Number: Name:

Exercise 11 (no submission required)

1) Derive the expressions for s_x and s_y for a spin-3/2 particle given in Eq. (361). Recall the properties of the ladder operators

$$s_\pm = s_x \pm i s_y, \quad s_\pm |s, m\rangle = \sqrt{s(s+1) - m(m \pm 1)}\, |s, m \pm 1\rangle\,,$$

with

$$|{\Uparrow}\rangle = |3/2, 3/2\rangle\,, \quad |{\uparrow}\rangle = |3/2, 1/2\rangle\,, \quad |{\downarrow}\rangle = |3/2, -1/2\rangle\,, \quad |{\Downarrow}\rangle = |3/2, -3/2\rangle\,.$$

2) Derive the projection operator $\mathcal{J}_0$ in Eq. (368) using the following procedure. The operator

$$X = \boldsymbol{s}_A \cdot \boldsymbol{s}_B$$

for two spin-3/2 particles has the eigenvalues

$$x_S = \frac{1}{2}\left[S(S+1) - \frac{15}{2}\right],$$

for the total-spin states with $S = 0, 1, 2, 3$. Since X has four distinct eigenvalues, it satisfies the minimal polynomial

$$(X - x_0)(X - x_1)(X - x_2)(X - x_3) = 0.$$

Hence every power X^n with $n \geq 4$ can be reduced to a linear combination of $1, X, X^2$, and X^3. Therefore, each projection operator can be written as a cubic polynomial in X:

$$\mathcal{J}_0 = a + bX + cX^2 + dX^3,$$

Determine the coefficients a, b, c, and d.
