Gauge-Higgs unification

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Standard Model | **Gauge-Higgs Unification**

$$\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} \begin{cases} \mathcal{L}^{\text{kin}}_{\text{Higgs}} \\ V^{\text{pot}}_{\text{Higgs}} \end{cases}$$

$$\left.\mathcal{L}_{\text{gauge}},\ \mathcal{L}^{\text{kin}}_{\text{Higgs}}\right\} \rightarrow \mathcal{L}^{5d}_{\text{gauge}}$$

$$+$$

$$\left.\mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}}\right\} \rightarrow \mathcal{L}^{5d}_{\text{fermion}}$$

$$\mathcal{L}^{5d}_{\text{gauge}} \Downarrow \quad V_{\text{eff}}(\Theta_H) \quad \Uparrow \mathcal{L}^{5d}_{\text{fermion}}$$

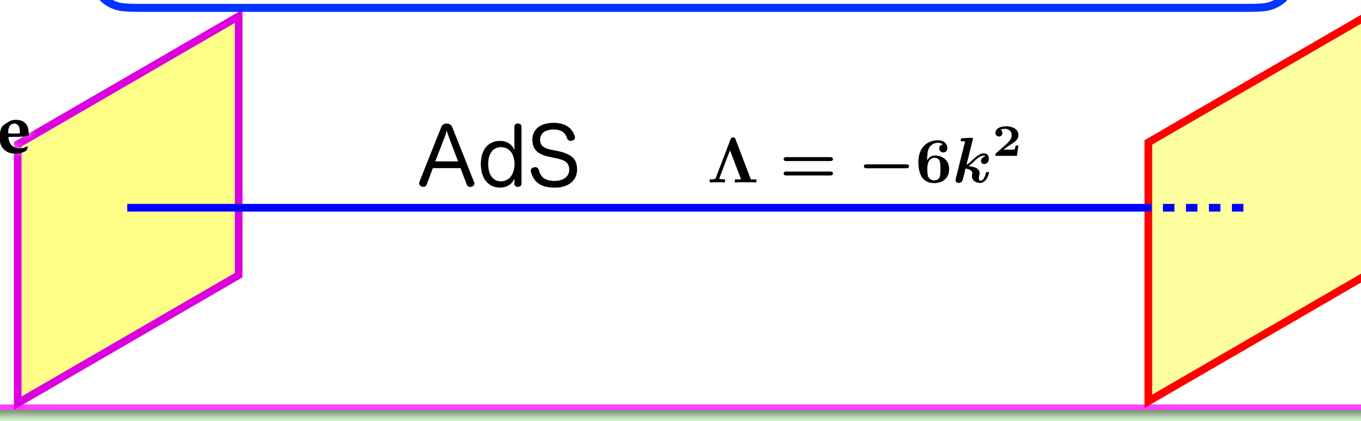
gauge principle

GUT inspired GHU in RS space

$$SO(5) \times U(1)_X \times SU(3)_C$$

UV brane

$$P_0 = \begin{pmatrix} -I_4 & \\ & 1 \end{pmatrix}$$



IR brane

$$P_1 = \begin{pmatrix} -I_4 & \\ & 1 \end{pmatrix}$$

$$\longrightarrow SO(4) \times U(1)_X \times SU(3)_C$$

$$\langle \hat{\Phi} \rangle \quad \longrightarrow SU(2)_L \times U(1)_Y \times SU(3)_C$$

AB phase θ_H

$$\longrightarrow U(1)_{\rm EM} \times SU(3)_C$$

$$P \exp \left\{ ig \oint dy \, A_y \right\} \sim e^{i\Theta_H(x)}$$

$$\Theta_H(x) = \theta_H + \frac{H(x)}{f_H}$$

Dynamical gauge sym breaking $V_{\rm eff}(\theta_H)$

Finite m_H generated at 1-loop

Solves gauge hierarchy problem.

W boson mass m_W

Flavor problems

- CKM matrix
- FCNC suppression
  - Neutrino oscillations
  - PMNS matrix

Y. Hosotani, "*An Introduction to Gauge-Higgs Unification*"

ISBN 978-981-98-0097-1, World Scientific Pub. (2025)

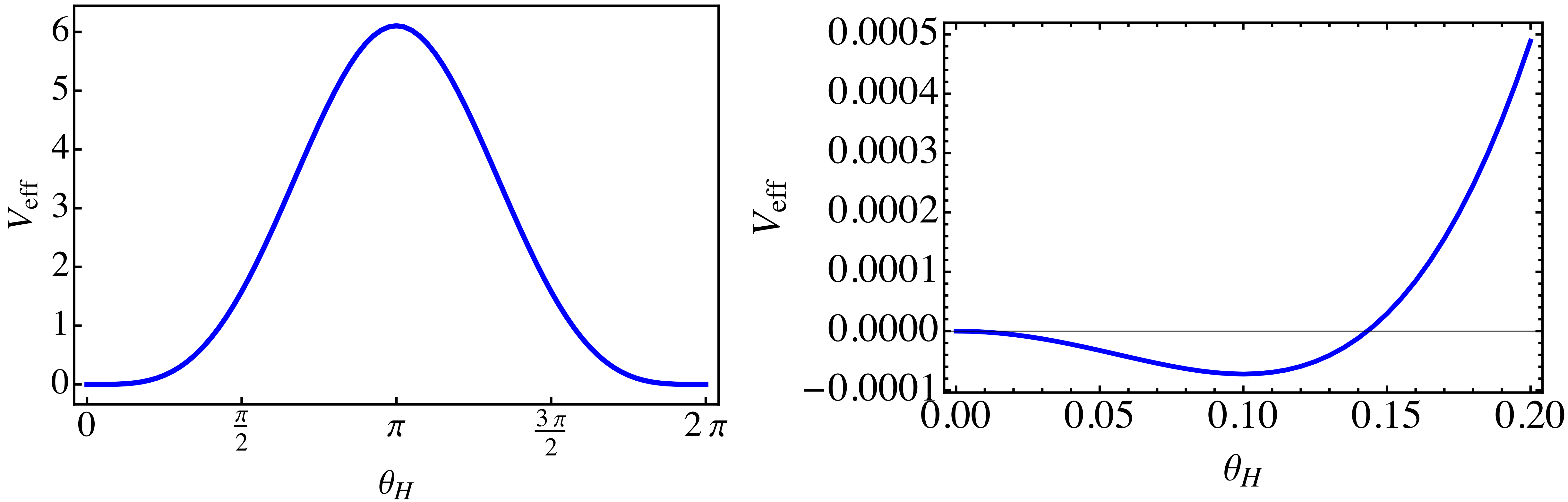
Matter

$\begin{pmatrix} u \\ d \\ u' \\ d' \end{pmatrix}$ $\begin{pmatrix} \nu \\ e \\ \nu' \\ e' \end{pmatrix}$

	B-model (GUT inspired)
Quark	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}$ $(\mathbf{3},\mathbf{1})^{+}_{-\frac{1}{3}}$ $(\mathbf{3},\mathbf{1})^{-}_{-\frac{1}{3}}$ $D^{\pm}$
Lepton	$(\mathbf{1},\mathbf{4})_{-\frac{1}{2}}$
Dark fermion	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}$ $(\mathbf{1},\mathbf{5})^{+}_{0}$ $(\mathbf{1},\mathbf{5})^{-}_{0}$ $(\mathbf{1},\mathbf{4})_{-\frac{1}{2}}$
Brane fermion	$(\mathbf{1},\mathbf{1})_{0}$ $\hat{\chi}$
Brane scalar	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$ $\hat{\Phi}$

$SU(3)_C \times SO(5) \times U(1)_X$ content

$$\frac{V_{\rm eff}(\theta_H)}{m_{\rm KK}^4/16\pi^6}$$



$$\theta_H^{\rm min} = 0.1 \ , \ m_{\rm KK} = 13\,{\rm TeV} \ , \ m_H = 125.1\,{\rm GeV}$$

EW sym breaking takes place.

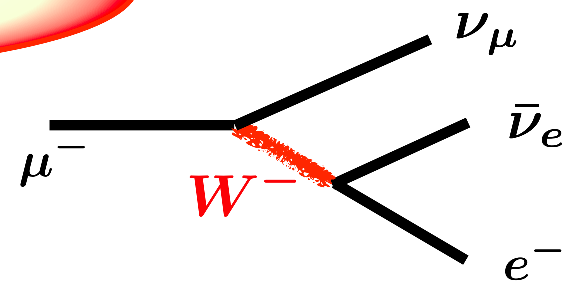
m_W

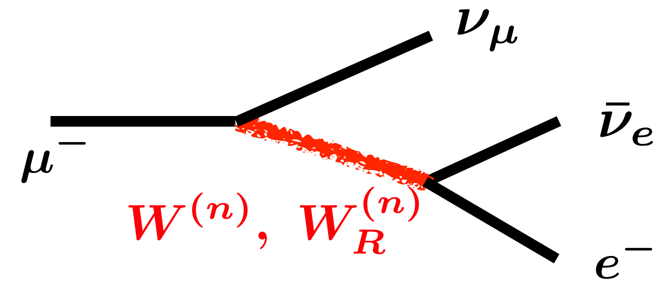
$$e = g_w \sin\theta_W^{\rm shell}$$

$$2S(1;\lambda_W, z_L)C'(1;\lambda_W, z_L) + \lambda_W \sin^2\theta_H = 0 \quad (m_W = k\lambda_W)$$

$$2S(1;\lambda_Z, z_L)C'(1;\lambda_Z, z_L) + \frac{\lambda_Z \sin^2\theta_H}{1-\sin^2\theta_W^{\rm shell}} = 0 \quad (m_Z = k\lambda_Z)$$

Muon decay



$+$


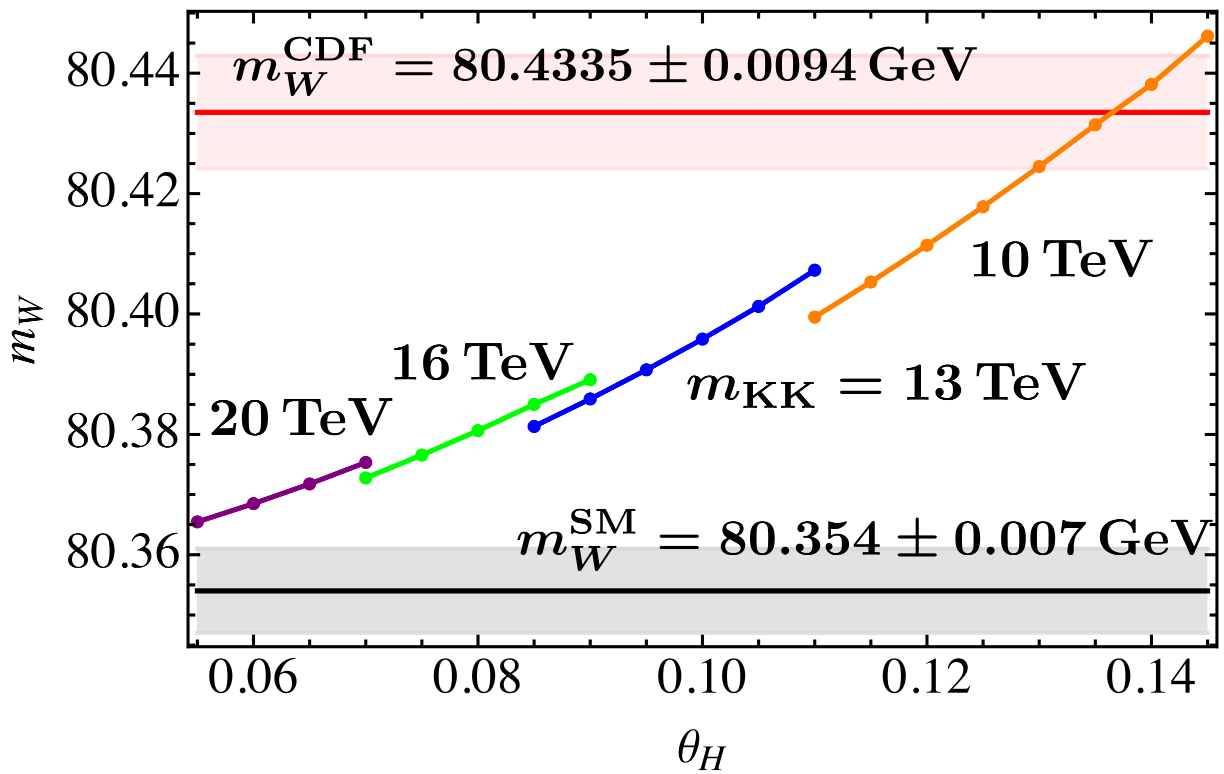
$+$ (loops)

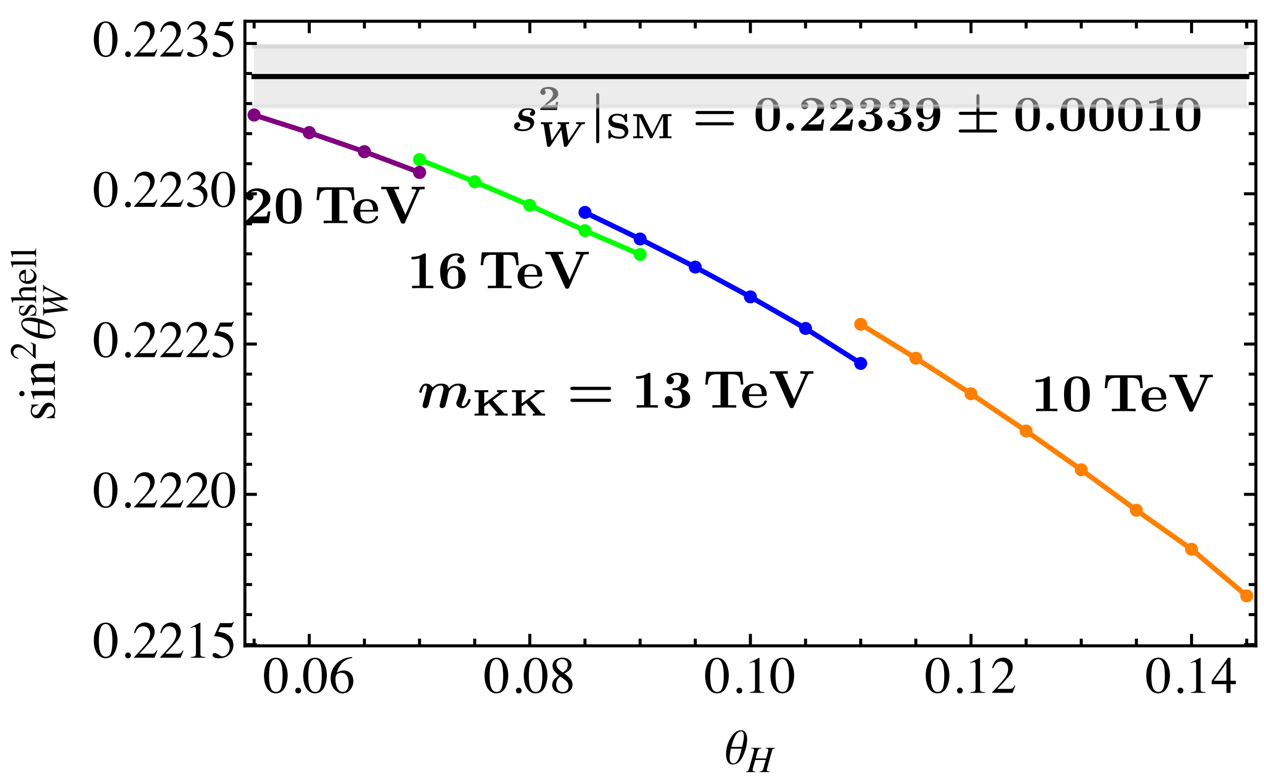
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2\sin^2\theta_W^{\rm shell}} \frac{\hat g^{W^{(0)}}_{\mu\nu_\mu,L}\hat g^{W^{(0)}}_{e\nu_e,L}}{m^2_{W^{(0)}}} (1+\Delta r_G)\,(1+\Delta r^{\rm loop}_{\rm GHU})$$

$$\Delta r_G = \frac{1}{\hat g^{W^{(0)}}_{\mu\nu_\mu,L}\hat g^{W^{(0)}}_{e\nu_e,L}} \sum_{n=1}^{\infty} \left\{ \hat g^{W^{(n)}}_{\mu\nu_\mu,L}\hat g^{W^{(n)}}_{e\nu_e,L}\left[\frac{m_{W^{(0)}}}{m_{W^{(n)}}}\right]^2 + \hat g^{W_R^{(n)}}_{\mu\nu_\mu,L}\hat g^{W_R^{(n)}}_{e\nu_e,L}\left[\frac{m_{W^{(0)}}}{m_{W_R^{(n)}}}\right]^2 \right\}$$

$$\Delta r^{\rm loop}_{\rm GHU} \sim \Delta r^{\rm loop}_{\rm SM}$$

m_W





$m_{\rm KK} = 13\,{\rm TeV} \quad 0.85 \le \theta_H \le 0.11$

$80.381\,{\rm GeV} \le m_W \le 80.407\,{\rm GeV}$

θ_H universality

Hosotani, Funatsu, Hatanaka, Orikasa, Yamatsu, PRD108, 115036 (2023). [2310.03276]

Flavor problem - quarks

$$\Psi^{\alpha}_{(3,4)} \quad \begin{pmatrix} u \\ d \\ u' \\ d' \end{pmatrix} \quad \theta_H$$

$\uparrow$ $SO(5)$

d

d'

$\updownarrow$ Brane int

D_d^+ $\quad (m_d, m_s, m_b)$

D_d^- $\quad \neq (m_u, m_c, m_t)$

$\Psi^{\pm\alpha}_{(3,1)} \qquad D_d^{\pm}$

Mass terms $D^{\pm}$ $\qquad m_{\alpha\beta}\, \overline{\Psi}^{+\alpha}_{(3,1)} \Psi^{-\beta}_{(3,1)} + \text{H.c.}$

$$\begin{pmatrix} d \\ d' \\ D_d^+ \\ D_d^- \end{pmatrix} \xleftrightarrow{\;m_{12},\, m_{21}\;} \begin{pmatrix} s \\ s' \\ D_s^+ \\ D_s^- \end{pmatrix} \xleftrightarrow{\;m_{23},\, m_{32}\;} \begin{pmatrix} b \\ b' \\ D_b^+ \\ D_b^- \end{pmatrix}$$

m_{13}, m_{31}

Mixing

Hosotani, Funatsu, Hatanaka, Orikasa, Yamatsu, PRD111, 095023 (2025). [2503.11136]

Perturbation theory

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + \frac{dz^2}{k^2}\right)$$

$$\boxed{(K + V)\Psi = \lambda\,\Psi}$$

$$(m = k\lambda)$$

$$\Psi = \frac{1}{z^2}\begin{pmatrix} d^1 \\ \vdots \\ D^3 \end{pmatrix} \qquad d^1 = \begin{pmatrix} d_L \\ d'_L \\ d_R \\ d'_R \end{pmatrix} \quad D^1 = \begin{pmatrix} D^+_{d\,L} \\ D^-_{d\,L} \\ D^+_{d\,R} \\ D^-_{d\,R} \end{pmatrix}$$

$$K = \begin{pmatrix} K_{d^1} & & & & & \\ & K_{d^2} & & & & \\ & & K_{d^3} & & & \\ & & & K_{D^1} & & \\ & & & & K_{D^2} & \\ & & & & & K_{D^3} \end{pmatrix}$$

$$\tilde{m}_{\alpha\beta} = \frac{m_{\alpha\beta}}{k}$$

$$V = \begin{pmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 0 & V_{12} & V_{13} \\ & & & V_{21} & 0 & V_{23} \\ & & & V_{31} & V_{32} & 0 \end{pmatrix} \qquad V_{\alpha\beta} = \frac{1}{z}\begin{pmatrix} & & & \tilde{m}_{\alpha\beta} \\ & & \tilde{m}^*_{\beta\alpha} & \\ & \tilde{m}_{\alpha\beta} & & \\ \tilde{m}^*_{\beta\alpha} & & & \end{pmatrix}$$

$$K\,|\psi_{\alpha(n)}\rangle = \lambda_{\alpha(n)}\,|\psi_{\alpha(n)}\rangle$$

Mass eigenstates

$$|\Psi^d\rangle = |\psi_{1(0)}\rangle + \sum_{\alpha=1}^{3}\sum_{n} c^d_{\alpha(n)}|\psi_{\alpha(n)}\rangle$$

$$|\Psi^s\rangle = |\psi_{2(0)}\rangle + \sum_{\alpha=1}^{3}\sum_{n} c^s_{\alpha(n)}|\psi_{\alpha(n)}\rangle$$

$$|\Psi^b\rangle = |\psi_{3(0)}\rangle + \sum_{\alpha=1}^{3}\sum_{n} c^b_{\alpha(n)}|\psi_{\alpha(n)}\rangle$$

W couplings - CKM

$$\mathcal{L}_{\rm int}^{W} = -i\frac{g_w}{\sqrt{2}} W_\mu^\dagger (\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^\mu \hat{g}_L^{Wq} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{H.c.}$$

$$\hat{g}_L^{Wq} = \begin{pmatrix} \hat{g}_L^{Wud} & \hat{g}_L^{Wus} & \hat{g}_L^{Wub} \\ \hat{g}_L^{Wcd} & \hat{g}_L^{Wcs} & \hat{g}_L^{Wcb} \\ \hat{g}_L^{Wtd} & \hat{g}_L^{Wts} & \hat{g}_L^{Wtb} \end{pmatrix}$$

$$(\hat{g}_L^{Wud}, \hat{g}_L^{Wus}, \hat{g}_L^{Wub}) = \sum_n (\delta_{n0} + c^d_{d^{(n)}}, c^s_{d^{(n)}}, c^b_{d^{(n)}})\, \hat{g}^W_{L\, u^{(0)} d^{(n)}}$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \frac{1}{\hat{g}_L^{W\nu_e e}}\, \hat{g}_L^{Wq}$$

CKM

$$\tilde{m}_{11} = \tilde{m}_{22} = \tilde{m}_{33} = 0.8$$

$$\tilde{m}_{12} = \tilde{m}_{21}^* = 1.18 \times 10^{-2}$$

$$\tilde{m}_{23} = \tilde{m}_{32}^* = 8.10 \times 10^{-5}$$

$$\tilde{m}_{13} = \tilde{m}_{31}^* = (1.52 - 3.74i) \times 10^{-6}$$

$$V_{\rm CKM} = \begin{pmatrix} 0.9758 & 0.2163 + 0.0076i & (1.532 - 3.372i) \times 10^{-3} \\ -0.2163 + 0.0075i & 0.9750 & (4.076 - 0.001i) \times 10^{-2} \\ (7.412 - 3.372i) \times 10^{-3} & (-4.106 - 0.075i) \times 10^{-2} & 0.9995 \end{pmatrix}$$

Unitarity triangle

	Exp	SM
$\alpha = \arg\left(-\dfrac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 89.97^\circ$	$(84.1^{+4.5}_{-3.8})^\circ$	91.6°
$\beta = \arg\left(-\dfrac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 22.50^\circ$		
$\sin 2\beta = 0.7071$	0.709 ± 0.011	0.713
$\gamma = \arg\left(-\dfrac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = 67.53^\circ$	$(65.7 \pm 3.0)^\circ$	65.7°

Z couplings

$$\mathcal{L}_{\rm int}^{Z} = -i\frac{g_w}{\cos\theta_W^0} Z_\mu \Bigg\{ (\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^\mu\, \hat{g}_L^{Zu} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} + (L \to R)$$

$$+(\bar{d}_L, \bar{s}_L, \bar{b}_L)\gamma^\mu\, \hat{g}_L^{Zd} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + (L \to R) \Bigg\}$$

$$\hat{g}^{Zu}_{L/R} = \begin{pmatrix} \hat{g}^{Zuu}_{L/R} & 0 & 0 \\ 0 & \hat{g}^{Zcc}_{L/R} & 0 \\ 0 & 0 & \hat{g}^{Ztt}_{L/R} \end{pmatrix} \qquad \hat{g}^{Zd}_{L/R} = \begin{pmatrix} \hat{g}^{Zdd}_{L/R} & \hat{g}^{Zds}_{L/R} & \hat{g}^{Zdb}_{L/R} \\ \hat{g}^{Zsd}_{L/R} & \hat{g}^{Zss}_{L/R} & \hat{g}^{Zsb}_{L/R} \\ \hat{g}^{Zbd}_{L/R} & \hat{g}^{Zbs}_{L/R} & \hat{g}^{Zbb}_{L/R} \end{pmatrix}$$

$$\hat{g}^{Z\beta\gamma}_{L/R} = \hat{g}^{Z\gamma\beta\,*}_{L/R} = \sum_{\alpha}\sum_{n,m} (\delta_{\alpha\beta}\delta_{n0} + c^{\beta\;*}_{\alpha(n)})\, \hat{g}^{Z}_{L/R\,\alpha^{(n)}\alpha^{(m)}} (\delta_{\alpha\gamma}\delta_{m0} + c^{\gamma}_{\alpha(m)})$$

$$\frac{1}{\hat{g}_L^{W\nu_e e}}\begin{pmatrix}\hat{g}_L^{Zuu}\\ \hat{g}_L^{Zcc}\\ \hat{g}_L^{Ztt}\end{pmatrix}=\begin{pmatrix}0.34606\\ 0.34606\\ 0.34639\end{pmatrix}\quad \frac{1}{\hat{g}_L^{W\nu_e e}}\begin{pmatrix}\hat{g}_R^{Zuu}\\ \hat{g}_R^{Zcc}\\ \hat{g}_R^{Ztt}\end{pmatrix}=\begin{pmatrix}-0.15394\\ -0.15394\\ -0.15361\end{pmatrix}$$

$$\frac{1}{\hat{g}_L^{W\nu_e e}}\begin{pmatrix}\hat{g}_L^{Z\,dd}\\ \hat{g}_L^{Z\,ss}\\ \hat{g}_L^{Z\,bb}\end{pmatrix}=\begin{pmatrix}-0.42293\\ -0.42291\\ -0.42303\end{pmatrix}\quad \frac{1}{\hat{g}_L^{W\nu_e e}}\begin{pmatrix}\hat{g}_R^{Z\,dd}\\ \hat{g}_R^{Z\,ss}\\ \hat{g}_R^{Z\,bb}\end{pmatrix}=\begin{pmatrix}0.07697\\ 0.07697\\ 0.07698\end{pmatrix}$$

SM $\sin^2\theta_W^{\mathrm{SM}}=0.2312$

$$\hat{g}^Z_{u\,L}=\ 0.3459\ ,\quad \hat{g}^Z_{u\,R}=-0.1541$$

$$\hat{g}^Z_{d\,L}=-0.4229\ ,\quad \hat{g}^Z_{d\,R}=\ 0.0771$$

FCNC

$$\hat{g}^{Z\beta\gamma}_{L/R} = \sum_{\alpha}\sum_{n,m}\left(\delta_{\alpha\beta}\delta_{n0} + c^{\beta\,*}_{\alpha^{(n)}}\right)\hat{g}^{Z}_{L/R\,\alpha^{(n)}\alpha^{(m)}}\left(\delta_{\alpha\gamma}\delta_{m0} + c^{\gamma}_{\alpha^{(m)}}\right)$$

$$\frac{1}{\hat{g}_L^{W\nu_e e}}\begin{pmatrix}\hat{g}_L^{Z\,ds}\\ \hat{g}_L^{Z\,db}\\ \hat{g}_L^{Z\,sb}\end{pmatrix} = \begin{pmatrix}(-7.4-0.6i)\times 10^{-7}\\ (-2.2+6.2i)\times 10^{-7}\\ -7.3\times 10^{-6}\end{pmatrix} \qquad \frac{1}{\hat{g}_L^{W\nu_e e}}\begin{pmatrix}\hat{g}_R^{Z\,ds}\\ \hat{g}_R^{Z\,db}\\ \hat{g}_R^{Z\,sb}\end{pmatrix} = \begin{pmatrix}-1.0\times 10^{-7}\\ (1.4+0.9i)\times 10^{-8}\\ (-9.0+0.2i)\times 10^{-8}\end{pmatrix}$$

$$\Delta m_M^{\rm tree} \sim \frac{m_M f_M^2 |\hat{g}_M^{Z\,d}|^2}{3m_Z^2} \qquad \begin{pmatrix}\Delta m_K^{\rm tree}\\ \Delta m_{B_d}^{\rm tree}\\ \Delta m_{B_s}^{\rm tree}\end{pmatrix} \sim \begin{pmatrix}2.7\times 10^{-19}\\ 3.3\times 10^{-18}\\ 8.7\times 10^{-16}\end{pmatrix}\ \mathrm{GeV}$$

$$\begin{pmatrix}m_{K_L^0} - m_{K_S^0}\\ m_{B_H^0} - m_{B_L^0}\\ m_{B_{sH}^0} - m_{B_{sL}^0}\end{pmatrix} \sim \begin{pmatrix}(3.484\pm 0.006)\times 10^{-15}\\ (3.336\pm 0.013)\times 10^{-13}\\ (1.1693\pm 0.0004)\times 10^{-11}\end{pmatrix}\ \mathrm{GeV}$$

FCNCs are naturally suppressed.

Neutrino oscillations

Leptons

$$\Psi^{\alpha}_{(1,4)} \quad \begin{pmatrix} \nu \\ e \\ \nu' \\ e' \end{pmatrix} \quad \theta_H$$

$$\nu'^{\alpha}$$

Brane int

$$\hat{\chi}^{\alpha}$$

$$m_{B_\alpha} \overline{\hat{\chi}}^{\alpha} \nu'^{\alpha}_{R} + \text{h.c.}$$

$\hat{\chi}^{\alpha} = (\hat{\chi}^{\alpha})^c$ on the UV brane

$$\delta(y) \frac{1}{2} \Big\{ \overline{\hat{\chi}}^{\alpha} \gamma^{\mu} \partial_{\mu} \hat{\chi}^{\alpha} + M_{\alpha\beta} \overline{\hat{\chi}}^{\alpha} \hat{\chi}^{\beta} \Big\}$$

Majorana masses $\quad M = M^{\dagger} = M^{t}$

Mixing

Y. Hosotani, arXiv:2507.08321

Seesaw mechanism

$$\hat{\chi}^{\alpha} = \begin{pmatrix} \xi_\alpha \\ \eta_\alpha \end{pmatrix} = \begin{pmatrix} \eta_\alpha^c \\ -\xi_\alpha^c \end{pmatrix}$$

Without mixing

(m_e, m_μ, m_τ)

$$m_{\nu_\alpha} \sim \frac{m_{\ell_\alpha}^2 M_\alpha}{(|c_{\ell_\alpha}| - \frac{1}{2})\, m_{B_\alpha}^2}$$

$(c_e, c_\mu, c_\tau) = (-1.0068, -0.7930, -0.6754)$

$$\frac{i}{2}(\nu_L^{c\dagger}, \nu_R^{\prime\dagger}, \eta^{c\dagger}) \begin{pmatrix} 0 & m_\ell & 0 \\ m_\ell & 0 & m_B \\ 0 & m_B & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^{\prime c} \\ \eta \end{pmatrix}$$

$$\Rightarrow \; m_\nu \sim \frac{m_\ell^2 M}{m_B^2}$$

Inverse seesaw

Neutrino oscillations

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}_L = U_{\rm PMNS} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}_L \qquad \{m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\}$$

In GHU

$$F^{1/2}\, M\, F^{1/2}\, \vec{p}_j = m_{\nu_j}\, \vec{p}_j$$

GHU seesaw

$$F = \begin{pmatrix} F_e & & \\ & F_\mu & \\ & & F_\tau \end{pmatrix} \qquad F_e = \frac{m_e^2}{(|c_e| - \frac{1}{2})\, m_{B_e}^2}$$

$$U_{\rm PMNS} = \begin{pmatrix} \vec{p}_1 & \vec{p}_2 & \vec{p}_3 \end{pmatrix} \qquad \delta_{CP} = 0 \ \text{ or } \ \pi$$

$$M = F^{-1/2} \Big\{ \sum_{j=1}^{3} m_{\nu_j}\, \vec{p}_j \otimes \vec{p}_j{}^\dagger \Big\} F^{-1/2} \qquad M^\dagger = M = M^*$$

Example

Normal ordering (NO) NuFit-6.0 arXiv:2410.05380 (JHEP)

$$\Delta m^2_{21} \sim 7.49 \times 10^{-5}\,\mathrm{eV}^2 \;,\quad \Delta m^2_{31} \sim 2.53 \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2\theta_{12} \sim 0.307 \;,\quad \sin^2\theta_{23} \sim 0.561 \;,\quad \sin^2\theta_{13} \sim 0.022 \;,\quad \delta_{CP} \sim \pi$$

$$U_{\mathrm{PMNS}} \sim \begin{pmatrix} 0.823 & 0.548 & -0.148 \\ -0.275 & 0.613 & 0.741 \\ 0.497 & -0.569 & 0.655 \end{pmatrix}$$

GHU

$$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = (1, 8.71, 50.3)\,\mathrm{meV}$$

$$(F_e, F_\mu, F_\tau) = (1, 3, 10) \times 10^{-18}$$

$$(m_{B_e}, m_{B_\mu}, m_{B_\tau}) = (6.83 \times 10^5, 1.10 \times 10^8, 1.32 \times 10^9)\,\mathrm{GeV}$$

$$M = \begin{pmatrix} 4.40 & -1.63 & -2.28 \\ -1.63 & 10.3 & 3.89 \\ -2.28 & 3.89 & 2.47 \end{pmatrix} \times 10^6\,\mathrm{GeV}$$

Summary

GUT inspired SO(5)xU(1)xSU(3) GHU

$$m_W^{\rm SM} < m_W(\theta_H) < m_W^{\rm CDF}$$

CKM matrix

FCNCs are naturally suppressed.

Neutrino oscillations

PMNS matrix in Normal Ordering