1 Introduction: Hadrons, resonances, and entanglement

1.1 Hadrons and QCD

- **Hadrons**: particles that interact via the strong interaction
- Fundamental theory of the strong interaction: **QCD**

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \bar{q}(i\not{D} - m_q)q \tag{1}$$

  SU(3) color gauge theory of quarks and gluons

- Observed hadrons [1]
  - qqq baryons $(p, n, \Lambda, \cdots)$: about 170 states
  - $q\bar{q}$ mesons $(\pi, K, \eta, \cdots)$: about 210 states
- Quarks have three colors: r, g, b
  All observed hadrons are color singlets (color-neutral states)
  No explicit constraint in QCD forbids colored states
  → Problem of **color confinement**
- Quarks have six flavors: u, d, s, c, b, t
  Flavor quantum numbers of conventional hadrons are described by qqq or $q\bar{q}$
  No explicit constraint in QCD forbids multiquark configurations such as $\bar{q}\bar{q}qq$, $\bar{q}qqqq$, $\cdots$
  → Problem of **exotic hadrons**

1.2 Exotic hadrons and resonances

- Recent observations of exotic hadron candidates [2, 3]
  Only ~ 10 candidates out of ~ 380 hadrons
- Pentaquark candidates $P_c(4312), P_c(4440), P_c(4457)$ [4, 5] (Fig. 1, left)

$$P_c \to J/\psi(\bar{c}c) + p(uud)$$

  Minimal quark content[1]: $\bar{c}cuud$

- Tetraquark candidate T_{cc} [6, 7] (Fig. 1, right)

$$T_{cc} \to D^0(\bar{u}c)D^{*+}(\bar{d}c) \to D^0D^0\pi^+$$

  Minimal quark content: $cc\bar{u}\bar{d}$

[1] In principle, P_c could be described by uud, but it is highly unnatural to explain their masses without a $\bar{c}c$ pair.

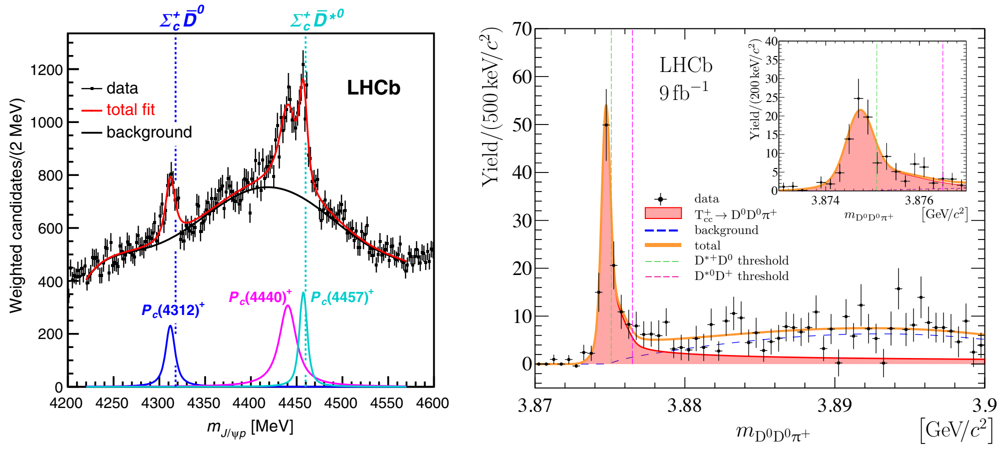

Figure 1: Examples of exotic hadron candidates. Left: spectrum of the pentaquark candidates P_c, adapted from R. Aaij *et al.* (LHCb collaboration), Phys. Rev. Lett. **122**, 222001 (2019). Right: spectrum of the tetraquark candidate T_{cc}, adapted from R. Aaij *et al.* (LHCb collaboration), Nature Phys. **18**, 751 (2022).

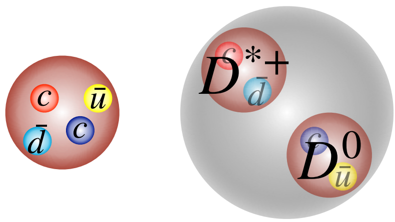

Figure 2: Schematic illustration of a multiquark state (left) and a hadronic molecule (right).

- Exotic hadrons are observed in the spectra of decay products

$$P_c \to J/\psi + p$$
$$T_{cc} \to D^0 + D^0 + \pi^+$$

  Unstable states → treated as **resonances** in hadron scattering
  In fact, only ∼ 20 out of ∼ 380 hadrons are stable

- Various possible internal structures
  - Multiquarks: compact color-singlet states
    $cc\bar{u}\bar{d}$ for T_{cc} (Fig. 2, left)
  - Hadronic molecules: weakly bound states of hadrons
    D^0D^{*+} for T_{cc} (Fig. 2, right)

- States with the same quantum numbers can mix with each other:

$$|T_{cc}\rangle = c_1 |cc\bar{u}\bar{d}\rangle + c_2 |D^0 D^{*+}\rangle + \cdots \tag{2}$$

How can we determine the weights c_i?
Is such a decomposition well-defined?
What is a resonance "state" $|T_{cc}\rangle$?

- Plan of the first half of this lecture
  - §2 (21 April): Resonances ↔ eigenstates with complex energies
  - §3 (28 April): Definition of the scattering amplitude
  - §4 (12 May): Resonances ↔ poles of the scattering amplitude
  - §5 (19 May): Feshbach resonances (bound states embedded in the continuum)
  - §6 (26 May): Nonrelativistic effective field theory
  - §7 (9 June): Compositeness and weak-binding relation

1.3 Hadron scattering and quantum entanglement

- Study of hadron scattering
Important for extracting resonance information
Provides basic information on hadronic molecules

- Prototype of hadron scattering: **nuclear force** (nucleon–nucleon scattering)

- Nucleons have internal degrees of freedom: **spin** and **isospin**
  - Spin $S = 1/2$: $|\uparrow\rangle, |\downarrow\rangle$
  - Isospin $I = 1/2$: $|p\rangle, |n\rangle$

- Two-nucleon states: combinations of spin and isospin
Many possible scattering channels: $|p\uparrow p\uparrow\rangle, |p\uparrow n\downarrow\rangle, \cdots$

- Symmetry in QCD: $\mathrm{SU}(2)_S \times \mathrm{SU}(2)_I$
The interaction is invariant under independent rotations of spin and isospin
Some processes are forbidden, and different processes are related to each other
→ Scattering can be described with a small number of independent components

- Low-energy s-wave scattering can be described by only **two channels**
  - ${}^1S_0$ $(S = 0, I = 1)$

- $^3S_1$ ($S = 1$, $I = 0$)

These channels are independent of each other

- Empirical properties of nuclear forces
  - Similar behavior in the $S = 0$ and $S = 1$ channels
    Spin-flavor SU(4) symmetry [8, 9]
  - Large scattering lengths (near the unitary limit)
    Nonrelativistic conformal symmetry [10]

$\rightarrow$ Suggestive of **emergent symmetries** beyond QCD

- Quantum entanglement in two-spin systems
  - Product state: no correlation between spins

$$|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle \tag{3}$$

  - Entangled state (Bell state): correlated spins

$$|S = 0\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \tag{4}$$

- **Entanglement entropy (EE)**
  A quantitative measure of quantum correlation
- Scattering as a quantum operation
  The scattering matrix S acts on the initial state:

$$|\text{out}\rangle = S\,|\text{in}\rangle \tag{5}$$

  S can generate entanglement between nucleons
- **Entanglement suppression** [11]
  Condition: scattering does not generate entanglement
  $\rightarrow$ Constraints on scattering amplitudes
  $\rightarrow$ Explanation of **emergent symmetries** in nuclear forces
- Plan of the second half of this lecture
  - §8 (16 June): Spin-1/2 states
  - §9 (23 June): Entanglement entropy

– §10 (30 June): Classical and quantum correlations

– §11 (7 July): Bell's inequality

– §12 (14 July): Entanglement power of the S matrix

– §13 (21 July): Nuclear forces and entanglement suppression

– §14 (28 July): Generalization to spin and flavor

1.4 References

- Resonances in quantum mechanics
  Textbook：羽田野-井村 [12], A. Bohm [13], N. Moiseyev [14]
  Review article：Ashida-Gong-Ueda [15]

- Scattering theory
  Textbook：J.R. Taylor [16], R.G. Newton [17], 永江-兵藤 [18]
  Review article：Hyodo-Niiyama [19]

- Exotic hadrons and compositeness
  Review article：T. Hyodo [20], Kinugawa-Hyodo [21]
  物理学会誌：兵藤哲雄 [22]
  数理科学：兵藤哲雄 [23]

- Quantum information science
  Textbook：石坂-小川-河内-木村-林 [24], Nielsen-Chuang [25]