

$$A_\mu = \frac{i}{2} \left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) = -\frac{\partial_\mu \phi}{\sqrt{2} f} + \mathcal{O}(p^3)$$