic relaxation energy (R_x)

Total biding energy and atomic relaxation energy

$$\mathscr{H} = \sum_{i=1}^{Z} \left[\boldsymbol{\alpha}_i \cdot \boldsymbol{p}_i + (\beta - 1)\, m_e\right] + \sum_{i=1}^{Z} V_{\text{nuc}}(\boldsymbol{r}_i) + \sum_{i<j=1}^{Z} \frac{\alpha}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$$

$$\Psi = \frac{1}{\sqrt{Z!}} \begin{vmatrix} \psi_1(\boldsymbol{r}_1) & \dots & \psi_1(\boldsymbol{r}_Z) \\ \vdots & \ddots & \vdots \\ \psi_Z(\boldsymbol{r}_1) & \dots & \psi_Z(\boldsymbol{r}_Z) \end{vmatrix} \qquad \langle \psi_i | \psi_j \rangle = \int \psi_i^\dagger(\boldsymbol{r}) \psi_j(\boldsymbol{r}) d\boldsymbol{r} = \delta_{ij}$$

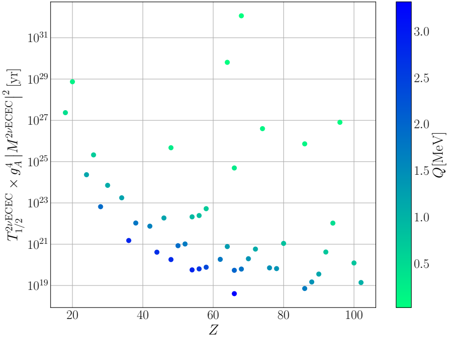
$$R_{\text{x}} = B_{\text{gs}}(Z) - B_{\text{x}}(Z) = \langle \Psi_{\text{gs}} | \mathscr{H} | \Psi_{\text{gs}} \rangle - \langle \Psi_{\text{x}} | \mathscr{H} | \Psi_{\text{x}} \rangle$$

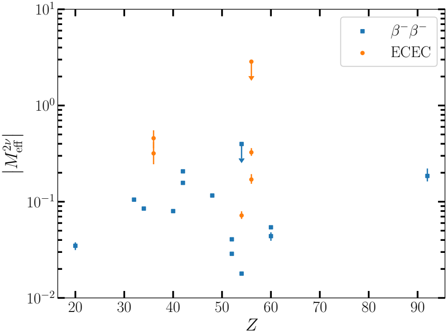


*V. A. Sevestrean, O. Nițescu, S. Ghinescu and S. Stoica, Phys. Rev. A **108**, 012810 (2023)*

Systematic study of 2νECEC*

- all s-wave electrons can be captured (prior only K and L_1)
- realistic screening for the electron w.f.
- capture fraction for all nuclei undergoing 2νECEC
- NMEs (pn-QRPA and NSM), Taylor expansion formalism, Pauli blocking effect, atomic relaxation energies for 2νECEC ^{124}Xe

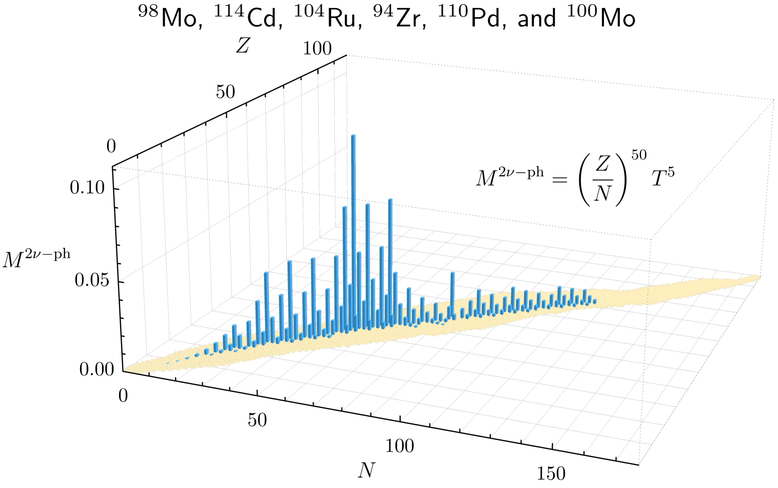




*O. Nițescu, S. Ghinescu, S. Stoica and F. Šimkovic, Universe **10** (2), 98 (2024).*

*O. Nițescu, S. Ghinescu, V. Sevestrean, M. Horoi, F. Šimkovic and S. Stoica, Journal of Physics G **51** (12), 125103 (2024).*

$2\nu\beta\beta$-decay semi-empirical formula



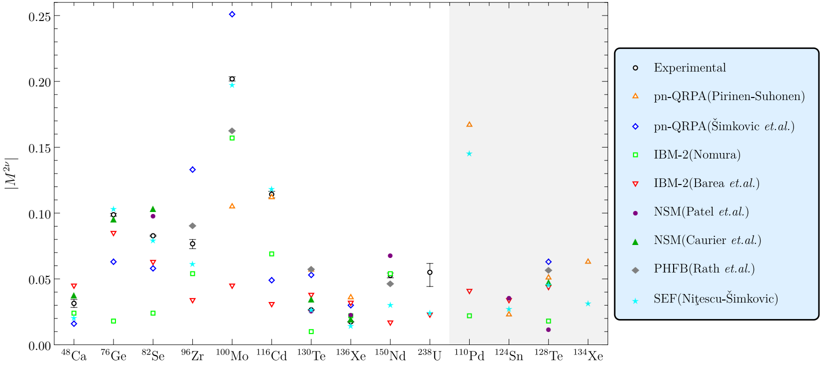
O. Nițescu and F. Šimkovic arXiv:2407.10422 (2024)

$2\nu\beta\beta$-decay NMEs: Nuclear Models vs SEF*

$$M^{2\nu-ph} = \left(\frac{Z_f}{N_f}\right)^{\alpha}\left(\frac{\Delta_{\rm pn}}{1-\beta_</\beta_>}\right)^{\gamma}(T_f)^{\sigma}\quad \text{for } 2\nu\beta\beta\text{-decay}$$

$$\alpha = 46.94,\ \gamma = 0.22,\ \sigma = 4.90\ \left(T_f \to T_f + 1\ \text{ for } 2\nu\text{ECEC}\right)$$

$$\beta_< = \min(\beta^Z, \beta^{Z+2}),\quad \beta_> = \max(\beta^Z, \beta^{Z+2}),\quad \Delta_{\rm pn} = \Delta_p^Z\ \Delta_n^Z\ \Delta_p^{Z+2}\ \Delta_n^{Z+2}.$$



$2\nu\beta\beta$-decay interesting pairs: ^{94,96}Zr, ^{98,100}Mo, ^{114,116}Cd, ^{128,130}Te and ^{134,136}Xe.

Conclusions

Radiative and exchange corrections in $2\nu\beta\beta$-decay:

- a shift of $\sim$10 keV in the summed electron spectrum which may be important for SM and BSM experimental investigations; (CUORE)

Phase shifts in the angular correlation for $2\nu\beta\beta$-decay:

- a 7% increase in $K^{2\nu}$; electrons are most likely emitted in the same direction when one of them is below 2 keV;

Two-neutrino double electron capture:

- inclusion of all s-wave electrons (up to 10% increase in the decay rate); NMEs (pn-QRPA and NSM), Pauli blocking effect, atomic relaxation energies for 2νECEC ^{124}Xe; (XENON and LUX)

Semi-empirical formula for $2\nu\beta\beta$-decay

- our $2\nu\beta\beta$-decay SEF provides the smallest χ^2; multilevel modeling by $T_f \to T_f + 1$ for 2νECEC; ratio of around 2 for the NMEs of the $2\nu\beta\beta$-decay of ^{94,96}Zr, ^{98,100}Mo, ^{114,116}Cd, ^{128,130}Te and ^{134,136}Xe pairs.

Results: 2νECEC of ^{124}Xe

atomic relaxation energies

Decay Chanel	R_{xy} (keV)
KK	64.62
KL_1	37.05
KL_2	36.76
KL_3	36.47
KM_1	32.98
KM_2	32.85
KM_3	32.79
KM_4	32.55
KM_5	32.54
KN_1	32.11
KN_2	32.06
KN_3	32.05
KN_4	31.97
KN_5	31.96
KO_1	31.93
KO_2	31.92
KO_3	31.92
L_1L_1	10.04
L_1L_2	9.69
L_1L_3	9.42
L_2L_2	9.40
L_2L_3	9.11
L_3L_3	8.84
L_1M_1	6.01
Other	< 6

capture fractions

XENON-1T data (all shells from K to N_5 have been assumed)		
Decay Chanel	Capture fractions (%)	R_{xy} (keV)
KK	72.4	64.3
$KL_{1,2,3}$	20.0	36.7–37.3
$KM_{1,2,3,4,5}$	4.3	32.9–33.3
$KN_{1,2,3,4,5}$	1.0	32.3–32.4
$L_{1,2,3}L_{1,2,3}$	1.4	8.8–10.0
Other	0.8	< 10
This work (only shells with $\kappa = -1$ from K to O_1 have been assumed)		
Decay Chanel	Capture fraction (%)	R_{xy} (keV)
KK	74.2	64.6
KL_1	19.0	37.1
KM_1	3.9	33.0
KN_1	0.9	32.1
KO_1	0.1	31.9
L_1L_1	1.2	10.0
Other	0.7	< 10

*O. Nițescu, S. Ghinescu, V. Sevestrean, M. Horoi, F. Šimkovic and S. Stoica, Journal of Physics G **51** (12), 125103 (2024).*

Results: SEF for $2\nu\beta\beta$-decay

	$M^{2\nu-\mathrm{th}}$										$M^{2\nu-\mathrm{exp}}$	$M^{2\nu-\mathrm{ph}}$	
	QRPA	QRPA	IBM	IBM	IBM	NSM	NSM	PHFB	FSQP	ET		SEF	SSD
Fitted													
^{48}Ca	–	0.016	0.069	0.024	0.045	–	0.039	–	–	–	0.0314 ± 0.0030	0.022	–
^{76}Ge	–	0.063	0.083	0.018	0.085	–	0.097	–	0.083	0.085	0.0987 ± 0.0010	0.105	–
^{82}Se	–	0.058	0.072	0.024	0.063	0.099	0.105	–	0.103	0.156	0.0828 ± 0.0005	0.081	–
^{96}Zr	–	0.133	0.058	0.054	0.034	–	–	0.092	0.072	–	0.0770 ± 0.0040	0.063	–
^{100}Mo	0.105	0.251	0.197	0.157	0.045	–	–	0.164	0.154	0.179	0.2019 ± 0.0016	0.199	0.174
^{116}Cd	0.112	0.049	0.089	0.069	0.031	–	–	–	0.088	0.137	0.1142 ± 0.0027	0.120	0.148
^{130}Te	0.057	0.053	0.035	0.010	0.038	0.027	0.036	0.059	0.027	0.034	0.0265 ± 0.0003	0.028	–
^{136}Xe	0.036	0.030	0.056	0.022	0.032	0.024	0.021	–	–	–	0.0174 ± 0.0002	0.016	–
^{150}Nd	–	–	0.077	0.054	0.017	0.069	–	0.048	–	–	0.0527 ± 0.0019	0.032	0.023
^{238}U	–	–	–	–	0.023	–	–	–	–	–	0.0550 ± 0.0110	0.026	–
χ^2/N	5170	2100	2845	2828	1773	470	686	3442	480	4774	–	30	233
Predicted													
^{110}Pd	0.167	–	–	0.022	0.041	–	–	–	0.233	0.211	< 2.61	0.147	–
^{124}Sn	0.023	–	–	–	0.034	0.037	–	–	–	–	–	0.029	–
^{128}Te	0.051	0.063	0.022	0.018	0.044	0.013	0.049	0.058	0.030	0.050	0.0366 ± 0.0007	0.047	0.015
^{134}Xe	0.063	–	–	–	–	–	–	–	–	–	< 1.25	0.033	–

O. Nițescu and F. Šimkovic arXiv:2407.10422 (2024)

Results: SEF for $2\nu\beta\beta$-decay

	$M^{2\nu-\mathrm{ph}}$									
	Previous models			Present models						LOOCV for SEF
Nucleus	Ren	Rajan	Pritychenko	A	B	C	D	SEF	$M^{2\nu-\mathrm{exp}}$	χ^2_ν (prediction)
Fitted										
^{48}Ca	0.030	0.035	0.037	0.008	0.012	0.020	0.023	0.022	0.0314 ± 0.0030	48 (0.022)
^{78}Ge	0.198	0.123	0.410	0.026	0.044	0.068	0.108	0.105	0.0987 ± 0.0010	41 (0.107)
^{82}Se	0.095	0.096	0.163	0.015	0.027	0.040	0.080	0.081	0.0828 ± 0.0005	35 (0.073)
^{96}Zr	0.060	0.063	0.008	0.027	0.049	0.058	0.068	0.063	0.0770 ± 0.0040	48 (0.063)
^{100}Mo	0.073	0.091	0.220	0.045	0.079	0.110	0.197	0.199	0.2019 ± 0.0016	47 (0.183)
^{116}Cd	0.071	0.093	0.121	0.031	0.058	0.081	0.129	0.120	0.1142 ± 0.0027	49 (0.121)
^{130}Te	0.077	0.068	0.032	0.006	0.014	0.016	0.029	0.028	0.0265 ± 0.0003	32 (0.030)
^{136}Xe	0.030	0.055	0.017	0.004	0.010	0.012	0.016	0.016	0.0174 ± 0.0002	40 (0.015)
^{150}Nd	0.030	0.061	0.203	0.009	0.021	0.025	0.027	0.032	0.0527 ± 0.0019	29 (0.031)
^{238}U	0.024	–	0.046	0.005	0.014	0.010	0.020	0.026	0.0550 ± 0.0110	48 (0.026)
χ^2_ν (ν)	5651 (9)	24981 (2)	17632 (7)	5450 (8)	3171 (8)	2017 (7)	67 (7)	43 (7)		
Predicted										
^{110}Pd	0.135	0.160	0.270	0.047	0.084	0.115	0.143	0.147	< 2.61	
^{124}Sn	0.098	–	0.019	0.009	0.018	0.025	0.029	0.029	–	
^{128}Te	0.067	–	0.042	0.014	0.030	0.037	0.045	0.047	0.0366 ± 0.0007	
^{134}Xe	–	–	0.086	0.010	0.022	0.025	0.033	0.033	< 1.25	

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