

$\Lambda(1405)$ as a hadronic molecule



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2020, Sep. 28th 1



Introduction

- Structure of “unstable” resonance?



Structure of $\Lambda(1405)$ resonance

- Accurate $\bar{K}N$ scattering amplitude

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);

M. Tanabashi, *et al.* (Particle Data Group), PRD 98, 030001 (2018)

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

- $\bar{K}N$ compositeness

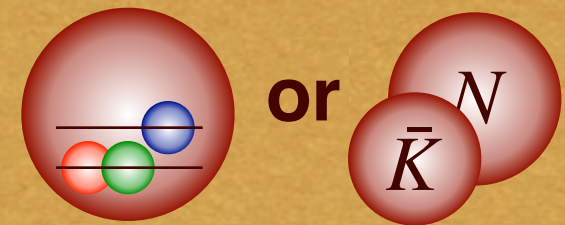
Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

T. Hyodo, JPS journal Vol. 75 No. 8, 478 (2020)

T. Kinugawa, T. Hyodo, in preparation



Summary



Classification of hadrons

Observed hadrons

PDG2018 : <http://pdg.lbl.gov/>

Only **color singlet** states are observed.

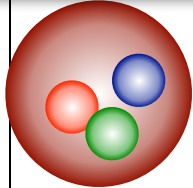
—> Color confinement problem

Flavor quantum numbers are described by $qqq/q\bar{q}$.

Why no $qqq\bar{q}\bar{q}$, $qqqqq\bar{q}$, ... states (**exotic hadrons**)?

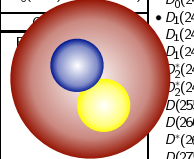
—> Exotic hadron problem, as nontrivial as confinement!

$\Lambda(2700)$ $13/2^+$ **	$\Lambda(1710)$ $1/2^-$ *	$\Sigma(3000)$ *
$\Lambda(1800)$ $1/2^-$ ***	$\Lambda(1810)$ $1/2^+$ ***	$\Sigma(3170)$ *
$\Lambda(1820)$ $5/2^+$ ****	$\Lambda(1830)$ $5/2^-$ ****	
$\Lambda(1890)$ $3/2^+$ ****	$\Lambda(2000)$ *	
$\Lambda(2020)$ $7/2^+$ *	$\Lambda(2100)$ $7/2^-$ ****	
$\Lambda(2110)$ $5/2^+$ ***	$\Lambda(2325)$ $3/2^-$ *	
$\Lambda(2350)$ $9/2^+$ ***	$\Lambda(2585)$ **	



~ 150 baryons

$a_1(1640)$ $1^-(1^-)$ *	$a_0(2430)$ $1^-(6^-)$ *	$D_0^*(2400)^0$ $1/2(0^+)$	BOTTOM, CHARMED ($B = C = \pm 1$)	$\chi_{B1}(1P)$ $0^-(1^-)$ *
$f_2(1640)$ $0^+(2^{++})$	$f_0(2510)$ $0^+(6^{++})$	$D_0^*(2400)^{\pm}$ $1/2(0^+)$	$B_c^{\pm}(2S)^{\pm}$ $0^-(?)$	$\chi_{B2}(1P)$ $0^+(2^{++})$
$\omega(1650)$ $0^-(1^-)$		$D_1(2420)^0$ $1/2(1^+)$		$\eta_B(2S)$ $0^+(0^-)$
$\omega_3(1670)$ $0^-(3^-)$		$D_1(2420)^{\pm}$ $1/2(1^+)$		$\eta_B(2S)$ $0^-(1^-)$
$\pi_2(1670)$ $1^-(2^-)$		$D_2(2460)^0$ $1/2(2^+)$		$\eta_B(2P)$ $0^+(0^+)$
		$D_2(2460)^{\pm}$ $1/2(2^+)$		$\eta_B(2P)$ $0^+(1^+)$
		$D(2550)^0$ $1/2(0^-)$		$h_B(2P)$ $0^-(1^-)$
		$D(2600)$ $1/2(?)$		$\chi_{B2}(2P)$ $0^+(2^{++})$
		$D^*(2640)^{\pm}$ $1/2(?)$		$\chi(3S)$ $0^-(1^-)$
		$D(2750)$ $1/2(?)$		$\chi_{B1}(3P)$ $0^+(1^+)$



~ 210 mesons

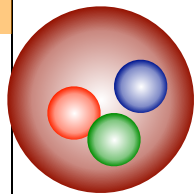
All ~ 360 hadrons emerge from single QCD Lagrangian.

Unstable states via strong interaction

Stable/unstable hadrons

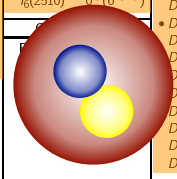
PDG2018 : <http://pdg.lbl.gov/>

p	$1/2^+$ ****	$\Delta(1232)$	$3/2^+$ ****	Σ^+	$1/2^+$ ****	Ξ^0	$1/2^+$ ****	Λ_c^+	$1/2^+$ ****
n	$1/2^+$ ****	$\Delta(1600)$	$3/2^+$ ***	Σ^0	$1/2^+$ ****	Ξ^-	$1/2^+$ ****	$\Lambda_c(2595)^+$	$1/2^-$ ***
$N(1440)$	$1/2^+$ ****	$\Delta(1620)$	$1/2^-$ ****	Σ^-	$1/2^+$ ****	$\Xi(1530)$	$3/2^+$ ****	$\Lambda_c(2625)^+$	$3/2^-$ ***
$N(1520)$	$3/2^-$ ****	$\Delta(1700)$	$3/2^-$ ****	$\Sigma(1385)$	$3/2^+$ ****	$\Xi(1620)$	*	$\Lambda_c(2765)^+$	*
$N(1535)$	$1/2^-$ ****	$\Delta(1750)$	$1/2^+$ *	$\Sigma(1480)$	*	$\Xi(1690)$	***	$\Lambda_c(2880)^+$	$5/2^+$ ***
$N(1650)$	$1/2^-$ ****	$\Delta(1900)$	$1/2^-$ **	$\Sigma(1560)$	**	$\Xi(1820)$	$3/2^-$ ***	$\Lambda_c(2940)^+$	***
$N(1675)$	$5/2^-$ ****	$\Delta(1905)$	$5/2^+$ ****	$\Sigma(1580)$	$3/2^-$ **	$\Xi(1950)$	***	$\Sigma_c(2455)$	$1/2^+$ ****
$N(1680)$	$5/2^+$ ****	$\Delta(1910)$	$1/2^+$ ****	$\Sigma(1620)$	$1/2^-$ *	$\Xi(2030)$	$\geq 5/2^-$ ***	$\Sigma_c(2520)$	$3/2^+$ ***
$N(1685)$	*	$\Delta(1920)$	$3/2^+$ ***	$\Sigma(1660)$	$1/2^+$ ***	$\Xi(2120)$	**	$\Sigma_c(2800)$	***
$N(1700)$	$3/2^-$ ***	$\Delta(1930)$	$5/2^-$ ***	$\Sigma(1670)$	$3/2^-$ ****	$\Xi(2250)$	*	Ξ_c^+	$1/2^+$ ***
$N(1710)$	$1/2^+$ ***	$\Delta(1940)$	$3/2^-$ **	$\Sigma(1690)$	**	$\Xi(2370)$	**	Ξ_c^0	$1/2^+$ ***
$N(1720)$	$3/2^+$ ****	$\Delta(1950)$	$7/2^+$ ****	$\Sigma(1730)$	$3/2^+$ **	$\Xi(2500)$	*	Ξ_c^+	$1/2^+$ ***
$N(1860)$	$5/2^+$ **	$\Delta(2000)$	$5/2^+$ **	$\Sigma(1750)$	$1/2^-$ ***			Ξ_c^0	$1/2^+$ ***
$N(1875)$	$3/2^-$ ***	$\Delta(2150)$	$1/2^-$ *	$\Sigma(1770)$	$1/2^+$ *	Ω^-	$3/2^+$ ****	$\Xi_c(2645)$	$3/2^+$ ***
$N(1880)$	$1/2^+$ **	$\Delta(2200)$	$7/2^-$ *	$\Sigma(1775)$	$5/2^-$ ****	$\Omega(2250)$	***	$\Xi_c(2790)$	$1/2^-$ ***
$N(1895)$	$1/2^-$ **	$\Delta(2300)$	$9/2^+$ **	$\Sigma(1840)$	$3/2^+$ **	$\Omega(2380)$	**	$\Xi_c(2815)$	$3/2^-$ ***
$N(1900)$	$3/2^+$ ***	$\Delta(2350)$	$5/2^-$ *	$\Sigma(1880)$	$1/2^+$ **	$\Omega(2470)$	**	$\Xi_c(2930)$	*
$N(1990)$	$7/2^+$ **	$\Delta(2390)$	$7/2^+$ *	$\Sigma(1900)$	$1/2^-$ *			$\Xi_c(2980)$	***
$N(2000)$	$5/2^+$ **	$\Delta(2400)$	$9/2^-$ **	$\Sigma(1915)$	$5/2^+$ ****			$\Xi_c(3055)$	***
$N(2040)$	$3/2^+$ **	$\Delta(2420)$	$11/2^+$ ****	$\Sigma(1940)$	$3/2^+$ **			$\Xi_c(3080)$	***
$N(2060)$	$5/2^-$ **	$\Delta(2750)$	$13/2^-$ **	$\Sigma(1940)$	$3/2^-$ ***			$\Xi_c(3123)$	*
$N(2100)$	$1/2^+$ *	$\Delta(2950)$	$15/2^+$ **	$\Sigma(2000)$	$1/2^-$ *			Ω_c^0	$1/2^+$ ***
$N(2120)$	$3/2^-$ **			$\Sigma(2030)$	$7/2^+$ ****			$\Omega_c(2770)^0$	$3/2^+$ ***
$N(2190)$	$7/2^-$ ****	Λ	$1/2^+$ ****	$\Sigma(2070)$	$5/2^+$ *			Ξ_{cc}^+	*
$N(2220)$	$9/2^+$ ****	$\Lambda(1405)$	$1/2^-$ ****	$\Sigma(2080)$	$3/2^+$ **			Λ_b^0	$1/2^+$ ***
$N(2250)$	$9/2^-$ ****	$\Lambda(1520)$	$3/2^-$ ****	$\Sigma(2100)$	$7/2^-$ **			$\Lambda_b(5912)^0$	$1/2^-$ ***
$N(2300)$	$1/2^+$ **	$\Lambda(1600)$	$1/2^+$ ***	$\Sigma(2250)$	***			$\Lambda_b(5920)^0$	$3/2^-$ ***
$N(2570)$	$5/2^-$ **	$\Lambda(1670)$	$1/2^-$ ****	$\Sigma(2455)$	**			Σ_b	$1/2^+$ ***
$N(2600)$	$11/2^-$ ***	$\Lambda(1690)$	$3/2^-$ ****	$\Sigma(2620)$	**			Σ_b	$3/2^+$ ***
$N(2700)$	$13/2^+$ **	$\Lambda(1710)$	$1/2^+$ *	$\Sigma(3000)$	*			Ξ_b^0	$1/2^+$ ***
		$\Lambda(1800)$	$1/2^-$ ***	$\Sigma(3170)$	*			Ξ_b^-	$1/2^+$ ***
		$\Lambda(1810)$	$1/2^+$ ***					$\Xi_b(5935)$	$1/2^+$ ***
		$\Lambda(1820)$	$5/2^+$ ****					$\Xi_b(5945)^0$	$3/2^+$ ***
		$\Lambda(1830)$	$5/2^-$ ****					$\Xi_b(5955)$	$3/2^+$ ***
		$\Lambda(1890)$	$3/2^+$ ****					Ω_b	$1/2^+$ ***
		$\Lambda(2000)$	*						
		$\Lambda(2020)$	$7/2^+$ *						
		$\Lambda(2050)$	$3/2^-$ *						
		$\Lambda(2100)$	$7/2^-$ ****						
		$\Lambda(2110)$	$5/2^+$ ***						
		$\Lambda(2325)$	$3/2^-$ *						
		$\Lambda(2350)$	$9/2^+$ ***						
		$\Lambda(2585)$	**						



~ 150 baryons

LIGHT UNFLAVORED ($S = C = B = 0$)		STRANGE ($S = \pm 1, C = B = 0$)		CHARMED, STRANGE ($C = S = \pm 1$)		$c\bar{c}$ $F_c(F_c)$	
$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$	$F_c(F_c)$
π^+	$1^-(0^-)$	$\rho(1680)$	$0^-(1^-)$	K^+	$1/2(0^-)$	D_s^+	$0(0^-)$
π^0	$1^-(0^-)$	$\rho(1690)$	$1^+(3^-)$	K^0	$1/2(0^-)$	D_s^0	$0(0^-)$
η	$0^+(0^-)$	$\rho(1700)$	$1^+(1^-)$	K_S^0	$1/2(0^-)$	D_{s1}^+	$0(0^+)$
η'	$0^+(0^+)$	$\rho(1700)$	$1^-(2^+)$	K_L^0	$1/2(0^-)$	D_{s1}^0	$0(0^+)$
$\phi(500)$	$0^+(0^+)$	$\phi(1710)$	$0^+(0^+)$	$K_S^0(800)$	$1/2(0^+)$	$D_{s1}(2460)^+$	$0(1^+)$
$\rho(770)$	$1^+(1^-)$	$\eta(1760)$	$0^+(0^-)$	$K^*(892)$	$1/2(1^-)$	$D_{s1}(2536)^+$	$0(1^+)$
$\omega(782)$	$0^-(1^-)$	$\eta(1760)$	$1^-(1^-)$	$K^*(1270)$	$1/2(1^+)$	$D_{s2}(2573)$	$0(2^+)$
$\eta(958)$	$0^+(0^+)$	$\pi(1800)$	$1^-(0^-)$	$K_2^*(1400)$	$1/2(1^+)$	$D_{s1}(2700)^+$	$0(1^-)$
$\phi(980)$	$0^+(0^+)$	$\phi(1810)$	$0^+(2^+)$	$K_2^*(1410)$	$1/2(1^-)$	$D_{s1}(2860)^+$	$0(2^+)$
$a_0(980)$	$1^-(0^+)$	$X(1835)$	$2^2(2^+)$	$K_2^*(1430)$	$1/2(1^-)$	$D_{s1}(3040)^+$	$0(2^+)$
$\phi(1020)$	$0^-(1^-)$	$X(1840)$	$2^2(2^+)$	$K_2^*(1430)$	$1/2(0^+)$		
$h_1(1170)$	$0^-(1^+)$	$\phi(1850)$	$0^-(3^-)$	$K_2^*(1430)$	$1/2(2^-)$	BOTTOM ($B = \pm 1$)	
$b_1(1235)$	$1^+(1^+)$	$\eta(1870)$	$0^+(2^-)$	$K_2^*(1460)$	$1/2(0^-)$	B^+	$1/2(0^-)$
$a_1(1260)$	$1^-(1^+)$	$\pi_2(1880)$	$1^-(2^+)$	$K_2^*(1580)$	$1/2(2^-)$	B^0	$1/2(0^-)$
$f_2(1270)$	$0^+(2^+)$	$\phi(1900)$	$0^+(1^-)$	$K_2^*(1630)$	$1/2(2^+)$	$B^+/\bar{B}^0$ ADMIXTURE	$1/2(0^-)$
$f_1(1285)$	$0^+(1^+)$	$f_2(1910)$	$0^+(2^+)$	$K_2^*(1650)$	$1/2(1^+)$	$B^+/\bar{B}^0/\bar{B}_s^0/b$ -baryon	$1/2(0^-)$
$\eta(1295)$	$0^+(0^-)$	$\phi(1950)$	$0^+(2^+)$	$K_2^*(1680)$	$1/2(1^-)$	ADMIXTURE	$1/2(0^-)$
$\pi(1300)$	$1^-(0^-)$	$\phi(1990)$	$1^+(3^-)$	$K_2^*(1770)$	$1/2(2^-)$	V_b and V_{cb} CKM	$1/2(0^-)$
$\phi_2(1320)$	$1^-(2^+)$	$f_2(2010)$	$0^+(2^+)$	$K_2^*(1780)$	$1/2(3^-)$	Matrix Elements	$1/2(0^-)$
$f_0(1370)$	$0^+(0^+)$	$f_0(2020)$	$0^+(0^+)$	$K_2^*(1820)$	$1/2(2^-)$	B^+	$1/2(1^-)$
$h_1(1380)$	$1^-(1^+)$	$a_1(2040)$	$1^-(4^+)$	$K_2^*(1830)$	$1/2(0^-)$	$B_1(5721)^+$	$1/2(1^+)$
$\pi_1(1400)$	$1^-(1^+)$	$f_4(2050)$	$0^+(4^+)$	$K_2^*(1850)$	$1/2(0^+)$	$B_1(5721)^0$	$1/2(1^+)$
$\eta(1405)$	$0^+(0^-)$	$\pi_2(2100)$	$1^-(2^+)$	$K_2^*(1980)$	$1/2(2^+)$	$B_2(5732)$	$1/2(2^+)$
$f_1(1420)$	$0^+(1^+)$	$f_0(2100)$	$0^+(0^+)$	$K_2^*(2045)$	$1/2(4^+)$	$B_2(5747)^+$	$1/2(2^+)$
$\omega(1420)$	$0^-(1^-)$	$f_2(2150)$	$0^+(2^+)$	$K_2^*(2250)$	$1/2(2^-)$	$B_2(5747)^0$	$1/2(2^+)$
$f_2(1430)$	$0^+(2^+)$	$\mu(2170)$	$1^-(1^-)$	$K_2^*(2320)$	$1/2(3^+)$	$B(5970)^+$	$1/2(2^+)$
$a_0(1450)$	$1^-(0^+)$	$\phi(2170)$	$0^-(1^-)$	$K_2^*(2380)$	$1/2(5^-)$	$B(5970)^0$	$1/2(2^+)$
$\phi(1450)$	$0^-(1^-)$	$f_2(2200)$	$0^+(0^+)$	$K_2^*(2500)$	$1/2(4^-)$		
$\eta(1475)$	$0^+(0^-)$	$f_2(2225)$	$0^+(2^+)$	$K_2^*(3100)$	$1/2(4^-)$	BOTTOM, STRANGE ($B = \pm 1, S = \pm 1$)	
$f_0(1500)$	$0^+(0^+)$	$\eta(2250)$	$0^+(0^-)$			B_s^+	$0(0^-)$
$f_1(1510)$	$0^+(1^+)$	$\rho(2250)$	$1^+(3^-)$	CHARMED ($C = \pm 1$)		B_s^0	$0(0^-)$
$f_2(1525)$	$0^+(2^+)$	$f_2(2300)$	$0^+(2^+)$	D^+	$1/2(0^-)$	B_c^+	$0(1^-)$
$f_2(1565)$	$0^+(2^+)$	$f_2(2300)$	$0^+(4^+)$	D^0	$1/2(0^-)$	B_c^0	$0(1^-)$
$\rho(1570)$	$1^+(1^+)$	$f_2(2330)$	$0^+(0^+)$	$D^-(2007)^0$	$1/2(1^-)$	$B_c^-(5840)^0$	$0(2^+)$
$h_1(1595)$	$0^-(1^+)$	$f_2(2340)$	$0^+(2^+)$	$D^-(2010)^+$	$1/2(1^-)$	$B_c^-(5850)$	$0(2^+)$
$\pi_1(1600)$	$1^-(1^+)$	$\rho(2350)$	$1^+(5^-)$	$D^-(2017)^+$	$1/2(1^-)$	BOTTOM, CHARMED ($B = C = \pm 1$)	
$\Lambda(1640)$	$1^-(1^+)$	$a_6(2450)$	$1^-(6^+)$	$D_0^-(2400)^+$	$1/2(0^+)$	B_c^+	$0(0^-)$
$f_2(1640)$	$0^+(2^+)$	$f_2(2510)$	$0^+(6^+)$	$D_0^-(2420)^+$	$1/2(1^+)$	$B_c(25)^+$	$1/2(2^+)$
$\rho(1645)$	$0^+(2^+)$			$D_0^-(2460)^+$	$1/2(2^+)$		
$\omega(1650)$	$0^-(1^-)$			$D_0^-(2460)^+$	$1/2(1^+)$		
$\omega_3(1670)$	$0^-(3^-)$			$D_0^-(2550)^0$	$1/2(0^-)$		
$\pi_2(1670)$	$1^-(2^+)$			$D_0^-(2600)$	$1/2(2^+)$		
				$D^-(2640)^+$	$1/2(2^+)$		
				$D(2750)$	$1/2(2^+)$		

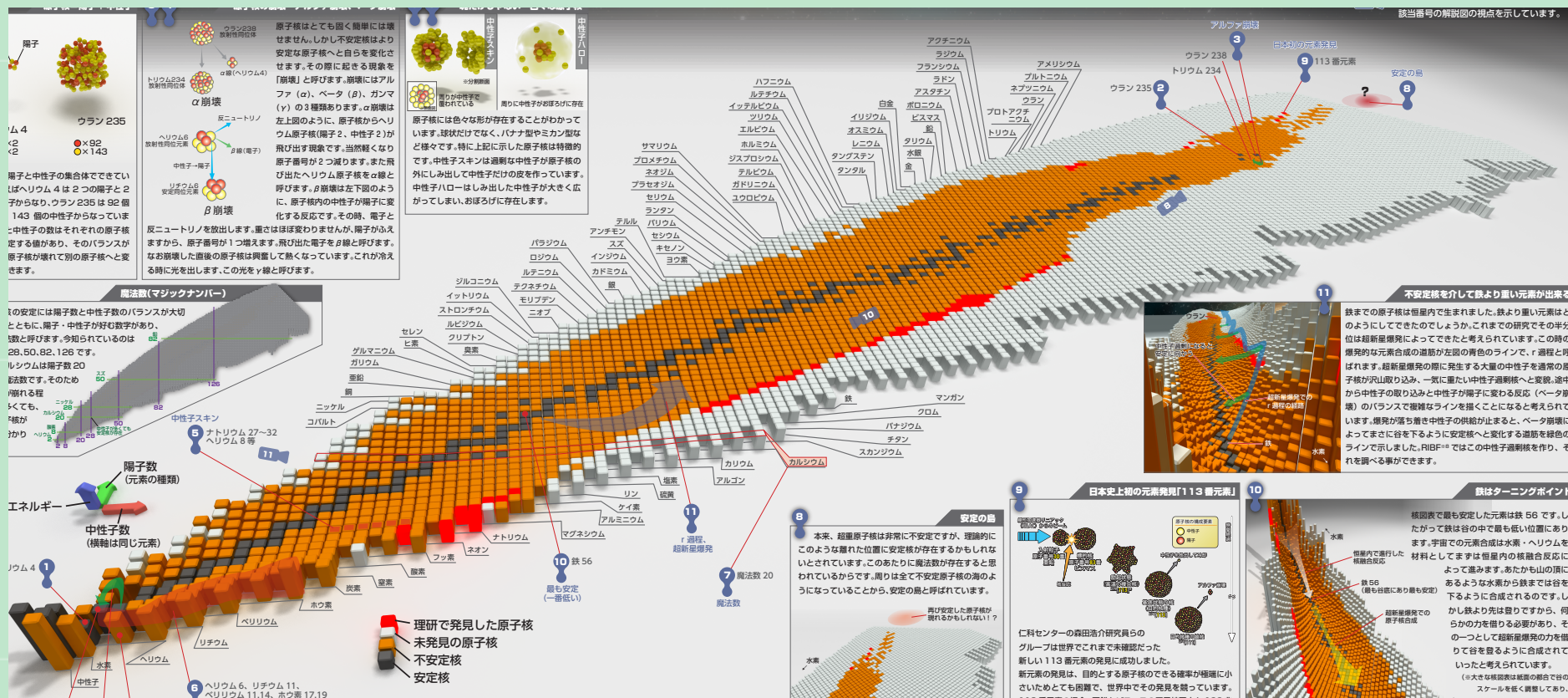


~ 210 mesons

Most of hadrons are **unstable** (above two-hadron threshold)

Relation to unstable nuclei

Stable nuclei (~300), unstable nuclei (~2000)



<https://www.nishina.riken.jp/enjoy/kakuzu/index.html>

Structure of unstable nuclei

- clustering, halo nuclei, Efimov effect, ...

Nature of resonances

Theoretical treatment for **unstable** hadrons

- **resonances** in hadron-hadron scattering
- **pole** of the scattering amplitude $\longleftrightarrow$ “eigenstate”
- analytic continuation: unique

Resonance as an “eigenstate” of Hamiltonian

- **complex energy**

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

Von G. Gamow, z. Zt. in Göttingen.

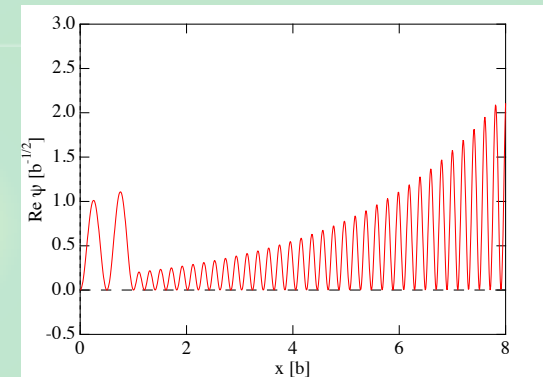
Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + i \frac{\hbar \lambda}{4\pi},$$

wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- diverging wave function
- complex expectation value (norm, $\langle r^2 \rangle$)
- interpretation problem





Introduction

- Structure of “unstable” resonance?



Structure of $\Lambda(1405)$ resonance

- **Accurate $\bar{K}N$ scattering amplitude**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);

M. Tanabashi, *et al.* (Particle Data Group), PRD 98, 030001 (2018)

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

- **$\bar{K}N$ compositeness**

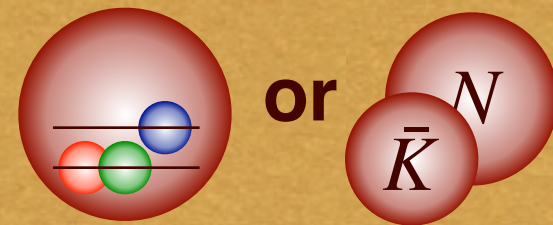
Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

T. Hyodo, JPS journal Vol. 75 No. 8, 478 (2020)

T. Kinugawa, T. Hyodo, in preparation



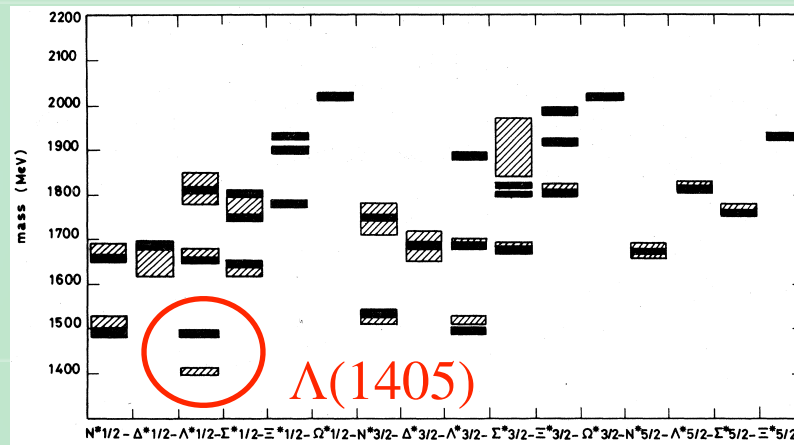
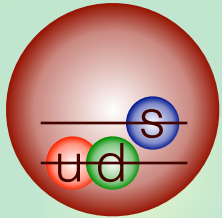
Summary



$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture $\rightarrow$ exotic candidate

N. Isgur and G. Karl, Phys. Rev. D18, 4187 (1978)

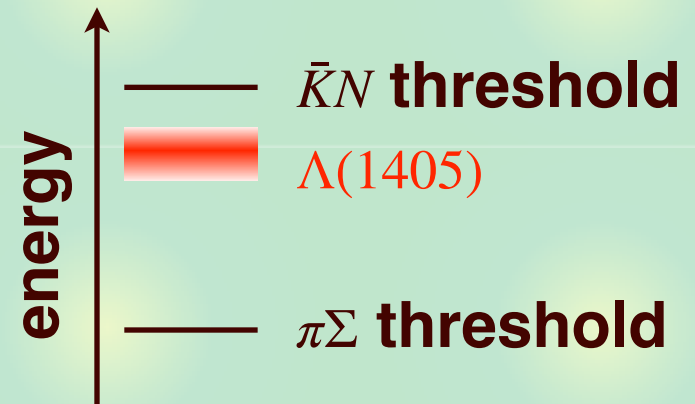
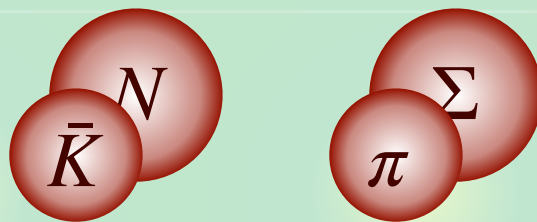


— : theory

▨ : experiment

Resonance in coupled-channel scattering

- coupling to MB states



Detailed analysis of $\bar{K}N$ - $\pi\Sigma$ scattering is necessary.

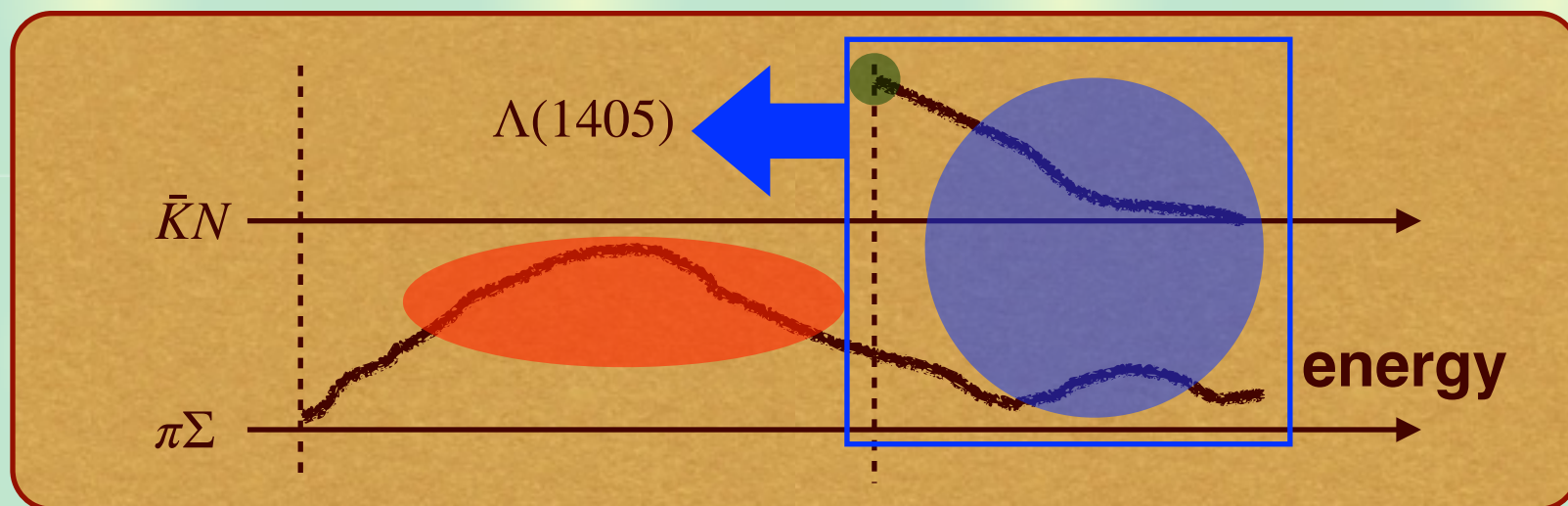
Strategy for $\bar{K}N$ interaction

Above the $\bar{K}N$ threshold : direct constraints

- K^-p **total cross sections** (old data)
- $\bar{K}N$ **threshold branching ratios** (old data)
- K^-p **scattering length** (new data : SIDDHARTA)

Below the $\bar{K}N$ threshold: indirect constraints

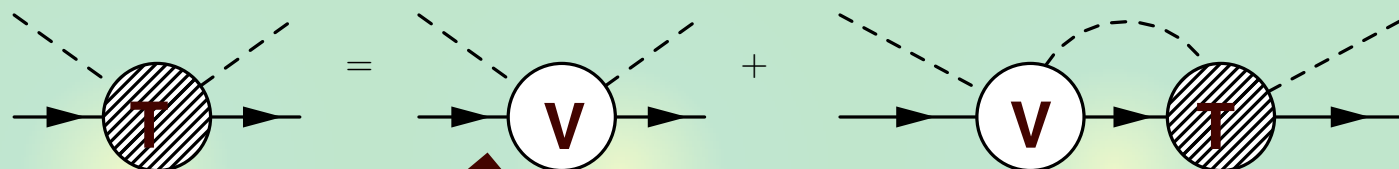
- $\pi\Sigma$ **mass spectra** (new data : LEPS, CLAS, HADES, ...)



Construction of the realistic amplitude

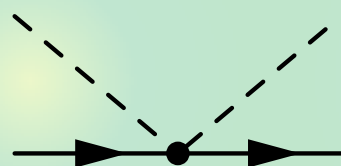
Chiral SU(3) coupled-channels ($\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, K\Xi$) approach

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)



Chiral perturbation theory

1) TW term

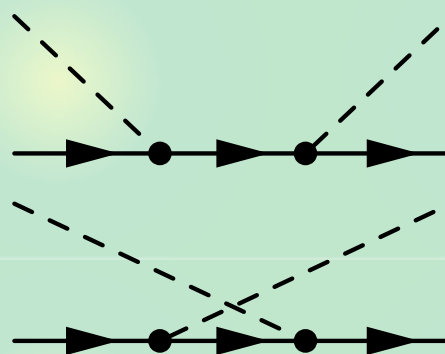


$\mathcal{O}(p)$

6 cutoffs

TW model

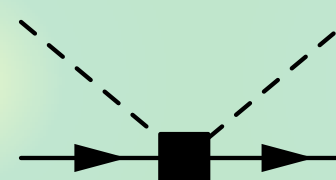
2) Born terms



$\mathcal{O}(p)$

TWB model

3) NLO terms



$\mathcal{O}(p^2)$

7 LECs

NLO model

Best-fit results

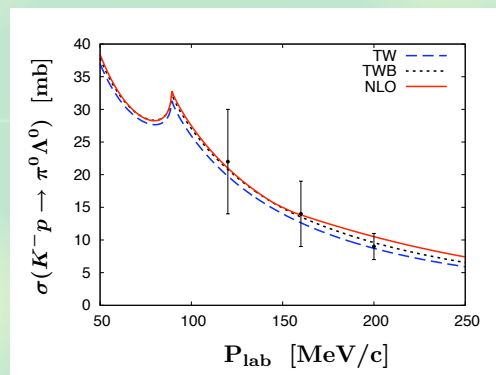
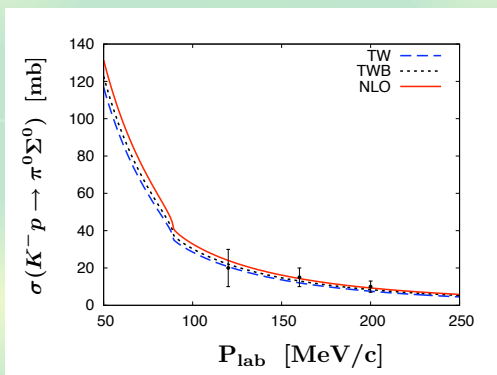
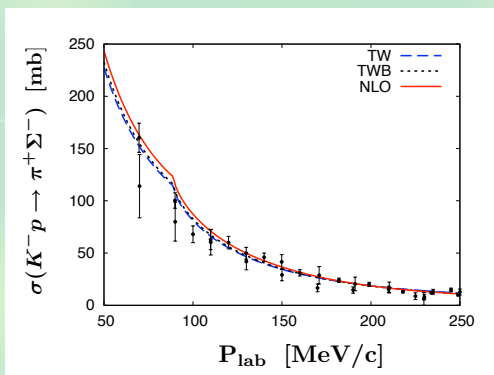
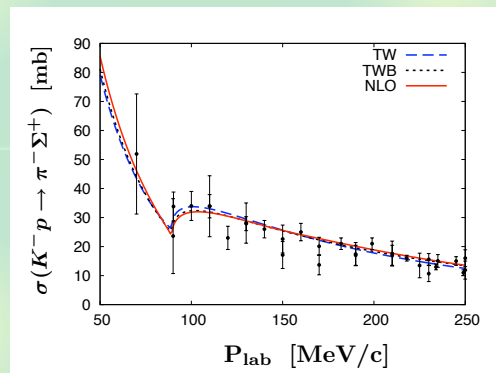
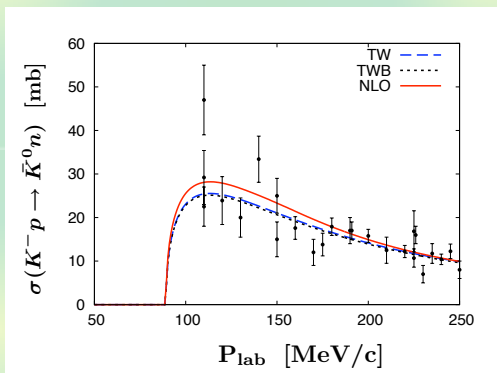
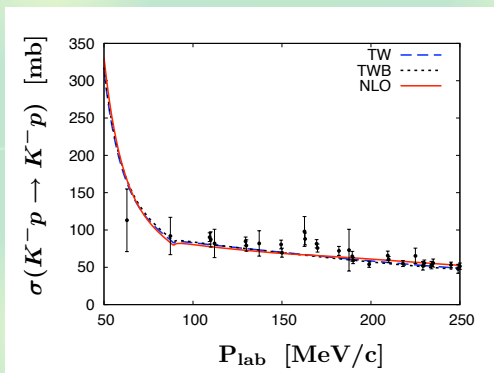
K at rest

	TW	TWB	NLO	Experiment
ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$ [10]
Γ [eV]	495	514	591	$541 \pm 89 \pm 22$ [10]
γ	2.36	2.36	2.37	2.36 ± 0.04 [11]
R_n	0.20	0.19	0.19	0.189 ± 0.015 [11]
R_c	0.66	0.66	0.66	0.664 ± 0.011 [11]
$\chi^2/\text{d.o.f}$	1.12	1.15	0.96	

} **SIDDHARTA**

} **Branching ratios**

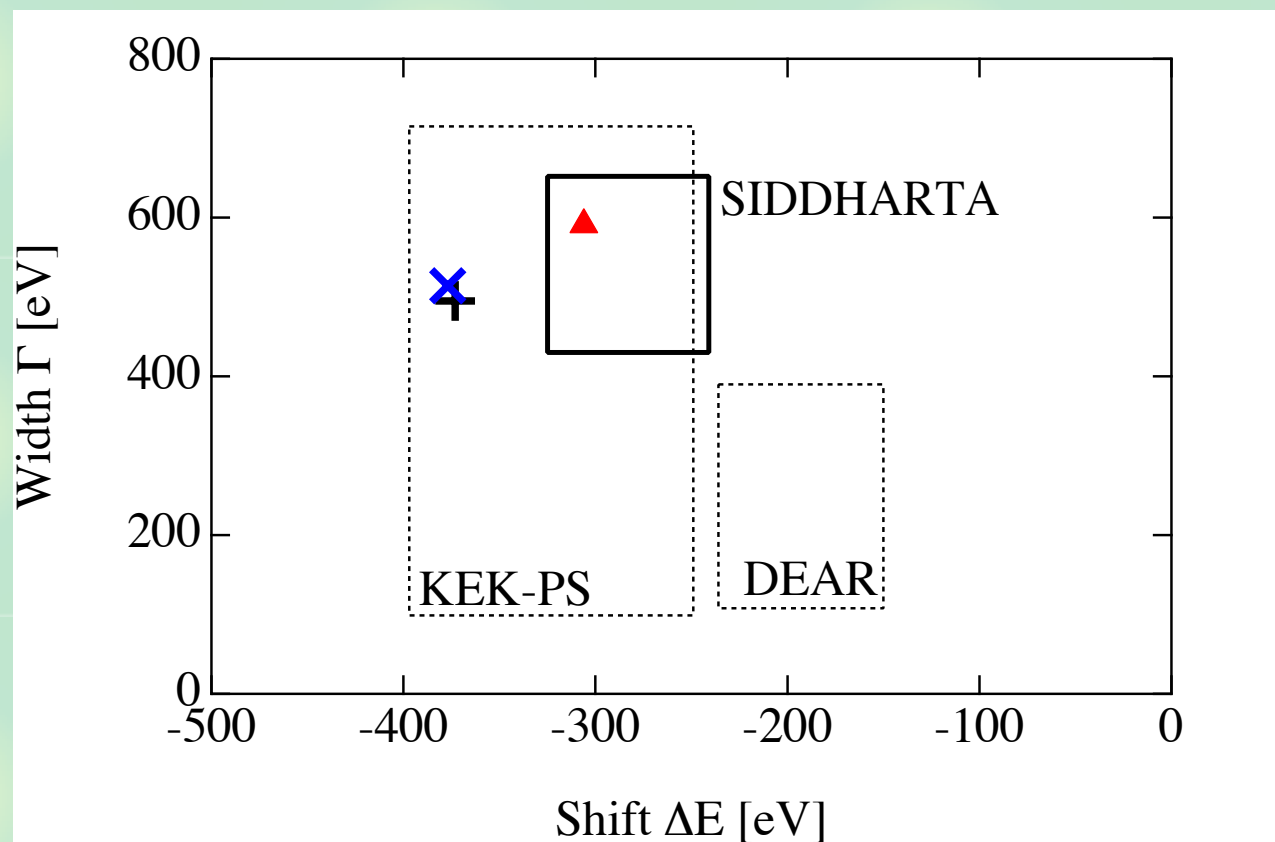
K^-p cross sections



Accurate description of all existing data ($\chi^2/\text{d.o.f} \sim 1$)

Comparison with SIDDHARTA

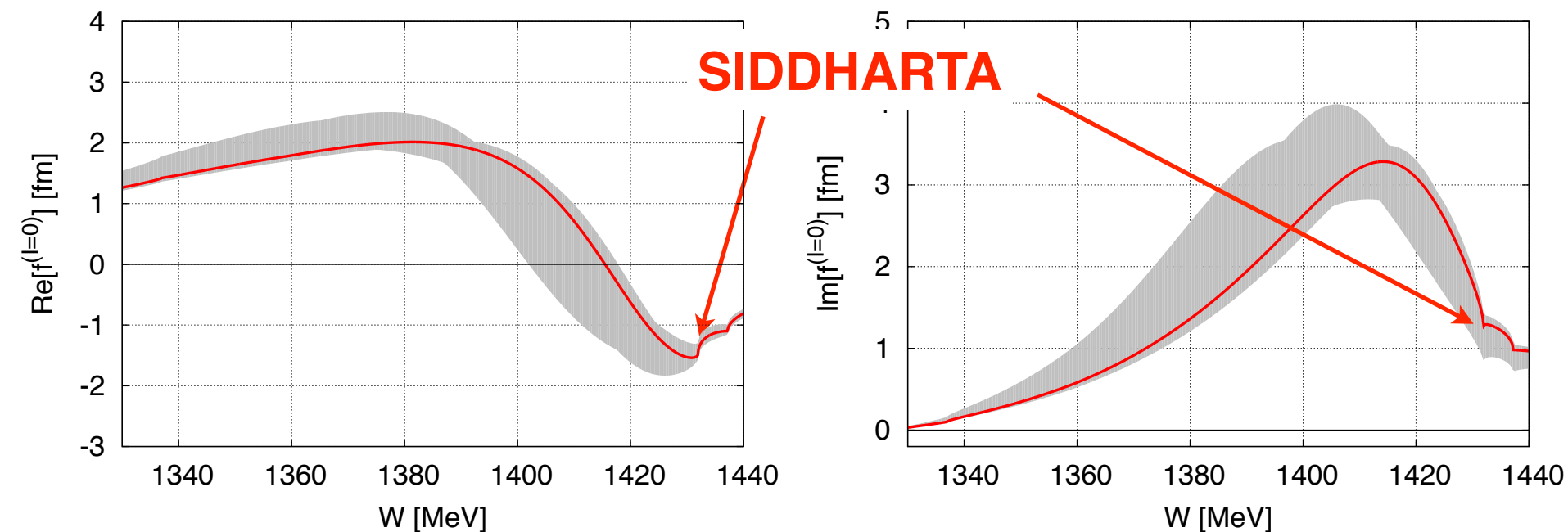
	TW	TWB	NLO
$\chi^2/\text{d.o.f.}$	1.12	1.15	0.957



TW and **TWB** are reasonable, while best-fit requires **NLO**.

Subthreshold extrapolation

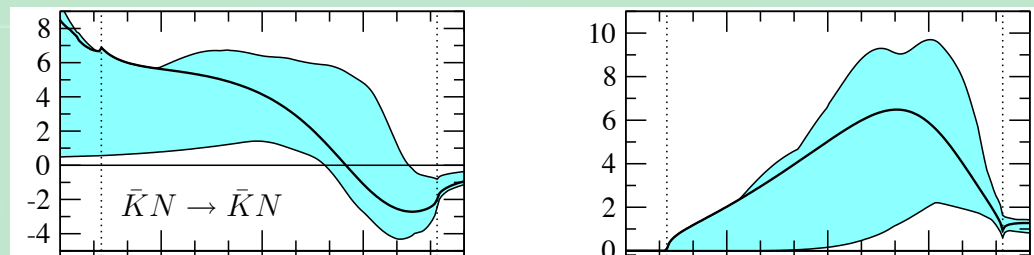
Uncertainty of $\bar{K}N \rightarrow \bar{K}N(I=0)$ amplitude below threshold



Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise,
NPA 954, 41 (2016)

- c.f. without SIDDHARTA

R. Nissler, Doctoral Thesis (2007)



SIDDHARTA is essential for **subthreshold** extrapolation.

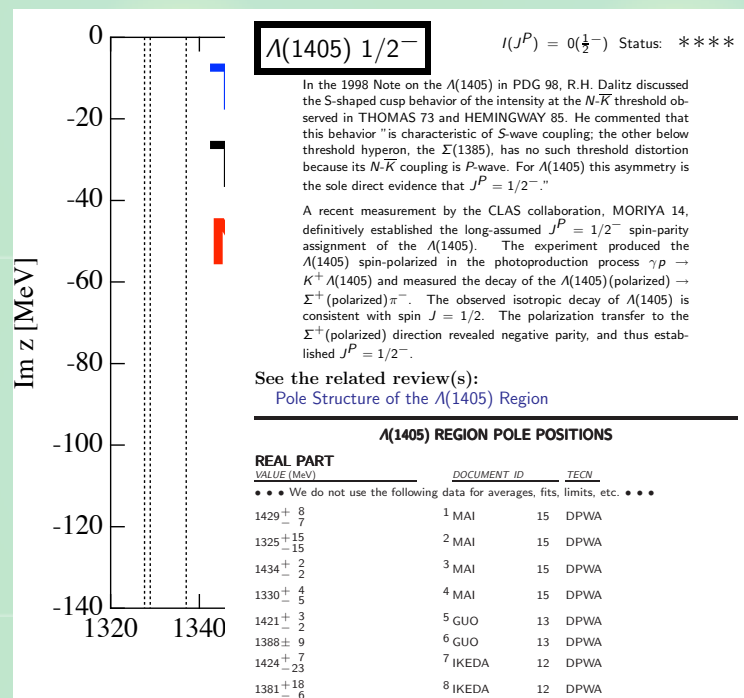
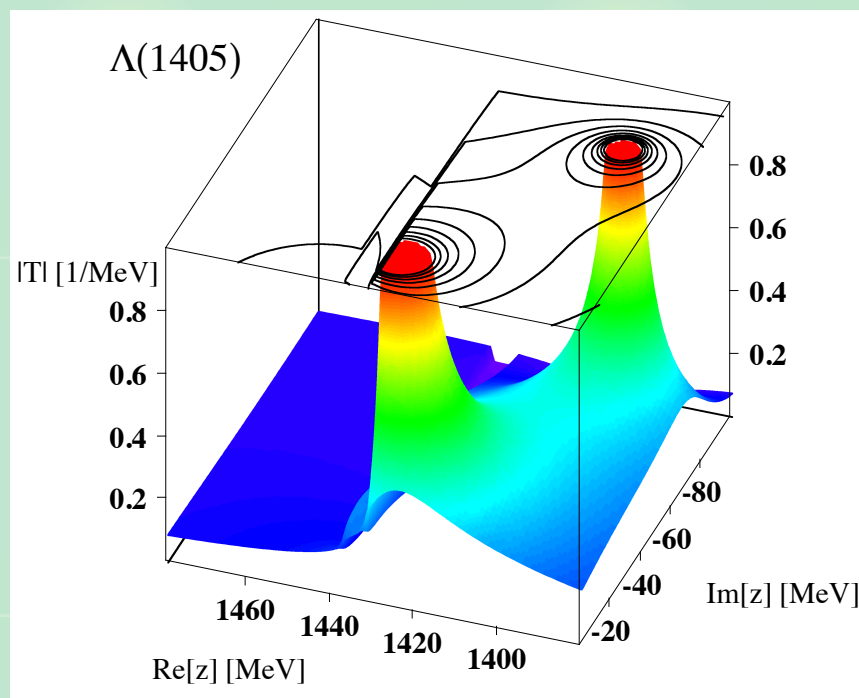
Extrapolation to complex energy: two poles

Two poles : superposition of two eigenstates

J.A. Oller, U.G. Meissner, PLB 500, 263 (2001);

D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meissner, NPA 723, 205 (2003);

U.G. Meissner, Symmetry 12, 981 (2020)



NLO analysis confirms the two-pole structure.

The results are adopted in PDG.

M. Tanabashi, *et al.*, PRD 98, 030001 (2018), <http://pdg.lbl.gov/>

New data : K^-p correlation function

K^-p total cross sections

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

- Old bubble chamber data

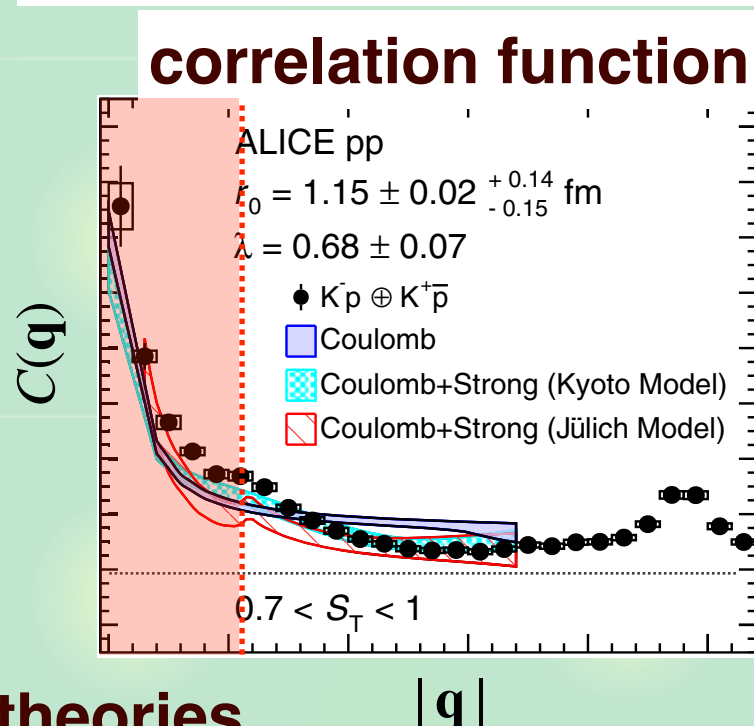
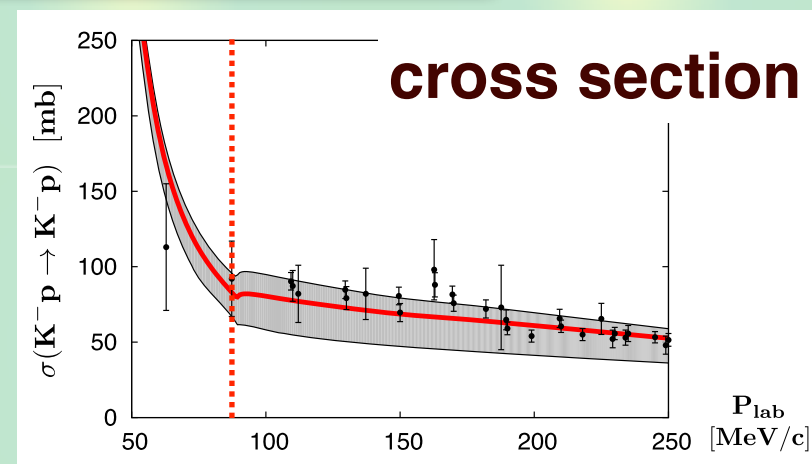
K^-p correlation function

ALICE collaboration, PRL 124, 092301 (2020)

$$C(q) = \frac{N_{K^-p}(\mathbf{p}_{K^-}, \mathbf{p}_p)}{N_{K^-}(\mathbf{p}_{K^-})N_p(\mathbf{p}_p)}$$

- Excellent **precision** ($\bar{K}^0_n$ cusp)
- Low-energy data **below** $\bar{K}^0_n$

—> important constraint on $\Lambda(1405)$ theories



Prediction from chiral SU(3) dynamics

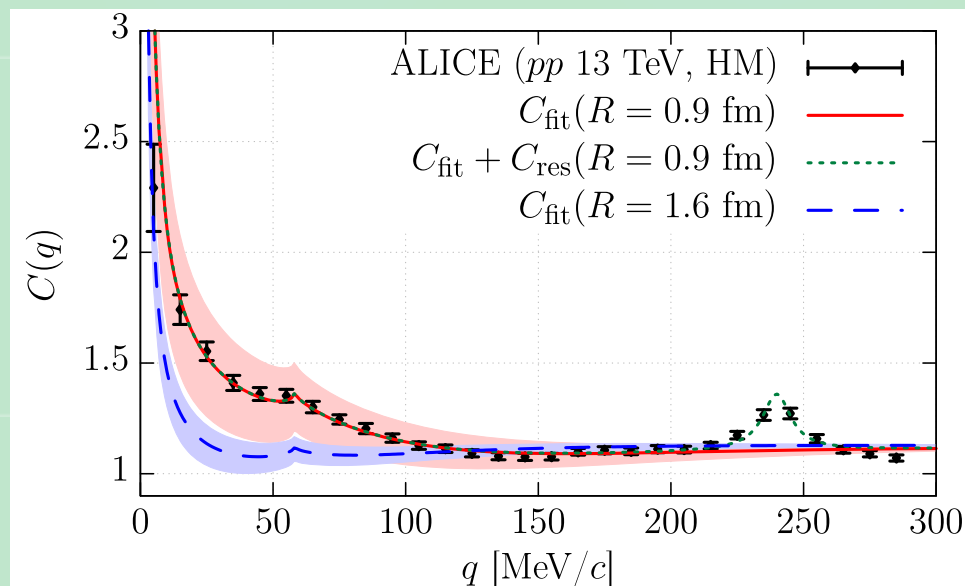
Theoretical calculation of $C(q)$

$$C(q) \simeq \int d^3\mathbf{r} S(\mathbf{r}) |\Psi_q^{(-)}(\mathbf{r})|^2$$

- wave function $\Psi_q^{(-)}(\mathbf{r})$: coupled-channel $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential

K. Miyahara, T. Hyodo, W. Weise, PRC98, 025201 (2018)

- source function $S(\mathbf{r})$: determined by K^+p data



Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

Correlation function is well reproduced.



Introduction

- Structure of “unstable” resonance?



Structure of $\Lambda(1405)$ resonance

- Accurate $\bar{K}N$ scattering amplitude

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);

M. Tanabashi, *et al.* (Particle Data Group), PRD 98, 030001 (2018)

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

- $\bar{K}N$ compositeness

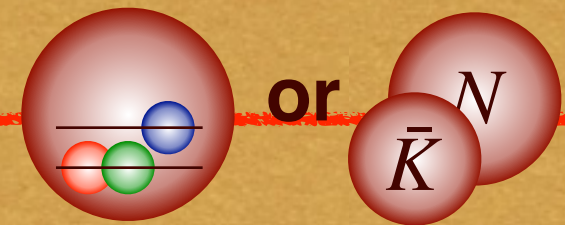
Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

T. Hyodo, JPS journal Vol. 75 No. 8, 478 (2020)

T. Kinugawa, T. Hyodo, in preparation



Summary



Compositeness of hadrons



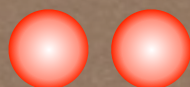
Structure of a given resonance (pole)?



Weak binding relation for stable bound states

S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness X
threshold channel



or

“Elementariness” Z
other contributions



observables (a_0, B)



Effective field theory $\rightarrow$ description of low-energy scattering amplitude, generalization to **unstable** resonances

Weak-binding relation for stable states

Compositeness X of s-wave weakly bound state ($R \gg R_{\text{typ}}$)

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

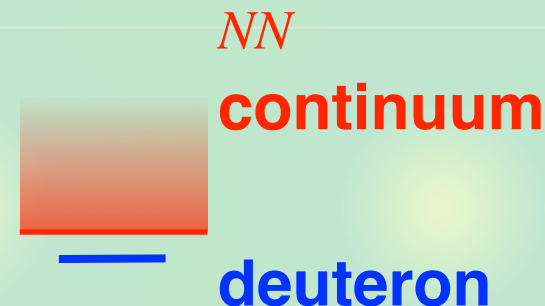
$$|d\rangle = \sqrt{X} |NN\rangle + \sqrt{1-X} |\text{others}\rangle$$

range of interaction

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

↓
↑

scattering length
radius of state



- Deuteron is NN composite : $a_0 \sim R \Rightarrow X \sim 1$
- Internal structure from **observable** (a_0, B)

Problem: applicable only for stable states

Effective field theory

Low-energy scattering with near-threshold bound state

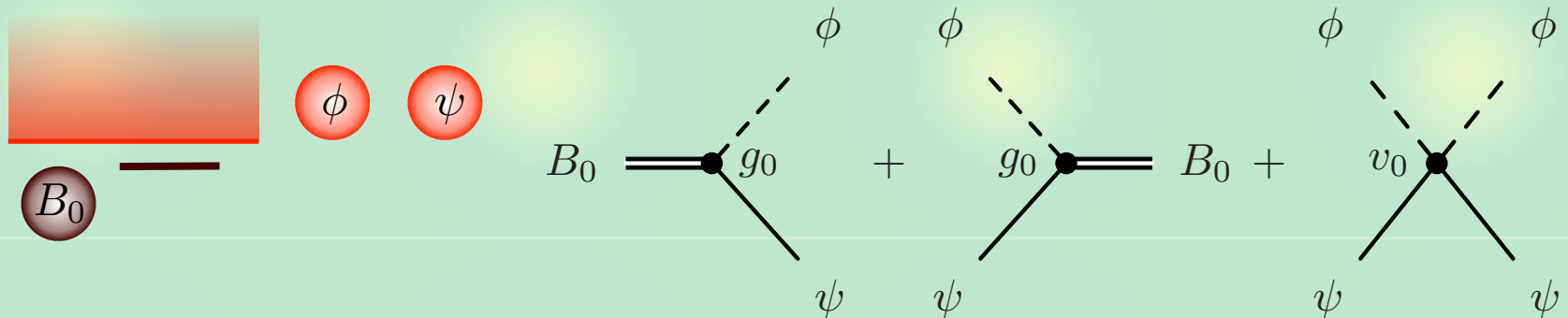
- Nonrelativistic EFT with contact interaction

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M} \nabla \psi^\dagger \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^\dagger \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \omega_0 B_0^\dagger B_0 \right]$$

$$H_{\text{int}} = \int d\mathbf{r} \left[g_0 \left(B_0^\dagger \phi \psi + \psi^\dagger \phi^\dagger B_0 \right) + v_0 \psi^\dagger \phi^\dagger \phi \psi \right]$$



- **cutoff** : $\Lambda \sim 1/R_{\text{typ}}$ (interaction range of microscopic theory)
- At low momentum $p \ll \Lambda$, interaction $\sim$ contact

Compositeness and “elementariness”

Eigenstates

$$H_{\text{free}} |B_0\rangle = \omega_0 |B_0\rangle, \quad H_{\text{free}} |\mathbf{p}\rangle = \frac{\mathbf{p}^2}{2\mu} |\mathbf{p}\rangle$$

free (discrete + continuum)

$$(H_{\text{free}} + H_{\text{int}}) |B\rangle = -B |B\rangle$$

full (bound state)

- normalization of $|B\rangle$ + completeness relation

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\mathbf{p}}{(2\pi)^3} |\mathbf{p}\rangle\langle \mathbf{p}|$$

- projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\mathbf{p}}{(2\pi)^3} |\langle \mathbf{p}|B\rangle|^2$$

“elementariness”

compositeness

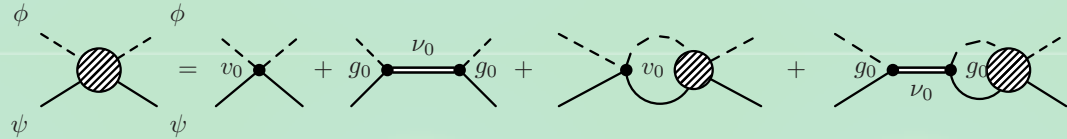


Z, X : real and nonnegative $\rightarrow$ interpreted as **probability**

Weak binding relation

$\psi\phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi} \frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E - \omega_0}, \quad G(E) = \frac{1}{2\pi^2} \int_0^\Lambda dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness $X \leftarrow v(E), G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R = \sqrt{2\mu B}$ expansion of scattering length a_0

$$a_0 = -f(E=0) = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\} \text{renormalization dependent}$$

renormalization independent

If $R \gg R_{\text{typ}}$, correction terms neglected: $X \leftarrow (a_0, B)$

Introduction of decay channel

Introduce decay channel

$$H'_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right]$$

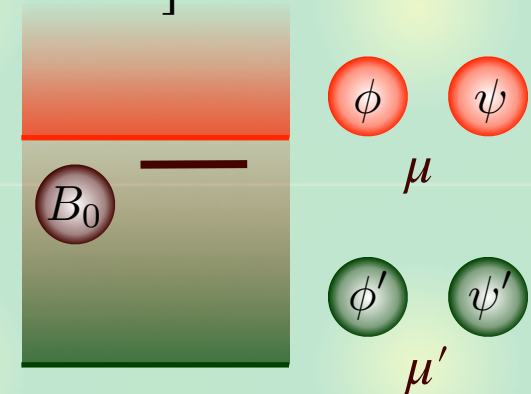
$$H'_{\text{int}} = \int d\mathbf{r} \left[g'_0 \left(B_0^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} B_0 \right) + \nu'_0 \psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + \nu_0^t (\psi^{\dagger} \phi^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} \phi \psi) \right]$$

Quasi-bound state : complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H |QB\rangle = -E_{QB} |QB\rangle, \quad E_{QB} \in \mathbb{C}$$

$$\nu_{\psi} + \nu_{\phi} = \nu$$



Generalized relation : **correction** from threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004), ...

If $|R| \gg (R_{\text{typ}}, \ell)$, **correction terms neglected:** $X \leftarrow (a_0, E_{QB})$

Evaluation of compositeness

Generalized weak-binding relation

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_{QB}) determinations by several groups

- neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, $X \sim 1$ with small $U/2$ (complex nature)

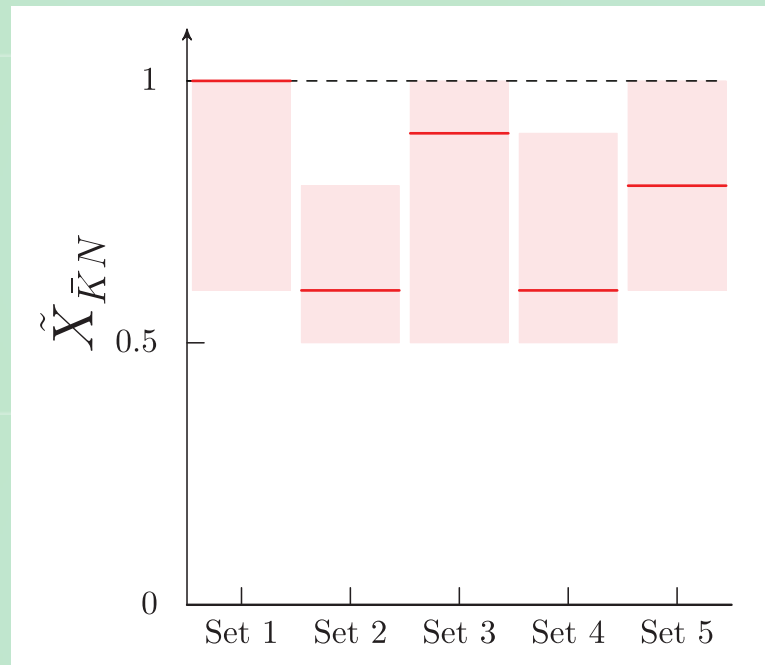
$\Lambda(1405)$: $\bar{K}N$ composite dominance $\leftarrow$ observables

Uncertainty estimation

Estimation of correction terms: $|R| \sim 2 \text{ fm}$

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O} \left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- ρ meson exchange picture: $R_{\text{typ}} \sim 0.25 \text{ fm}$
- energy difference from $\pi\Sigma$: $\ell \sim 1.08 \text{ fm}$



$\bar{K}N$ composite dominance holds even **with correction terms.**

Correction term and zero range limit

What happens if $R_{\text{typ}} \rightarrow 0$?

$$a_0 = R \frac{2X}{1+X}$$

- Limit $\Lambda \rightarrow \infty$ can be taken in renormalizable EFT

- EFT with only ψ, ϕ fields should have $X = 1$

$$\Rightarrow a_0 = R$$

- “effective range model” gives $a_0 \neq R$: contradiction?

E. Braaten, M. Kusunoki, D. Zhang, *Annals Phys.* **323**, 1770 (2008)

R_{typ} should be either R_{int} or length scale in the amplitude

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left(\frac{R_{\text{typ}}}{R} \right) \right\}, \quad R_{\text{typ}} = \max(R_{\text{int}}, |r_e|, \dots)$$

- relevant to system with large $|r_e|$

T. Kinugawa, T. Hyodo, in preparation

Summary



Structure of unstable resonance is **nontrivial**.



Pole structure of the $\Lambda(1405)$ region is now well constrained by the experimental data. Nominal $\Lambda(1405)$ can be a superposition of **two states**.

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);

M. Tanabashi, *et al.* (Particle Data Group), PRD 98, 030001 (2018)

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)



Generalized weak-binding relation shows that high-mass pole of $\Lambda(1405)$ is dominated by **molecular $\bar{K}N$** component.

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

T. Hyodo, JPS journal Vol. 75 No. 8, 478 (2020)

T. Kinugawa, T. Hyodo, in preparation