

$$P_i e^{i\pi S_{in}} = P_f e^{i\pi S_{fn}}$$