Symmetry & Spontaneous Breaking

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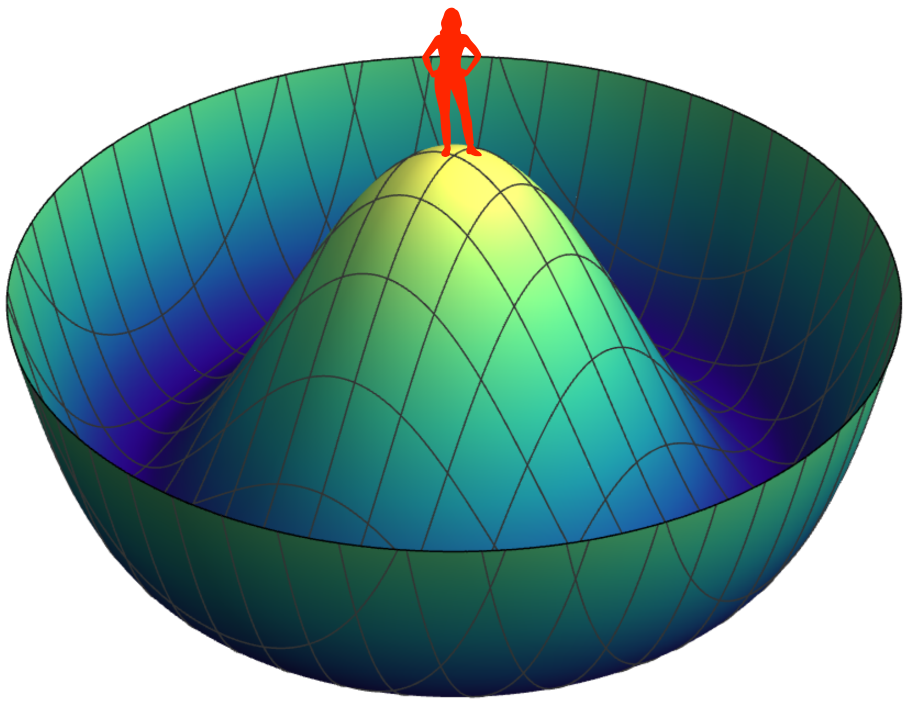
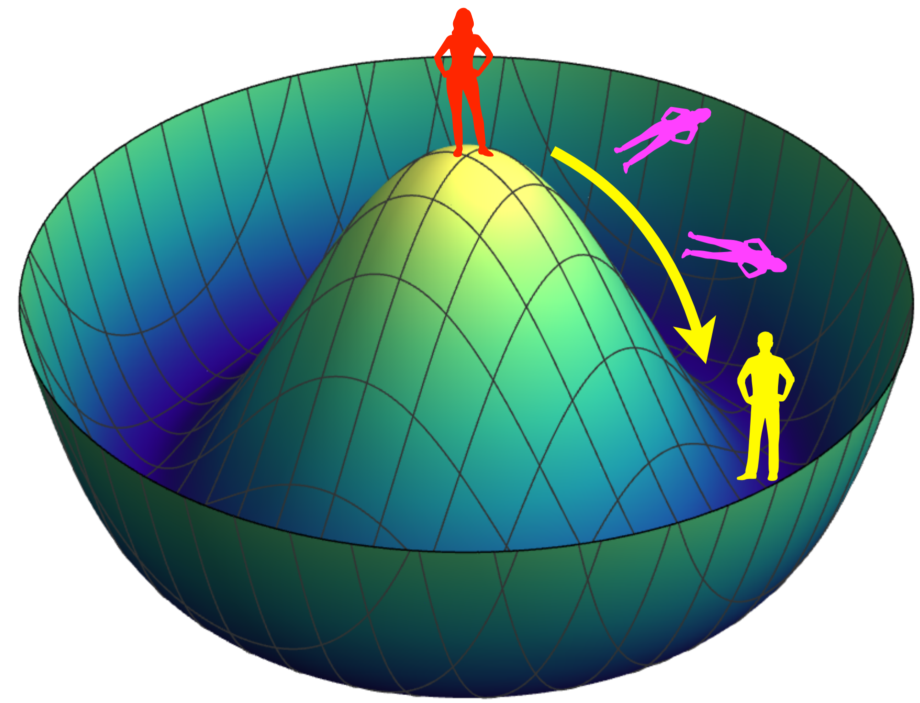
1. Introduction

Symmetry





Spontaneous Symmetry Breaking

Why symmetry in physics ?

Why spontaneous symmetry breaking ?

Symmetry is the heart of physics laws.

Lecture 1

1. Introduction
2. Relativity
3. Field theory
4. U(1) Goldstone model
5. SSB in magnetism
6. Higgs mechanism
7. Unification and SM

Lecture 2

8. Superconductivity - BCS (1957)
9. Superconductivity - Nambu (1959)
10. Chiral symmetry
11. Nambu-Jona-Lasinio model
12. Pions as NG bosons
13. Summary

2. Relativity

Newton equation

$$m_j \frac{d^2\vec{r}_j}{dt^2} = \vec{F}_j = -\nabla_j\{U + U_{\rm ext}\}$$

$$U = \sum_{j<k} V(|\vec{r}_j - \vec{r}_k|)$$

When $U_{\rm ext} = 0$

invariant under translations, rotations

invariant under Galilean transformations

⇓

Galileo's relativity principle

⇓

Einstein's special relativity

Principle of relativity = Universality

Physics laws take the same form irrespective of

Space position & direction

Time

Moving or not

Absolute symmetry

3. Field theory

Condensed matter physics	many body
Statistical physics	quasi-particle creation/annihilation
$\vdots$	$\vdots$
Nuclear physics	relativistic
Particle physics	particle creation/annihilation

$$\psi(x) = \psi(\vec{x}, t), \quad (\psi_{e\uparrow}\ , \psi_{e\downarrow})$$

$$A_\mu(x), \quad F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\nu = (\vec{E}, \vec{B})$$

… successful so far …

Equations of motion $\longleftarrow$ **Action** $I = \int d^4x\, \mathcal{L}$

$\delta I = 0$

Action principle

Spacetime symmetry

Internal symmetry

global	baryon no sym., chiral symmetry
local	gauge symmetry

Physics is simple at the fundamental level.

$\Updownarrow$

Symmetry

4. U(1) Goldstone model

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V[\Phi]$$

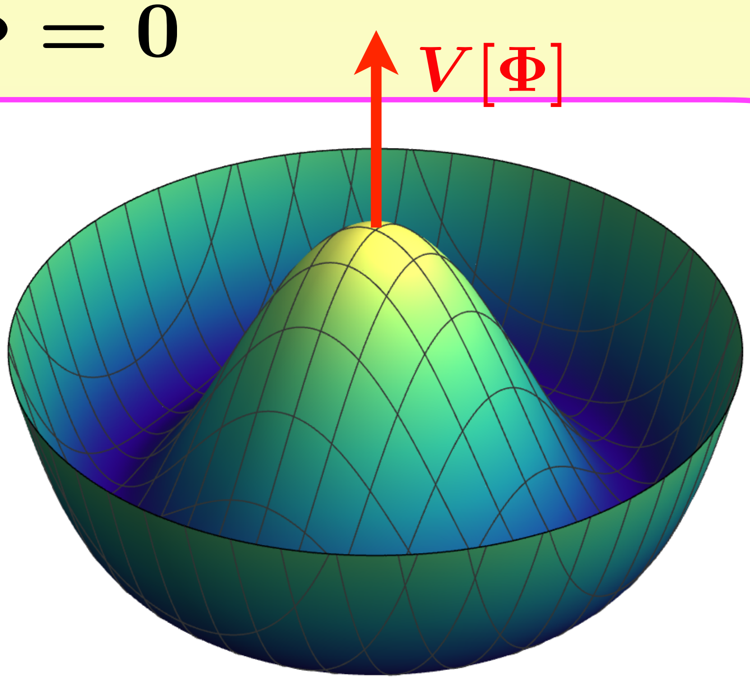
$$V[\Phi] = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\left(\partial_\mu \partial^\mu + m^2 + 2\lambda\, \Phi^\dagger \Phi\right)\Phi = 0$$

$m^2 < 0$

$$\rightarrow \quad V = \lambda \left\{ \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 - \frac{v^4}{4} \right\}$$

$$v = \sqrt{\frac{-m^2}{\lambda}}$$



Hamiltonian

$$H = \int d^3x \left\{ \Pi^\dagger \Pi + \nabla \Phi^\dagger \nabla \Phi + V[\Phi] \right\} \qquad (\Pi = \dot{\Phi})$$

$$H = \int d^3x \left\{ \Pi^\dagger \Pi + \nabla \Phi^\dagger \nabla \Phi + V[\Phi] \right\}$$

$U(1)$ inv. $\quad \Phi \to \Phi' = e^{i\alpha}\Phi$

$$\partial_\mu j^\mu(x) = 0 \quad \text{where} \quad j^\mu(x) = i\left(\partial^\mu \Phi^\dagger \Phi - \Phi^\dagger \partial^\mu \Phi\right)$$

U(1) charge $\qquad Q = \int d^3x\, j^0(t, \vec{x})$

$$\frac{dQ}{dt} = \int d^3x\, \partial_0 j^0 = -\int d^3x\, \mathrm{div}\, \vec{j} = 0$$

or $\qquad \dfrac{dQ}{dt} = \dfrac{i}{\hbar}[H, Q] = 0$

$U(1)$ inv $\longrightarrow$ $[H, Q] = 0$

Q is conserved. : symmetry of H

$U(1)$ transformation

$$U(\alpha) = e^{-i\alpha Q}$$

$$U(\alpha)^\dagger \Phi(x) U(\alpha) = e^{i\alpha}\, \Phi(x)$$

$|\Psi_G\rangle$: $|$ground state$\rangle$ or $|$vacuum$\rangle$

If $U(\alpha)|\Psi_G\rangle = e^{i\beta}\,|\Psi_G\rangle$

$$\langle\Psi_G|U(\alpha)^\dagger \Phi(x) U(\alpha)|\Psi_G\rangle = \langle\Psi_G|\Phi(x)|\Psi_G\rangle$$

$$= e^{i\alpha}\,\langle\Psi_G|\Phi(x)|\Psi_G\rangle$$

$$\rightarrow \quad \langle\Psi_G|\Phi(x)|\Psi_G\rangle = 0$$

$\langle\Psi_G|\Phi(x)|\Psi_G\rangle \neq 0$ $\Rightarrow$

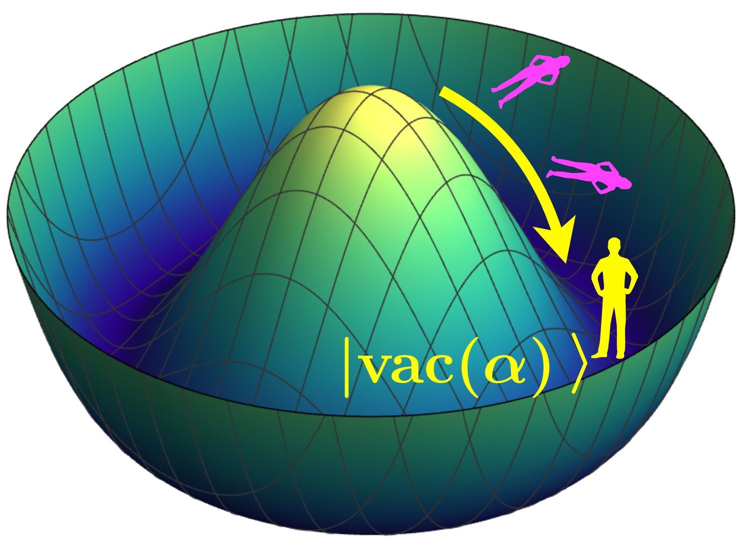
$|\Psi_G\rangle$ is not invariant under U(1) transformation.

is not an eigenstate of Q.

$$V = \lambda\left\{\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 - \frac{v^4}{4}\right\} \qquad v = \sqrt{\frac{-m^2}{\lambda}}$$

$$\langle \mathrm{vac}(\alpha)|\Phi|\mathrm{vac}(\alpha)\rangle = e^{i\alpha} v/\sqrt{2}$$

$$|\mathrm{vac}(\alpha)\rangle = U(\alpha)\,|\mathrm{vac}(0)\rangle$$



Hamiltonian is invariant under a transformation of symmetry Q,

whereas $|\Psi_G\rangle$ is not invariant.

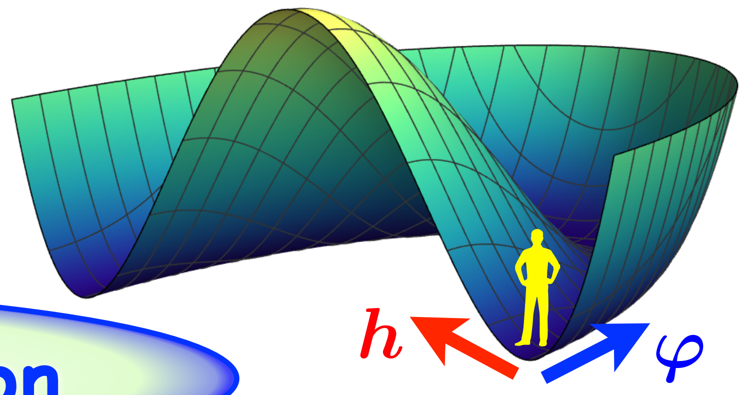
Symmetry Q is spontaneously broken.

$$\Phi = \frac{1}{\sqrt{2}}(v + h(x))e^{i\varphi(x)/v}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h - \lambda v^2 h^2 + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi + \cdots$$

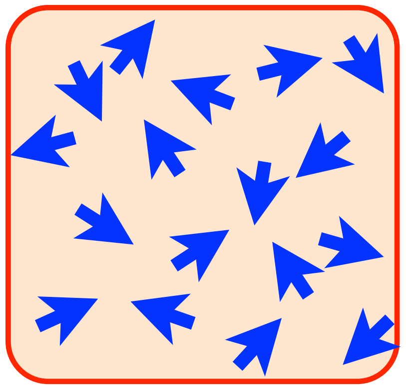
φ : massless

Nambu-Goldstone boson

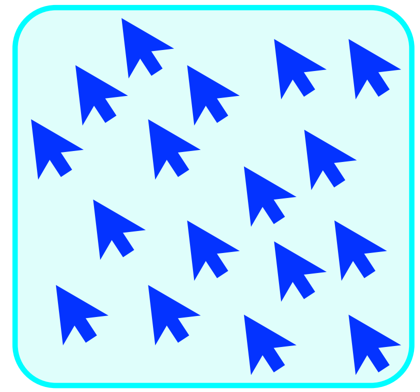


5. SSB in magnetism

Hamiltonian is rotationally invariant.

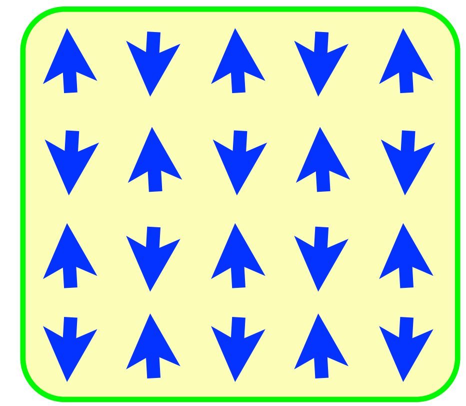


para-magnet



ferro-magnet

$\langle \vec{M} \rangle \neq 0$



anti-ferro-magnet

Spin waves — NG bosons

ferro-magnet : $\epsilon(k) = ak^2$

anti-ferro-magnet : $\epsilon(k) = b\,k \qquad (S = \frac{1}{2})$

6. Higgs mechanism

U(1) Goldstone model + gauge interactions

$$\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi - V[\Phi] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (D_\mu = \partial_\mu - ieA_\mu)$$

Gauge invariance :
$$\begin{cases} \Phi \to \Phi' = e^{ie\Lambda(x)} \Phi \\ A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda(x) \end{cases}$$

$$\Phi = \frac{1}{\sqrt{2}} (v + h(x)) e^{i\varphi(x)/v}$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu h \partial^\mu h - \lambda v^2 h^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ \frac{1}{2} e^2 v^2 (A_\mu - \frac{1}{ev} \partial_\mu \varphi)(A^\mu - \frac{1}{ev} \partial^\mu \varphi) + \cdots$$

Unitary gauge

$$e\Lambda(x) = -\frac{\varphi(x)}{v} \qquad \varphi' = 0\,, \quad A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu \varphi$$

$$\mathcal{L}' = \frac{1}{2}\partial_\mu h \partial^\mu h - \lambda v^2 h^2 - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_V^2 A'_\mu A'^\mu + \cdots$$

$$(m_V = ev)$$

$$\partial_\mu F^{\mu\nu} - m_V^2 A^\nu = 0$$

drop ʻ

$$\Rightarrow \partial_\nu A^\nu = 0 \ \Rightarrow (\partial^2 - m_V^2)\, A_\mu = 0$$

massive spin 1

No of physical degrees of freedom

$m^2 > 0$, $\langle \Phi \rangle = 0$	A_μ : massless gauge field	2	4
	$\Phi = \Phi_1 + i\Phi_2$	2	
$m^2 < 0$, $\langle \Phi \rangle \neq 0$	A_μ : massive vector field	3	4
	$\Phi : \begin{cases} h & \text{massive} \\ \varphi & \text{absorbed by } A_\mu \end{cases}$	1	

7. Unification and SM

EM int.	Maxwell	$U(1)_{\mathrm{EM}}$	$\left.\right\}$ $SU(2)_L \times U(1)_Y$
Weak int.	Fermi		
Strong int.	QCD	$SU(3)_C$	

$SU(2)_L \times U(1)_Y \to U(1)_{EM}$

$$\mathcal{L}^{\mathrm{gauge}} = -\frac{1}{2}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \quad A_\mu = \frac{1}{2}\tau^a A^a_\mu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

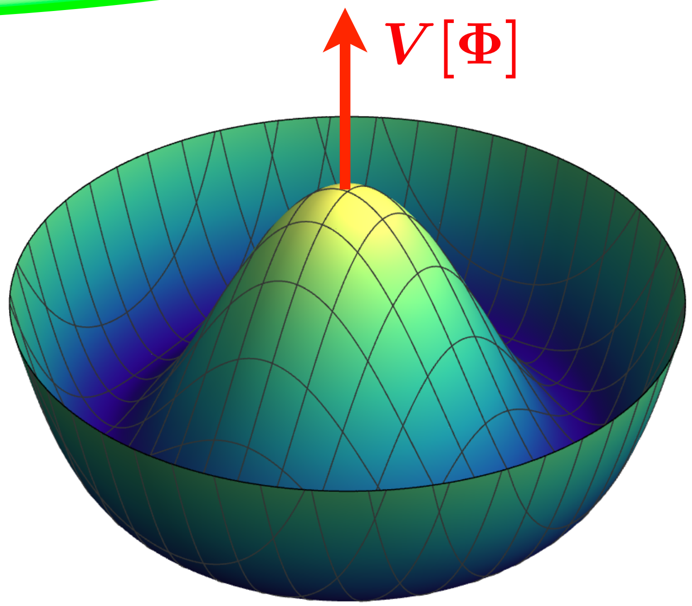
$$\mathcal{L}^{\mathrm{Higgs}} = (D_\mu\Phi)^\dagger D^\mu\Phi - V[\Phi] \qquad \Phi: \; SU(2)_L \text{ doublet}$$

$$D_\mu\Phi = \Big\{\partial_\mu - i\big(gA_\mu + Yg'B_\mu\big)\Big\}\Phi \qquad (Y = \tfrac{1}{2})$$

$$V[\Phi] = m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$SU(2)_L \times U(1)_Y \to U(1)_{EM}$

$$V[\Phi] = m^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$



$m^2 < 0$

$$\Rightarrow \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{-m^2}{\lambda}}$$

$U(1)_{\mathrm{EM}}$ is unbroken.

$$\Phi \to e^{i\alpha_{\mathrm{EM}}(x) Q_{\mathrm{EM}}} \Phi$$

$$Q_{\mathrm{EM}} = I_3 + Y \quad = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ for } \Phi$$

Gauge fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \pm i A_\mu^2)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu^\gamma \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$m_\gamma = 0\,, \quad m_W = \tfrac{1}{2} g v\,, \quad m_Z = \frac{m_W}{\cos\theta_W}$$

EM int. $U(1)_{\mathrm{EM}}$

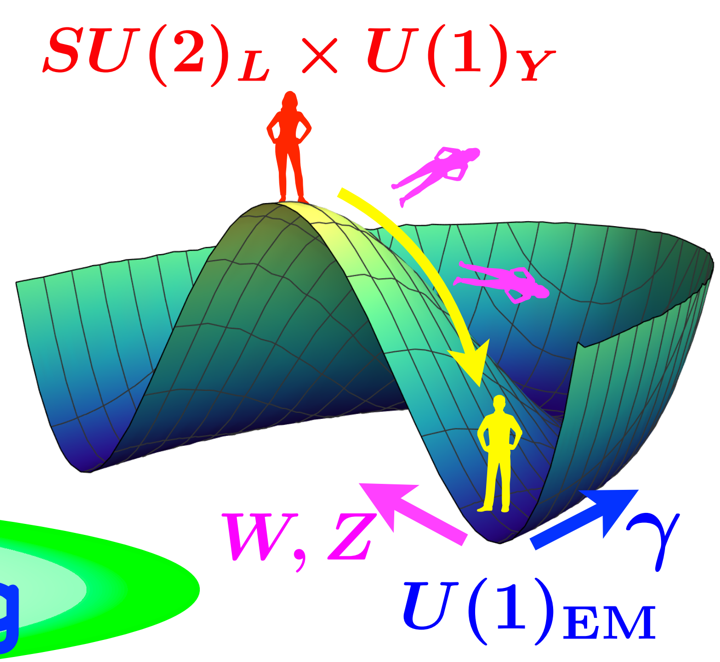
Weak int.

Strong int. $SU(3)_C$

$$\left.\begin{array}{l} U(1)_{\mathrm{EM}} \\ \text{Weak int.} \end{array}\right\} SU(2)_L \times U(1)_Y$$

Unification of forces

Spontaneous Symmetry Breaking



GUT

$$\left.\begin{array}{c} SU(5) \\ SO(10) \end{array}\right\} \to SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{\mathrm{EM}}$$

8. Superconductivity - BCS (1957)

At $T < T_c$, electric currents flow with no resistivity.

Cooper pairs

BCS ground state: $|\Psi_{\rm BCS}(\alpha)\,\rangle = \prod \left\{ u_p + e^{i\alpha} v_p c_e^\dagger(\vec{p},\uparrow) c_e^\dagger(-\vec{p},\downarrow) \right\} |0\,\rangle$

Not an eigenstate of electron number/electric charge.

Is $U(1)_{\rm EM}$ spontaneously broken?

or

$$|\Psi_N\,\rangle = \int d\alpha\, e^{-iN\alpha/2}\, |\Psi_{\rm BCS}(\alpha)\,\rangle$$ eigenstate of electron no.

$$|\Psi\,\rangle = \sum \sqrt{w_N}\, |\Psi_N\,\rangle$$

Quasi-particles

$$|\Psi_{\mathrm{BCS}}(\alpha)\rangle = \prod \left\{ u_p + e^{i\alpha} v_p c_e^\dagger(\vec{p},\uparrow) c_e^\dagger(-\vec{p},\downarrow) \right\} |0\rangle$$

Bogoliubov (1958)

$$\xi(\vec{p},\uparrow) = u_p c_e(\vec{p},\uparrow) - e^{i\alpha} v_p c_e^\dagger(-\vec{p},\downarrow)$$

$$\xi(-\vec{p},\downarrow) = u_p c_e(-\vec{p},\downarrow) + e^{i\alpha} v_p c_e^\dagger(\vec{p},\uparrow)$$

$$(u_p^2 + v_p^2 = 1)$$

$\Rightarrow$ $\xi(\vec{p},\uparrow)|\Psi_{\mathrm{BCS}}(\alpha)\rangle = 0$

Energy gap $\Delta(p)$ $\Rightarrow$ **Superconductivity**

Bogoliubov transformation

$$c_e(\vec{p},\uparrow) = a \qquad \{a, a^\dagger\} = \{b, b^\dagger\} = 1, \quad \text{others} = 0$$
$$c_e(-\vec{p},\downarrow) = b$$

$$\xi(\vec{p},\uparrow) = u_p a - v_p b^\dagger \qquad |u_p|^2 + |v_p|^2 = 1$$
$$\xi(-\vec{p},\downarrow) = u_p b + v_p a^\dagger$$

$$\{\xi(\vec{p},\uparrow), \xi^\dagger(\vec{p},\uparrow)\} = \{u_p a - v_p b^\dagger, u_p^* a^\dagger - v_p^* b\} = |u_p|^2 + |v_p|^2 = 1$$

$$\{\xi(\vec{p},\uparrow), \xi(-\vec{p},\downarrow)\} = \{u_p a - v_p b^\dagger, u_p b + v_p a^\dagger\} = u_p v_p - v_p u_p = 0$$

$$\xi(\vec{p},\uparrow)(u_p + v_p a^\dagger b^\dagger)|0\rangle = (u_p a - v_p b^\dagger)(u_p + v_p a^\dagger b^\dagger)|0\rangle$$
$$= (u_p v_p \underbrace{a\, a^\dagger}_{1 - a^\dagger a} b^\dagger - v_p u_p\, b^\dagger)|0\rangle = 0$$

Quasi-particles

$$|\Psi_{\mathrm{BCS}}(\alpha)\rangle = \prod \left\{ u_p + e^{i\alpha} v_p c_e^\dagger(\vec{p},\uparrow) c_e^\dagger(-\vec{p},\downarrow) \right\} |0\rangle$$

Bogoliubov (1958)

$$\xi(\vec{p},\uparrow) = u_p c_e(\vec{p},\uparrow) - e^{i\alpha} v_p c_e^\dagger(-\vec{p},\downarrow)$$

$$\xi(-\vec{p},\downarrow) = u_p c_e(-\vec{p},\downarrow) + e^{i\alpha} v_p c_e^\dagger(\vec{p},\uparrow)$$

$$(u_p^2 + v_p^2 = 1)$$

$\Rightarrow$ $\xi(\vec{p},\uparrow)|\Psi_{\mathrm{BCS}}(\alpha)\rangle = 0$

Energy gap $\Delta(p)$ $\Rightarrow$ **Superconductivity**

The BCS ground state is a vacuum state of quasi-particles.

A quasi-particle is a mixture of a particle and a hole. U(1) sym is broken.

An energy gap is generated.

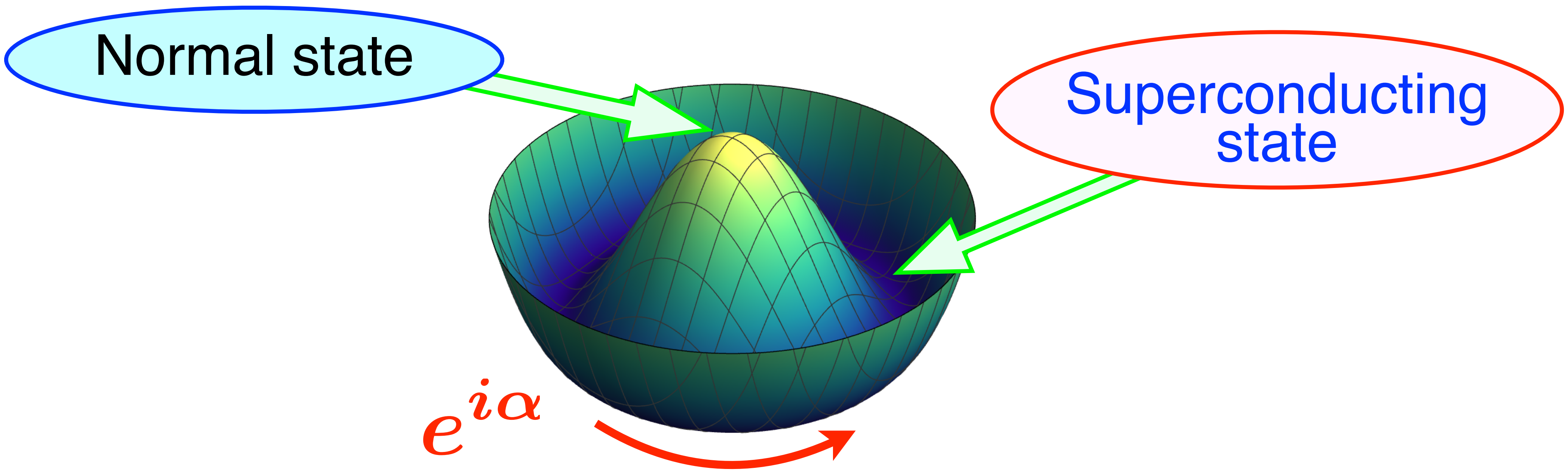
Cooper pair condensation

$$\langle \Psi_{\mathrm{BCS}}(\alpha) | \psi_{\downarrow}(x) \psi_{\uparrow}(x) | \Psi_{\mathrm{BCS}}(\alpha) \rangle = e^{i\alpha} \sum u_p v_p$$

$$= \Psi_{\mathrm{GL}}(\alpha)$$

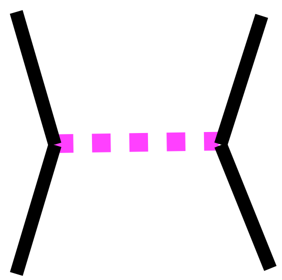
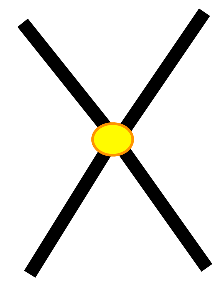
Ginzburg-Landau order parameter

Energy density $\mathcal{E}(\Psi_{\mathrm{GL}})$



9. Superconductivity – Nambu (1959)

$$H = H_0 + H_{\rm int}\ , \quad H_{\rm int} = H_{\rm int}^{\rm phonon} + H_{\rm int}^{\rm Coulomb}$$

$H_{\rm int} =$  $\sim$  $\sim \int d^3x\, g\, \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$

$$[H, Q] = 0 \ : \ U(1) \text{ inv}$$

Generalized Hartree-Fock approx. (Gorkov)

$$H = \underbrace{(H_0 + H_s)}_{H_0'} + \underbrace{(H_{\rm int} - H_s)}_{H_{\rm int}'}$$

$$H_s = \int d^3x\, g \left\{ \langle \psi_\uparrow^\dagger \psi_\downarrow^\dagger \rangle \psi_\downarrow \psi_\uparrow + \psi_\uparrow^\dagger \psi_\downarrow^\dagger \langle \psi_\downarrow \psi_\uparrow \rangle \right\}$$

Breaks U(1) sym.

Self-consistency: $H_{\rm int}' = H_{\rm int} - H_s \sim 0$

Nambu representation

$$\Psi(x) = \begin{pmatrix} \psi_\uparrow(x) \\ \psi_\downarrow^\dagger(x) \end{pmatrix} \qquad \Psi(\vec{p}) = \begin{pmatrix} c_e(\vec{p},\uparrow) \\ c_e^\dagger(-\vec{p},\downarrow) \end{pmatrix}$$

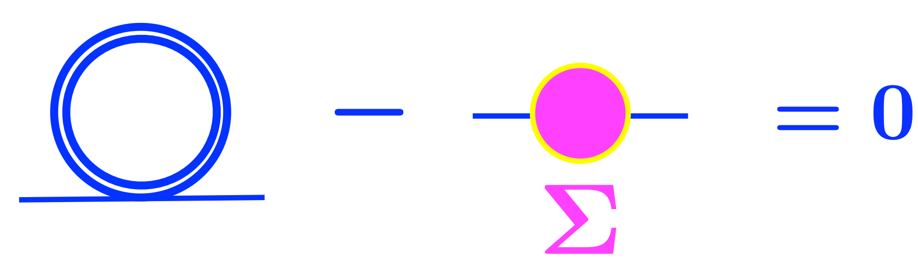
$$H_0' = \sum_{\vec{p}} \Psi^\dagger(\vec{p}) \left\{ \epsilon(\vec{p})\,\tau_3 + \underbrace{\chi(\vec{p})\,\tau_3 + \phi(\vec{p})\,\tau_1}_{\Sigma} \right\} \Psi(\vec{p})$$

Quasi-particle $\qquad \xi(\vec{p},\uparrow)\ ,\ \xi^\dagger(-\vec{p},\downarrow)$

$$E(\vec{p}) = \pm\sqrt{(\epsilon+\chi)^2 + \phi^2} \qquad \text{gap} \ = 2\,\phi\big|_{\text{Fermi surface}}$$

Self-consistency:

$$H_{\text{int}}' = H_{\text{int}} - H_s \sim 0$$



Gap equation

Nambu's analogy

Superconductivity: gap → *U(1) symmetry is broken.*

$$\left\{(\epsilon+\chi)(\vec{p})\,\tau_3+\phi(\vec{p})\,\tau_1\right\}\Psi(\vec{p})=E\,\Psi(\vec{p}) \qquad \Psi(\vec{p})=\begin{pmatrix} c_e(\vec{p},\uparrow) \\ c_e^\dagger(-\vec{p},\downarrow)\end{pmatrix}$$

⇕

Dirac equation: mass → *What symmetry is broken ?*

$$\left\{\vec{p}\,\vec{\sigma}\begin{pmatrix} I_2 & \\ & -I_2\end{pmatrix}+m\begin{pmatrix} & I_2 \\ I_2 & \end{pmatrix}\right\}\psi=E\,\psi \qquad \psi=\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$$

Mass is a gap !

10. Chiral symmetry

U(1) : $\psi \to e^{i\alpha}\psi$

Chiral U(1) : $\psi \to e^{i\alpha\gamma^5}\psi$ $\quad \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} \to \begin{pmatrix} e^{-i\alpha}\,\psi_L \\ e^{+i\alpha}\,\psi_R\end{pmatrix}$

$$\overline{\psi}\,p_\mu\gamma^\mu\,\psi = \psi_R^\dagger(p_0+\vec{p}\,\vec{\sigma})\psi_R + \psi_L^\dagger(p_0-\vec{p}\,\vec{\sigma})\psi_L$$

$$\left.\begin{aligned}\overline{\psi}\,\psi &= \psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L \\ \overline{\psi}\,\gamma^5\psi &= \psi_L^\dagger\psi_R - \psi_R^\dagger\psi_L\end{aligned}\right\}\ \text{Not invariant under chiral U(1)}$$

In $\mathcal{L}_0 = \overline{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$

mass term breaks chiral U(1) inv.

11. Nambu-Jona-Lasinio model (1961)

Dynamical mass generation

Superconductivity: gap ⇒ *U(1) symmetry is broken.*

$$\left\{(\epsilon+\chi)(\vec{p})\,\tau_3+\phi(\vec{p})\,\tau_1\right\}\Psi(\vec{p})=E\,\Psi(\vec{p})$$

Dirac equation: mass ⇒ *Chiral symmetry is broken.*

$$\left\{\vec{p}\,\vec{\sigma}\begin{pmatrix} I_2 & \\ & -I_2 \end{pmatrix}+m\begin{pmatrix} & I_2 \\ I_2 & \end{pmatrix}\right\}\psi=E\,\psi$$

Spontaneous breaking of chiral sym.

→ **Dynamical mass generation**

In $\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$

mass term breaks chiral U(1) inv.

We imagine that fundamental interactions of hadrons are chiral symmetric.

Chiral symmetry is spontaneously broken by dynamics.

QCD

4-fermi int.: $\mathcal{L}_{\rm NJL} = \overline{\psi}\, i\gamma^\mu \partial_\mu \psi + \mathcal{L}_{\rm int}$

$$\mathcal{L}_{\rm int} = g_0 \left\{ \left(\overline{\psi}\psi\right)^2 - \left(\overline{\psi}\gamma^5\psi\right)^2 \right\}$$

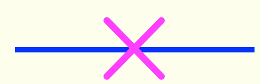
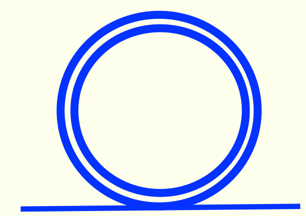
$$= 4g_0\, \psi_L^\dagger \psi_R \psi_R^\dagger \psi_L$$

$$\mathcal{L}_{\mathrm{NJL}} = \overline{\psi}\, i\gamma^{\mu}\partial_{\mu}\psi + \mathcal{L}_{\mathrm{int}}$$

$$= \underbrace{(\mathcal{L}_0 + \mathcal{L}_s)}_{\mathcal{L}'_0} + \underbrace{(\mathcal{L}_{\mathrm{int}} - \mathcal{L}_s)}_{\mathcal{L}'_{\mathrm{int}}}$$

$$-m\overline{\psi}\psi$$

Self-consistency:

$\mathcal{L}'_{\mathrm{int}} \sim 0 \;\Rightarrow$ m  $+$  $= 0$

$$m\left\{1 - \frac{ig_0}{2\pi^4}\int \frac{d^4p}{p^2 - m^2 + i\epsilon}\right\} = 0$$

Non-trivial solution $(m \neq 0)$ for $0 < \dfrac{2\pi^2}{g_0\Lambda^2} < 1$

NJL vacuum

Massless: $i\gamma^\mu\partial_\mu\,\psi^{(0)}(x) = 0$

$$\psi^{(0)}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2p}}\sum_{s=\pm1}\left\{a^{(0)}(\vec{p},s)u^{(0)}(\vec{p},s)e^{-ipx} + b^{(0)\dagger}(\vec{p},s)v^{(0)}(\vec{p},s)e^{+ipx}\right\}$$

Massive: $(i\gamma^\mu\partial_\mu - m)\,\psi^{(m)}(x) = 0$

$$\psi^{(m)}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E}}\sum_{s}\left\{a^{(m)}(\vec{p},s)u^{(m)}(\vec{p},s)e^{-ipx} + b^{(m)\dagger}(\vec{p},s)v^{(m)}(\vec{p},s)e^{+ipx}\right\}$$

$\left\{a^{(m)}(\vec{p},s), b^{(m)\dagger}(\vec{p},s)\right\}$ **are related to** $\left\{a^{(0)}(\vec{p},s), b^{(0)\dagger}(\vec{p},s)\right\}$ **by**

$$\psi^{(m)}(x) = e^{i\alpha\gamma^5}\psi^{(0)}(x) \quad \text{for} \quad x_0 = 0$$

Quarks = quasi-particles

$$a^{(m)}(\vec{p},s) = e^{-is\alpha}u_p\, a^{(0)}(\vec{p},s) + e^{is\alpha}v_p\, b^{(0)\dagger}(-\vec{p},s)$$

$$b^{(m)}(-\vec{p},s) = e^{-is\alpha}u_p\, b^{(0)}(-\vec{p},s) - e^{is\alpha}v_p\, a^{(0)\dagger}(\vec{p},s)$$

$$u_p = \sqrt{\frac{1}{2}\left(1+\frac{p}{E}\right)}\,,\;\; v_p = \sqrt{\frac{1}{2}\left(1-\frac{p}{E}\right)}\,,\;\; E = \sqrt{p^2+m^2}$$

NJL vacuum

chiral pairs

$$|\Omega^{(m)}(\alpha)\rangle = \prod_{\vec{p},s}\left\{u_p - e^{2is\alpha}v_p a^{(0)\dagger}(\vec{p},s) b^{(0)\dagger}(-\vec{p},s)\right\}|0\rangle$$

$$\Rightarrow \quad \begin{bmatrix} a^{(m)}(\vec{p},s) \\ b^{(m)}(\vec{p},s) \end{bmatrix} |\Omega^{(m)}(\alpha)\rangle = 0$$

$$\langle\Omega^{(m)}(\alpha)|\,\psi_R^{(m)\dagger}\psi_L^{(m)}\,|\Omega^{(m)}(\alpha)\rangle \neq 0$$

chiral condensates

Superconductivity

BCS ground state

Cooper pairs

$$|\Psi_{\mathrm{BCS}}(\alpha)\,\rangle = \prod \left\{ u_p + e^{i\alpha} v_p c_e^\dagger(\vec{p},\uparrow) c_e^\dagger(-\vec{p},\downarrow) \right\} |0\,\rangle$$

gap

Quasi-particle:

$$\xi(\vec{p},\uparrow) = u_p c_e(\vec{p},\uparrow) - e^{i\alpha} v_p c_e^\dagger(-\vec{p},\downarrow)$$

NJL vacuum

chiral pairs

$$|\Omega^{(m)}(\alpha)\,\rangle = \prod_{\vec{p},s} \left\{ u_p - e^{2is\alpha} v_p a^{(0)\dagger}(\vec{p},s) b^{(0)\dagger}(-\vec{p},s) \right\} |0\,\rangle$$

mass generation

Nucleon/quark:

$$a^{(m)}(\vec{p},s) = e^{-is\alpha} u_p\, a^{(0)}(\vec{p},s) + e^{is\alpha} v_p\, b^{(0)\dagger}(-\vec{p},s)$$

12. Pions as NG bosons

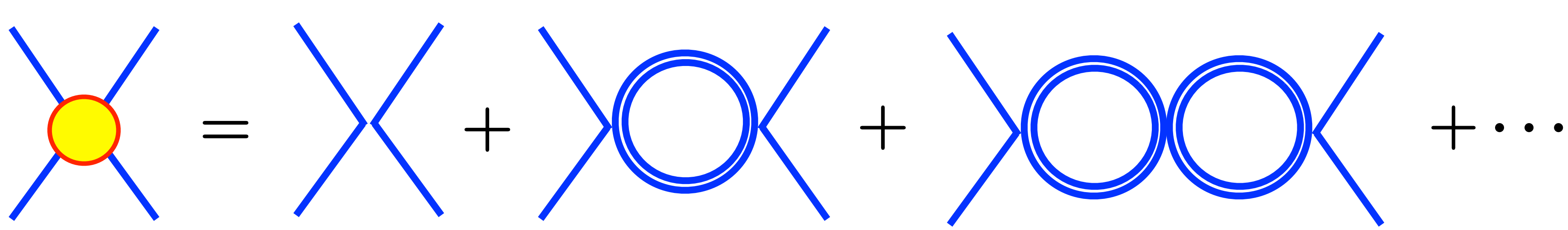
$\mathcal{L}_{\rm NJL}$: Chiral symmetry is spontaneously broken.

Conservation of $\quad j^{5\mu} = \overline{\psi}\gamma^{\mu}\gamma^{5}\psi\ , \quad \partial_{\mu} j^{5\mu} = 0$

$$\langle p'|j^{5\mu}|p\rangle = F(q^2)\overline{u}(p')\left\{\gamma^{\mu}\gamma^{5} + \frac{2mq^{\mu}}{q^2}i\gamma^{5}\right\}u(p)$$

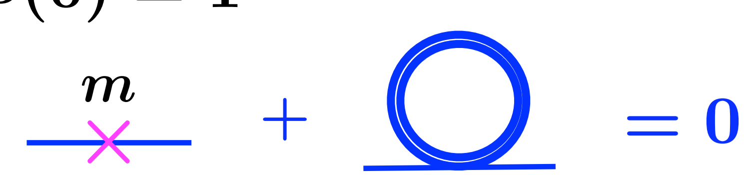
Nambu-Goldstone theorem

Massless 0^{-} pion



$$= 2g_0 i\gamma^5 \frac{1}{1-J_P(q^2)}\gamma^5$$

$$J_P(0) = 1$$



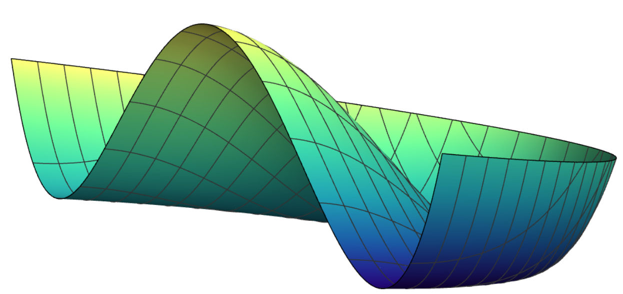
In real world

$$\begin{cases} \gamma^5 \times \text{flavor } SU(2)_F \text{ or } SU(3)_F \text{ symmetry is approximate.} \\ \text{Small bare quark masses } m_u, m_d \end{cases}$$ $\longleftarrow$ Higgs mechanism in SM

massless NG boson -> small mass $m_\pi^2 \ll m^2$

$$f_\pi^2 m_\pi^2 = \frac{m_u + m_d}{2} \langle \overline{u}u + \overline{d}d \rangle$$

Gell-Mann, Oaks, Renner 1968



Chiral condensates

$$\langle \overline{q}q \rangle \sim (250\,\mathrm{MeV})^3$$

lattice simulation JLQCD-TWQCD 2007

13. Summary

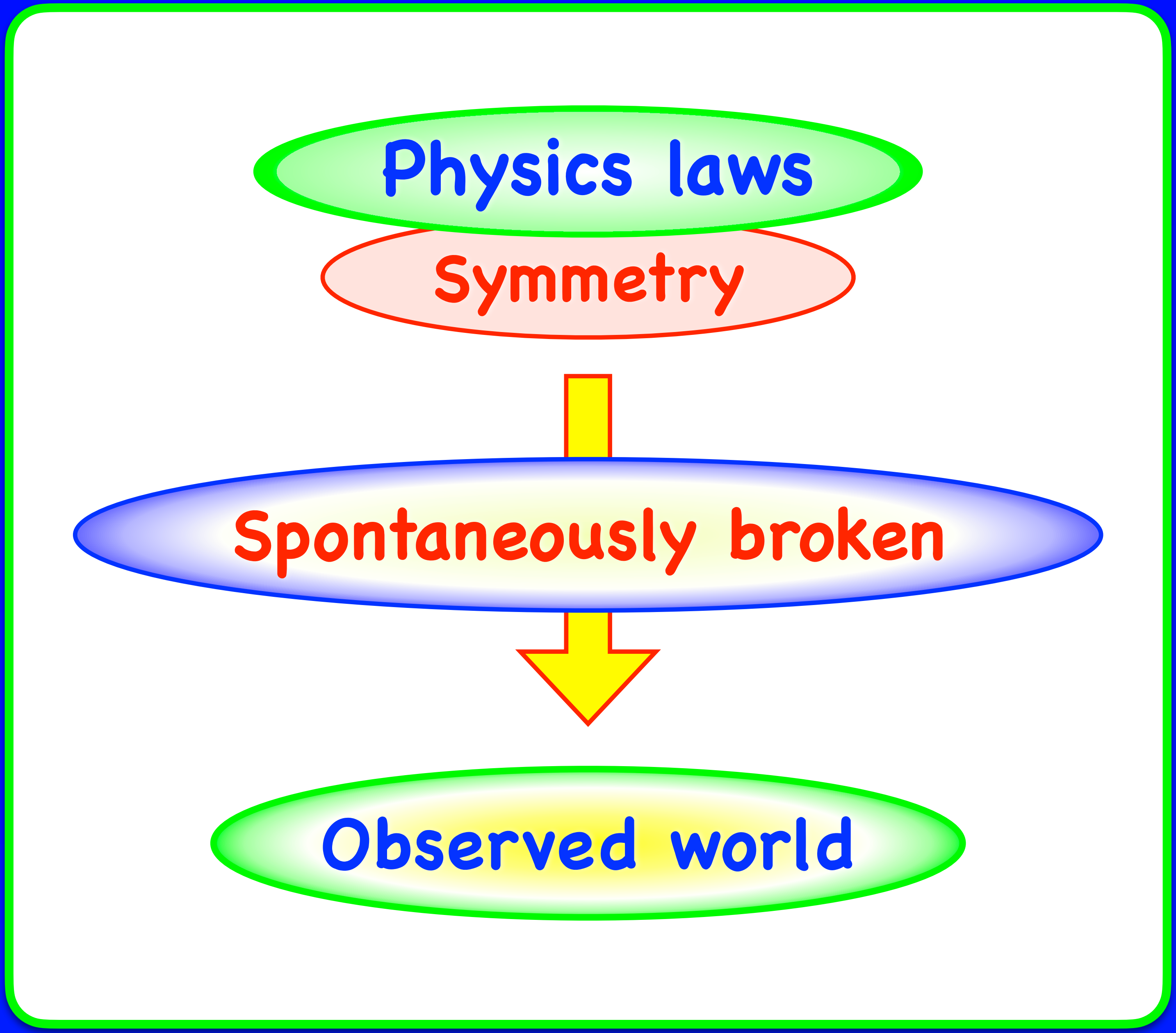
Physics is simple at the fundamental level.

⇕

Physics laws have Symmetry

Symmetry

is manifest or hidden in Nature.



Vacuum is not empty.

Particles = quasi-particles

Unification

of forces

Superconductivity

Chiral symmetry

breaking