PRECISION NEUTRINOLESS $\beta\beta$ DECAY NMEs

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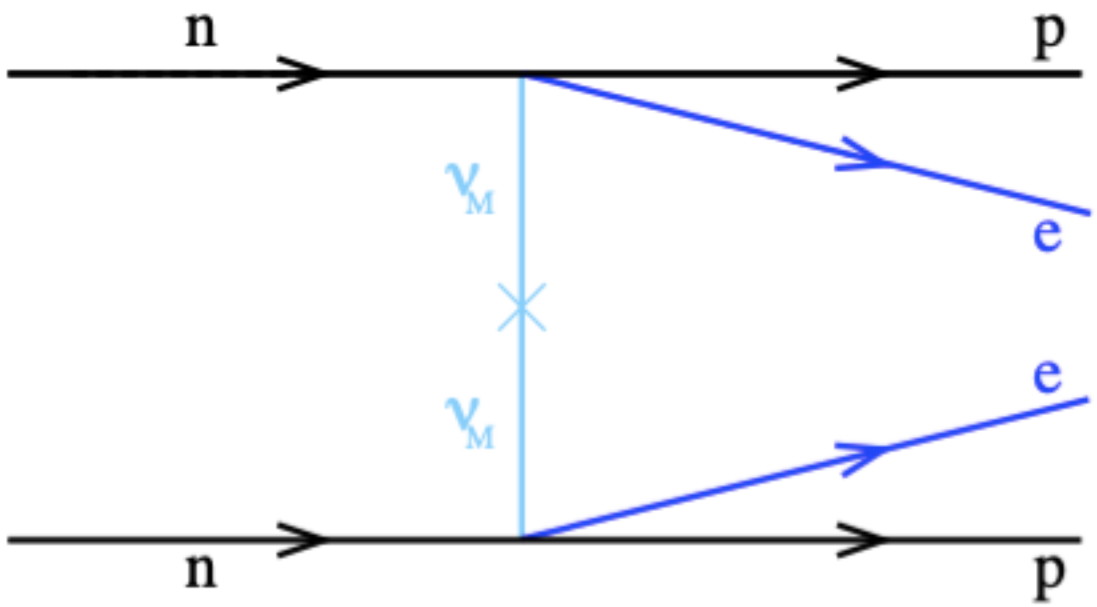


Engel, et al. Rep. Prog. Phys. 80, 046301, 2017

MOTIVATION

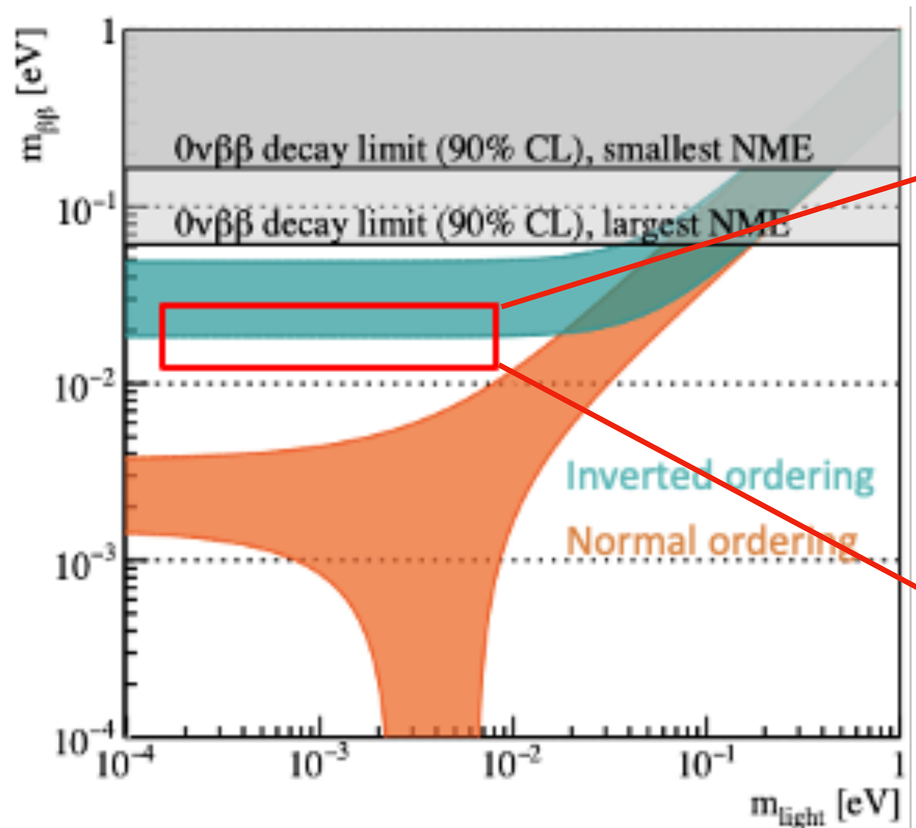
$g_A \equiv$ Axial coupling

$G_{0\nu} \equiv$ Phase-space factor

$M^{0\nu} \equiv$ **Nuclear matrix elements**

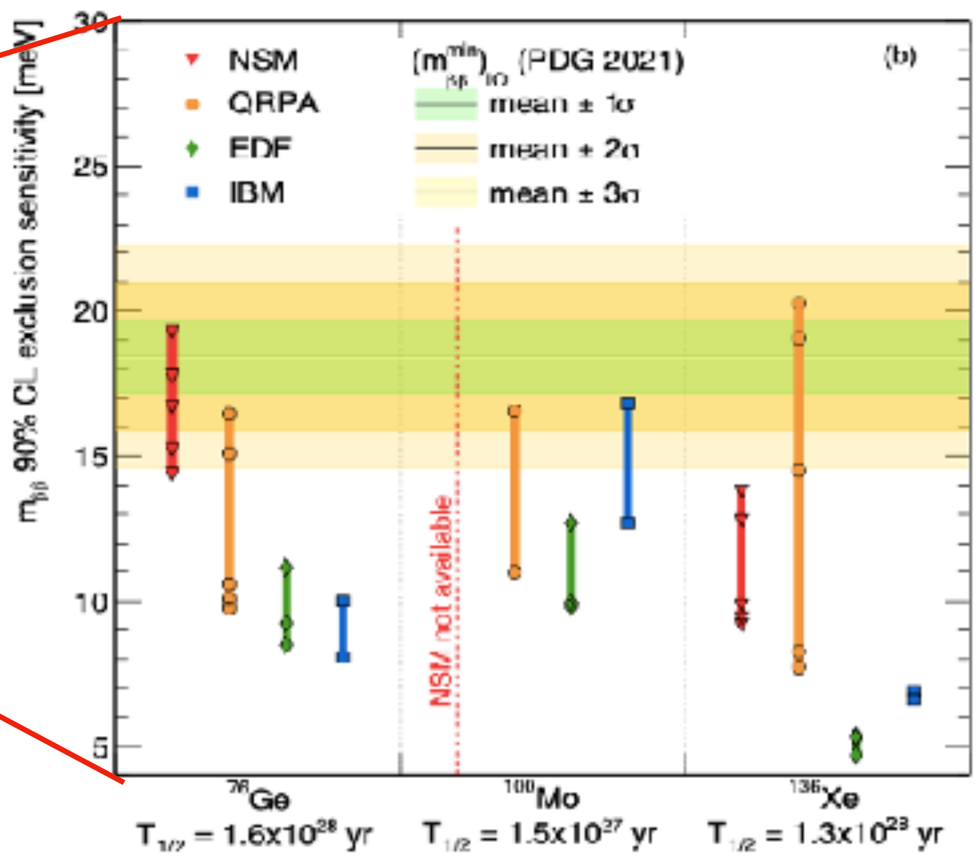
$$(T^{0\nu}_{1/2})^{-1} = g_A^4 G_{0\nu} \,|\, M^{0\nu} \,|^2 m_{\beta\beta}^2$$

$$m_{\beta\beta} = \sum_{j=light} U_{ej} m_j \equiv$$ Effective neutrino mass



Agostini et al. Rev. Mod. Phys. 95, 025002, 2023

Next decade experiments



Agostini et al. Phys. Rev. C 104, L042501, 2021

NUCLEAR MATRIX ELEMENTS

$$M^{0\nu} = \langle 0_f^+ || \sum_{a,b} H_k \hat{O}_k \tau_a^- \tau_b^- || 0_i^+ \rangle$$

Solve the many-body Schrödinger equation

- Nuclear Shell Model (NSM)
- Quasi-particle random phase approximation (QRPA)

$|0^+_{i(f)}\rangle \equiv$ Initial (final) wavefunctions

Obtain decay operators

- Chiral Effective Field Theory (χEFT)

$H_k(r) \equiv$ Neutrino potentials

$\hat{O}_k \equiv$ Spin-space operators

$k \equiv \mathrm{GT}, \mathrm{F}, \mathrm{T}$

$\tau^- \equiv$ Ladder isospin operator

NUCLEAR SHELL MODEL

We want to solve the **Schrödinger equation** ⟶ $H_{eff}|0^+\rangle = E|0^+\rangle$

$H_{eff} = \hat{T} + \hat{U} + \hat{V}_{eff} \equiv$ Effective Hamiltonian

$$U(r) = \frac{1}{2} m_N r^2 \omega^2 + D\vec{l}\cdot\vec{l} + C\vec{l}\cdot\vec{s}$$

$V_{eff} \equiv$ **Phenomenological** effective potential

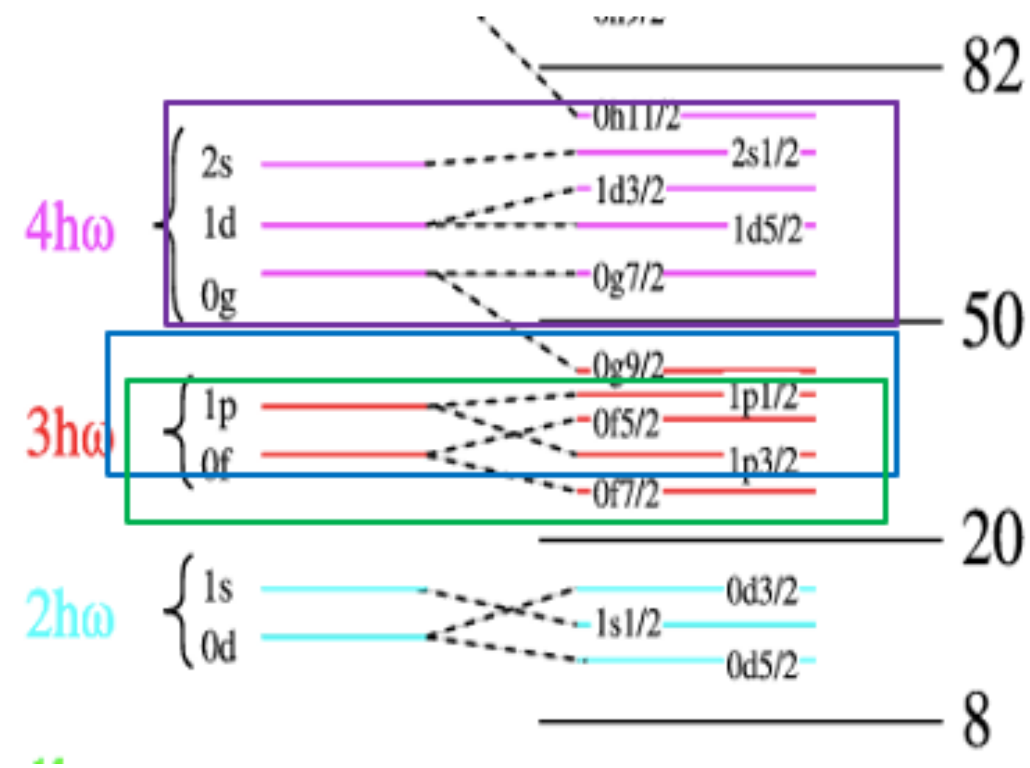
$r = |r_a - r_b| \equiv$ Distance between nucleons

Orbitals:
- EMPTY ORBITALS
- **VALENCE SPACE** ⟶ **Effective interactions**
- CORE

Valence Space (Magic Numbers):

^{48}Ca : 20 – 40 ^{76}Ge, ^{82}Se : 28 – 50

^{124}Sn, ^{130}Te, ^{136}Xe : 50 – 82



https://oer.physics.manchester.ac.uk/NP/Notes/Notes/Notesse23.xht

QUASI-PARTICLE RANDOM PHASE APPROXIMATION (QRPA)

$|QRPA\rangle = |0^+\rangle \longrightarrow$ Reference state

- $n = 0 \rightarrow 2$ harmonic oscillators shells **above** the **fermi level**

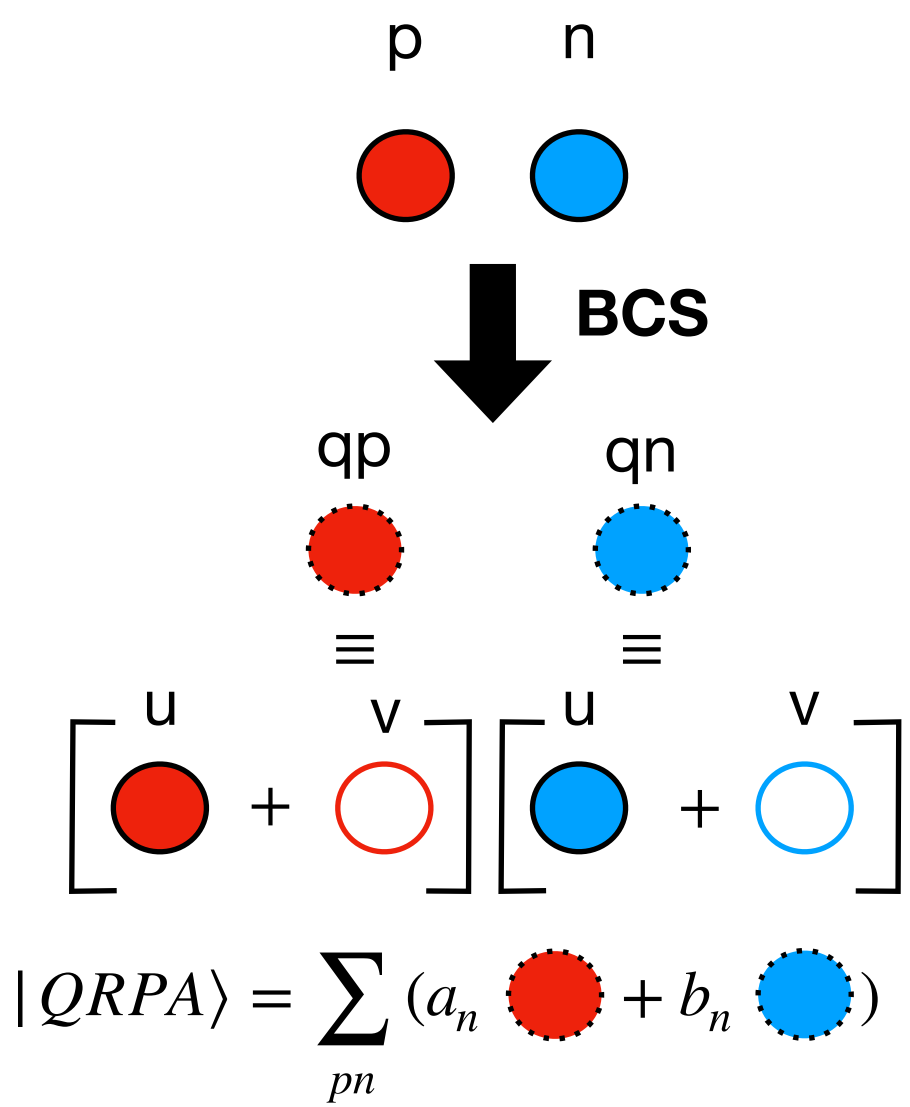
$\downarrow$

^{76}Ge, ^{82}Se : 18 orbitals

^{96}Zr, ^{100}Mo : 25 orbitals

^{116}Cd, ^{124}Sn, ^{130}Te, ^{136}Xe : 26 orbitals

Less complex correlations between nucleons



Credit, Jokiniemi

J. Suhonen, Springer-Verlag, Berlin Heidelberg, 2007

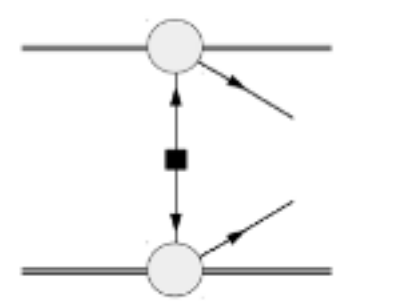
χEFT DIAGRAMS

$$M^{0\nu} = M^{0\nu}_{\mathrm{L}} + M^{0\nu}_{\mathrm{S}} + M^{0\nu}_{\mathrm{usoft}} + M^{0\nu}_{\mathrm{loops}}$$

$q \equiv$ Transferred momentum

Long-range (L)
$(q \sim 100\,\mathrm{MeV} + q - \text{dependent N}^2\text{LO})$

Leading Order (LO)



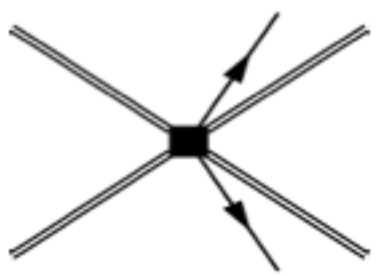
$q -$ dependent $\mathrm{N^2LO}$ → **Finite Size Corrections (FNS)**

Dipole form factors: $g_A^2(q^2)$ and $g_V^2(q^2)$
↓
Regulators

Short-range (S)
$(q >> 100\,\mathrm{MeV})$

already computed

Jokiniemi et al. Phys. Let. B 823, 136720, 2021



↓

$g_\nu^{NN} \equiv$ Nucleon-Nucleon coupling

$\Lambda \equiv$ Gaussian regulator

Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

χEFT DIAGRAMS

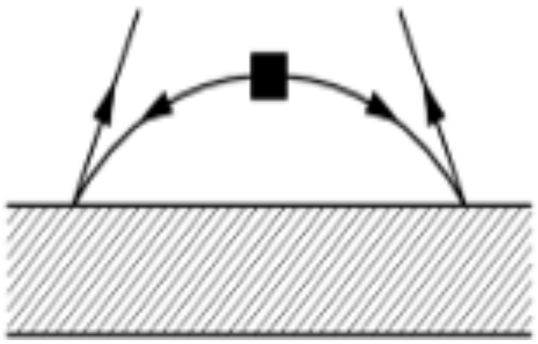
$$M^{0\nu} = M^{0\nu}_{\rm L} + M^{0\nu}_{\rm S} + M^{0\nu}_{\rm usoft} + M^{0\nu}_{\rm loops}$$

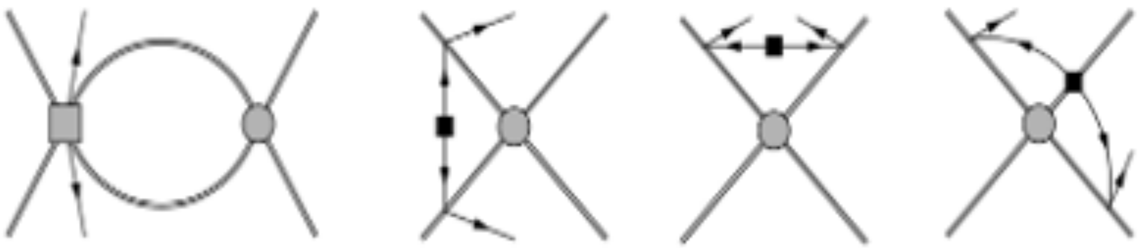
New terms

Ultrasoft $(q << 100\,\mathrm{MeV})$

Loop terms

Next-to-Next-to-Leading Order (N²LO)





Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

ULTRASOFT NME

Study of the **intermediate states** dependence of the **total ultrasoft NMEs**

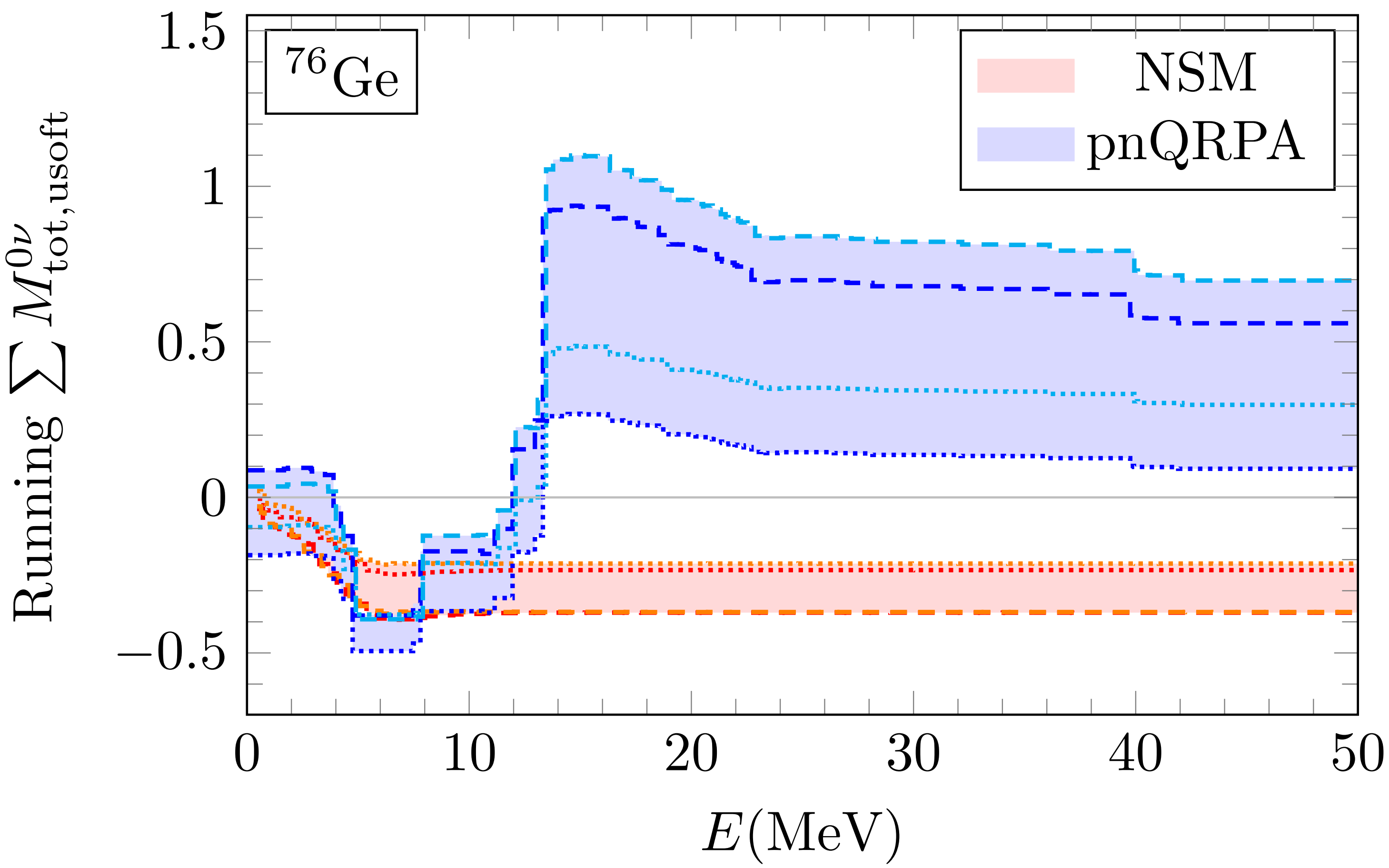
$$M^{0\nu}_{\text{tot,usoft}} = M^{0\nu}_{\text{usoft}} + M^{0\nu}_{\text{loops,usoft}}$$

$$M^{0\nu}_{\text{usoft}} \propto \langle 0^+_f \,|\, \text{GT} \,|\, 1^+_n \rangle \langle 1^+_n \,|\, \text{GT} \,|\, 0^+_i \rangle \cdot \ln(\mu_{\text{us}})$$

$\mu_{us} \equiv$ Ultrasoft scale

Different behaviour between models around **10 MeV**

→ **Different sign**



DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

Main uncertainty → $\mu_{\text{us}} = \dfrac{m_\pi}{2} \ldots 2m_\pi$

CLOSURE VS NON-CLOSURE

Non-closure: $|J_n^\pi\rangle \equiv$ Intermediate states

$$M^{0\nu}_{\text{non-cl}} \propto \frac{\langle 0_f^+ | J_\mu(x) | J_n^\pi \rangle \langle J_n^\pi | J^\mu(y) | 0_i^+ \rangle}{q(q + E_n - \frac{1}{2}(E_f + E_i))}$$

Closure:

$$M^{0\nu}_{\text{L}} \propto \frac{\langle 0_f^+ | J_\mu(x) J^\mu(y) | 0_i^+ \rangle}{q^2}$$

$$q \simeq k_F \simeq 100\,\text{MeV}$$

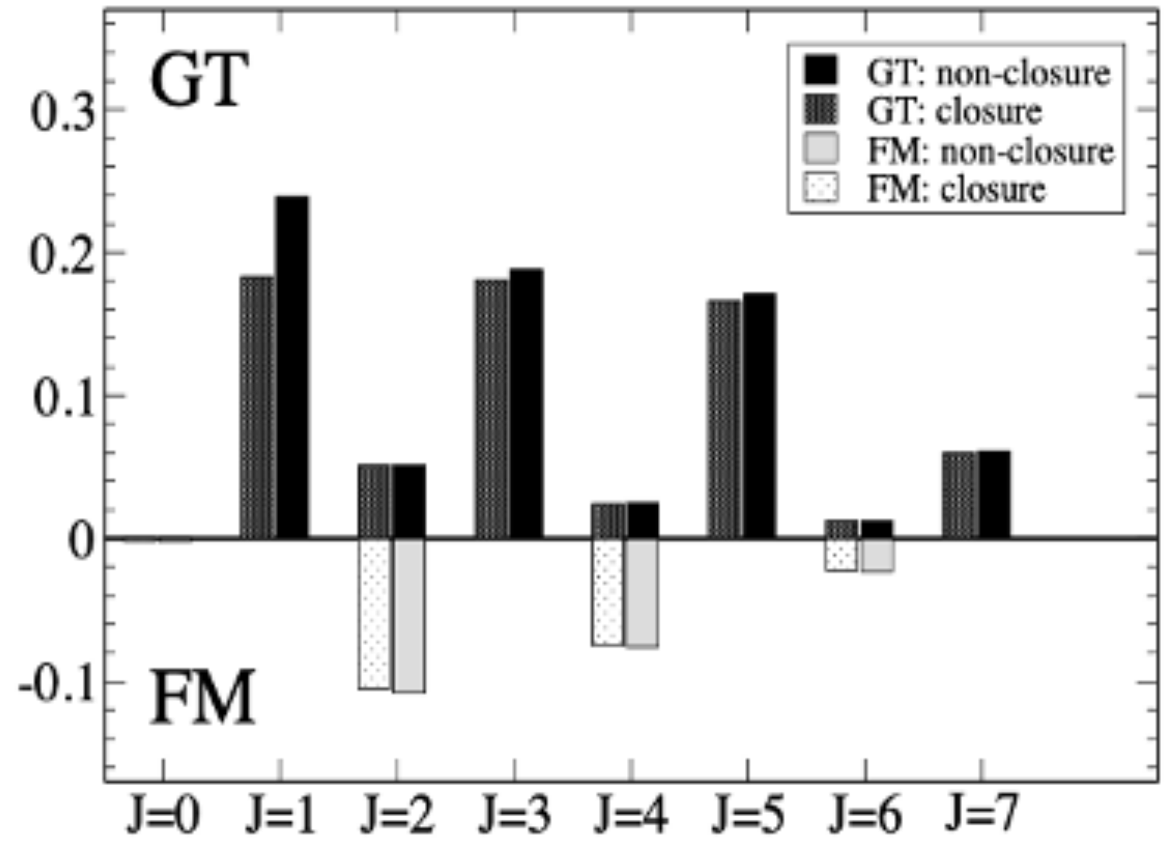
$$E_n - \frac{1}{2}(E_i + E_f) \to 0 \to (\chi\text{EFT})$$

$$J_\mu J^\mu = h_{\text{GT}}(q^2) + h_F(q^2) + h_T(q^2)$$

Main difference from $\mathbf{GT}\,|\,\mathbf{1^+}\rangle$ contributions

↓

Same dependence as $M^{0\nu}_{\text{usoft}}$



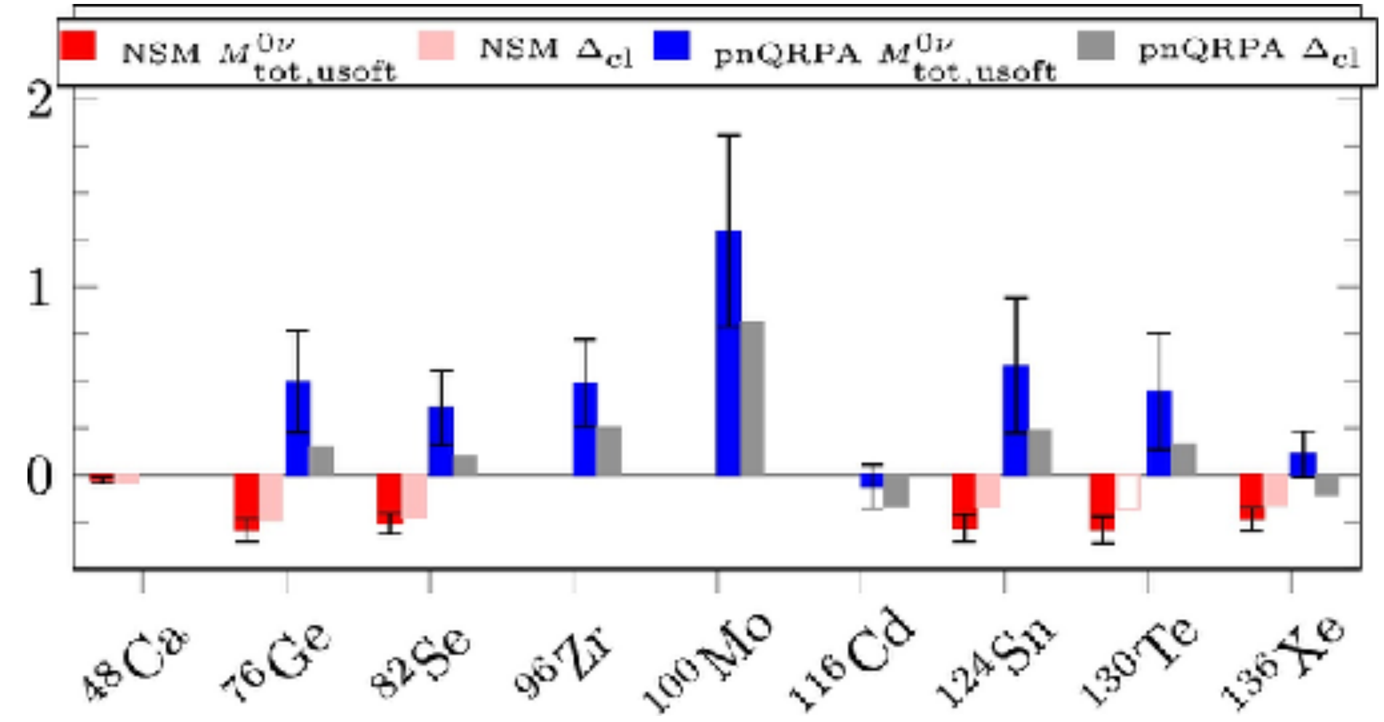
Sen'kov and Horoi Phys. Rev. C 88, 064312, 2013

CLOSURE VS NON-CLOSURE

According to χEFT the **ultrasoft term** must be the **main contribution beyond the closure approximation**

$$M^{0\nu}_{\mathrm{non-cl}} - M^{0\nu}_{\mathrm{L}} = \Delta_{\mathrm{cl}} \sim M^{0\nu}_{\mathrm{usoft}}$$

Cirigliano, et al. Phys. Rev. C 97, 065501, 2018



DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

Agreement in the **sign** between Δ_{cl} and $M^{0\nu}_{\mathrm{usoft}}$

NSM: Good agreement with the χEFT prediction

QRPA: Midler agreement with the χEFT prediction

→ Important **uncertainties** due to μ_{us} dependence

LOOP NME

$$M^{0\nu}_{\text{loops,soft}} = M^{0\nu}_{AA,\text{loops}} + M^{0\nu}_{VV,\text{loops}} + M^{0\nu}_{CT,\text{loops}}$$

Important contributions in **short-range distances** (high momenta)

Similar behaviour between models

Main dependence ⟶ Λ

$\Lambda = 349$ MeV $\Lambda = 550$ MeV

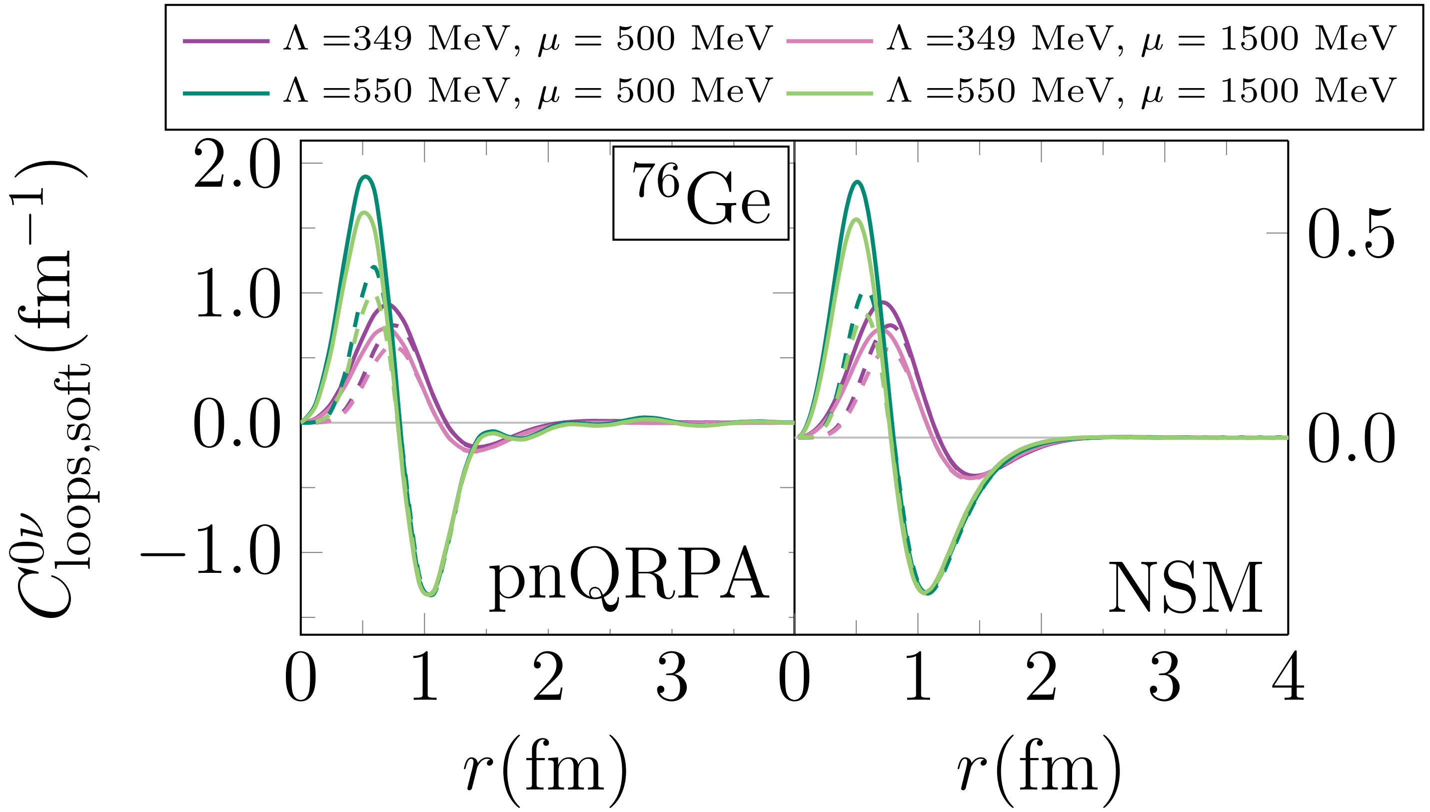
Most reliable: $\Lambda = 349$ MeV

Dependences: $H_{eff}, \mu, \Lambda \ldots$

$\Lambda \equiv$ Cutoff

$\mu \equiv$ Renormalization scale

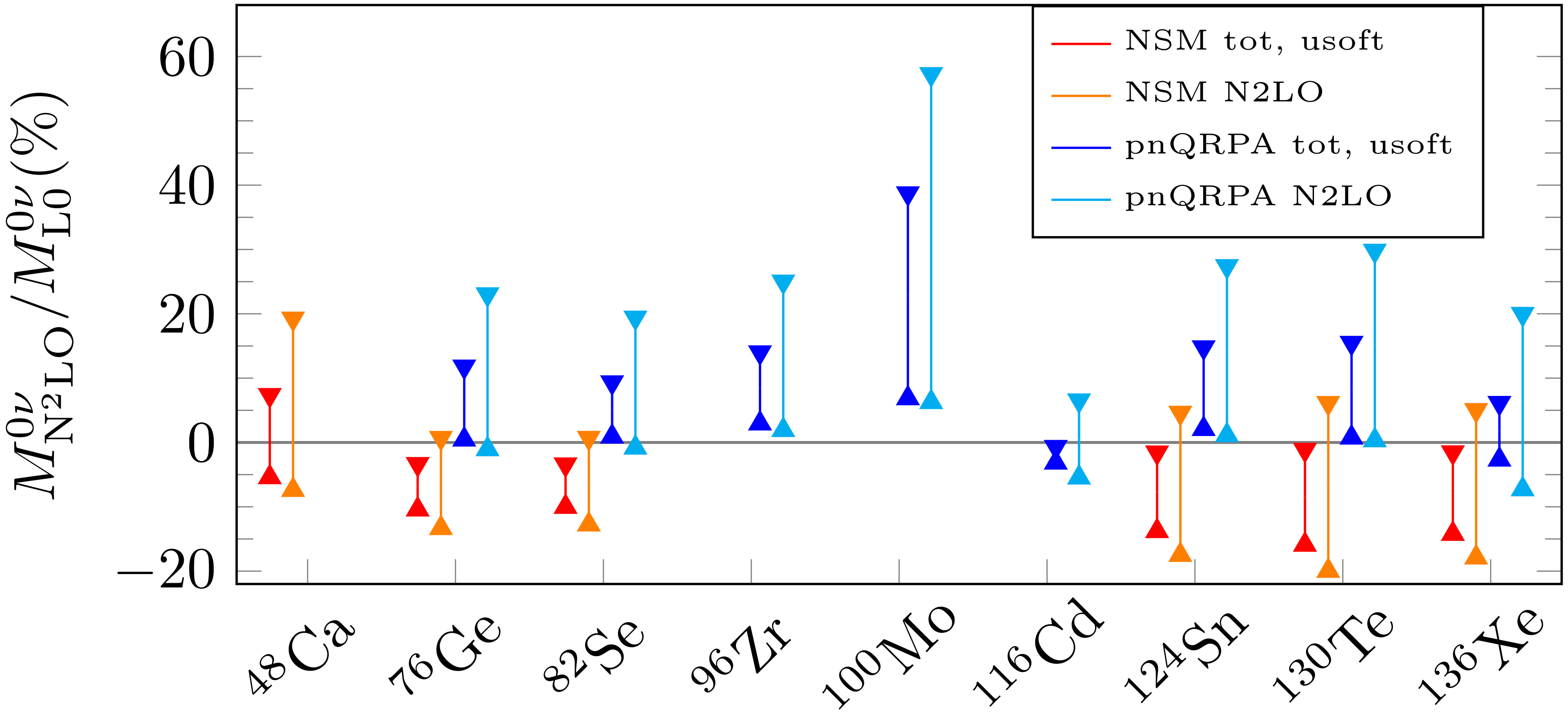
$$M^{0\nu}_{\text{loops,soft}} = \int_0^\infty C^{0\nu}_{\text{loops,soft}}(r)dr$$



DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

TOTAL N²LO NMEs

$$M^{0\nu}_{\mathrm{N^2LO}} = M^{0\nu}_{\mathrm{tot,usoft}} + M^{0\nu}_{\mathrm{loops,soft}}$$



DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

χEFT expectations $\sim$ (5-10) %

$M^{0\nu}_{\mathrm{N^2LO}}/M^{0\nu}_{\mathrm{LO}}$:

NSM $\lesssim$ **20 %**

Central values:

Total ultrasoft: −(5-10) %

Total N²LO: −(5-10) %

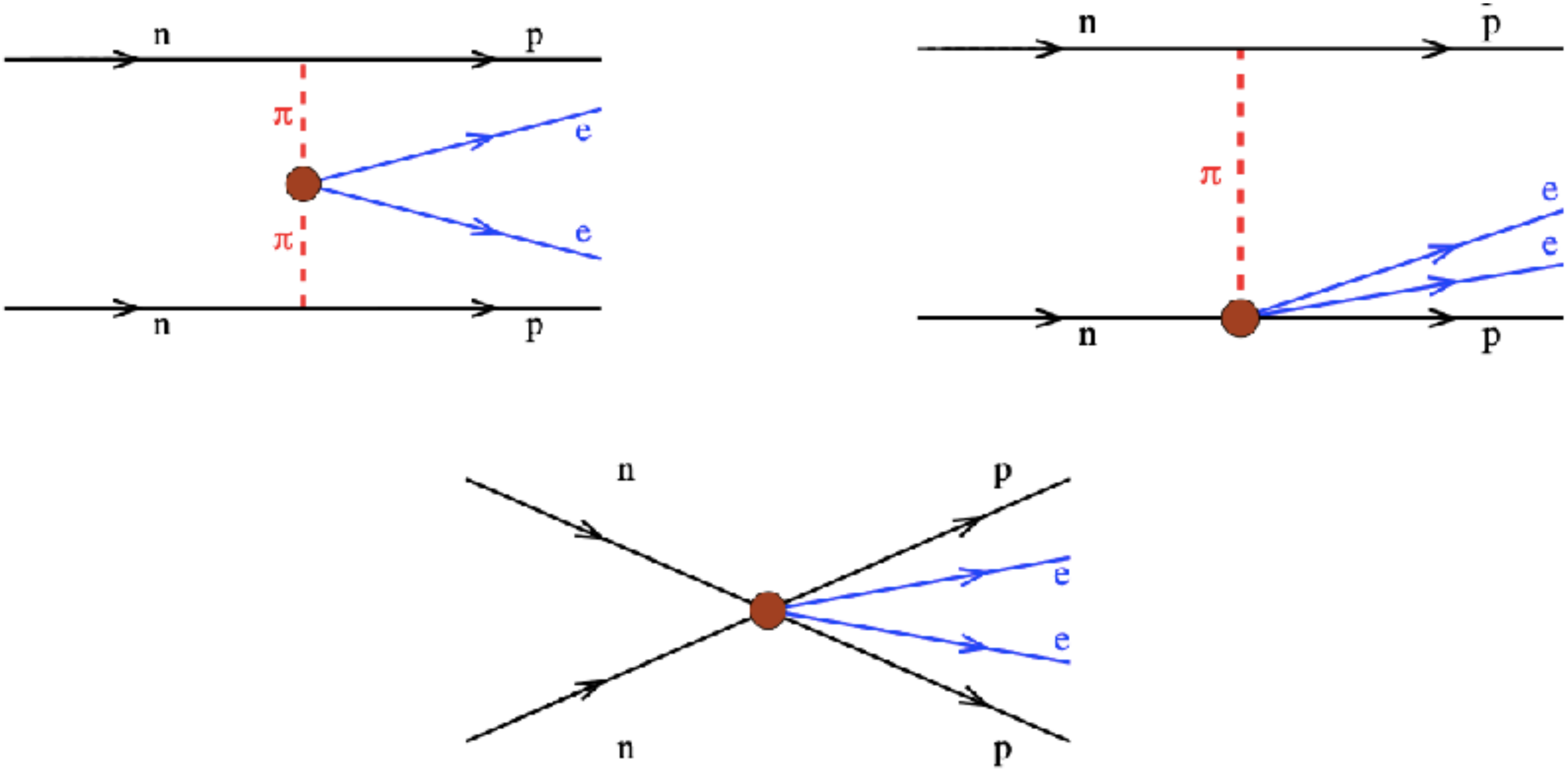
QRPA $\lesssim$ **30 %**

Central values:

Total ultrasoft: +(5-10) %

Total N²LO: +(10-15) %

HEAVY-ν EXCHANGE



Engel, et al. Rep. Prog. Phys. 80, 046301, 2017

HEAVY-ν EXCHANGE: MASS DEPENDENCE

$$M^{0\nu}_{\rm sterile}(m_i)\left\{\begin{array}{ll} M^{0\nu}_{\rm L,<}(m_i)+M^{0\nu}_{\rm S}(m_i)+M^{0\nu}_{\rm usoft}(m_i), & m_i<100\,\text{MeV}\\ M^{0\nu}_{\rm L}(m_i)+M^{0\nu}_{\rm S}(m_i), & 100\,\text{MeV}\le m_i<2\,\text{GeV}\\ M^{0\nu}_{9}(m_i), & 2\,\text{GeV}\le m_i\end{array}\right\}\ \text{LO}$$

$m_i\equiv$ **Sterile ν mass**

m_i dependence :

Change propagator

$$M^{0\nu}_{\rm L}\propto\frac{1}{q^2+m_i^2}\rightarrow M^{0\nu}_{\rm L,<}(m_i)=M^{0\nu}_{\rm L}(m_i)-m_i\left[\frac{d}{dm_i}M^{0\nu}_{\rm L}(m_i)\right]_{m_i=0}$$

To avoid double counting when treating different momentum regions

$$M^{0\nu}_{\rm S}(m_i)\propto g^{NN}_{\nu}(m_i)$$

Cirigliano, Dekens, Urrutia, arXiv:2412.10497

$$M^{0\nu}_{\rm usoft}(m_i)=m_i\cdot M^{0\nu}_{\rm usoft}+\mathcal{O}(\Delta E/m_i)$$

$$M^{0\nu}_{9}\propto\frac{1}{m_i^2}\rightarrow$$ **3 LECs:** $g_1^{\pi\pi}$, $g_1^{\pi N}$ and g_1^{NN} →

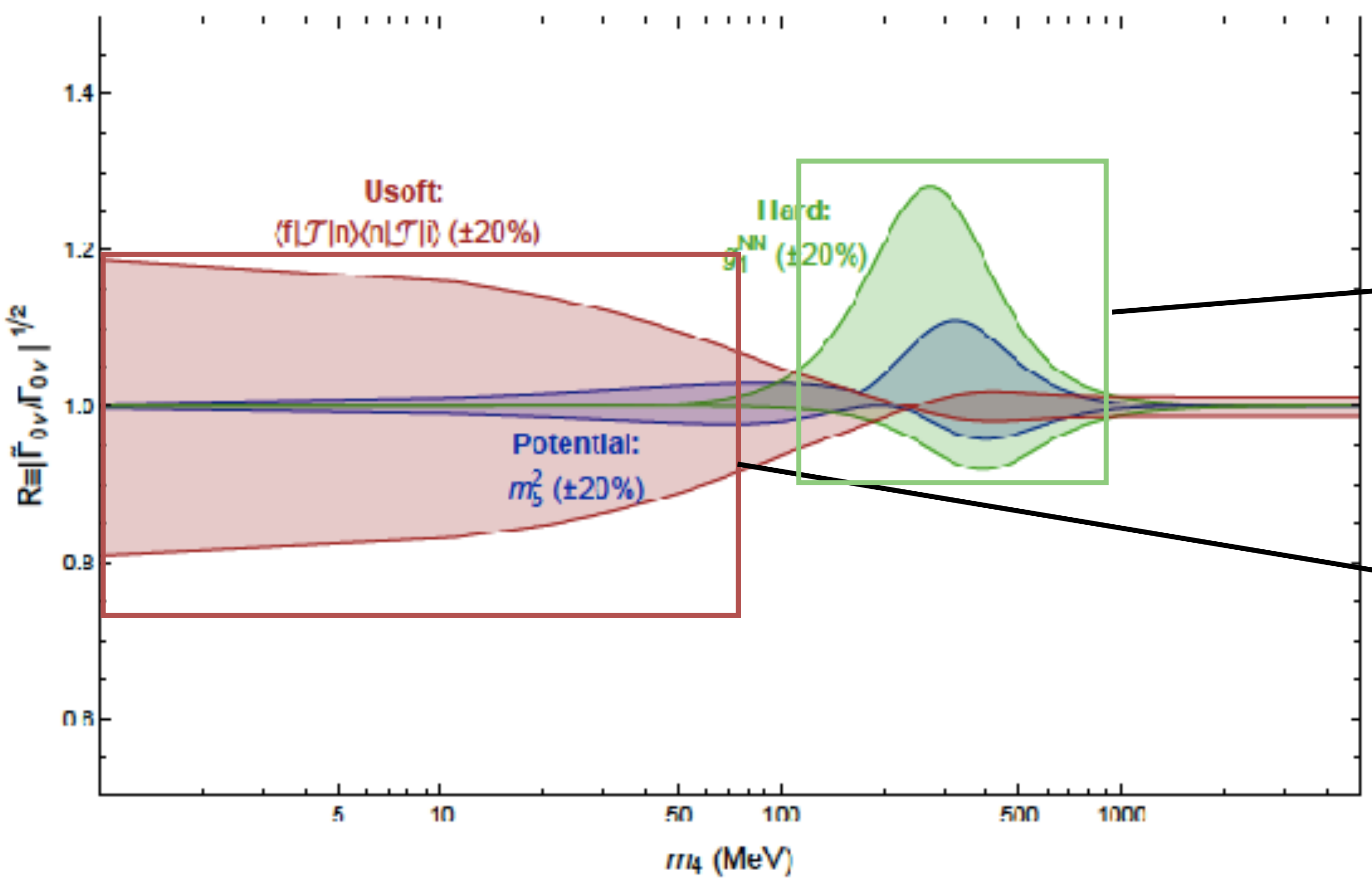
The sterile ν are integrated out at the quark level

Dekens, de Vries, DC et al, JHEP09(2024)201

HEAVY-ν EXCHANGE: ULTRASOFT ENHANCEMENT

Potential: $M_{\mathrm{L}}^{0\nu}(m_i)$

Hard: $M_{\mathrm{S}}^{0\nu}(m_i)$ **Usoft:** $M_{\mathrm{usoft}}^{0\nu}(m_i)$



$M_{\mathrm{S}}^{0\nu}(m_i)$ represents the main contribution together with $M_{\mathrm{L}}^{0\nu}(m_i)$

$M_{\mathrm{usoft}}^{0\nu}(m_i)$ **is the main contribution at LO**

Dekens, de Vries, DC et al, JHEP09(2024)201

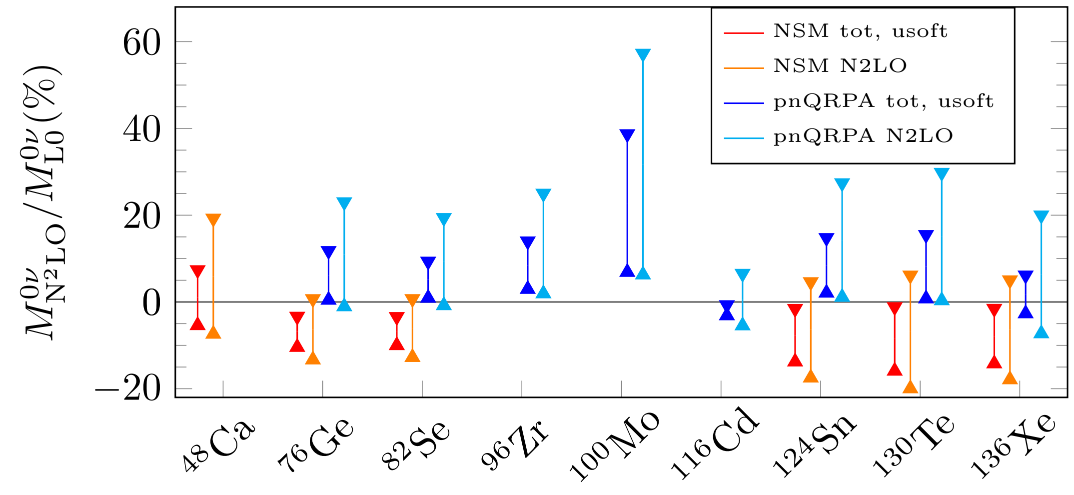
SUMMARY AND OUTLOOK

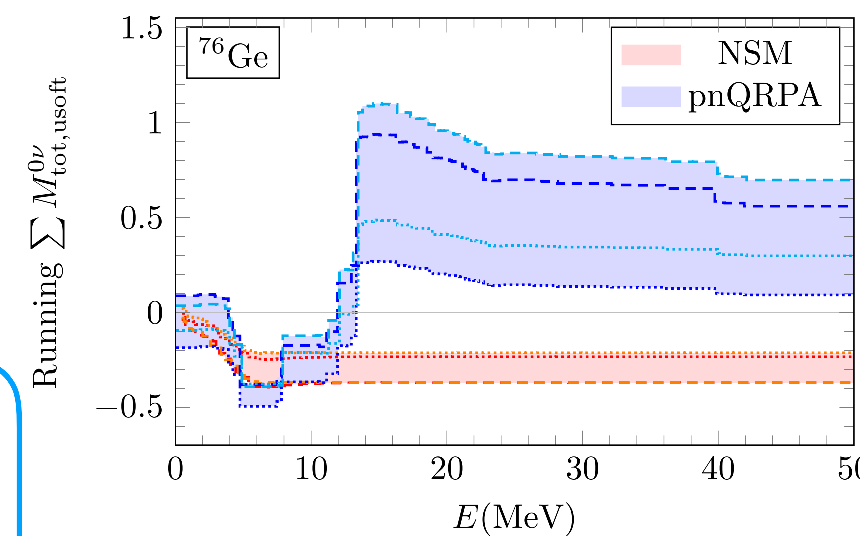
- **N²LO $0\nu\beta\beta$ NMEs**: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{130}Te and ^{136}Xe between NSM and QRPA
- Typically the **N²LO terms** are:

$$|M^{0\nu}_{N^2LO}/M^{0\nu}_{LO}| \leq 20\,\%\ \text{NSM}$$

$$|M^{0\nu}_{N^2LO}/M^{0\nu}_{LO}| \leq 30\,\%\ \text{QRPA}$$

- **Ultrasoft** goes up to **LO** for m_i smaller than 100 MeV





- Use **χ − Hamiltonians** to compute the wavefunctions
- Extend the **χEFT** study to **N³LO corrections**: two-body currents
- Study more **$0\nu\beta\beta$ mechanisms**

THANK YOU FOR YOUR ATTENTION!

Neutrinoless $\beta\beta$ decay nuclear matrix elements complete up to N^2LO in heavy nuclei

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DC, Jokiniemi, Menéndez, Phys.Lett.B 860 (2025)

Neutrinoless double beta decay rates in the presence of light sterile neutrinos

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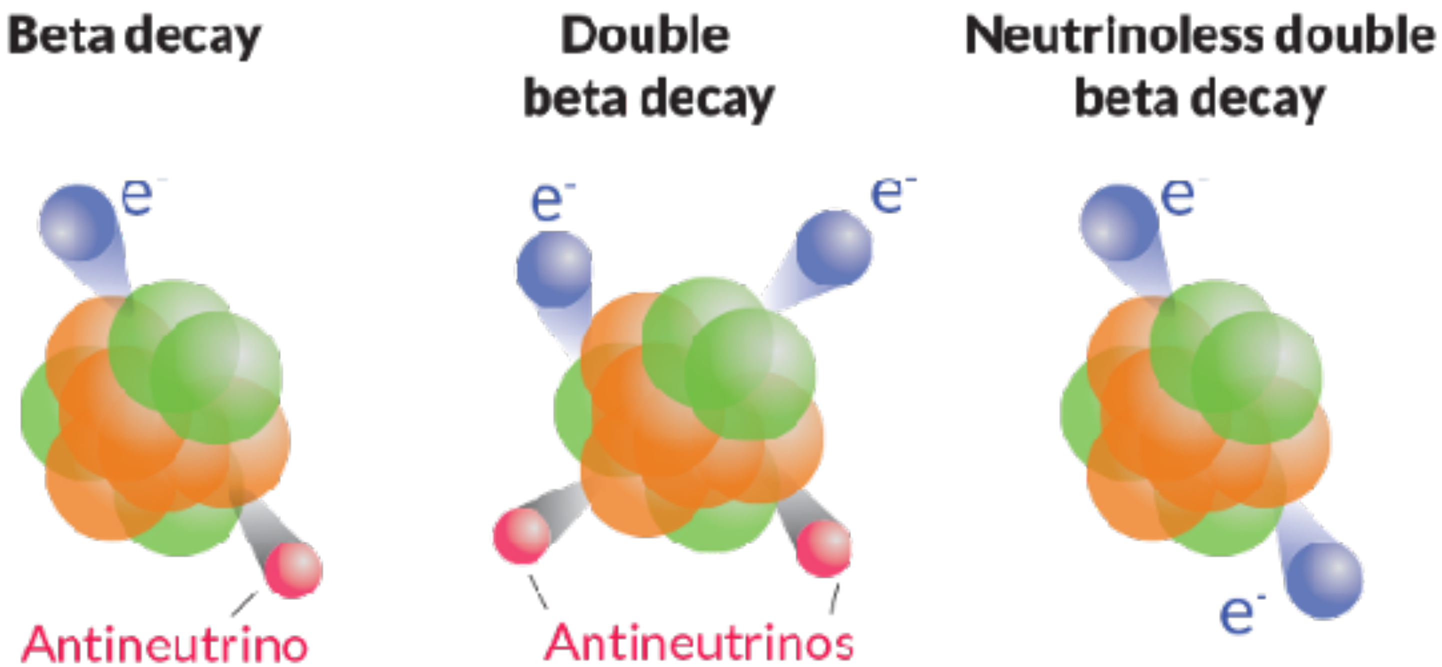
Dekens, de Vries, DC et al, JHEP09(2024)201

BACKUP SLIDES

NEUTRINOLESS $\beta\beta$ DECAY

$$0\nu\beta\beta : 2n \rightarrow 2p + 2e^-$$



https://www.sciencenews.org/article/quest-identify-nature-neutriros-alter-ego-heating

New Information:

- Absolute **mass of neutrino**
- Neutrino as **Majorana fermion**

- **Lepton Number Violation**
  (Beyond Standard Model Physics)
- **Baryon-Antibaryon assymetry**

NEUTRINOLESS $\beta\beta$ DECAY

^{76}Ge : β **decay forbidden** $(Q_\beta < 0)$

Energy difference between **odd-odd nuclei** and **even-even nuclei** ⟶ Pairing interaction

$\beta\beta$ **decay allowed** : $^{76}Ge \rightarrow {}^{76}Se \ (Q_{\beta\beta} > 0)$

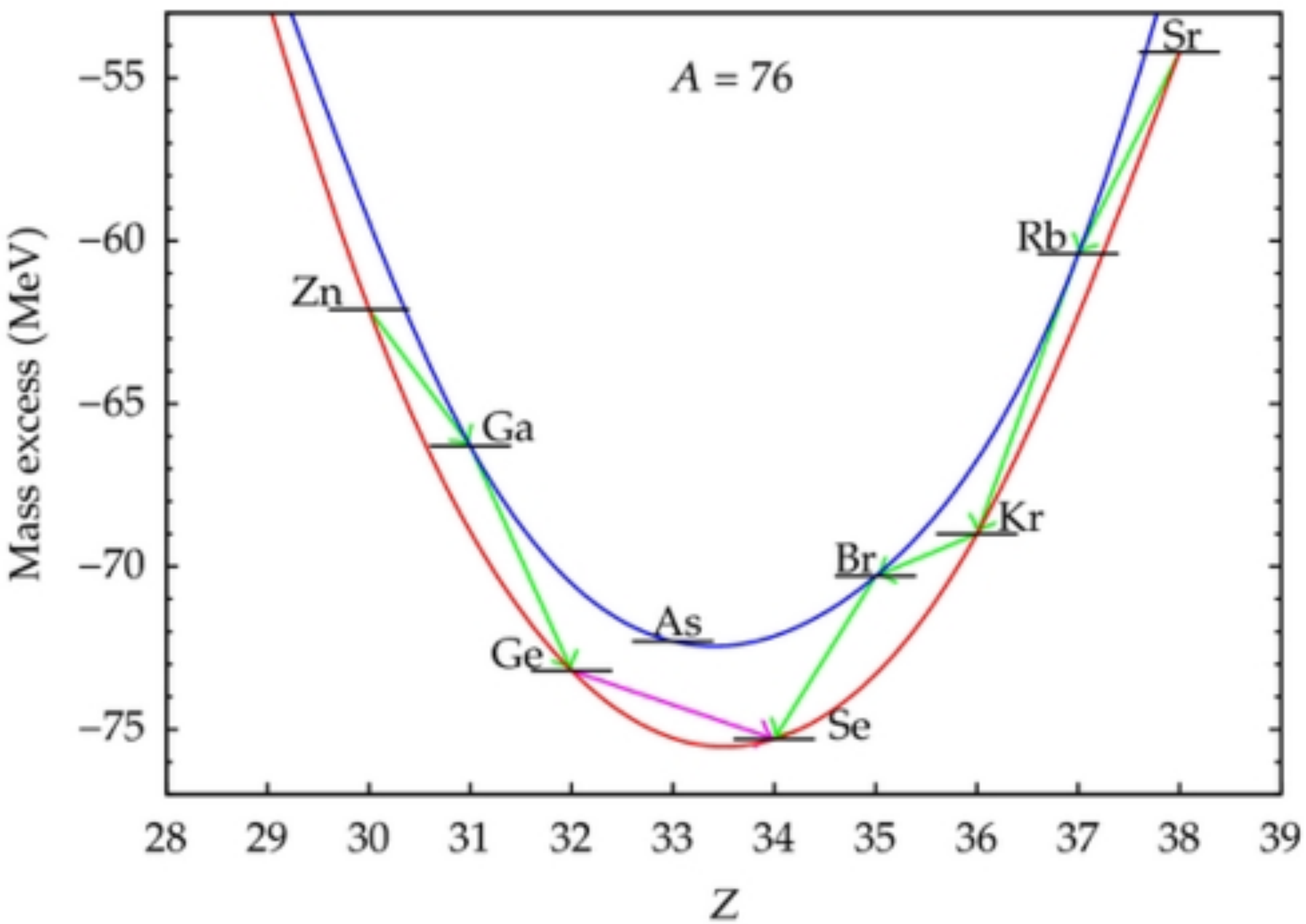
$$Q_{\beta\beta} = E_i - E_f - 2m_e$$

$Q_{\beta\beta} \equiv Q$ value for $\beta\beta$ decay

$m_e \equiv$ electron mass

$E_i \equiv$ initial energy state

$E_f \equiv$ final energy state



Giuliani, et al. Adv. High Energy Phys, 2012, 857016, 2012.

χEFT

Cirigliano et al. JHEP, **12**, 097, 2018

LEADING ORDER

$$M^{0\nu}_{\rm L} = M^{0\nu}_{\rm L,GT} + M^{0\nu}_{\rm L,F} + M^{0\nu}_{\rm L,T}$$

Part of the **N²LO** corrections can be added as a **q-dependence** in the axial, g_A and vector, g_V couplings

$$M^{0\nu}_{\rm S} \propto g^{NN}_{v} e^{-q^2/\Lambda^2}$$

$g^{NN}_{v} \equiv$ Nucleon-Nucleon coupling

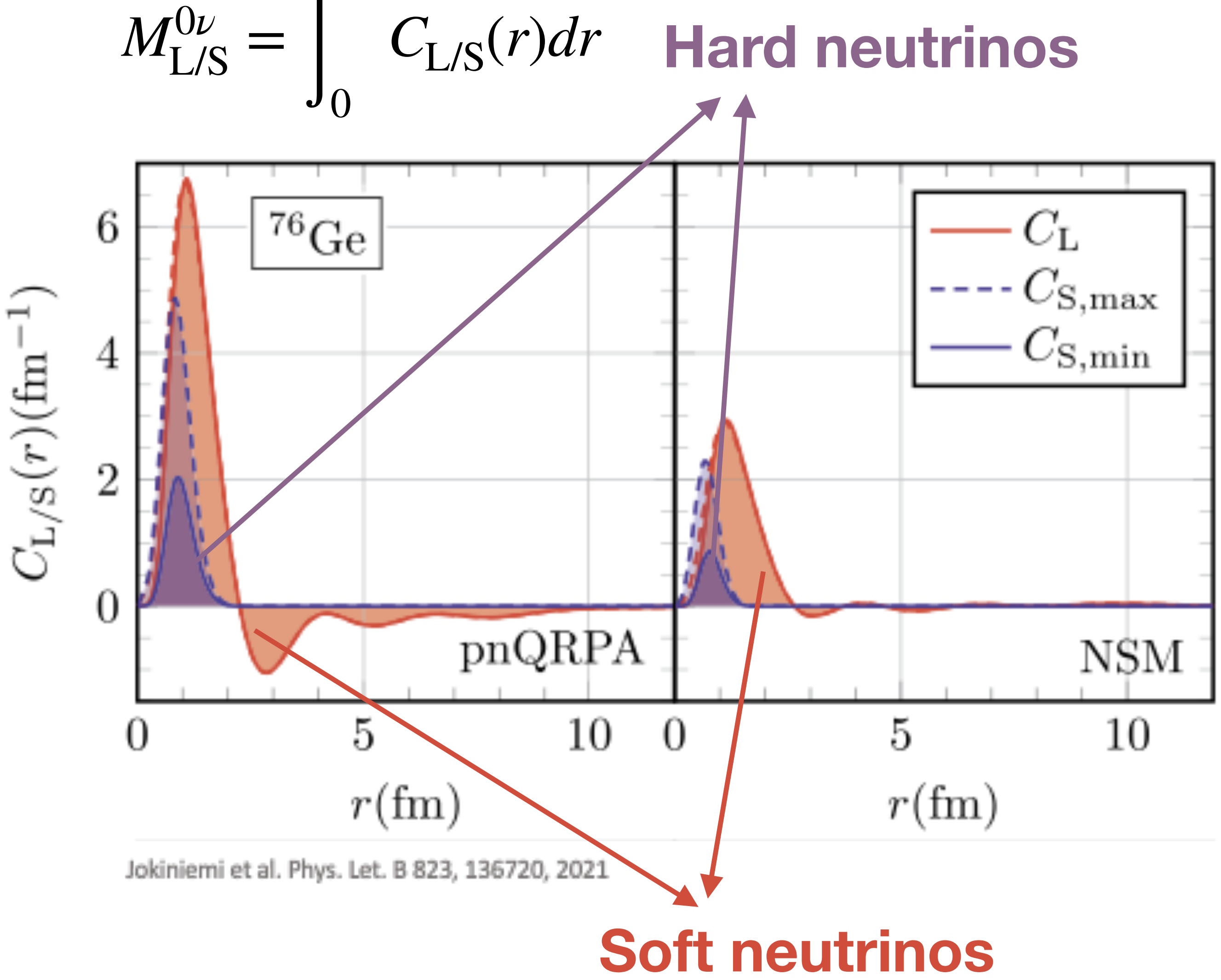
$\Lambda \equiv$ Cutoff

NSM: $M^{0\nu}_{\rm S}/M^{0\nu}_{\rm L} = +(15-50)\,\%$

QRPA: $M^{0\nu}_{\rm S}/M^{0\nu}_{\rm L} = +(30-80)\,\%$

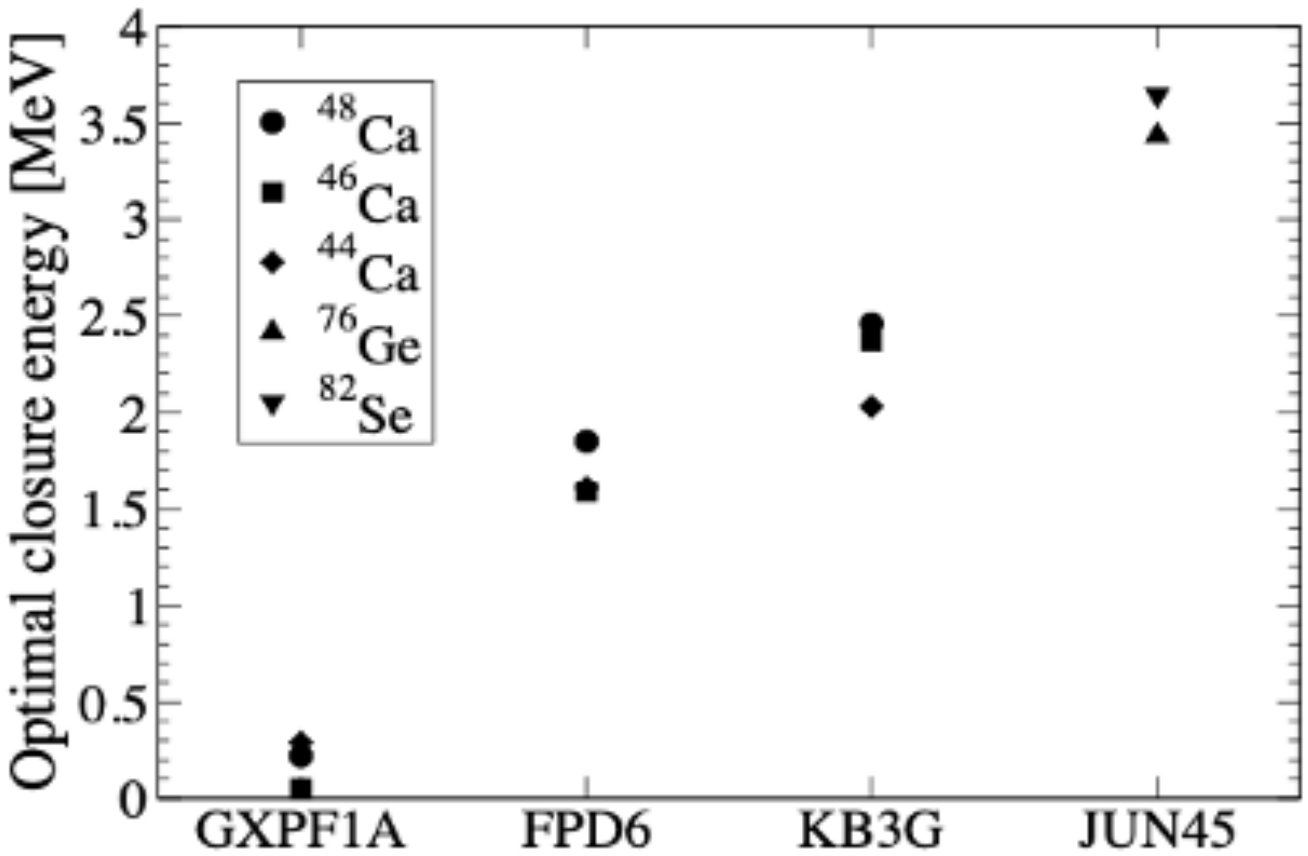
Jokiniemi et al. Phys. Let. B 823, 136720, 2021

$$M^{0\nu}_{\rm L/S} = \int_0^\infty C_{\rm L/S}(r)dr$$



Jokiniemi et al. Phys. Let. B 823, 136720, 2021

CLOSURE ENERGIES



A. Neacsu and M. Horoi Phys. Rev. C **91**, 024309, 2015

Nucleus	Interaction	$\langle E_n \rangle$ (MeV)
^{48}Ca	KB3G	2.5
^{48}Ca	GXPF1A	0.23
^{76}Ge	JUN45	3.5
^{82}Se	JUN45	3.6
^{136}Xe	GCN5082	3.7

S. Sarkar et al. arXiv:2406.13417v1. 2024

PHENOMENOLOGICAL INTERACTIONS

$A = 48$

- KB3G
- GXPF1A

KUO-BROWN interaction

⟶ Mass dependence
Monopole modifications

BONN-C potential

⟶ Two-body matrix elements from $A = 47 - 66$

$A = 76 - 82$

- JUN45
- GCN2850

BONN-C potential

⟶ 133 two-body matrix elements
4 single-particle energies with $A = 63 - 96$

G-matrix
Fit to 300 energy levels

$A = 130 - 136$

- GCN5082
- QX5082

Same as GCN2850

BONN-C potential

⟶ Binding energies pf 157 low-lying yrast states from $^{102\text{-}132}Sn$

NMEs for sterile neutrinos

$$M_{\mathrm{L}}^{0\nu}(m_i) = M_{\mathrm{GT}}(m_i) - \frac{M_{\mathrm{F}}(m_i)}{g_A^2} + M_{\mathrm{T}}(m_i)$$

$$M_{\mathrm{S}}^{0\nu}(m_i) = 2g_\nu^{NN}(m_i)m_\pi^2\frac{M_{\mathrm{F,sd}}}{g_A^2}$$

$$M_9^{0\nu}(m_i) = 2\eta(\mu_0, m_i)\frac{m_\pi^2}{m_i^2}[\frac{5g_1^{\pi\pi}}{6}(M_{\mathrm{GT}}^{PP} + M_{\mathrm{T}}^{PP}) + \frac{g_1^{\pi N}}{2}(M_{\mathrm{GT}}^{AP} + M_{\mathrm{T}}^{AP}) - \frac{2g_1^{NN}}{g_A^2}M_{\mathrm{F,sd}}]$$

$$M_{\mathrm{usoft}}^{0\nu}(m_i) = R_A m_i \sum_n \langle 0_f^+ | \tau^- \boldsymbol{\sigma} | 1_n^+ \rangle \langle 1_n^+ | \tau^- \boldsymbol{\sigma} | 0_i^+ \rangle + \mathcal{O}(\Delta E/m_i)$$