Renormalization of the electroweak current within the realistic shell model

NME2025

RCNP Workshop for "Theoretical and Experimental Approaches for Nuclear Matrix Elements of Double Beta decay"







Istituto Nazionale di Fisica Nucleare

Giovanni De Gregorio

The renormalization of the shell-model Gamow-Teller operator starting from effective field theory for nuclear systems L.Coraggio, N. Itaco, <u>G. De Gregorio</u>, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, M. Viviani, Physical Review C **109**, 014301 (2024)

Forbidden β decays within the realistic shell model <u>G. De Gregorio</u>, R. Mancino, L.Coraggio, N. Itaco, Physical Review C **110**, 014324 (2024)

Renormalization of στ matrix elements

Gamow-Teller transitions (β-decay, EC, 2νββ,charge-exchange) are hindered from expected values based on sum rules derived by nuclear structure models.

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Gamow-Teller transitions (β-decay, EC, 2vββ,charge-exchange) are hindered from expected values based on sum rules derived by nuclear structure models.

Quenching of $\sigma\tau$ matrix elements is quite a general phenomenon in nuclear-structure physics.

$$g_A = g_A^{eff} = q g_A$$

Renormalization of $\sigma\tau$ matrix elements

Z. Phys. A - Atomic Nuclei 332, 413417 (1989)



$$g_A = g_A^{eff} = q \ g_A$$

Martinez-Pinedo et al. PRC53 2602(1996)



J. Suhonen, ACTA PHYSICA POLONICA B, 3 (2018)

Mass range	$g_{ m A}^{ m eff}$
Full $0p$ shell	$1.03^{+0.03}_{-0.02}$
0p-low $1s0d$ shell	1.18 ± 0.05
Full $1s0d$ shell	$0.96^{+0.03}_{-0.02}$
	1.0
$A = 41 - 50 \ (1p0f \text{ shell})$	$0.937^{+0.019}_{-0.018}$
1p0f shell	0.98
⁵⁶ Ni	0.71
$A = 52-67 \ (1p0f \text{ shell})$	$0.838^{+0.021}_{-0.020}$
$A = 67-80 \ (0f_{5/2}1p0g_{9/2} \text{ shell})$	0.869 ± 0.019
A = 63-96 (1p0f0g1d2s shell)	0.8
$A = 76-82 \ (1p0f0g_{9/2} \text{ shell})$	0.76
$A = 90-97 \ (1p0f0g1d2s \text{ shell})$	0.60
¹⁰⁰ Sn	0.52
$A = 128 - 130 \ (0g_{7/2} 1d_{2s} 0h_{11/2} \text{ shell})$	0.72
$A = 130 - 136 (0g_{7/2} 1d_{2s} 0h_{11/2} \text{ shell})$	0.94
$A = 136 \ (0g_{7/2} 1d2s 0h_{11/2} \text{ shell})$	0.57

Quenching of $\sigma\tau$ matrix elements & $0\nu\beta\beta$ decay

The inverse of the $0\nu\beta\beta$ -decay half-life is proportional to the squared nuclear matrix element (NME). This evidences the relevance to calculate the NME $(M^{0\nu})$



$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} |M^{0\nu}|^2 \langle m_{\nu} \rangle^2 \propto g_A^4$$

- G^{0ν} → phase space factor
 ⟨m_ν⟩ = |∑_k m_k U_{ek}|, effective mass of the Majorana neutrino U_{ek}being the lepton mixing matrix

Quenching of στ matrix elements: truncated model space

Two main sources:

1) LIMITED MODEL SPACE

Nuclear Structure calculations are carried out in truncated model spaces -> effective Hamiltonians and operators



Shell model: A well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei



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The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

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 $V_{NN} (+V_{NNN}) \downarrow$ $MANY BODY PERTURBATION THEORY \downarrow$ H_{eff}

Realistic shell-model calculations starting from a nuclear Hamiltonian and decay operators derived consistently

 $H \rightarrow H_{eff}$

$$H|\psi_{\nu}\rangle = E_{\nu}|\psi_{\nu}\rangle \rightarrow H_{eff}|\varphi_{\alpha}\rangle = E_{\nu}|\varphi_{\alpha}\rangle$$

 $|\varphi_{\alpha}\rangle$ = eigenvectors obtained diagonalizing H_{eff} in the reduced model space $|\varphi_{\alpha}\rangle$ = P $|\psi_{\nu}\rangle$

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 $\langle \varphi_{\nu} | \Theta | \varphi_{\lambda} \rangle \neq \langle \Psi_{\nu} | \Theta | \Psi_{\lambda} \rangle$

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We then require an effective operator Θ_{eff} defined as follows

$$\Theta_{eff} = \sum_{\nu\lambda} |\varphi_{\nu}\rangle \langle \Psi_{\nu}|\Theta|\Psi_{\lambda}\rangle \langle \varphi_{\lambda}|$$

 $\left\langle \varphi_{\nu} \middle| \Theta_{eff} \middle| \varphi_{\lambda} \right\rangle = \left\langle \Psi_{\nu} \middle| \Theta \middle| \Psi_{\lambda} \right\rangle$

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$$\Theta_{eff} = \sum_{\nu\lambda} |\varphi_{\nu}\rangle \langle \Psi_{\nu}|\Theta|\Psi_{\lambda}\rangle \langle \varphi_{\lambda}| \qquad \langle \varphi_{\nu}|\Theta_{eff}|\varphi_{\lambda}\rangle = \langle \Psi_{\nu}|\Theta|\Psi_{\lambda}\rangle$$

$$\Theta^{GT} = g_A \sigma \tau^{\pm} \rightarrow \Theta^{GT}_{eff} = g_A^{eff} \sigma \tau^{\pm} \qquad \Theta^{E2} = er^2 Y_{\mu}^2 \rightarrow \Theta^{E2}_{eff} = e_{eff} r^2 Y_{\mu}^2$$

Quenching of $\sigma\tau$ matrix elements: theory

Two main sources:

2) NON-NUCLEONIC DEGREES OF FREEDOM

Processes in which the weak probe prompts a meson to be exchanged between two nucleons

→ meson-exchange two-body currents (2BC)



H. Hyuga and A. Arima, J. Phys. Soc. Jpn. Suppl. 34, 538 (1973)

Quenching of στ matrix elements: meson exchange currents

In the 80s starting from OBEP models two-nucleon meson-exchange current operators have been constructed consistently as required by the continuity equation for vector currents and the PCAC.

Nowadays, EFT provides a powerful approach where both nuclear potentials and two-body electroweak currents (2BC) may be consistently constructed, the latter appearing as subleading corrections to the one-body GT operator *στ*[±]

Quenching of $\sigma\tau$ **matrix elements: meson exchange currents**

Nowadays, EFT provides a powerful approach where both nuclear potentials and two electroweak currents may be consistently constructed, the latter appearing as subleading corrections to the one-body GT operator *o*^{*t*}



Nuclear potential

Electroweak axial currents

The axial current J_A

The matrix elements of the axial current J_A are derived through a chiral expansion up to N3LO, and employing the same LECs as in 2NF and 3NF

$$J_A^{LO} = -g_A \sum_i \sigma_i \tau_{i,\pm}$$

$$J_A^{N^2LO} = \frac{g_A}{2m_N^2} \sum_i K_i \times (\sigma_i x K_i) \tau_{i,\pm}$$

$$J_A^{N^3LO}(1PE; \mathbf{k}) = \sum_{i < j} \frac{g_A}{2f_\pi^2} \Big\{ 4c_3 \,\tau_j \mathbf{k} \,+\, \left(\,\tau_i \times \tau_j \right)_{\pm} \times \Big[\Big(c_4 + \frac{1}{4m} \sigma_i \,\times\, \mathbf{k} \,- \frac{i}{2m} K_i \Big) \Big] \Big\} \,\sigma_j \cdot \mathbf{k} \, \frac{1}{\omega_k^2}$$

$$J_{A}^{N^{3}LO}(CT; \mathbf{k}) = \sum_{i < j} z_{0} \left(\tau_{i} \times \tau_{j}\right)_{\pm} (\sigma_{i} \times \sigma_{j})$$
$$z_{0} = \frac{g_{A}}{2f_{\pi}^{2}m_{N}^{2}} \left[\frac{m_{N}}{4g_{A}\Lambda_{\chi}}c_{D} + \frac{m_{N}}{3}(c_{3} + 2c_{4}) + \frac{1}{6}\right]$$



A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C 93, 015501 (2016)

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A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C 93, 015501 (2016)

Two-body contribution to $\sigma \tau$: light nuclei

GT nuclear matrix elements of the β -decay of *p*-shell nuclei have been calculated with Green's function Monte Carlo (GFMC) and no-core shell model (NCSM) methods, including contributions from 2BC

S. Pastore et al., Phys. Rev. C 97 022501(R) (2018)

P. Gysbers et al., Nat. Phys. 15 428 (2019)

$^{0.96}$ 1 1.04	0.96 1 1.04	0.96 1 1.04	$0.96 \ 1 \ 1.04$
3 H β -decay	⁶ He β-decay	Be ε-cap(gs)	Be ϵ -cap(ex)
○ •		○ ●	
□ ■		■ ●	□
◆ •		◆ ●	◆◆
⁸ Li β-decay	⁸ B β-decay	⁸ He β-decay	¹⁰ C β-decay
● ● ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■			•

G. B. King et al., Phys. Rev. C 102 025501 (2020)

Two-body contribution to *στ*: Medium-mass nuclei

The contribution of 2BC improves systematically the agreement between theory and experiment

In-Medium SRG

Gysbers et al. Nature Phys. 15 428 (2019)

A proper treatment of nuclear correlations and consistency between GT two-body currents and 3N forces, derived in terms of ChPT, describes microscopically the "quenching puzzle"

Details of the Calculations

- Nuclear Hamiltonian: Entem-Machleidt N3LO two-body potential plus N2LO three-body potential ($\Lambda = 500 \text{ MeV}$)
- Axial current J_A calculated at N3LO in ChPT
- Heff obtained calculating the Q-box up to the 3rd order in V_{NN} (up to 2p-2h core excitations) and up to the 1st order in V_{NNN}
- Effective operators are consistently derived by way of the MBPT
- fp-shell nuclei: four proton and neutron orbitals outside ⁴⁰Ca: 0f7/2, 0f5/2, 1p3/2, 1p1/2
- fpg-shell nuclei: four proton and neutron orbitals outside ⁵⁶Ni: 0f5/2, 1p3/2, 1p1/2, 0g9/2

RSM calculations, starting from ChPT two- and three-body potentials and two-body meson-exchange currents for spectroscopic and spin-isospin dependent observables of ⁴⁸Ca, ⁷⁶Ge, ⁸²Se

⁴⁸Ca: GT Strength

Charge exchange experiments

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma}B_{exp}(GT)$$

$$B_{th}(GT) = \frac{\left|\left\langle\varphi_f\right||J_A|\left|\varphi_i\right\rangle\right|^2}{2J_i + 1}$$

GT fp shell Nuclei

GT matrix elements of 60 experimental decays of 43 fp-shell nuclei

$$\sigma = \sqrt{\frac{\sum_{i} (x_i - \hat{x_i})^2}{n}}$$

Fpg shell nuclei: GT strength

Presentation title

$$M_{GT}^{2\nu} = \sum_{k} \frac{\langle 0_f^+ ||\vec{\sigma} \cdot \tau^-||k\rangle \langle k||\vec{\sigma} \cdot \tau^-||0_i^+\rangle}{E_k + E_0}$$

Blue: bare J_A at LO in ChPT (namely the GT operator g_A)

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Green: effective J_A at LO in ChPT

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Green: effective J_A at LO in ChPT

Black: bare J_A at N3LO in ChPT

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Green: effective J_A at LO in ChPT

Black: bare J_A at N3LO in ChPT

Red effective J_A at N3LO in ChPT

Conclusions and persectives (1st part)

Conclusions

- Correlations + electroweak 2BC \implies quite good description of $\sigma\tau$ observables
- 2BC introduce ~ 20% reduction of GT matrix elements

Perspectives

- Meson-exchange two-body currents for the M1 transitions
- Calculations for heavier-mass systems (100Mo, 130Te, 136Xe)
- Calculating $0\nu\beta\beta$ decay $M^{0\nu}$ including also the LO contact term

The total half-life of the β decay is expressed in terms of the k-th partial decay half-life as

where $\kappa = 6144$ s

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 \tilde{C} is the integrated shape function, whose integrand defines the β -decay energy spectrum

$$\tilde{C} = \int_{1}^{w_0} p_e w_e (w_0 - w_e)^2 F(Z, w_e) C_n(w_e) dw_e$$

- $F(Z, w_e) = F_0(Z, w_e)L_0(Z, w_e)$, F is the Fermi function and takes into account the distortion of the electron wave function by the nuclear charge and L_0 accounts for the finite size effect.
- w_e is the electron energy

where $\kappa = 6144$ s

• $C_n(w_e)$ is the shape factor of the n-th forbidden transition which depends on the nuclear matrix elements (NMEs) of the decay operators.

H. Behrens and W. Büring, Nuclear Physics Al62 (1971) 11 1-144

G. De Gregorio, R. Mancino, L.Coraggio, N. Itaco, Physical Review C 110, 014324 (2024)

$$\tilde{C} = \int_{1}^{w_0} p_e w_e (w_0 - w_e)^2 F(Z, w_e) C_n(w_e) dw_e$$

In general C_n is function of W_e .

For allowed transition:
$$C_n(w_e) = \text{Const} = B(\text{GT}) = g_A^2 \frac{|\langle f|| \sum_k \sigma_k \tau_k^- ||i\rangle|^2}{2J_i + 1}$$

Beta spectrum is insensitive to B(GT)

The β decay spectrum

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Beta spectrum is insensitive to Lth-unique decays

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Beta spectrum is sensitive to Lth-nonunique decays

J. Kostensalo and J. Suhonen, Phys. Rev. C 96, 024317 (2017)

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J. Kostensalo and J. Suhonen, Phys. Rev. C 96, 024317 (2017)

- 1. One can extract the empirical value of q from the β spectrum on forbidden non-unique decays.
- 2. Alternatively, having a microscopic theory without free parameters, β spectra offer a further benchmark of the theoretical framework.

M. Paulsen et al. Phys. Rev. C 110, 055503 - 2024

Details of the Calculations

Calculations for nuclei in the neighborhood of $0\nu\beta\beta$ candidates above 78Ni core:

- Second forbidden non-unique gs to gs β -decay of ⁹⁴Nb and ⁹⁹Tc
- Fourth forbidden non-unique gs to gs β -decay of ¹¹³Cd and ¹¹⁵In

Interaction

 $V_{\text{low-k}}$ of CD-Bonn with Λ =2.6 fm⁻¹+ Coulomb force for protons

TBME +SP energies of H_{eff} & Θ_{eff} (a) third order +3-body correlations

One major proton and neutron shell above ⁷⁸Ni

$$ft = \frac{\kappa}{\tilde{c}} \int_{1}^{w_0} p_e w_e (w_0 - w_e)^2 F(Z, w_e) \, dw_e$$

	Bare	Effective	Ехр
⁹⁴ Nb	11.30	11.58	11.95 (7)
⁹⁹ Tc	11.580	11.876	12.325 (12)
¹¹³ Cd	21.902	22.493	23.127 (14)
¹¹⁵ ln	21.22	21.64	22.53 (3)

G. De Gregorio, R. Mancino, L.Coraggio, N. Itaco, Physical Review C 110, 014324 (2024)

P. Belli, et al., Phys. Rev. C 76, 064603 (2007).

G. De Gregorio, R. Mancino, L. Coraggio, N. Itaco, Physical Review C 110, 014324 (2024)

L. Pagnanini, et al., arXiv:2401.16059

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- Using the bare operator, but introducing a quenching factor of the axial constant to improve the reproduction
 of the experimental logfts, there is a distortion of the shape of the energy spectra that affects the agreement
 with the observed ones.

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Perspectives

• Study of forbidden β decays in other mass regions, close to nuclei that are candidates for detecting the 0vββ decay

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Perspectives

- Study of forbidden β decays in other mass regions, close to nuclei that are candidates for detecting the 0vββ decay
- Tackle the forbidden β-decay problem starting from the derivation of electroweak currents by way of the chiral Perturbation theory

Presentation title

14th International Spring Seminar on Nuclear Physics: "Cutting-edge developments in nuclear structure physics"

Main topics

- •New frontiers in nuclear many-body theory and nuclear forces
- •Experimental progress in nuclear structure studies
- Nuclear physics for testing fundamental physics
- •Nuclear astrophysics
- •Quantum computing and machine learning for nuclear physics

https://agenda.infn.it/event/42803/

19th-23th May 2025