$0\nu\beta\beta$ decay within Relativistic Configuration-interaction Density functional (ReCD) theory

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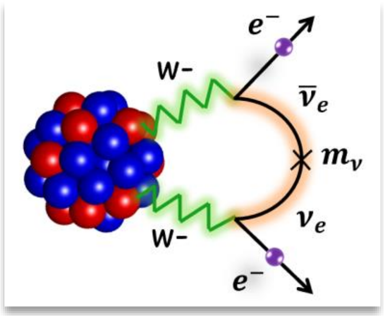
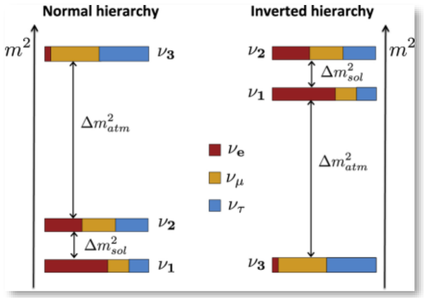
Outline

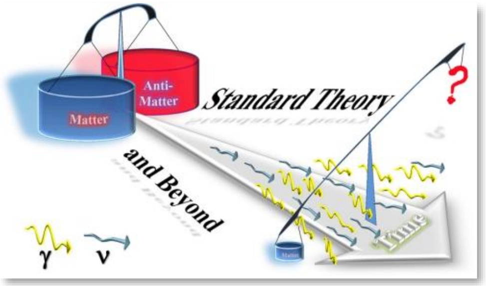
- $0\nu\beta\beta$ decay and the nuclear matrix element (NME)
- Basic concepts of the ReCD theory
- Correlation between $0\nu\beta\beta$ decay and double GT transition
- Reducing the uncertainty in short-range NME
- Summary

$0\nu\beta\beta$ decay and NMEs

- Violation of **lepton number**
- **Majorana nature** of neutrinos
- Neutrino **mass scale** and **hierarchy**
- **Matter dominance** in the Universe







Theoretical study on $0\nu\beta\beta$-decay NMEs build a bridge between **experimental data** and **underlying new physics**

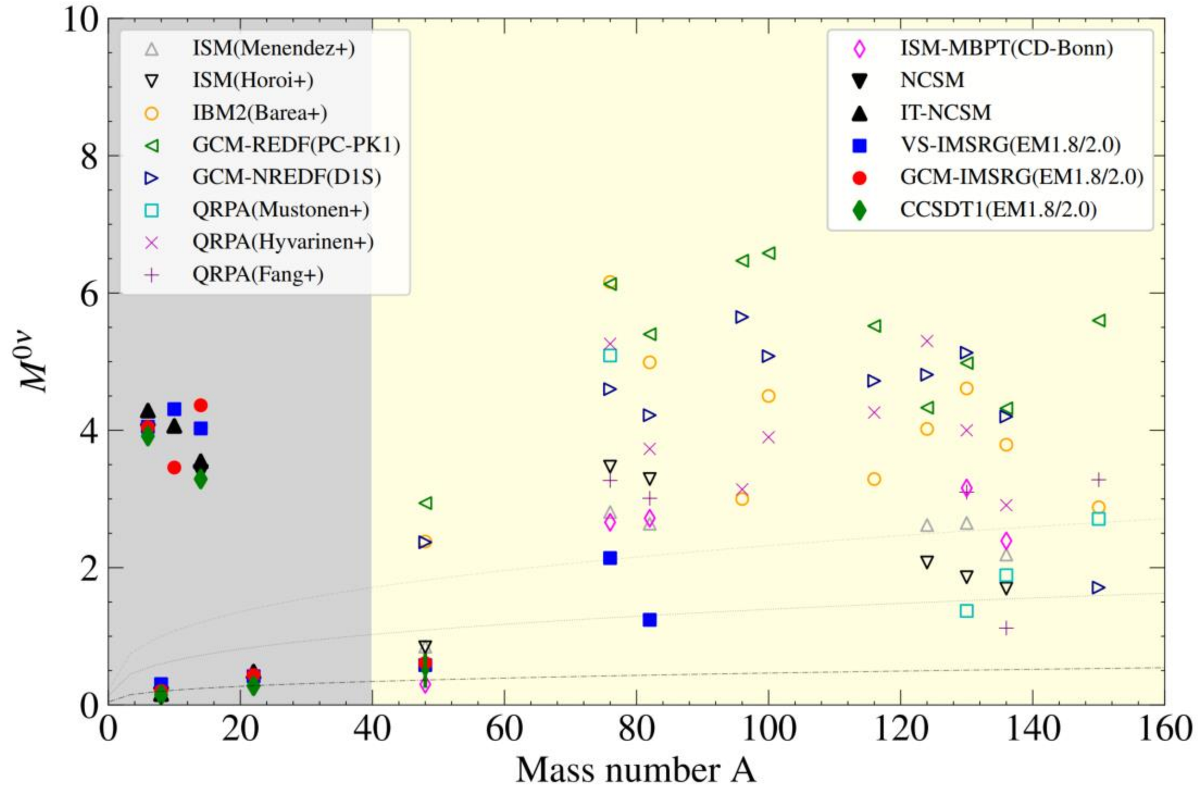
- $0\nu\beta\beta$ decay half-life: $[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}(Q,Z)|M^{0\nu}|^2\left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2$

Phase space factor $G^{0\nu}(Q,Z)$ Effective neutrino mass: $\langle m_{\beta\beta}\rangle = \sum_i |U^2_{ei}|m_i$

Nuclear matrix element (NME): $M^{0\nu} = \langle\Psi_f|\hat{O}^{0\nu}|\Psi_i\rangle$ $\Rightarrow$ **Calculated theoretically**

Uncertainty of the NMEs

$$M^{0\nu} = \langle \Psi_f | \hat{O}^{0\nu} | \Psi_i \rangle$$



Yao, Meng, Niu, Ring, PPNP 126, 103965 (2022)

The predicted NMEs differ by a factor of more than three

- On the nuclear wavefunction side:
  - ✓ Limited model space
  - ✓ Missing correlations
- On the decay operator side:
  - ✓ Beyond closure approximation (10%)
  - ✓ High-order one-body currents (20%)
  - ✓ Two-body currents (10%)
  - ✓ **Contact decay operator (15% to 70%)**

Things we have done

- On nuclear wavefunction side:
  - ✓ Establishing a shell model approach based on the density functional theory ⇒ ReCD theory
  - ✓ Exploring the relationship between $0\nu\beta\beta$ decay and double Gamow-Teller transition

- On decay operator side:
  - ✓ Fully relativistic evaluations of $0\nu\beta\beta$ decay NMEs based on relativistic wavefunctions from ReCD theory and leading-order decay operator from the relativistic chiral EFT ⇒ Reducing uncertainty in the short-range NME

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ReCD theory

Density functional theory (DFT)
- ✓ Very large model space
- ✓ Suitable for almost all nuclei
- ✓ **Misses some correlations**
- ✓ **Overestimates the NMEs**

+

Configuration-interaction shell model (CISM)
- ✓ **Limited model space**
- ✓ Suitable for medium heavy nuclei
- ✓ Considers sufficient many-body correlations
- ✓ **Underestimates the NMEs**

= ReCD theory

1. A self-consistent relativistic DFT calculation

State $|\Phi_0\rangle$ with minimum energy in the PES

2. Construction of intrinsic configuration space

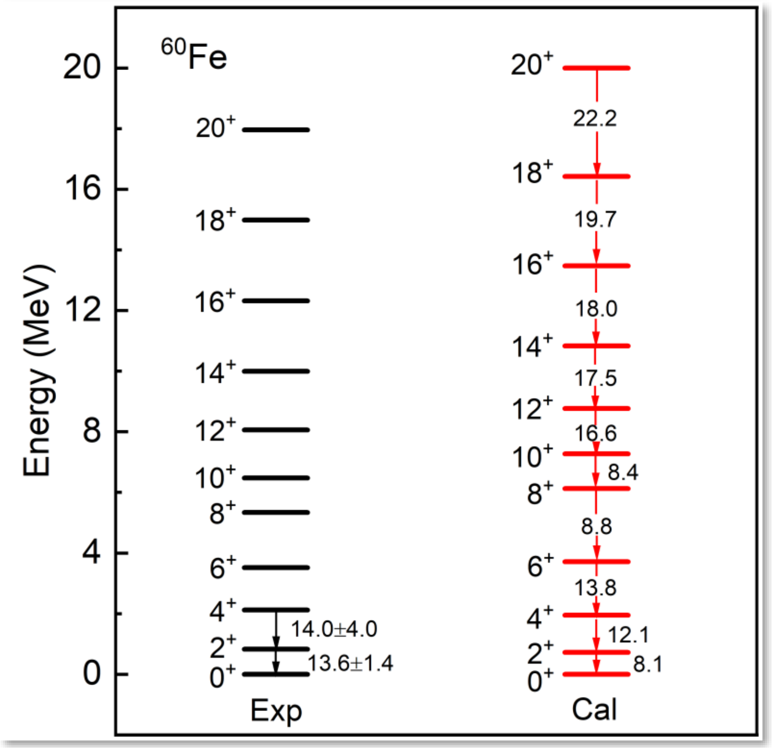
Quasiparticle states on top of $|\Phi_0\rangle$

3. Angular momentum projection

Restoration of rotational symmetry

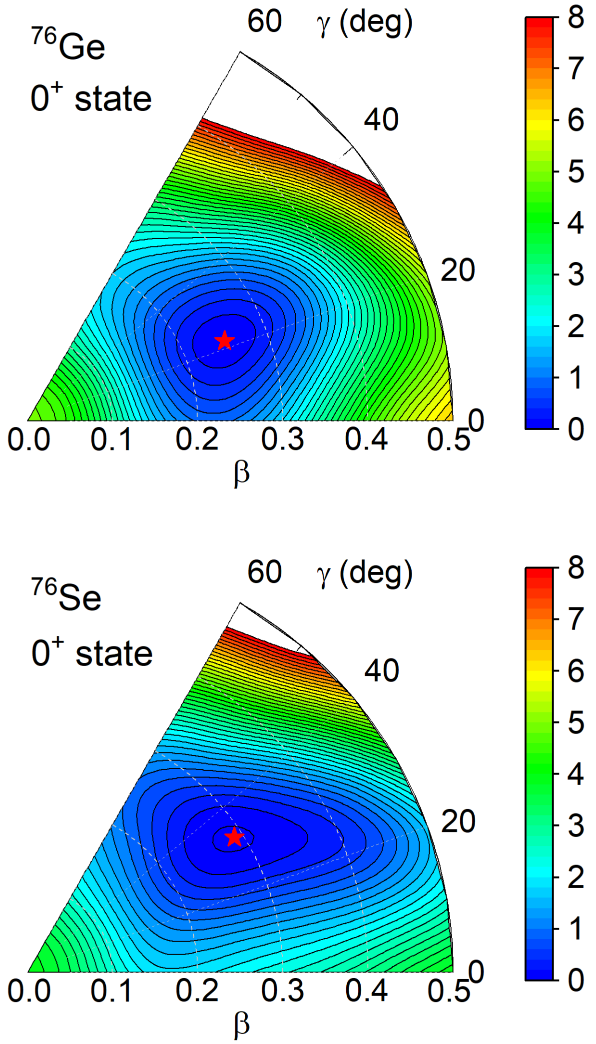
4. Shell model diagonalization

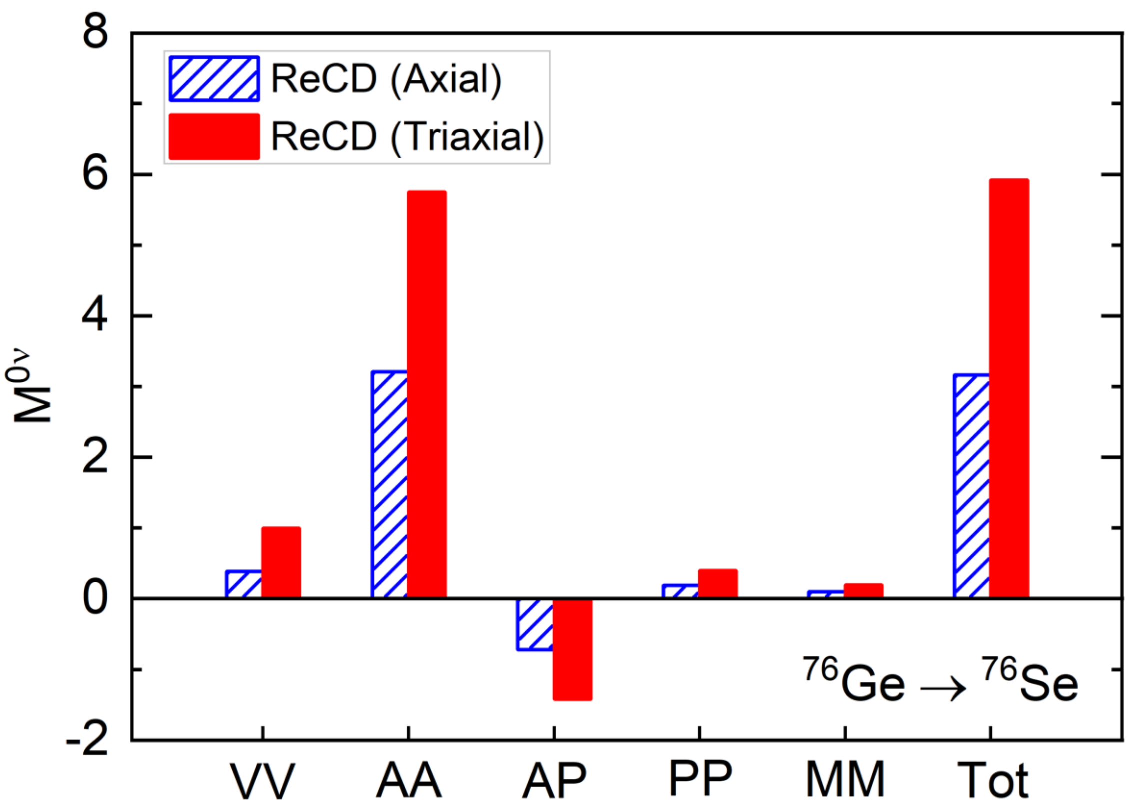
Configuration mixing or interaction based on DFT



P. W. Zhao, J. Meng, P. Ring, Phys. Rev. C 94, 041301(R) (2016); **Y. K. Wang**, P. W. Zhao, J. Meng, Phys. Rev. C 105, 054311 (2022)

Both axial and triaxial degrees of freedom are included in the ReCD theory





Y. K. Wang, P. W. Zhao, J. Meng, Science Bulletin 69, 2017-2020 (2024)

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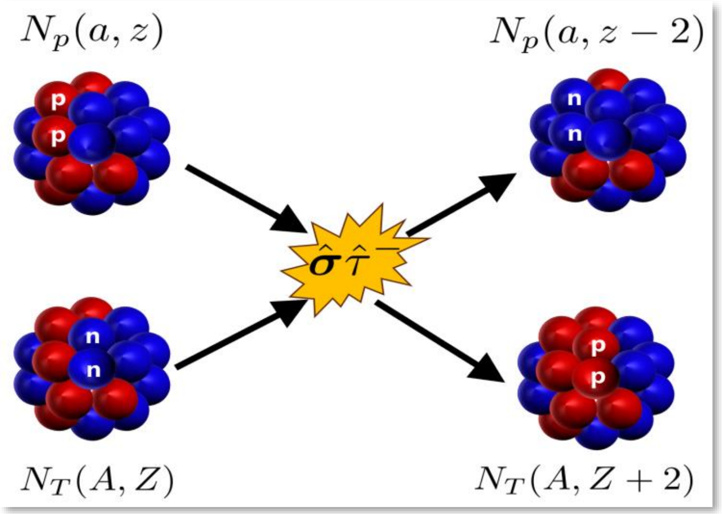
Why double Gamow-Teller (DGT) transition?

- DGT cross section can be factorized into three parts: reaction factor, DGT-transition NMEs of initial state and the final state

Santopinto et al., PRC 98, 061601(R) (2018)

$\Rightarrow$ Determining DGT-transition NMEs by the experimental cross section



$$N_T(A,Z)+N_p(a,z)\to N_T(A,Z+2)+N_p(a,z-2)$$

$0\nu\beta\beta$ decay: $\langle\Psi_F|\hat{O}^{0\nu}|\Psi_I\rangle$

$$\hat{O}^{0\nu}\propto h^{0\nu}(r_{12})\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\tau_1^+\tau_2^+$$

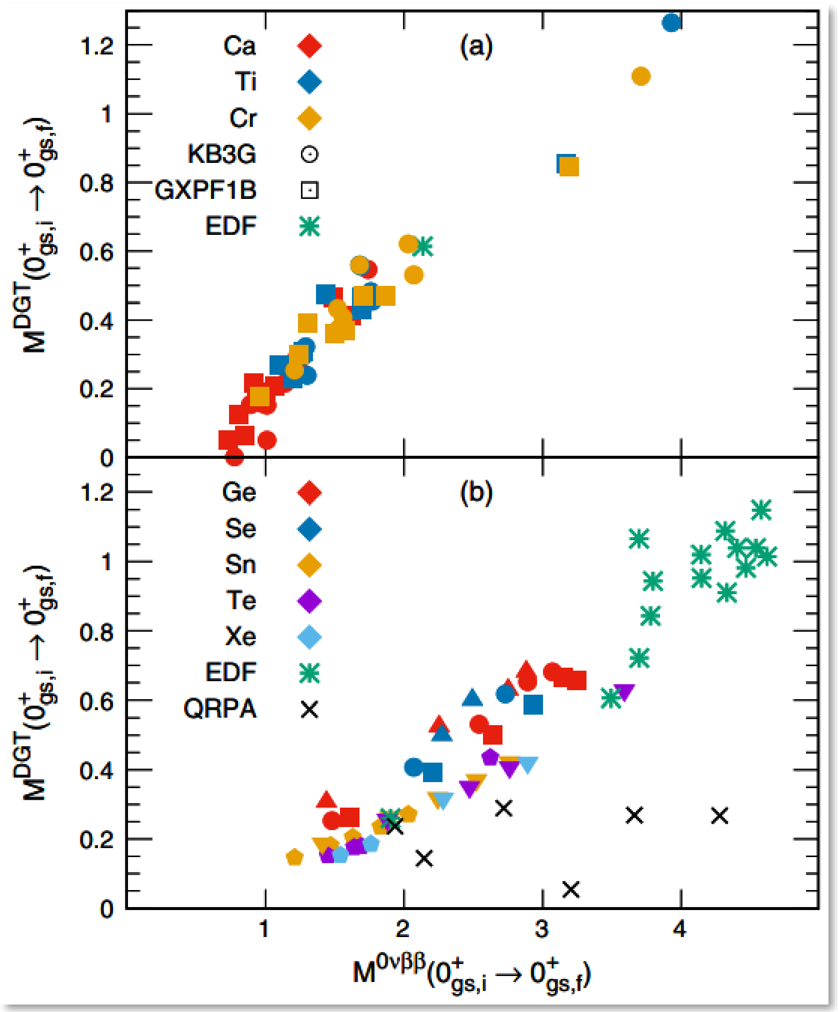
$\Longleftrightarrow$

DGT trans.: $\langle\Psi_F|\hat{O}^{\mathrm{DGT}}|\Psi_I\rangle$

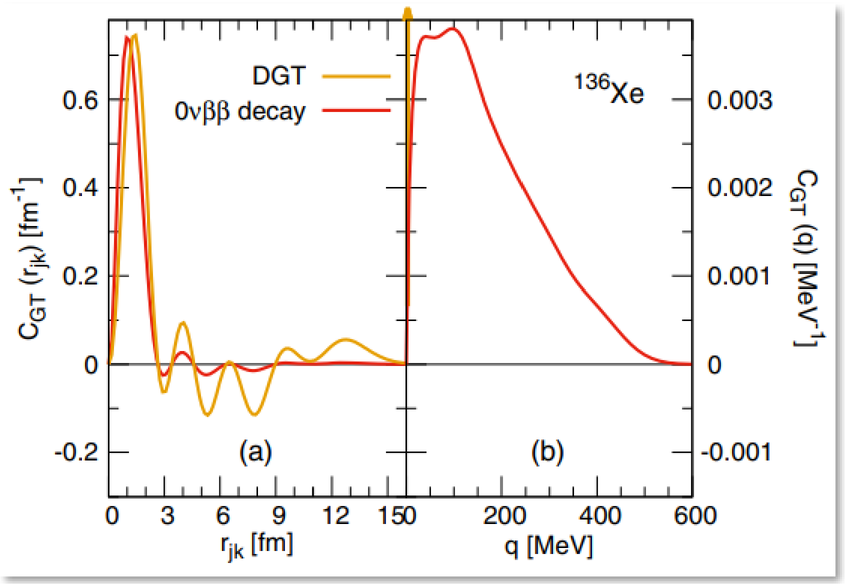
$$\hat{O}^{\mathrm{DGT}}\propto\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\tau_1^+\tau_2^+$$

Constraining the $0\nu\beta\beta$ decay NME by DGT transitions?

DGT transition and $0\nu\beta\beta$ decay in CISM



A good linear correlation is observed

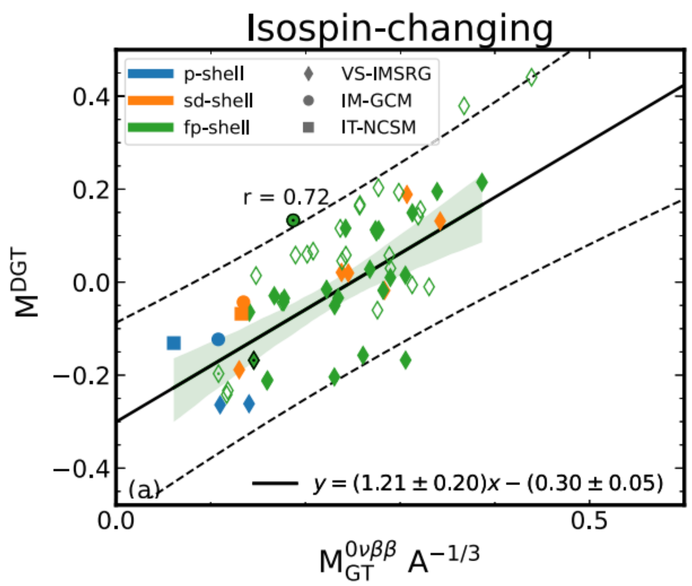


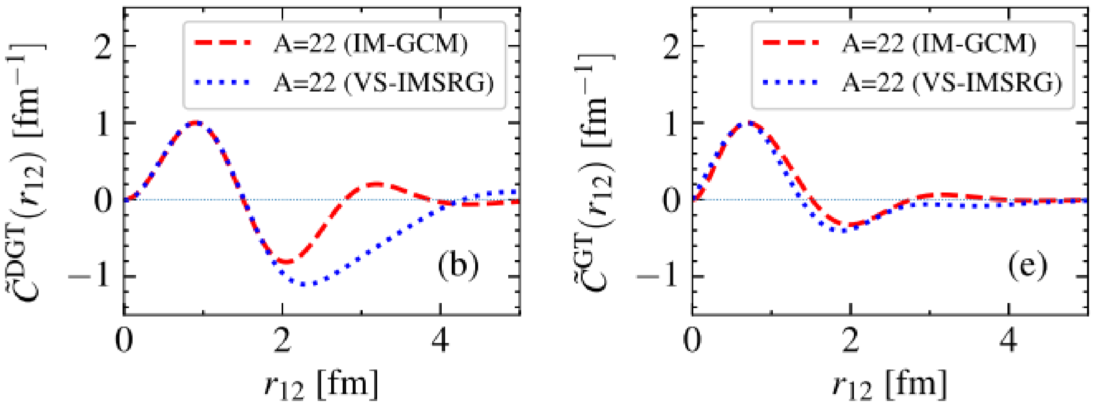
$$M^{\alpha} = \int dr_{12} C^{\alpha}(r_{12}), r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$$

The short-range character of both DGT and $0\nu\beta\beta$ decay matrix elements can explain the simple linear relation between them. References [72,73] showed that if an operator only probes the short-range physics of low-energy states, the corresponding matrix elements factorize into a universal operator-dependent constant times a state-dependent number common to all short-range operators.

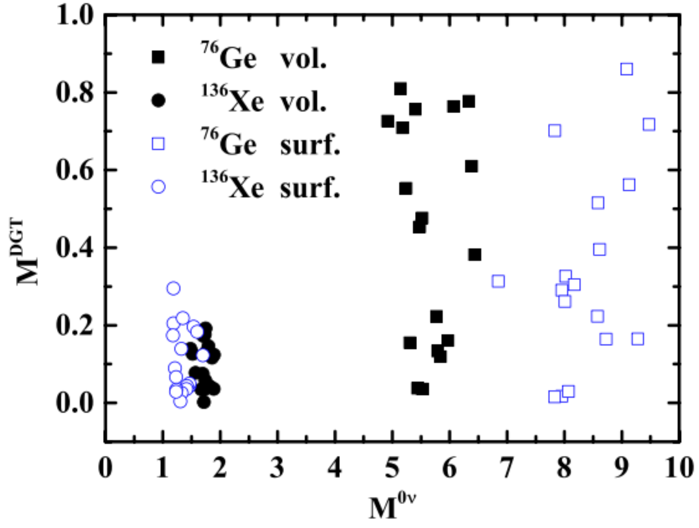
Shimizu, Menéndez, Yako, Phys. Rev. Lett. 120, 142502 (2018)

DGT transition and $0\nu\beta\beta$ decay in ab initio and QRPA methods





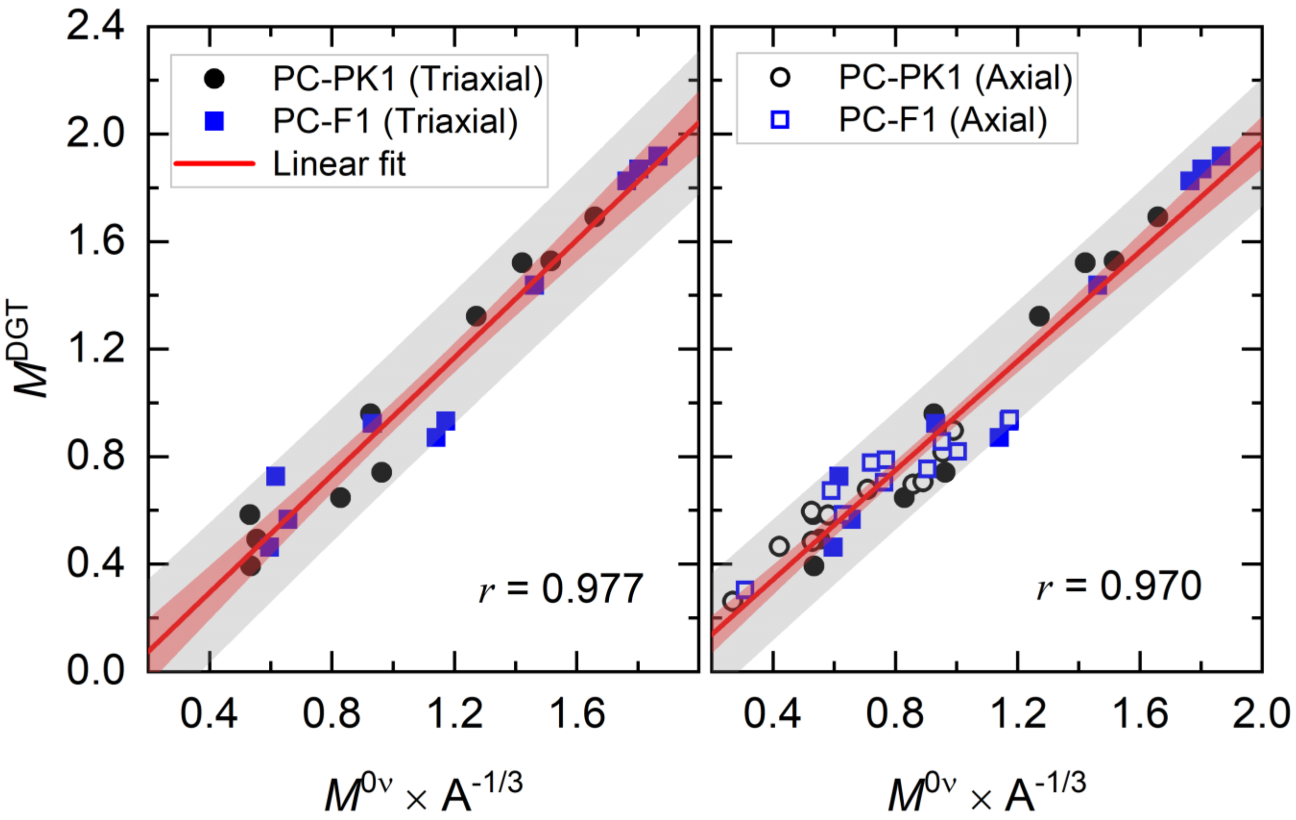
Yao et al., Phys. Rev. C 106, 014315 (2022)



Lv et al., Phys. Rev. C 108, L051304 (2023)

- The linear correlation between $0\nu\beta\beta$ decay and DGT transition is weaker in IMSRG and IMGCM
- No clear correlation appears QRPA calculations

$0\nu\beta\beta$ decay and DGT transition in ReCD theory



Ten nuclei:

^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te, ^{136}Xe

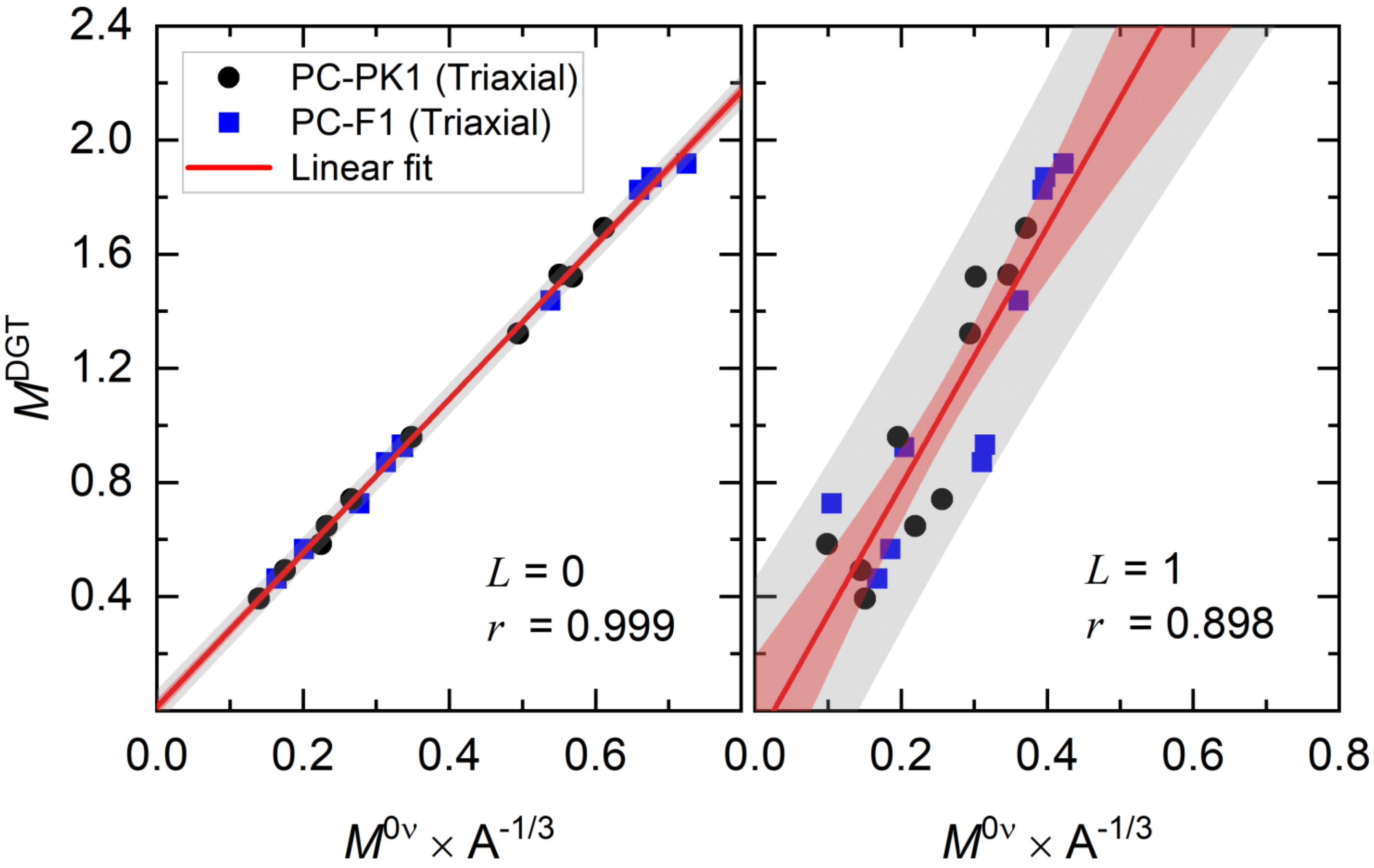
Axial and triaxial deformations

Full model space

- A strong linear correlation between $0\nu\beta\beta$ decay and DGT transition is demonstrated
- The linear correlation is robust against nuclear deformations

Y. K. Wang, P. W. Zhao, J. Meng, PLB 855, 138796 (2024)

Decomposition of the $0\nu\beta\beta$-decay NMEs



$$\hat{\mathcal{O}}^{0\nu} = \sum_L \hat{\mathcal{O}}^{0\nu}_L$$

$$M^{0\nu} = \sum_L M^{0\nu}_L$$

- The **leading order term** $M^{0\nu}_{L=0}$ correlated strongly with $M^{\rm DGT}$, while the correlation between $M^{0\nu}_{L=1}$ and $M^{\rm DGT}$ is much weaker

Y. K. Wang, P. W. Zhao, J. Meng, PLB 855, 138796 (2024)

Decomposition of the $0\nu\beta\beta$-decay NMEs

$$\mathcal{O}^{AA}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \int \frac{d^3q}{(2\pi)^3} H(q) e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}$$

$e^{\pm i\boldsymbol{q}\cdot\boldsymbol{r}}$ expanded by Y_{LM}

$$\mathcal{O}^{AA}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{2}{\pi}\int q^2 dq H(q) \sum_{LM} [j_L(qr_1)Y_{LM}(\hat{\boldsymbol{r}}_1)][j_L(qr_2)Y^*_{LM}(\hat{\boldsymbol{r}}_2)]$$

$L = 0$

$$\mathcal{O}^{AA}_{L=0}(r_1,r_2) = \frac{1}{2\pi^2}\int q^2 dq H(q) j_0(qr_1) j_0(qr_2)$$

Integrating over q

$$\mathcal{O}^{AA}_{L=0}(r_1,r_2) \approx \frac{1}{2}\left[X_1(r_1)Y_1(r_2) + X_1(r_2)Y_1(r_1)\right]$$

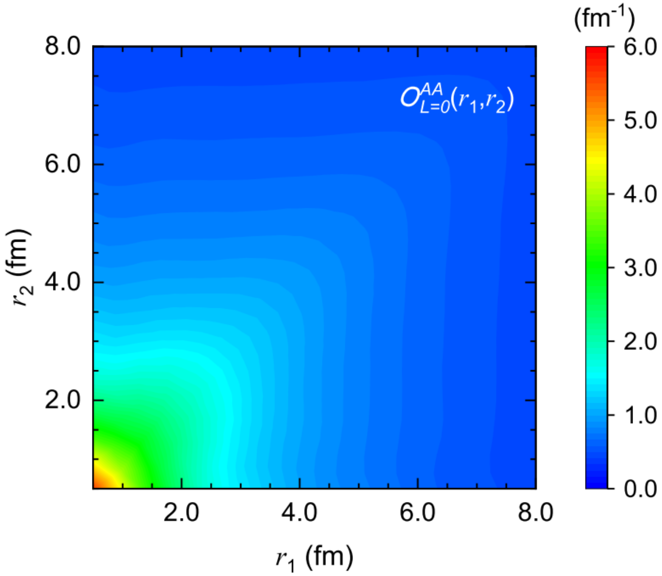
$|nljm\rangle$ basis

$$\langle 13|\mathcal{O}^{AA}_{L=0}(r_1,r_2)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2|24\rangle \approx \langle n_1l_1|X_1(r_1)|n_2l_2\rangle\langle n_3l_3|Y_1(r_2)|n_4l_4\rangle \times \langle j_1m_1|\boldsymbol{\sigma}_1|j_2m_2\rangle\langle j_3m_3|\boldsymbol{\sigma}_2|j_4m_4\rangle\delta_{n_1n_2}\delta_{n_3n_4}\delta_{l_1l_2}\delta_{l_3l_4}$$

$$\langle 13|\mathcal{O}^{\mathrm{DGT}}|24\rangle = -\frac{1}{\sqrt{3}}\langle j_1m_1|\boldsymbol{\sigma}_1|j_2m_2\rangle\langle j_3m_3|\boldsymbol{\sigma}_2|j_4m_4\rangle\delta_{n_1n_2}\delta_{n_3n_4}\delta_{l_1l_2}\delta_{l_3l_4}$$

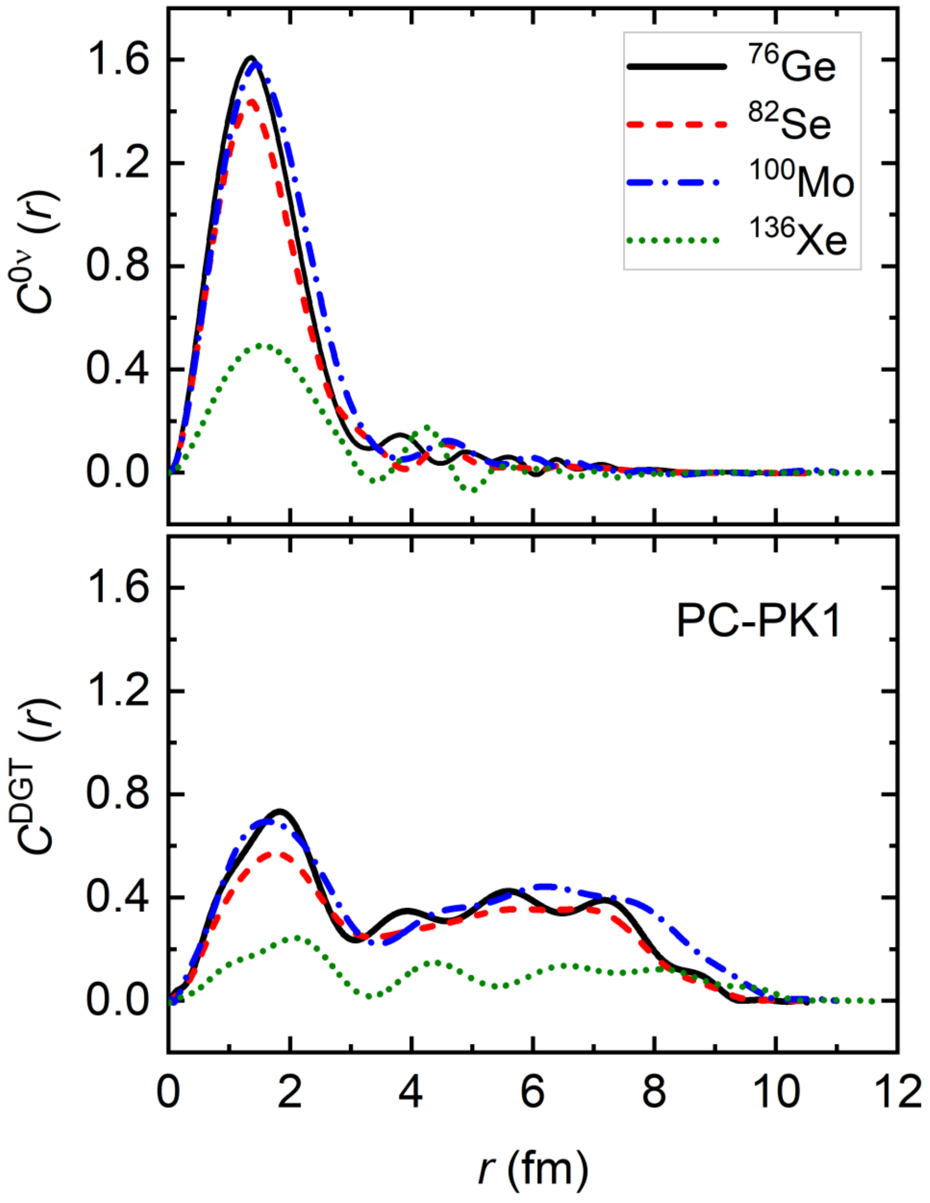
$$\sum_{1234}\langle 13|\mathcal{O}^{AA}(\boldsymbol{r}_1,\boldsymbol{r}_2)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2|24\rangle \hat{d}^\dagger_1\hat{d}^\dagger_3\hat{c}_4\hat{c}_2$$

Decay operator in axial-vector channel



Neutrino potential in (r_1, r_2) plane

Distribution of the NMEs



$$M^{\alpha} = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 C^{\alpha}(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

$\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2)$; $\boldsymbol{r} = (\boldsymbol{r}_1 - \boldsymbol{r}_2)$

$$M^{\alpha} = \int dr C^{\alpha}(r)$$

- $0\nu\beta\beta$-decay NMEs mainly distribute at the range of $r_{12} < 3$ fm $\Rightarrow$ short-range character
- The short-range character in not observed in the DGT-transition NMEs

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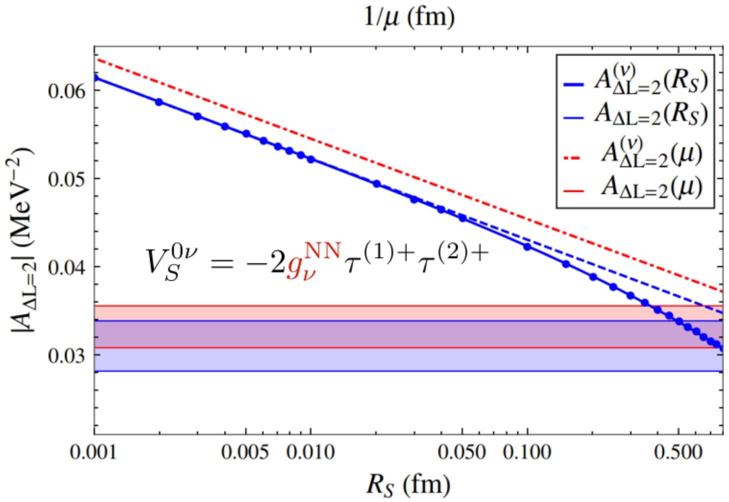
Leading-order decay operator derived from chiral EFT

Nonrelativistic chiral EFT



Cirigliano et al, PRL 120, 202001 (2018)

Ways to determine the g_ν^{NN}:

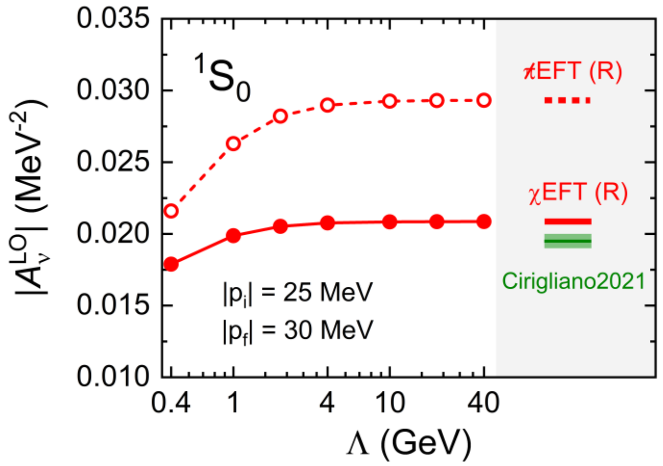
① Lattice QCD calculation

② Fitting the CIB coupling of nuclear force

③ Evaluated by generalized Cottingham model

Contribution from contact term varies from 15% to 70%

Relativistic chiral EFT



$$n(\boldsymbol{p}_i)n(-\boldsymbol{p}_i) \to p(\boldsymbol{p}'_f)p(-\boldsymbol{p}'_f)e(\boldsymbol{p}_e)e(-\boldsymbol{p}_e)$$

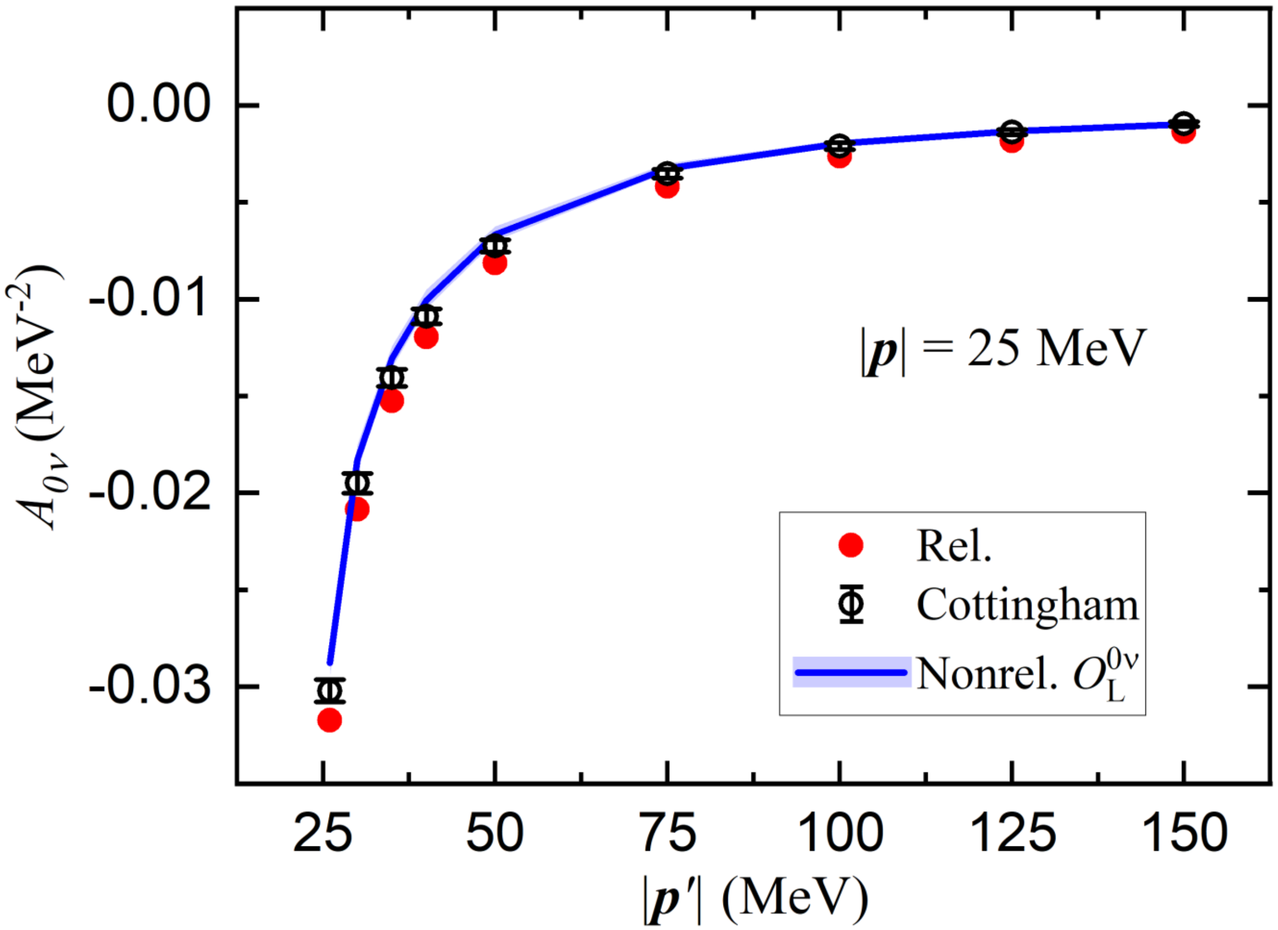
Y. L. Yang, P. W. Zhao, PLB 855, 138782 (2024)

- The obtained decay amplitude is slightly larger than that evaluated by the Cottingham model

Prediction of the NMEs for **experimentally relevant nuclei** based on the operator derived from the relativistic chiral EFT?

nn → ppee transition amplitudes

$n(\boldsymbol{p})n(-\boldsymbol{p}) \to p(\boldsymbol{p}')p(-\boldsymbol{p}')e(\boldsymbol{p}_e)e(-\boldsymbol{p}_e)$



Y. K. Wang, Y. L. Yang, P. W. Zhao, *in preparation*

- The leading-order chiral NN potential is regulated by $\exp(-\boldsymbol{q}^2/\Lambda)$
- The uncertainty in nonreal. results comes from the momentum cutoff in the range $\Lambda = 400$-600 MeV
- The short-range coupling g_ν^{NN} is determined by matching the nonrelativistic long-range amplitude to the targeted values from the Cottingham formula
- The difference between the relativistic and nonrelativistic long-range amplitude is about 10%

Low-energy coupling g_ν^{NN}

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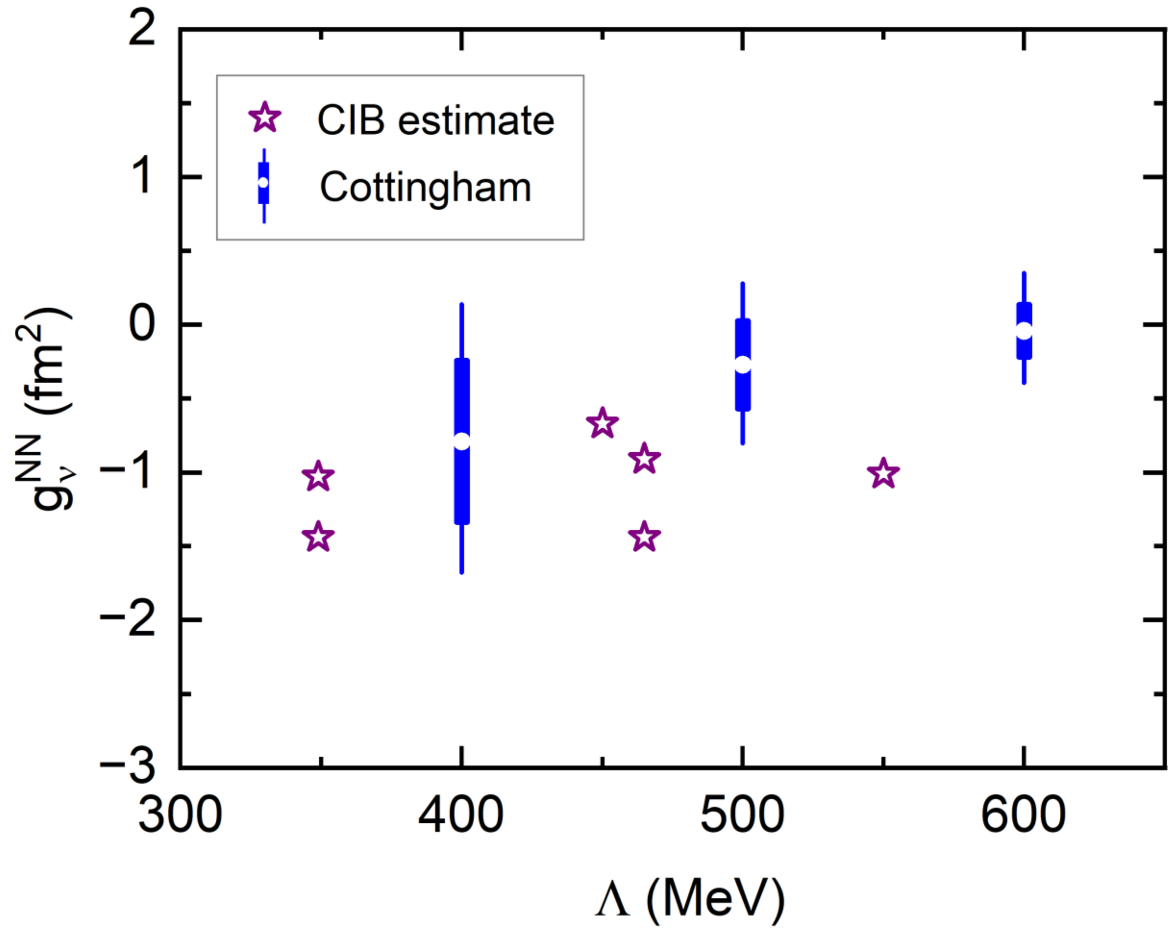
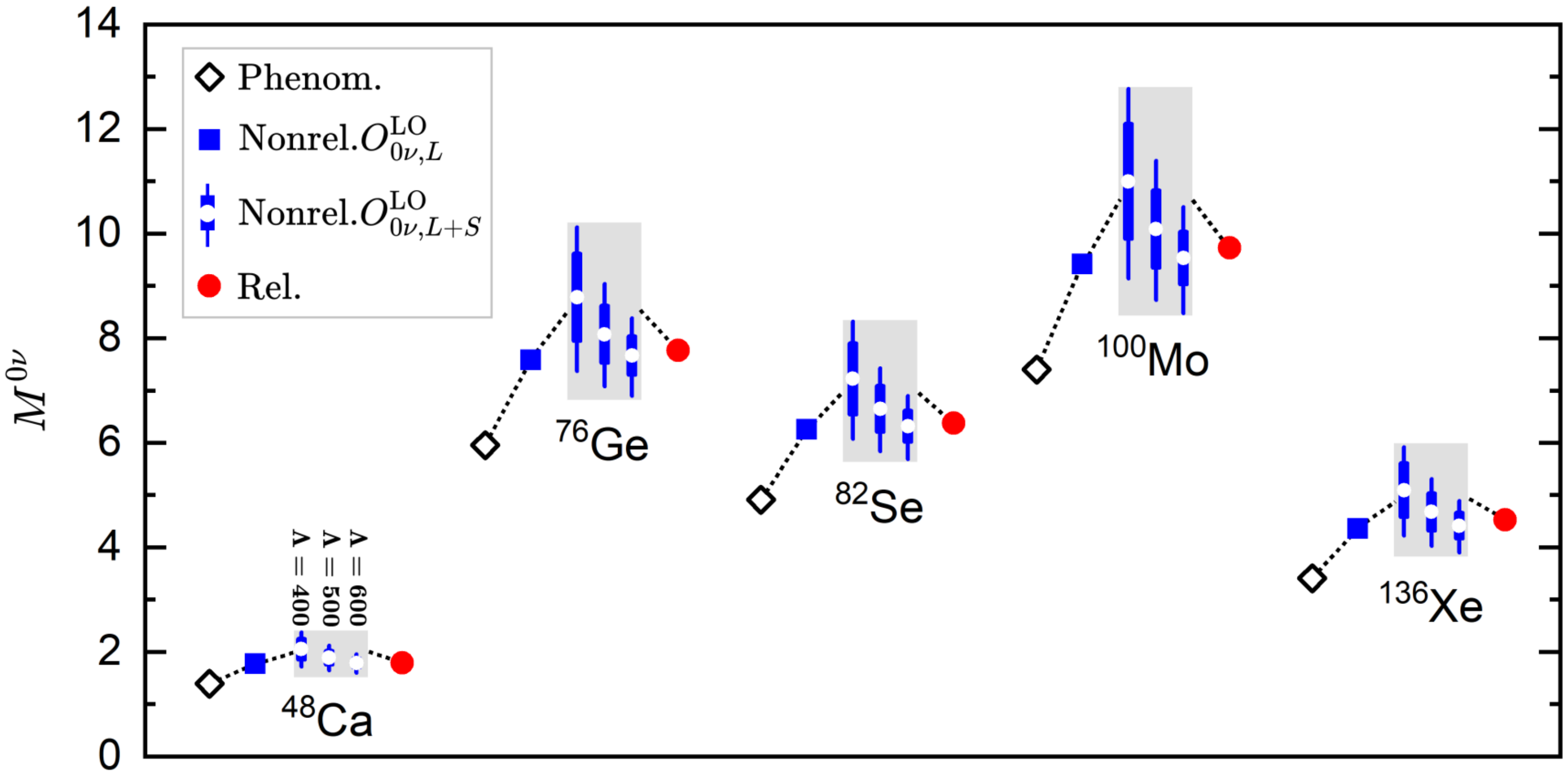


- Two sources of uncertainties: systematic error of the Cottingham formula, and the different choices of the kinematics $\boldsymbol{p}'$
- Our g_ν^{NN} values are generally consistent with the CIB estimates
- The uncertainties of g_ν^{NN} are quite large, even leading opposite signs

The CIB estimates from: Jokiniemi, Soriano, Menendez, PLB 823, 136720 (2021)

NMEs in experimentally relevant nuclei

ReCD theory ⇒ relativistic wavefunctions, relativistic EFT ⇒ renormalized leading-order operator



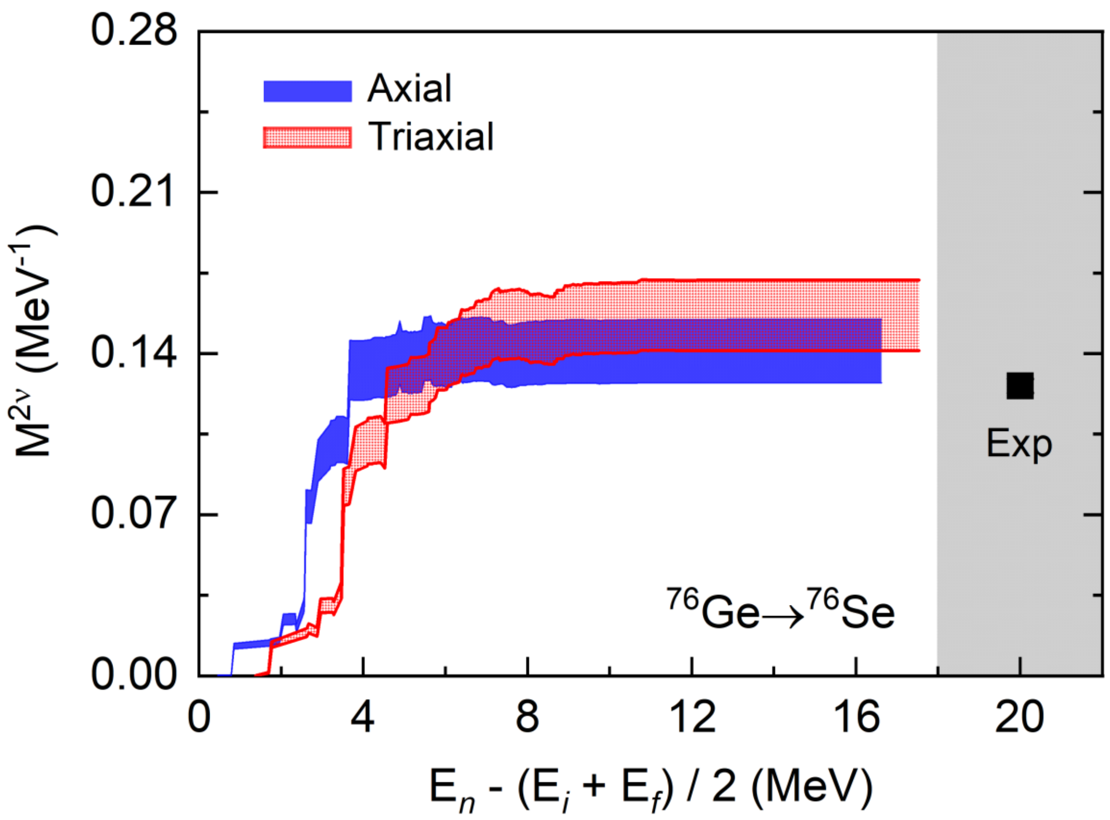
- The nonrelativistic NMEs exhibit large uncertainties
- The short-range term accounts for -10% to 35% of the NMEs from the long-range part
- Our relativistic NMEs lie within the uncertainty interval of nonrelativistic NMEs

Y. K. Wang, Y. L. Yang, P. W. Zhao, in preparation

Summary

- The Relativistic Configuration-interaction Density functional theory is established to evaluate the $0\nu\beta\beta$-decay NMEs:
  - ✓ Nuclear triaxiality plays important roles on $0\nu\beta\beta$ decay in ^{76}Ge
  - ✓ Double Gamow-Teller transition might be used to constrain the NMEs of $0\nu\beta\beta$ decay
  - ✓ Relativistic evaluation of the NEMs for experimentally relevant nuclei is performed based on the renormalized relativistic decay operator

Two-neutrino double beta decay



- All possible **odd-odd intermediate 1^+ states** needs to be considered (around 200 1^+ states are included).
- The two-body currents are not considered ⇒ quenching factor q ranging from **0.68 to 0.77 are adopted** in our calculations.

Y. K. Wang, P. W. Zhao, J. Meng, Science Bulletin 69, 2017-2020 (2024)