Number: Name:

Exercise 1 (deadline: April 28, 2026)

1) Consider one-dimensional quantum mechanics defined on $x \in [0, L]$. For $E > 0$, find the eigenfunctions $\phi_n(x)$ that satisfy the free Schrödinger equation with boundary conditions $\phi(0) = \phi(L) = 0$, and express them using an integer n:

$$-\frac{1}{2\mu}\frac{d^2}{dx^2}\phi(x) = E\phi(x).$$

2) For the eigenfunctions $\{\phi_n\}$, show that the momentum operator $p = -i\,d/dx$ is Hermitian, namely,

$$\int_0^L \phi_n^*(x) p\phi_m(x) dx = \int_0^L [p\phi_n(x)]^* \phi_m(x) dx.$$

3) For the eigenfunctions $\{\phi_n\}$, examine whether the operator p^3 is Hermitian (pay attention to boundary terms).
