Number: Name:

Exercise 3 (deadline: May 26, 10:30 am, 2026)

1) Consider a perturbative solution of the pole condition (94). Let $E^{(n)}$ denote the nth-order contribution in $\hat{V}_t$ (or equivalently in $F(\boldsymbol{q})$) to the eigenenergy E, and expand the eigenenergy as

$$E = E^{(0)} + E^{(2)} + E^{(4)} + \cdots .$$

From Eq. (95), we have

$$E^{(0)} = \epsilon_0.$$

Because $\hat{V}_t$ is off-diagonal, there are no odd-order contributions to the energy E. Substituting this expansion into the self-energy $\Sigma(E)$, show that $E^{(2)} = \Sigma(\epsilon_0)$.

2) For $F(\boldsymbol{q}) = F(q^2)$ (i.e., spherically symmetric, depending only on q^2) and $\epsilon_0 > 0$, evaluate the imaginary part of E in the perturbative approximation $E = \epsilon_0 + \Sigma(\epsilon_0)$. Assume that $F(q^2)$ vanishes sufficiently rapidly as $|q^2| \to \infty$.
