Effects of deformation and $np$-pairing correlations on the Gamow-Teller transition for $sd$-shell $N=Z$ nuclei by a deformed QRPA(DQRPA)

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- Formalism & Method
  : Deformed QRPA (quasi-particle random phase approximation) with np pairing correlations
- Results : GT transition of $^{24}\text{Mg}$, $^{28}\text{Si}$, and $^{32}\text{S}$
- Summary
Are the np-pairing correlations restricted only to the vicinity of the $N = Z$ nuclei?

The np-pairing correlations are important for $N \approx Z$ nuclei: protons and neutrons occupy identical orbitals and have maximal spatial overlap.
How about np-pairing correlations for light $N=Z$ nuclei such as $^{24}\text{Mg}$, $^{28}\text{Si}$, and $^{32}\text{S}$?

- The np-pairing correlations have been only taken into account at BCS process for Ge isotope.
  
  E. Ha et al. PRC 92,044315(2015)

- This work is a test of the DQRPA including the np-pairing on GT transition.

- GT strength distributions are shown to be sensitive to the nuclear shape.
  
  E. Ha et al. NPA 934,73(2015)
  E. Ha et al. JKPS 67,1142(2015)
  E. Ha et al. PRC 88,017603(2013)

- We investigate the effects of np-pairing correlations and deformation on the GT transition of $^{24}\text{Mg}$, $^{28}\text{Si}$, and $^{32}\text{S}$.

I. Deformation and $np$ pairing effects on the Gamow-Teller transitions for $^2\text{Mg}$, $^4\text{Mg}$, and $^{26}\text{Mg}$ in the deformed QRPA (submitted in PRC)

II. Effects of deformation and $np$ pairing correlations on the Gamow-Teller transition for $N=Z$ nuclei, $^{24}\text{Mg}$, $^{28}\text{Si}$, and $^{32}\text{S}$, by a deformed QRPA (submitted in PLB)
How to include the deformation

Deformed Woods-Saxon (WS) potential
(cylindrical WS, Damgaard et al. 1969)

\[ V(\ell) = \frac{-V_0}{1 + \exp\left(\frac{\ell}{a}\right)}, \quad V_{so} = -\lambda(\hbar/2mc)^2 \text{grad} V(\ell)(\vec{\sigma} \times \vec{p}) \]

\[ \ell(u, v; \beta_2, \beta_4) = \frac{\text{CS}(u, v)}{\left|\nabla_{u,v} S(u, v)\right|}, \quad z = Cu, \quad \rho = Cv \]

- \( \beta_2 \): quadrupole deformation parameter
- \( \beta_4 \): hexadecapole deformation parameter

- We can determine these two parameters by taking values giving the minimum ground state energy.

- To exploit G-matrix elements, which is calculated on the spherical basis, deformed bases are expanded in terms of the spherical bases.

\[ |\alpha \Omega_\alpha \rangle = \sum_a B_{a}^{\alpha} |a \Omega_\alpha \rangle, \]

Deformed SPS

Sph. HO w. f.
The different total angular momenta of the SP basis states would be mixed since the deformed SPS are expanded in terms of the spherical SP bases.
The progress of QRPA for GT transitions

- **Motivations**: Single particle state
- **Results**: Bardeen Cooper Schrieffer
- **Summary**: G-matrix (Bonn potential)

### Formalism

1. **SPS (WS)**
   - BCS (nn+pp)
   - pn-QRPA

2. **SPS (WS)**
   - BCS (nn+pp+np)
   - QRPA

3. **DSPS (DWS)**
   - DBCS (nn+pp)
   - pn-DQRPA

4. **DSPS (DWS)**
   - DBCS (nn+pp+pn+np)
   - DQRPA

**Parameter free !!!**
RPA eq. for charge exchange reaction

\[
\begin{pmatrix}
A_{\alpha\beta\gamma\delta}^{\text{npnp}}(K) & B_{\alpha\beta\gamma\delta}^{\text{npnp}}(K) \\
-A_{\alpha\beta\gamma\delta}^{\text{npnp}}(K) & -A_{\alpha\beta\gamma\delta}^{\text{npnp}}(K)
\end{pmatrix}
\begin{pmatrix}
\tilde{X}_m(\gamma p\delta n)K \\
\tilde{Y}_m(\gamma p\delta n)K
\end{pmatrix}
= \hbar \Omega_K^m
\begin{pmatrix}
\tilde{X}_m(\gamma p\delta n)K \\
\tilde{Y}_m(\gamma p\delta n)K
\end{pmatrix}
\]

: QRPA eq. w/o np-pairing

: DQRPA eq. with np-pairing

1, 2 = quasi particle
The ground state energy of $^{24}\text{Mg}$ obtained from WS+BCS

- Our $\beta_2$ values are quite consistent with the values extracted from E2 transition.

- $E_{\text{MF'}}$: mean field $E$ in quasiparticle

- $E_{\text{pair}}$: pairing $\beta_2$ in quasiparticle

\[ E_{\text{pair}} = \frac{\beta_2^2}{3R_0^2} \left[ \frac{\pi}{2} - \frac{1}{2} \right] \]

\[ (R_0 = 1.2A^{1/3}) \]

- Our $\beta_2$ values are quite consistent with the values extracted from E2 transition.

\[ E_{\text{MF'}} = -23.06 \text{MeV at } \beta_2 = 0.5, \beta_4 = 0.05 \]

\[ \beta_4 \text{ deformation turns out to contribute enough to be neglected in the GT strength distribution.} \]

- The pairing energy becomes larger at $\beta_2 = 0$ than oblate and prolate shape. This stems from the scattered SPS by the deformation which weakens the overlap of nucleon wave function for the pairing.
Empirical pairing gaps and deformation parameters

<table>
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<tr>
<th>Nucleus</th>
<th>$\beta_2^{Ours}$</th>
<th>$\beta_2^{E2}$</th>
<th>$\Delta_p^{emp}$</th>
<th>$\Delta_n^{emp}$</th>
<th>$\delta_{np}^{emp}$</th>
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<tr>
<td>$^{24}\text{Mg}$</td>
<td>0.5</td>
<td>0.605</td>
<td>3.123</td>
<td>3.193</td>
<td>1.844</td>
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<td>$^{28}\text{Si}$</td>
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<td>0.482</td>
<td>2.314</td>
<td>1.896</td>
<td>0.143</td>
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<tr>
<td>$^{32}\text{S}$</td>
<td>0.3</td>
<td>0.312</td>
<td>2.141</td>
<td>2.207</td>
<td>1.047</td>
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</table>

- Deformed WS parameter: optimal param.
- Particle model space: $N_{max} = 5$ (deformed basis) $N_{max} = 10$ (spherical basis)
The open shell (0d5/2 state) structure of $^{24}\text{Mg}$ in a spherical shape changes to a closed shell structure and repulsive particle-hole ($p-h$) interaction in the GT transition plays meaningful roles around Fermi surface.
The GT strength distribution is rearranged and fragmented due to deformation effect. The repulsive $p-h$ interaction shifts the GT excitation to high-lying states. It comes from the modified smearing of the Fermi surface due to deformation effect.

- The np pairing shifts low-lying GT excitation to a bit higher energy region and predicts some high-lying GT states around 27MeV.

- The GT strength distribution is rearranged and fragmented due to deformation effect. The repulsive $p-h$ interaction shifts the GT excitation to high-lying states.
Main contributions for the high-lying GT state at $E_{ex} = 27.87$ MeV.

\[
< K^+, m | \hat{\beta}_K^- | QRPA > \\
= \sum_{\alpha'' \rho \alpha \beta'' \rho} N_{\alpha'' \rho \alpha \beta'' \rho} \langle \alpha' \rho \alpha | \sigma_K | \beta'' \rho \beta > [u_{\alpha'' \rho} \nu_{\beta'' \rho} X_{(\alpha'' \rho \beta')K} + u_{\alpha'' \rho} \nu_{\beta'' \rho} Y_{(\alpha'' \rho \beta')K}]
\]

| configuration [asymptotic q.n.] | X | $| < p | \sigma_K | n > |$ | $u_p \nu_n$ | total |
|----------------------------------|----|-----------------|-----------------|------------|--------|
| $(\pi 3/2^-_1, \nu 1/2^-_1), [\pi 101 3/2, \nu 101 1/2]$ | 0.635 | 1.390 | 0.05 | 0.044 |
| $(\pi 3/2^+_2, \nu 5/2^+_1), [\pi 202 3/2, \nu 202 5/2]$ | 0.212 | 1.397 | 0.10 | 0.029 |
| $(\pi 1/2^-_1, \nu 3/2^-_1), [\pi 101 1/2, \nu 101 3/2]$ | 0.630 | 1.395 | 0.01 | 0.008 |

The largest contribution comes from $\pi 3/2^-_1$ (comes from $0p3/2$) state and $\nu 1/2^-_1$ (comes from $0p3/2$) state, which is located below the Fermi surface.
SPSE of neutron for $^{24}$Mg
Main contributions for the high-lying GT state at $E_{ex} = 27.87$ MeV.

- $u_p v_n$ is small but the $\lambda$ value becomes larger due to the overlap of wave function with np-pairing.
- The possibility of the **high-lying GT excitations** in the Fermi sea is increased enough to appear themselves in the GT spectrum.

\[ E_{ex} \propto \sqrt{(\epsilon_p - \lambda_p)^2 + \Delta_p^2} + \sqrt{(\epsilon_n - \lambda_n)^2 + \Delta_n^2} + \cdots \]
The closed shell (1s1/2 state) structure of $^{28}\text{Si}$ in a spherical shape changes to an open shell structure, which leads to a mixture of attractive particle-particle (p−p) and repulsive particle-hole (p−h) interaction.
If we consider np-pairing there is the high excited GT peak around 31MeV.

The open shell structure leads to a mixture of attractive p−p interaction and repulsive p−h interaction.

These mixings scatter the spectrum more widely than those by the spheroidal case.

This fact is easily confirmed by the experimental data.
The closed shell of $^{32}\text{S}$ becomes to another closed shell and the repulsive interaction scatters to some extent the GT excitation to high-lying excitations.
**GT strength for $^{32}$S**

**PRC 43, 50(1991)**

- $np$-pairing shifts low-lying GT excitation to a bit higher energy region and predicts some high-lying GT states. But the high-lying GT excitations are very weak compared to those for $^{24}$Mg and $^{28}$Si.

- Deformation effects scatter reasonably a peak strength to lower and higher excitations.
Summary

1. We included the np-pairing as well as the nn & pp-pairing correlations to the DQRPA and this formalism was applied to GT transition of \( N=Z \) nuclei.

2. The deformation effect turned out to be much larger than the np-pairing effect for \(^{24}\text{Mg}^{28}\text{Si}, \text{and}^{32}\text{S}\) nuclei.

3. This phenomena are also inferred from the shell evolution and the modified smearing of the Fermi surface due to the \( \beta_2 \) deformation effect.

4. Attractive force property of the np pairing correlations decreases Fermi energy gaps of neutrons and protons around Fermi surface and this mechanism shifts a bit the main GT peaks to some higher energy region by the repulsive p\(-\)h interaction.

5. np-pairing gives rise to some high-lying GT excited states by the increase of overlapped wave functions of neutrons and protons.

6. GT transitions of medium heavy nuclei are in progress.

7. We are doing research on the correlations between the deformation and np-pairing effect.
Thanks for your attention!!
Set of parameter values defined by the program according to the input value of the ICHOIC variable. The symbols P (N) refer to the protons (neutrons). The $\lambda$ values in the case of the Chepurnov parametrisation are defined by $\lambda = 23.8 \times (1 + 2 \times (N - Z)/A)$. Blomqv.-Wahlb. stands for Blomqvist and Wahlborn. The values of $r_0$ and $a$ are in fermi, $V_0$ in MeV, $\kappa$ and $\lambda$ dimensionless.

<table>
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<th>Parametrisation</th>
<th>$\lambda$ (P)</th>
<th>$\lambda$ (N)</th>
<th>$r_{0-so}$ (P)</th>
<th>$r_{0-so}$ (N)</th>
<th>$r_0$ (P)</th>
<th>$r_0$ (N)</th>
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<th>$V_0$</th>
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</table>
✓ $^{24,26}\text{Mg}$ are well known deformed nuclei.
GT strength for $^{24}\text{Mg}$ : SM cal.
**RPA (random phase approximation)**

RPA takes into account the **ground state correlations**, which is including particle-hole excitation in the ground state.

$$\langle QRPA | A, A^+ | QRPA \rangle \approx \langle BCS | A, A^+ | BCS \rangle$$
Used parameters in this work.

* $N_{\text{max}} = 5$: radial quantum number

* WS input parameters: Roland Nojarov param. by Y. Tanaka et al.

* Pairing gap: five-term mass formula

* $g_{pp}(g_{ph}) = 0.99(1.15)$ particle – particle(particle – hole) int. strength
Two-body interaction

Realistic two body interaction inside nuclei was taken by Brueckner g-matrix, which is a solution of the Bethe-Salpeter Eq., derived from the Bonn-CD potential for nucleon-nucleon interaction in free space.

\[
g(\omega)_{ab,cd} = V_{ab,cd} + V_{ab,cd} \frac{Q_p}{\omega - H_0} g(\omega)_{ab,cd}
\]

\(a,b,c,d\) : single particle states from the Woods-Saxon potential.
\(V_{ab,cd}\) : phenomenological nucleon-nucleon potential in free space.
**Gamow-Teller (GT) Transition**

$\Delta T=1$, $\Delta S=1$, $\Delta L=0$, $\Delta J=1^+$ ($K=0,1,-1$)

The GT($\mp$) strength functions & total strengths

$$B_{GT}^- (m) = \sum_{K=0, \pm 1} | < 1(K), m | \beta_K^- ||_{Q R P A} |^2,$$

$$B_{GT}^+ (m) = \sum_{K=0, \pm 1} | < 1(K), m | \beta_K^+ ||_{Q R P A} |^2,$$

$$S_{GT}^- = \sum_{K=0, \pm 1} \sum_m | < 1(K), m | \beta_K^- ||_{Q R P A} |^2,$$

$$S_{GT}^+ = \sum_{K=0, \pm 1} \sum_m | < 1(K), m | \beta_K^+ ||_{Q R P A} |^2.$$

**Half-life of $\beta$-decay**

Half-life of $\beta$ decay is

$$t_{1/2} = \frac{\kappa}{f_0 B(GT)},$$

where $\kappa = 6147s$, $f_0^\mp = \int_1^{E_0} F_0 (\pm Z_f, \epsilon) p \epsilon (E_0 - \epsilon)^2 d\epsilon$ (phase-space factor)

We calculate the half-life of allowed $\beta$-decay in even-even nuclei.