Compositeness of hadrons and near-threshold dynamics

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2014, Nov. 11th

Announcement

Long-term workshop held under Yukawa International Program for Quark-Hadron Sciences (YIPQS) and The Nishinomiya-Yukawa Memorial International Workshop

Hadrons and Hadron Interactions in QCD 2015

--- Effective Theories and Lattice ---

YIPQS YITP Kyoto

February 15 (Sun) - March 21 (Sat), 2015
Yukawa Institute for Theoretical Physics, Kyoto University, Japan

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Important dates and deadlines:

- Registration with local support and/or VISA request: November 30, 2014

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Dates: Feb - Mar 2015 (5 weeks)
Theme: EFT and lattice QCD
Abstract/support: by Nov. 30
Details: google "hhiqcd"

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- **Introduction: structure of hadrons**
- **Compositeness of hadrons and near-threshold bound states**

  S. Weinberg, Phys. Rev. 137, B672 (1965)
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

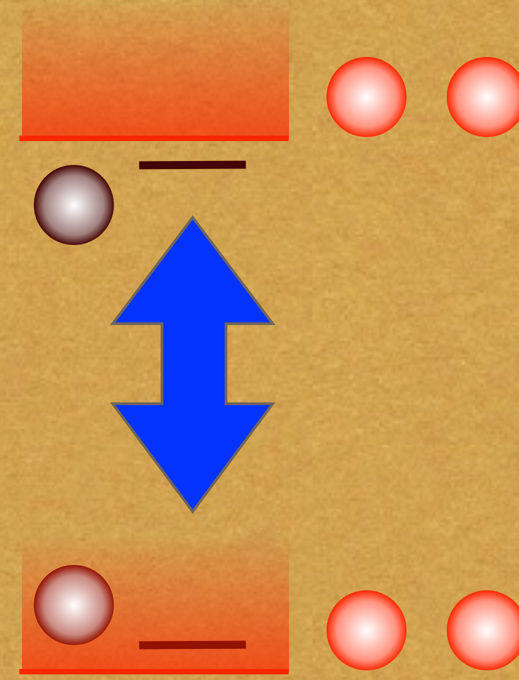
- **Near-threshold resonances**

  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

  Example: $\Lambda_c(2595)$ **(heavy!)**

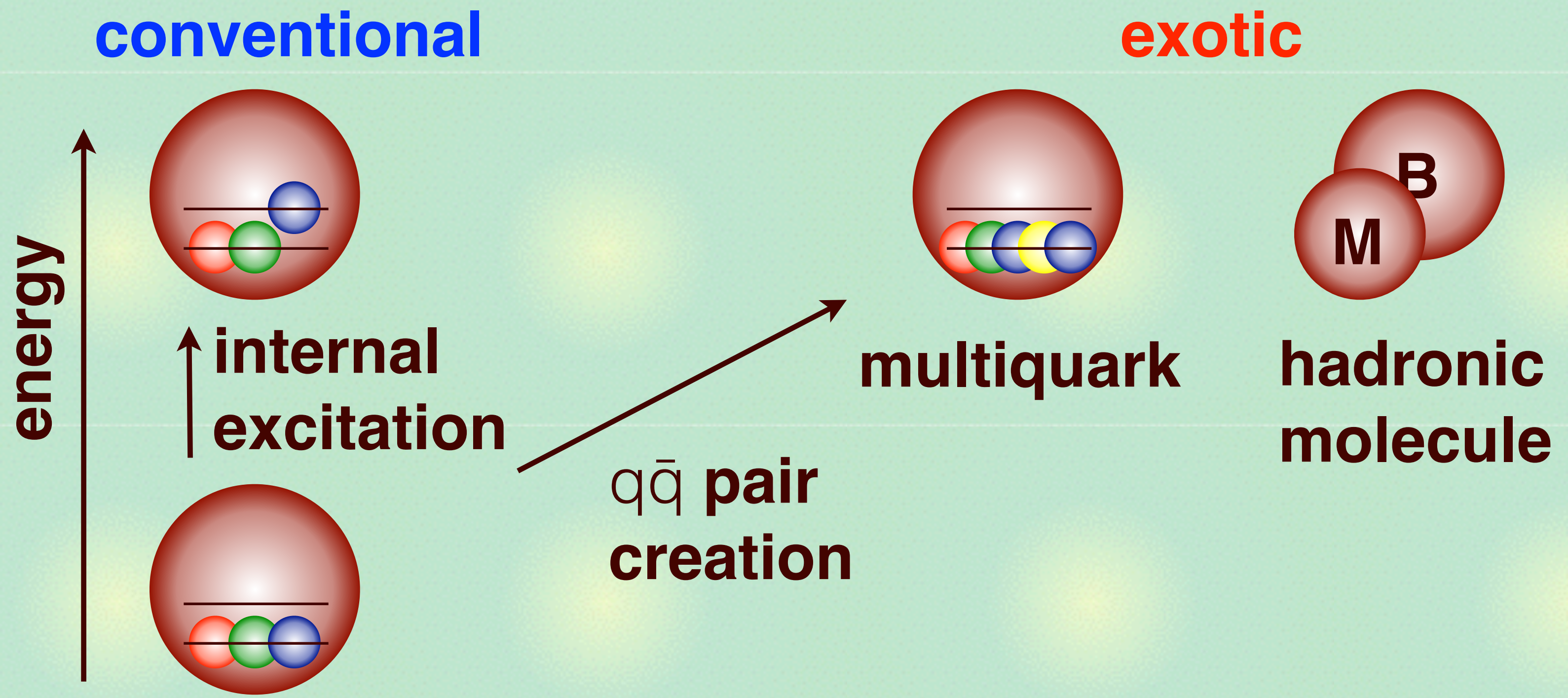
- **Near-threshold mass scaling**

  T. Hyodo, arXiv:1407.2372 [hep-ph], to appear in Phys. Rev. C



Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

$$|\Lambda(1405)\rangle = \underline{N_{3q}}|uds\rangle + \underline{N_{5q}}|uds\ q\bar{q}\rangle + \underline{N_{\bar{K}N}}|\bar{K}N\rangle + \cdots$$

Is this relevant strategy?

Ambiguity of definition of hadron structure

Decomposition of hadron "wave function"

$$|\,\Lambda(1405)\,\rangle = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\;q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

- **5q v.s. MB: double counting (orthogonality)?**

$$\langle\, udsq\bar{q}\,|\,\bar{K}N\,\rangle \neq 0$$

- **3q v.s. 5q: not clearly separated in QCD**

$$\langle\, uds\,|\,udsq\bar{q}\,\rangle \neq 0$$

- **hadron resonances: unstable, finite decay width**

$$|\,\Lambda(1405)\,\rangle = ?$$

How can we define the hadron structure?

What is the suitable basis to classify the hadron structure?

Strategy

Elementary or composite in terms of hadronic d.o.f., focusing on states near the lowest energy two-body threshold

elementary

- 6q **for deuteron**
- $c\bar{c}$ **for** X(3872)

composite

- NN **for deuteron**
- $\bar{D}D^*$ **for** X(3872)

- **orthogonality <— completeness relation**
- **normalization <— wave function normalization**
- **model dependence <— low energy universality**

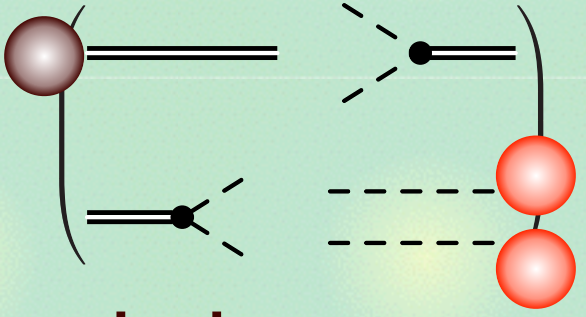
*** Basis must be asymptotic states (in QCD, hadrons).**

*** "Elementary" stands for any states other than two-body composite (CDD pole). Compact quark states, three-body, …**

Formulation

Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu}(+\hat{V}_{\rm sc}) \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



Elementariness by field renormalization constant

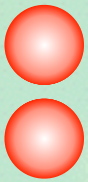
- Bound state normalization + completeness relation

$$\langle\,\Psi\,|\,\Psi\,\rangle = 1 \qquad 1 = |\,\psi_0\,\rangle\langle\,\psi_0\,| + \int d^3q|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|$$

$$1 = \underbrace{\left|\langle\,\Psi\,|\begin{pmatrix}|\,\psi_0\,\rangle \\ 0\end{pmatrix}\right|^2}_{} + \underbrace{\int d^3q \left|\langle\,\Psi\,|\begin{pmatrix}0 \\ |\,\boldsymbol{q}\,\rangle\end{pmatrix}\right|^2}_{} \equiv Z + X$$

bare state contribution

continuum contribution

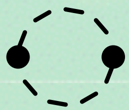


Z, X **: real and nonnegative --> probabilistic interpretation**

Weak binding limit

In general, Z is determined by the potential V.

$$Z(B)=\frac{1}{1-\frac{d}{dE}\int\frac{|\langle\psi_0|\hat{V}|\boldsymbol{q}\rangle|^2}{E-q^2/(2\mu)+i0^+}d^3q\Big|_{E=-B}}\equiv\frac{1}{1-\Sigma'(-B)}$$

$\Sigma(E)\sim$ 

In weak binding limit (R ≫ R_{typ}), Z is related to observables.

S. Weinberg, Phys. Rev. 137, B672 (1965)

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a=\frac{2(1-Z)}{2-Z}R+\mathcal{O}(R_{\text{typ}}),\quad r_e=\frac{-Z}{1-Z}R+\mathcal{O}(R_{\text{typ}}),$$

a : **scattering length,** r_e **: effective range**

R = $(2\mu B)^{-1/2}$ **: radius (binding energy)**

R_{typ} **: typical length scale of the interaction**

Criterion for the structure:

$$\begin{cases}a\sim R_{\text{typ}}\ll -r_e & \text{(elementary dominance)}, \quad Z\sim 1\\ a\sim R\gg r_e\sim R_{\text{typ}} & \text{(composite dominance)}. \quad Z\sim 0 \textbf{ (deuteron)}\end{cases}$$

Interpretation of negative effective range

For $Z > 0$, effective range is always negative.

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\rm typ}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\rm typ}),$$

$$\begin{cases} a \sim R_{\rm typ} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\rm typ} & \text{(composite dominance)}. \end{cases}$$

Simple attractive potential: $r_e > 0$

- only "composite dominance" is possible.

$r_e < 0$ **: energy- (momentum-)dependence of the potential**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

- pole term/Feshbach projection of coupled-channel effect

Negative r_e **—> something other than |p> : CDD pole**

Compositeness theorem

Exact $B \to 0$ **limit:**

If the s-wave scattering amplitude has a pole exactly at the threshold with a finite range interaction, then the field renormalization constant vanishes.

T. Hyodo, arXiv:1407.2372 [hep-ph], to appear in Phys. Rev. C

For bare state-continuum model (c**: nonzero constant)**

$$Z(B) = \frac{1}{1-\Sigma'(-B)} \approx \frac{1}{1 - c\frac{g_0^2}{\sqrt{B}}}$$

$\mathrm{Im}\,\Sigma(p^2/2\mu) \propto p^{2l+1}$

$Z(0)$ **vanishes for** $g_0 \neq 0$**. If** $g_0 = 0$**, no pole in the amplitude.**

For a local potential: poles in the effective range expansion

$$p_1 = i\sqrt{2\mu B}, \quad p_2 = -i\sqrt{2\mu B}\frac{2 - Z(B)}{Z(B)}$$

If $Z(0) \neq 0$**, then both** p_1 **and** p_2 **go to zero for** $B \to 0$

: contradict with simple pole at $p=0$

R.G. Newton, J. Math. Phys. 1, 319 (1960)

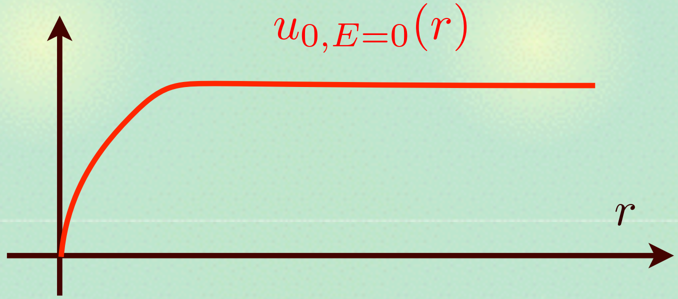
Interpretation of the compositeness theorem

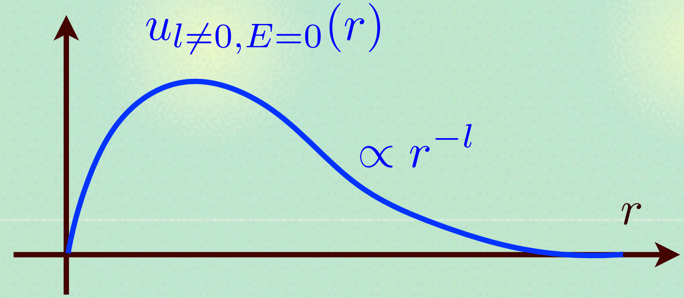
Z(B): **overlap of the bound state with bare state**

$$\left| \langle \Psi | \begin{pmatrix} | \psi_0 \rangle \\ 0 \end{pmatrix} \right|^2 + \int \left| \langle \Psi | \begin{pmatrix} 0 \\ | \boldsymbol{q} \rangle \end{pmatrix} \right|^2 d^3q = 1$$

- Z(B≠0)=0 **—> Bound state is completely composite.**

Two-body wave function at E=0: $u_{l,E=0}(r) \xrightarrow{r\to\infty} r^{-l}$





Z(0)=0: ~~**Bound state is completely composite.**~~
Composite component is infinitely large so that the fraction of any finite admixture of bare state is zero.

Generalization to resonances

Compositeness of bound states

$$Z(B) = \frac{1}{1 - \Sigma'(-B)}$$

Naive generalization to resonances:

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

$$\underline{Z(E_R)} = \frac{1}{1 - \Sigma'(-E_R)}$$

complex ↑ complex

- interpretation?

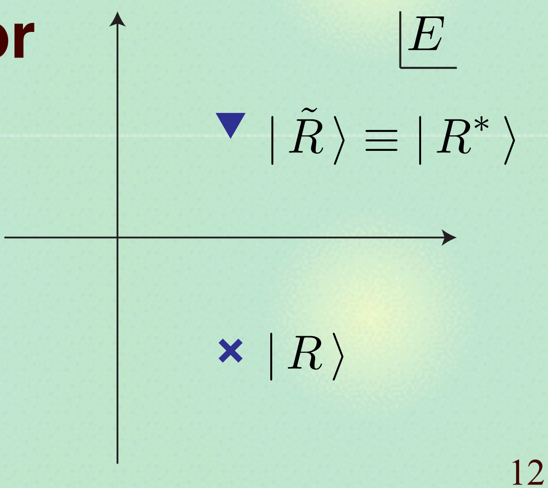
Normalization of resonances: Gamow vector

$$\langle R | R \rangle \to \infty, \quad \langle \tilde{R} | R \rangle = 1$$

$$1 = \underline{\langle \tilde{R} | B_0 \rangle \langle B_0 | R \rangle} + \int d\boldsymbol{p} \langle \tilde{R} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | R \rangle$$

complex

$$\langle \tilde{R} | B_0 \rangle = \langle B_0 | R \rangle \neq \langle B_0 | R \rangle^*$$



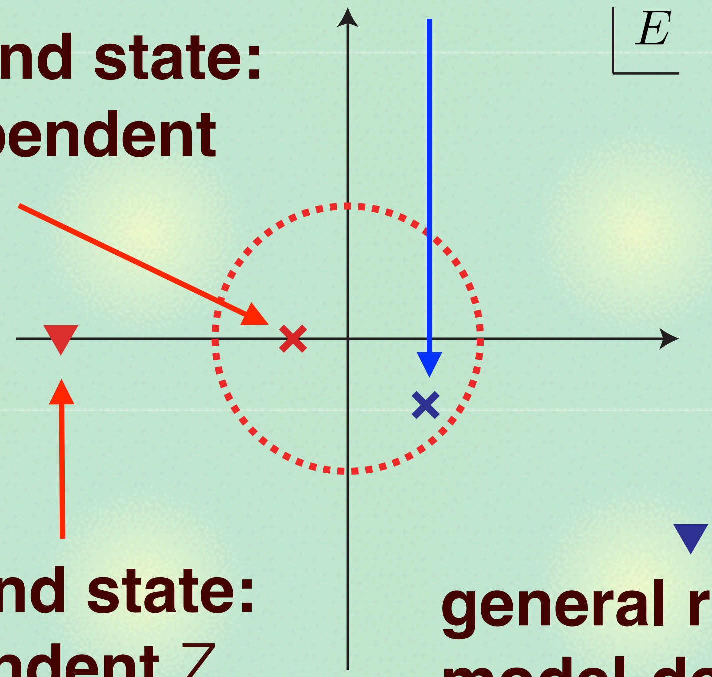
Near-threshold resonances

Weak binding limit for bound states

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables

What about near-threshold resonances (~ small binding)?

shallow bound state:
model-independent
structure



general bound state:
model-dependent Z

general resonance:
model-dependent Z

Poles in the effective range expansion

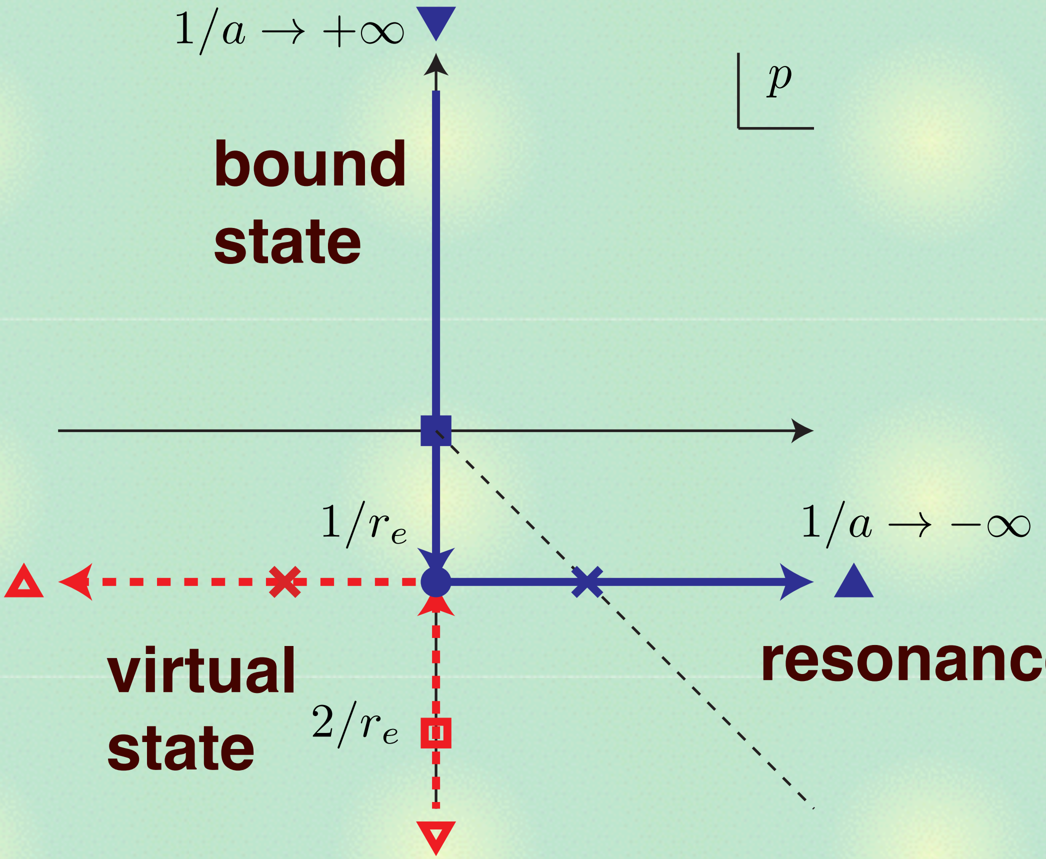
Near-threshold phenomena: effective range expansion

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013) with opposite sign of scattering length

$$f(p) = \left(-\frac{1}{a} + \frac{r_e^2}{2}p^2 - ip\right)^{-1}$$

$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a} - 1}$$

- pole trajectories with a fixed $r_e < 0$



Resonance pole position <--> (a, r_e)

Application: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering

- central values in PDG

$$E = 0.67 \text{ MeV}, \quad \Gamma = 2.59 \text{ MeV} \qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5 \text{ fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5 \text{ fm}$$

- field renormalization constant: complex

$$Z = 1 - 0.608i$$

Large negative effective range

<— substantial elementary contribution other than $\pi\Sigma_c$ (three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ is **not likely a $\pi\Sigma_c$ molecule**

Hadron mass scaling and threshold effect

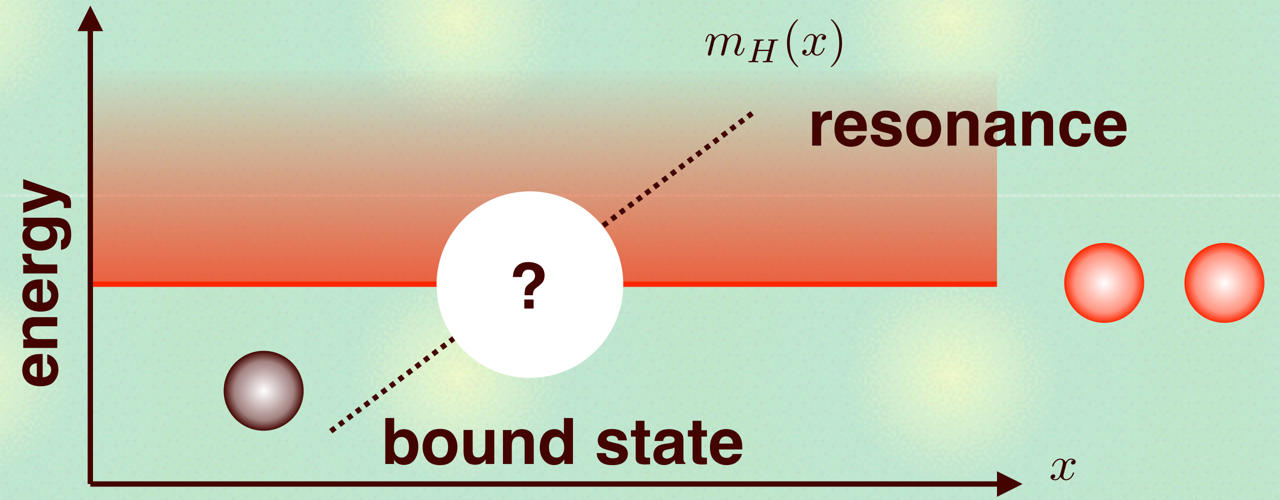
Systematic expansion of hadron masses

- ChPT: light quark mass m_q
- HQET: heavy quark mass m_Q
- large Nc: number of colors Nc

Hadron mass scaling

$$m_H(x); \quad x = \frac{m_q}{\Lambda}, \frac{\Lambda}{m_Q}, \frac{1}{N_c}$$

What happens at two-body threshold?



Formulation

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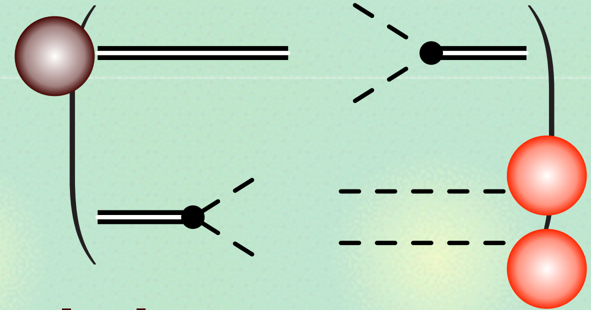
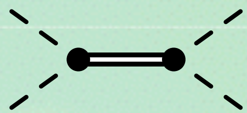
$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu}(+\hat{V}_{\rm sc}) \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$

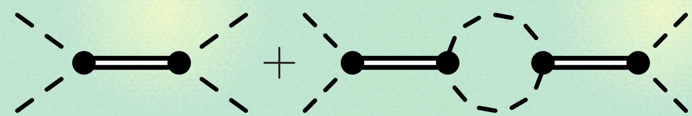


Equivalent single-channel scattering formulation

$$\hat{V}_{\rm eff}(E) = \frac{\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}}{E - M_0} \sim$$



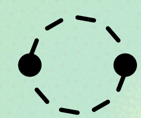
$$f(\boldsymbol{p}, \boldsymbol{p}', E) = -\frac{4\pi^2\mu\langle\,\boldsymbol{p}\,|\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{p}'\,\rangle}{E - M_0 - \Sigma(E)} \sim$$



Pole condition:

$$E_h - M_0 = \Sigma(E_h)$$

$$\Sigma(E) = \int \frac{\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|\hat{V}|\,\psi_0\,\rangle}{E - q^2/(2\mu) + i0^+} d^3q \sim$$

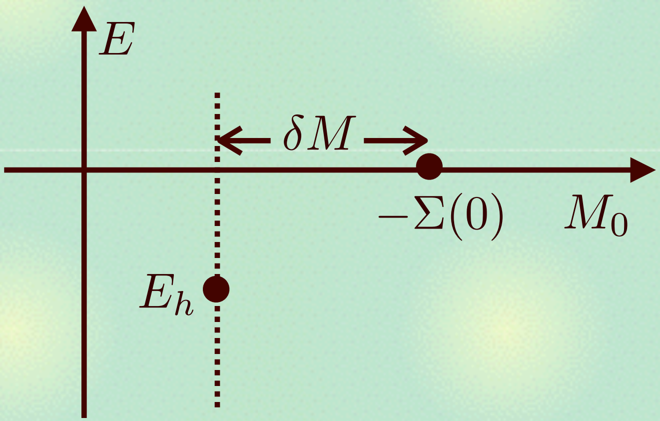


Question: How E_h behaves against M_0 around $E_h=0$?

Near-threshold bound state

Bound state condition around $E_h=0$

$$E_h + \Sigma(0) - \delta M = \Sigma(E_h)$$



Leading contribution of the expansion:

$$E_h = \frac{1}{1-\Sigma'(0)}\delta M = Z(0)\delta M, \quad \Sigma'(E) \equiv \frac{d\Sigma(E)}{dE}$$

Field renormalization constant

$$\left|\langle \Psi | \begin{pmatrix} |\psi_0\rangle \\ 0 \end{pmatrix}\right|^2 + \int \left|\langle \Psi | \begin{pmatrix} 0 \\ |\boldsymbol{q}\rangle \end{pmatrix}\right|^2 d^3q = 1$$

$Z(0)$ **vanishes for** $l=0$: **compositeness theorem**

$$E_h \propto \begin{cases} \mathcal{O}(\delta M^2) & l=0 \\ \delta M & l \neq 0 \end{cases}$$

Near-threshold bound state (general)

General argument by Jost function (Fredholm determinant)

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972)

$$f_l(p) = \frac{\not{f}_l(-p) - \not{f}_l(p)}{2ip\not{f}_l(p)}$$

pole (eigenstate) = Jost function zero

Expansion of the Jost function:

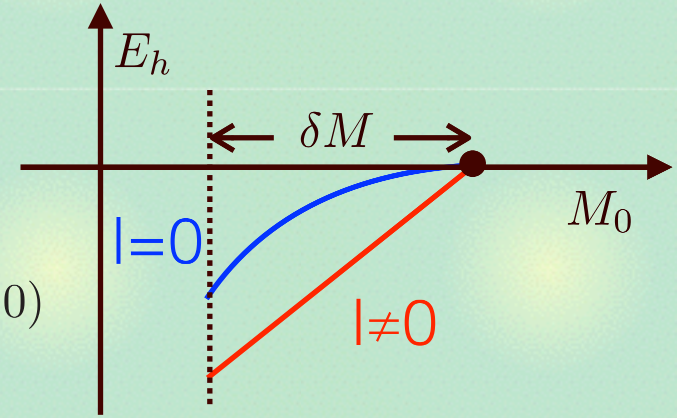
$$\not{f}_l(p) = \begin{cases} 1 + \alpha_0 + i\gamma_0 p + \mathcal{O}(p^2) & l = 0 \\ 1 + \alpha_l + \beta_l p^2 + \mathcal{O}(p^3) & l \neq 0 \end{cases}$$

- γ_0 **and** β_l **are nonzero for a general local potential**
- **zero at** $p=0$ **(**$1+\alpha_l=0$**) must be simple (double) for** $l=0$ **(**$l\neq 0$**)**

R.G. Newton, J. Math. Phys. 1, 319 (1960)

Near-threshold scaling:

$$1 + \alpha_l \sim \delta M \quad \Rightarrow \quad E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases} \quad (\delta M < 0)$$



General threshold behavior

Near threshold scaling:

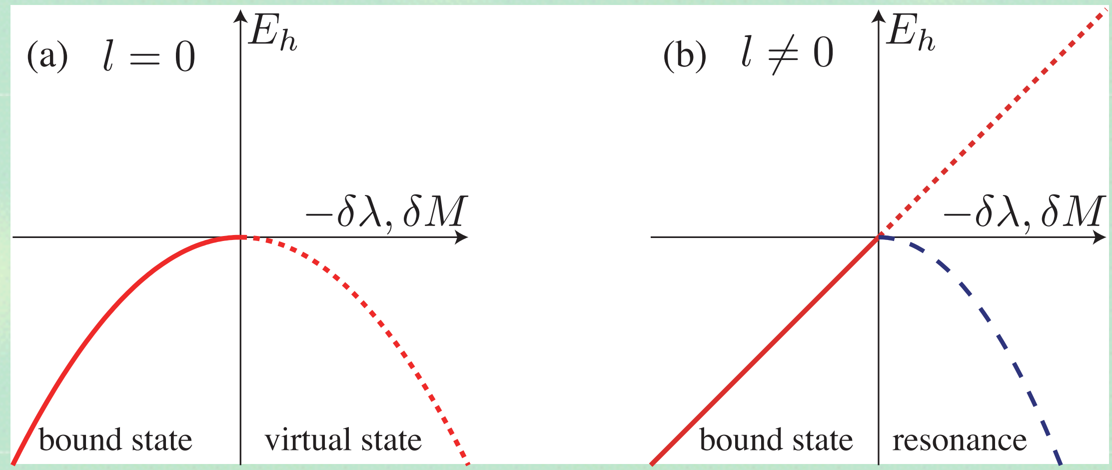
- $\delta M < 0$

$$E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases}$$

- $\delta M > 0$

$$E_h \propto -\delta M^2 \quad l = 0$$

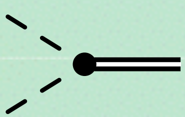
$$\begin{cases} \mathrm{Re}\ E_h \propto \delta M \\ \mathrm{Im}\ E_h \propto -(\delta M)^{l+1/2} \end{cases} \quad l \neq 0$$



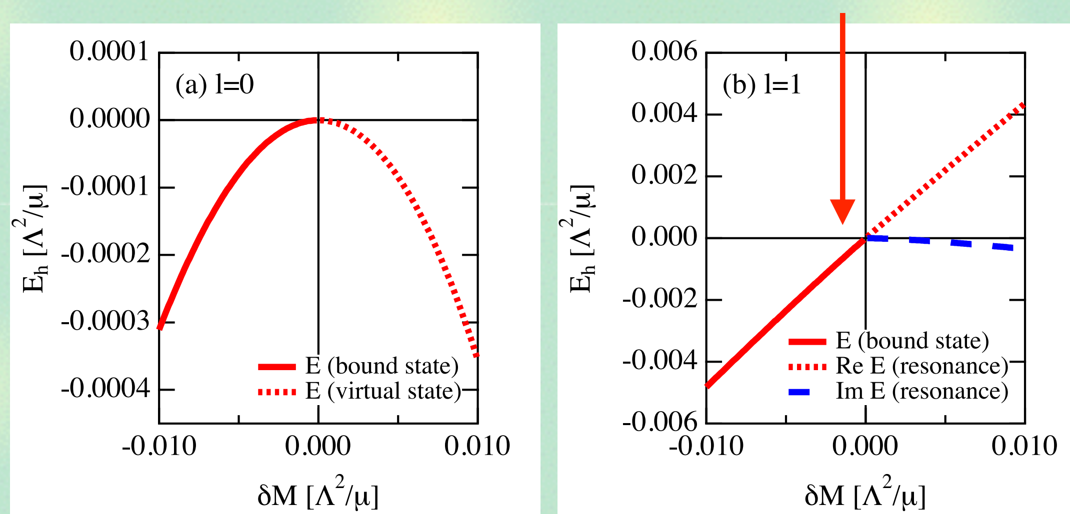
c.f. NN $^1S_0$

slope: Z(0)

Numerical calculation

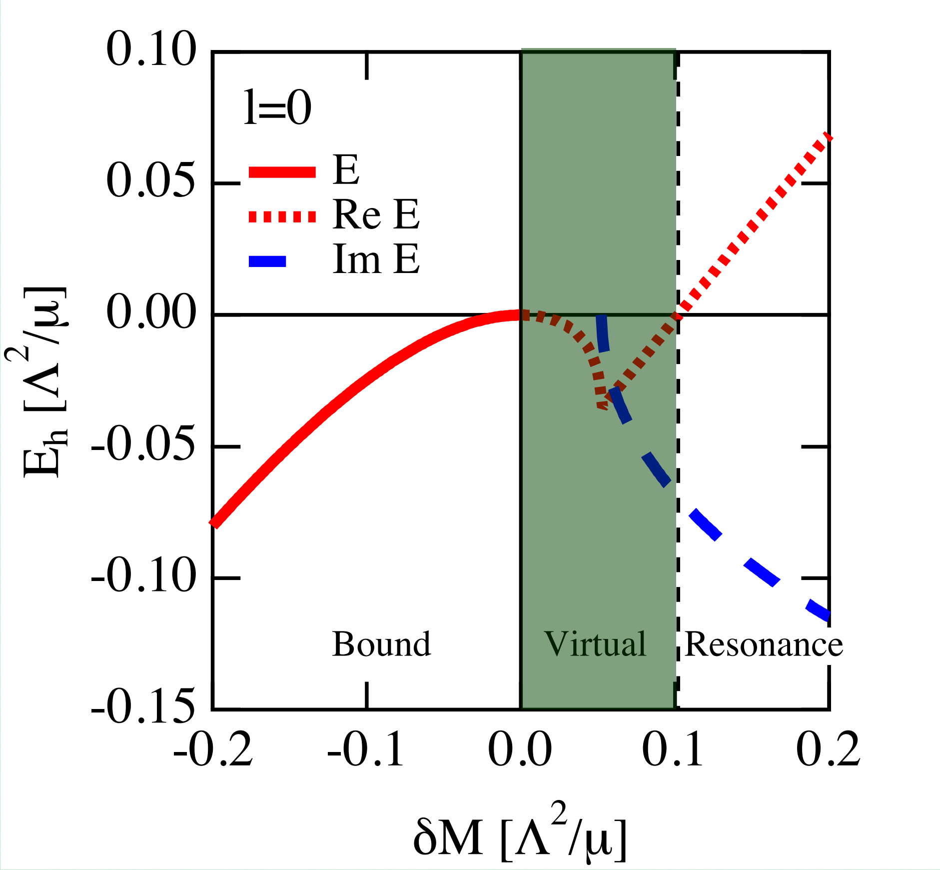


$$\langle \boldsymbol{q} | \hat{V} | \psi_0 \rangle = g_l |\boldsymbol{q}|^l \Theta(\Lambda - |\boldsymbol{q}|)$$



Chiral extrapolation across s-wave threshold

Scaling in wider energy region



Near-threshold scaling: nonperturbative phenomenon

—> Naive ChPT does not work. Resummation is needed.

c.f.) NN **sector,** $\bar{K}$N **sector, ...**

Summary

Compositeness of hadrons near threshold

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)
T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)
T. Hyodo, arXiv:1407.2372 [hep-ph], to appear in Phys. Rev. C

- Compositeness / elementariness
  - suitable classification for hadron structure
  - model independent in the weak binding limit
- Near-threshold resonance:
  - structure from effective range
- Near-threshold mass scaling:
  - caution on the chiral extrapolation