Hadronic molecules and their structure (part I)

Tetsuo Hyodo

Tokyo Metropolitan University

Clusters & Hierarchies

2022, Jun. 28th

Contents

Part I : Basics

- Chiral symmetry

S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory* (Springer, Berlin, 2012)

- Resonances

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972);
T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

Part II : Application

- $\bar{K}N$ scattering and $\Lambda(1405)$ resonance

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Observed hadrons (2020)

Particle Data Group (PDG) 2020 eddition

http://pdg.lbl.gov/

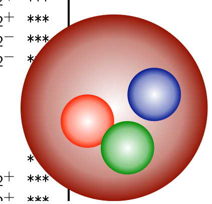
p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
Λ	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2110)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2012)^-$	$?^-$	***
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	*
$\Xi_c(2930)$		
$\Xi_c(2970)$		
$\Xi_c(3055)$		
$\Xi_c(3080)$		
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3120)^0$		***

Ξ_{cc}^{++}		***
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6227)$		***
Ω_b^-	$1/2^+$	***
$P_c(4312)^+$		*
$P_c(4380)^+$		*
$P_c(4440)^+$		*
$P_c(4457)^+$		*



~160 baryons

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$
$a_1(1420)$	$1^-(1^{++})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
• $a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	
• $\eta_2(1645$	
• $\omega(16$	
• ω_3	

	$I^G(J^{PC})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(0^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
• $\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
$a_0(1950)$	$1^-(0^{++})$
• $f_2(1950)$	$0^+(2^{++})$
• $a_4(1970)$	$1^-(4^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
$\pi_2(2005)$	$1^-(2^{-+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

ner States

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
$D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$0(0^-)$

$c\overline{c}$ (+ possibly non-$q\overline{q}$ states)

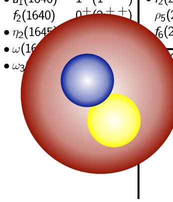
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
$c1(1P)$	$0^+(1^{++})$
$(1P)$	$0^-(1^{+-})$
$c2(1P)$	$0^+(2^{++})$
$(2S)$	$0^+(0^{-+})$
$(2S)$	$0^-(1^{--})$

$c\overline{c}$ continued

	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\overline{b}$ (+ possibly non-$q\overline{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$



~210 mesons

Observed hadrons (2022)

Particle Data Group (PDG) 2022 eddition

http://pdg.lbl.gov/

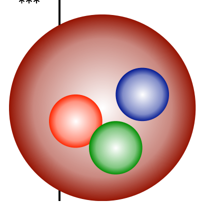
p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	**
$\Lambda(1520)$	$3/2^-$	**
$\Lambda(1600)$	$1/2^+$	**
$\Lambda(1670)$	$1/2^-$	**
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		*

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2110)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		**
Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(201$		
$\Omega(225$		
$\Omega(238$		
$\Omega(247$		

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2923)$		**
$\Xi_c(2930)$		**
$\Xi_c(2970)$	$1/2^+$	***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(3120)^0$		***
Ξ_{cc}^+		*
Ξ_{cc}^{++}		***

Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^-	$1/2^+$	***
Ξ_b^0	$1/2^+$	***
$\Xi'_b(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6100)^-$	$3/2^-$	***
$\Xi_b(6227)^-$		***
$\Xi_b(6227)^0$		***
Ω_b^-	$1/2^+$	***
$\Omega_b(6316)^-$		*
$\Omega_b(6330)^-$		*
$\Omega_b(6340)^-$		*
$\Omega_b(6350)^-$		*
$P_c(4312)^+$		*



~170 baryons

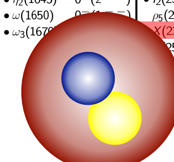
newly observed hadrons in 2 years!

LIGHT UNFLAVORED ($S=C=B=0$)	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(500)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	• $a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$X(1750)$	$?^-(1^{--})$
• $f_0(980)$	$0^+(0^{++})$	$\eta(1760)$	$0^+(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $\phi(1020)$	$0^-(1^{--})$	$f_2(1810)$	$0^+(2^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	$X(1835)$	$?^?(0^{-+})$
• $b_1(1235)$	$1^+(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $a_1(1260)$	$1^-(1^{++})$	• $\eta_2(1870)$	$0^+(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_1(1285)$	$0^+(1^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $\eta(1295)$	$0^+(0^{-+})$	$f_2(1910)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $a_4(1970)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2005)$	$1^-(2^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2020)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	• $f_4(2050)$	$0^+(4^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $a_0(1450)$	$1^-(0^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$\rho_3(2250)$	$1^+(3^{--})$
• $\pi_1(1600)$	$1^-(1^{-+})$	• $f_2(2300)$	$0^+(2^{++})$
• $a_1(1640)$	$1^-(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
$f_2(1640)$	$0^+(2^{++})$	$f_0(2330)$	$0^+(0^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$	• $f_2(2340)$	$0^+(2^{++})$
• $\omega(1650)$		$\rho_5(2350)$	$1^+(5^{--})$
• $\omega_3(167$		$X(2370)$	$?^?(?^{??})$
		(2510)	$0^+(6^{++})$

STRANGE ($S=\pm1$, $C=B=0$)	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
• $K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
• $K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
• $K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)$	$1/2(0^+)$
• $D_1(2420)$	$1/2(1^+)$
• $D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)$	$1/2(2^+)$
$D_0(2550)^0$	$1/2(0^-)$
$D_J^*(2600)^0$	$1/2(1^-)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D_2(2740)^0$	$1/2(2^-)$
• $D_3^*(2750)$	$1/2(3^-)$
$D_1^*(2760)^0$	$1/2(1^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C=\pm1$, $S=\pm1$) (+ possibly non-$q\bar{q}$ states)	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
$D_{s0}(2590)^+$	$0(0^-)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
• $D_{s3}^*(2860)^\pm$	$0(3^-)$
$X_0(2900)$	$?(0^+)$
$X_1(2900)$	$?(1^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$
BOTTOM ($B=\pm1$)	
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)$	$1/2(2^+)$
$B_J(5840)$	$1/2(?^?)$
$B_J(5970)$	$1/2(?^?)$
BOTTOM, STRANGE ($B=\pm1$, $S=\mp1$)	
B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$
$B_{sJ}(6063)^0$	$0(?^?)$
$B_{sJ}(6114)^0$	$0(?^?)$
BOTTOM, CHARMED ($B=C=\pm1$)	
• B_c^+	$0(0^-)$
• $B_c(2S)^\pm$	$0(0^-)$
$c\bar{c}$ (+ possibly non-$q\bar{q}$ states)	
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
$(1P)$	$0^+(0^{++})$
$(1P)$	$0^+(1^{++})$
$1P)$	$0^-(1^{+-})$
$(1P)$	$0^+(2^{++})$
$2S)$	$0^+(0^{-+})$
$S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$

$c\bar{c}$ continued	$I^G(J^{PC})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
$X(4630)$	$0^+(?^{?+})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c1}(4685)$	$0^+(1^{++})$
$\chi_{c0}(4700)$	$0^+(0^{++})$
$b\bar{b}$ (+ possibly non-$q\bar{q}$ states)	
• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$
OTHER	
Further States	



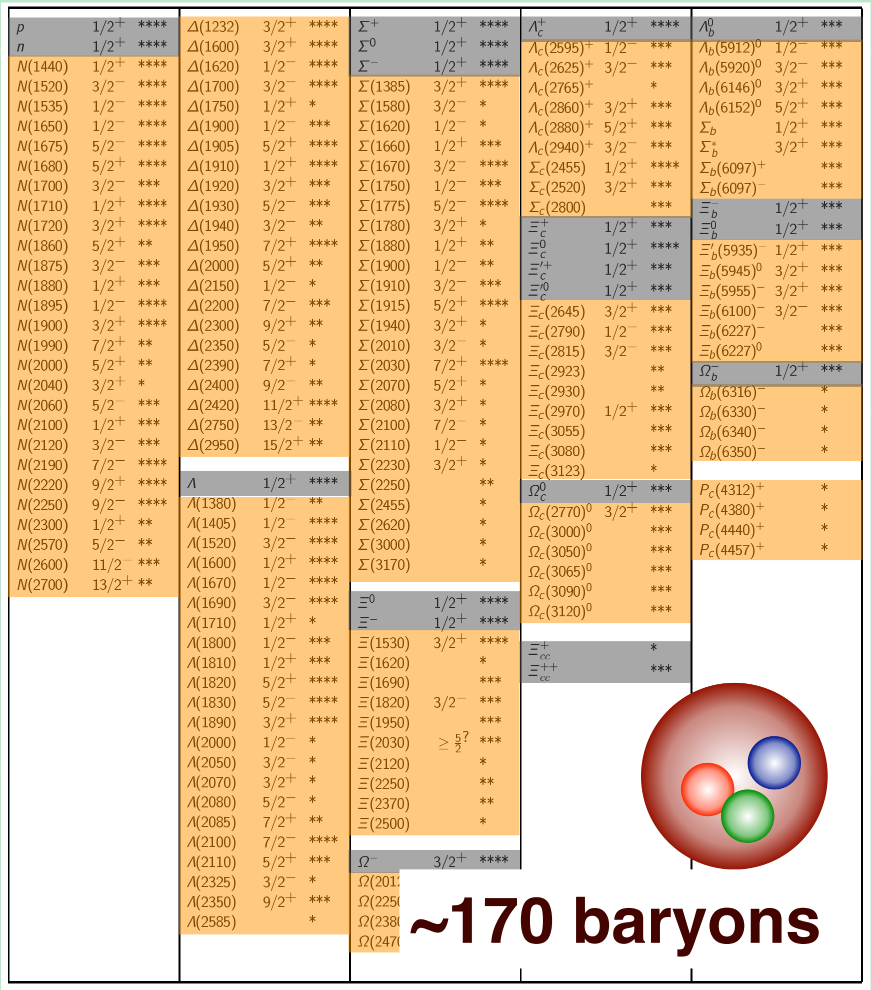
~210 mesons

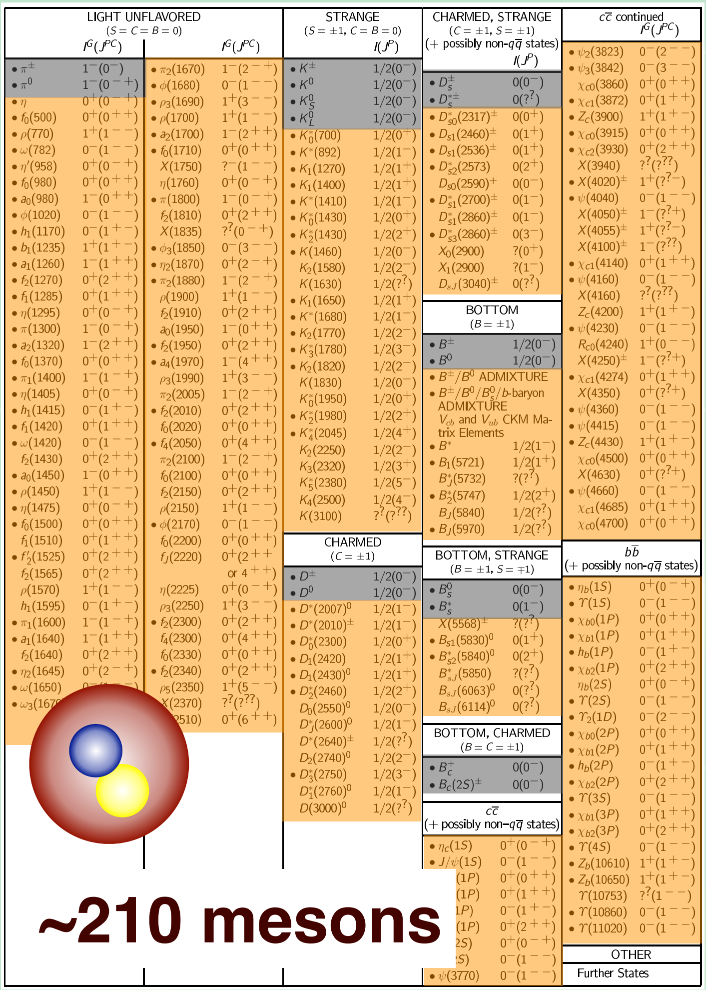
All ~ 380 hadrons emerge from single QCD Lagrangian

Unstable states via strong interaction

Stable/unstable hadrons

http://pdg.lbl.gov/

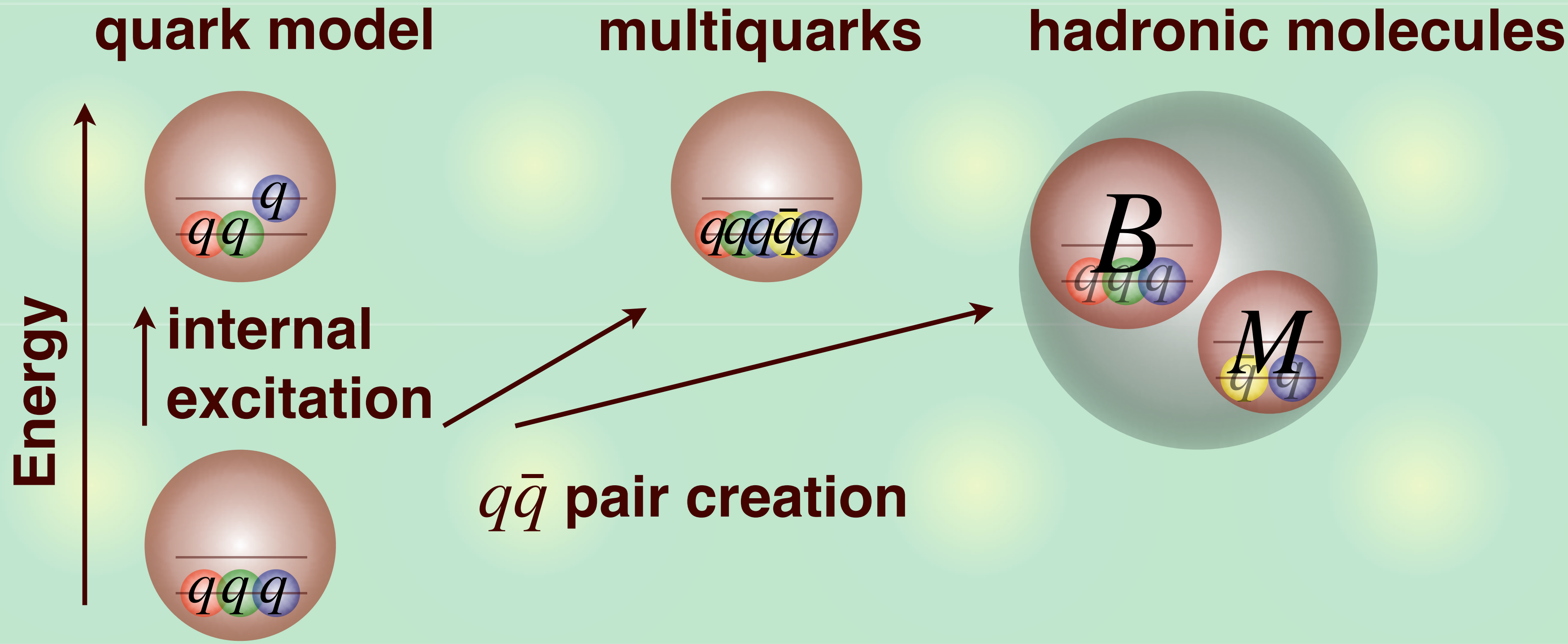




Most of hadrons are unstable (above two-hadron threshold)

Aim of this lecture

Various excitations of hadrons



Strategy :

- use **symmetry principle** to constrain hadron interactions
- treat unstable hadrons as **resonances** in scattering

Contents

Part I : Basics

- Chiral symmetry

S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory* (Springer, Berlin, 2012)

- Resonances

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972);
T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

Part II : Application

- $\bar{K}N$ scattering and $\Lambda(1405)$ resonance

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

QCD and isospin symmetry

QCD Lagrangian (u, d quark part)

$$\mathscr{L}_{\mathrm{QCD}} = \underbrace{\bar{q}(i\gamma^{\mu}D_{\mu})q}_{\text{kinetic term}} - \underbrace{\bar{q}Mq}_{\text{mass term}} + (\text{gluons, heavy quarks})$$

kinetic term **mass term**

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad \bar{q} = (\bar{u} \quad \bar{d}) = q^{\dagger}\gamma^{0}, \quad M = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$$

Flavor $\mathrm{SU}(2)_V$ (isospin) transformation $U^{\dagger}U = 1, \ \det U = 1$

$$q \to Uq, \quad \bar{q} \to \bar{q}U^{\dagger}$$

$m_u \simeq m_d$: $\mathscr{L}_{\mathrm{QCD}}$ **has an approximate $\mathrm{SU}(2)_V$ symmetry**

$$\bar{q}q \to \bar{q}U^{\dagger}Uq = \bar{q}q$$

Consequence: hadrons belog to isospin multiplets

$$N = (p, n), \quad \pi = (\pi^{+}, \pi^{0}, \pi^{-}), \quad \cdots$$

Chiral transformation

Decompose q into right-handed q_R and left-handed q_L

$$q = q_R + q_L, \quad q_R = P_R q, \quad q_L = P_L q, \quad P_{R,L} = (1 \pm \gamma_5)/2,$$

$$\bar{q} = \bar{q}_R + \bar{q}_L, \quad \bar{q}_R = \bar{q} P_L, \quad \bar{q}_L = \bar{q} P_R$$

- Chiral $\mathrm{SU}(2)_R \otimes \mathrm{SU}(2)_L$ transformation $R^\dagger R = L^\dagger L = 1$

$$q_R \to R q_R, \quad \bar{q}_R \to \bar{q}_R R^\dagger, \quad q_L \to L q_L, \quad \bar{q}_L \to \bar{q}_L L^\dagger$$

Kinetic term : q_R and q_L are separated —> chiral symmetry

$$\bar{q}_R (i\gamma^\mu D_\mu) q_R + \bar{q}_L (i\gamma^\mu D_\mu) q_L$$

Mass term : q_R and q_L are mixed —> chiral symmetry broken

$$-m\bar{q}_R q_L - m\bar{q}_L q_R, \quad \bar{q}_R q_L \to \bar{q}_R R^\dagger L q_L \neq \bar{q}_R q_L$$

- invariant if $R = L$ —> $\mathrm{SU}(2)_V$ is unbroken

Spontaneous symmetry breaking (SSB)

u, d **quark masses are much smaller than hadron scale**

—> approximately massless

—> QCD has (approximate) chiral symmetry

Order parameter of SSB : chiral condensate

$$\langle \bar{q}q \rangle \equiv \langle 0 | \bar{q}q | 0 \rangle$$

- $|0\rangle$ **: QCD vacuum**
- **Operator** $\bar{q}q = \bar{q}_L q_R + \bar{q}_R q_L$ **breaks chiral symmetry**
- **It is known that** $\langle \bar{q}q \rangle \neq 0$ **at low-energy hadron physics**

$$\mathrm{SU}(2)_R \otimes \mathrm{SU}(2)_L \to \mathrm{SU}(2)_V$$

Chiral symmetry is spontaneously broken by QCD vacuum

Consequence of SSB 1 : NG bosons

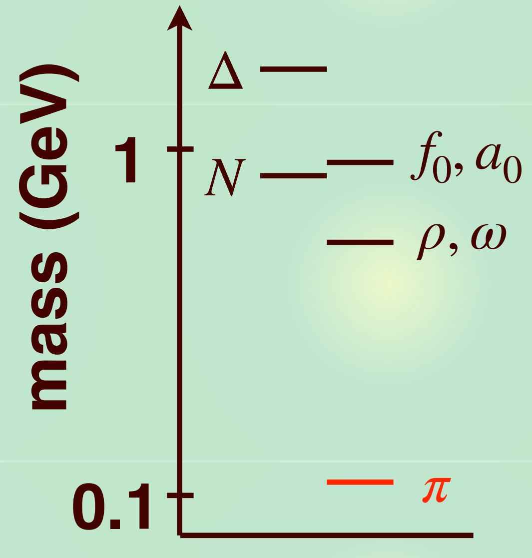
Appearance of massless Nambu-Goldstone (NG) bosons

Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); Phys. Rev. 124, 246 (1961),
J. Goldstone, Nuovo Cim. 19, 154 (1961),
J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127, 965 (1962)

- n_{NG} : # of NG bosons, n_{BS} : # of broken generators

$$n_{NG} = n_{BS}$$

- $\mathrm{SU}(2)_R \otimes \mathrm{SU}(2)_L \to \mathrm{SU}(2)_V$ case : $n_{BS} = 3$
- π has $I = 1$: 3 components
- π is much lighter than other hadrons



c.f.) in the absence of Lorentz invariance...

H. Watanabe and H. Murayama, Phys. Rev. Lett. 108, 251602 (2012),
Y. Hidaka, Phys. Rev. Lett. 110, 091601 (2013)

$$n_I + 2n_{II} = n_{BS}$$

Consequence of SSB 2 : low-energy theorems

Low-energy theorems : relations dictated by chiral symmetry

- derived by current algebra —> chiral perturbation theory

Gell-Mann Oakes Renner relation

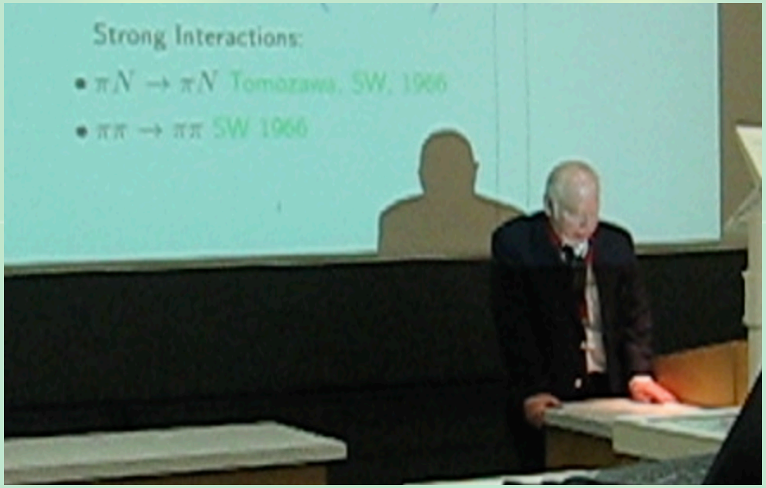
M. Gell-Mann, R.J. Oakes, B. Renner, Phys. Rev. 175, 2195 (1968)

$$m_\pi^2 f_\pi^2 = -m\langle \bar{q} q \rangle + \cdots$$

- f_π **: pion decay constant (** $\pi^+ \to \mu^+ \nu_\mu$ **)**

Weinberg-Tomozawa theorem

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966),
Y. Tomozawa, Nuovo Cim. A46, 707 (1966)



$$a \propto -\frac{m_\pi}{f_\pi^2} + \cdots$$

- $\pi\pi, \pi N$ **scattering lengths** a **<—** f_π

Weinberg-Tomozawa theorem for πN

Meson-baryon interaction $V_{\rm WT}$ and scattering length $a_{\rm WT}$

$$V_{\rm WT} = -\frac{Cm}{f^2} \quad \text{—> (Born approx.)} \quad a_{\rm WT} = -\frac{CmM}{8\pi(m+M)f^2}$$

- C : coupling strength determined by group theory

$$C^{\rm SU(2)} = -I_\alpha(I_\alpha+1) + 2 + I_T(I_T+1)$$

—> **sign** and **strength** of the interaction are fixed

- proportional to meson mass m

—> interaction vanishes in the chiral limit $m \to 0$

Channel	C	m [MeV]	$V_{\rm WT}$ [fm]	$a_{\rm WT}$ [fm]	$a_{\rm emp}$ [fm]
$\pi N(I=1/2)$	2	138	-3.2	0.22	0.240 ± 0.003
$\pi N(I=3/2)$	-1	138	1.6	-0.11	-0.122 ± 0.003

Empirical πN scattering lengths are well reproduced

Flavor SU(3) : $\bar{K}N$ system?

Meson-baryon interaction $V_{\rm WT}$ and scattering length $a_{\rm WT}$

$$V_{\rm WT} = -\frac{Cm}{f^2} \quad \text{—> (Born approx.)} \quad a_{\rm WT} = -\frac{CmM}{8\pi(m+M)f^2}$$

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$\pi N(I=3/2)$	-1	138	1.6	-0.11	-0.122 ± 0.003
$\bar{K}N(I=0)$	3	496	-12.1	0.63	$-1.70 + 0.68i$
$\bar{K}N(I=1)$	1	496	-4.0	0.21	$0.37 + 0.60i$
$\pi\Sigma(I=0)$	4	138	-6.4	0.46	none

- **Imaginary part <— decay to $\pi\Sigma, \pi\Lambda$**
- **Sign of $\mathrm{Re}\ a_{\bar{K}N(I=0)}$ <— $\Lambda(1405)$ below threshold**

$\bar{K}N(I=0)$ interaction is too strong; Born approx. is not valid

—> need for nonperturbative resummation (part II)

Contents

Part I : Basics

- Chiral symmetry

S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory* (Springer, Berlin, 2012)

- Resonances

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972);
T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

Part II : Application

- $\bar{K}N$ scattering and $\Lambda(1405)$ resonances

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Characterization of resonance

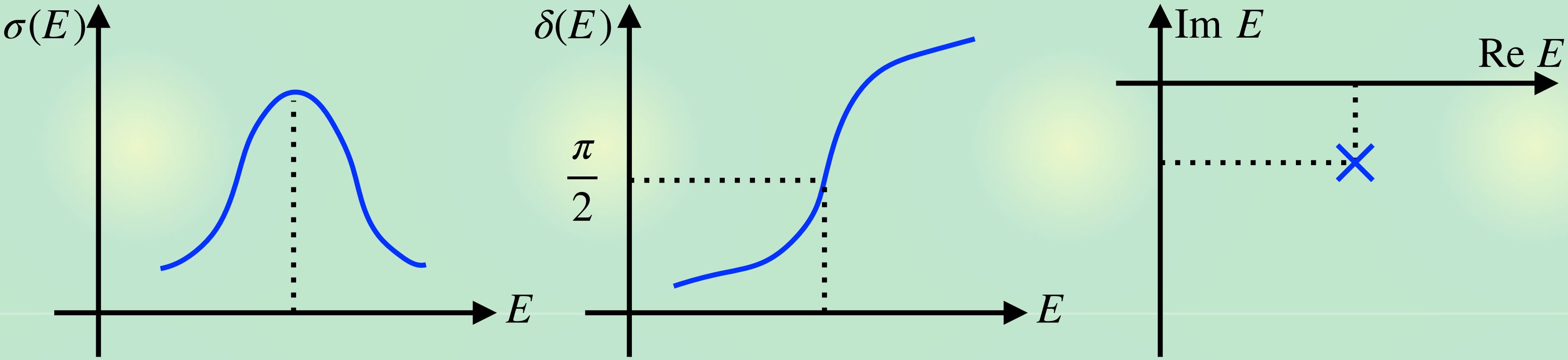
Various definitions of resonance

1) peak in spectra / cross sections

2) $\pi/2$ crossing of phase shift

3) Eigenstate of Hamiltonian : $H|R\rangle = E_R|R\rangle, \quad E_R \in \mathbb{C}$

4) Pole of the scattering amplitude



We will show that

- 3) and 4) are theoretically **unambiguous definitions**
- 1) and 2) agree with 3) and 4) in an idealized situation

Gamow theory

Resonance as “eigenstate” of Hamilotonian

- **Complex eigenenergy**

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

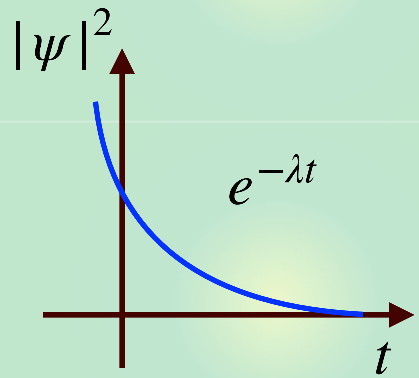
Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + \boxed{i\,\frac{h\,\lambda}{4\,\pi}},$$

wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- **time dependence : decreasing probability**

$$\psi = \Psi(q)\,.\,e^{+\frac{2\pi i E}{h}t}, \quad \propto e^{+2\pi i E_0 t/h} e^{-(\lambda/2)t}, \quad |\psi|^2 \propto e^{-\lambda t}$$



Eigenvalue of Hermitian operator should be real…

- **real in Hilbert space (~ square integrable functions)**

$$\int |\psi(\boldsymbol{r})|^2 d^3r < \infty$$

- **complex eigenvalue is allowed in extended space**

Square well potential

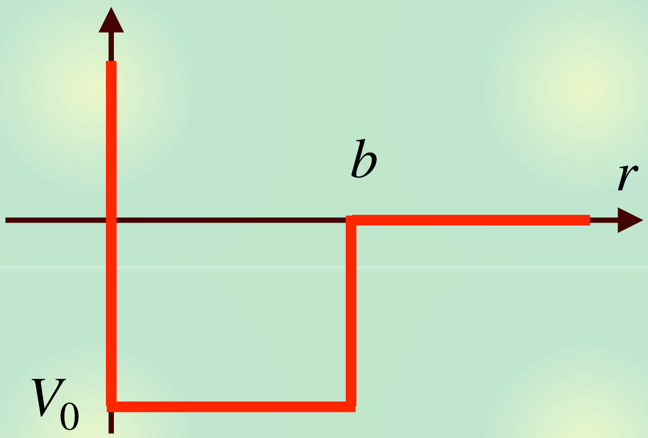
Schödinger equation for radial w.f. $\chi_\ell(r)$ in s wave ($\hbar = 1,\ m = 1$)

$$-\frac{1}{2}\frac{d^2\chi(r)}{dr^2} + V(r)\chi(r) = E\chi(r)$$

- w.f. : $\psi_{\ell,m}(\boldsymbol{r}) = \frac{\chi_\ell(r)}{r} Y_\ell^m(\hat{\boldsymbol{r}})$

- Square well potential

$$V(r) = \begin{cases} -V_0 & (0 \le r \le b) \\ 0 & (b < r) \end{cases}$$



- Scattering solution ($E > 0$, continuum, b.c. $\chi(r=0)=0$)

$$\chi(r) = \begin{cases} C\sin(\sqrt{2(E+V_0)}r) & (0 \le r \le b) \\ A^-(p)e^{-i\sqrt{2E}r} + A^+(p)e^{+i\sqrt{2E}r} & (b < r) \end{cases}$$

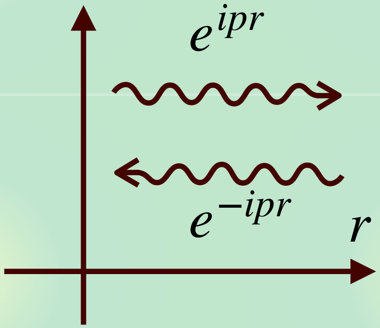
Note : scattering solution is not normalizable

Bound state solution

w.f. at $r \to \infty$ (eigenmomentum $p = \sqrt{2E}$)

$$\chi(r) \to A^{-}(p)e^{-ipr} + A^{+}(p)e^{+ipr}$$

incoming outgoing



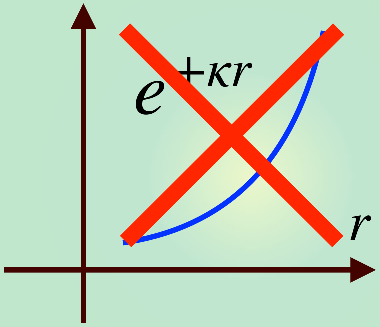
Bound state ($E < 0$, discrete eigenstate)

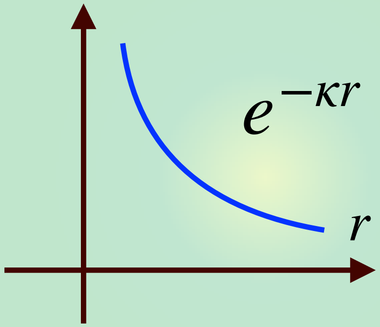
- **$p = \sqrt{2E}$ is pure imaginary**

$$p = i\kappa, \quad \kappa = \sqrt{2|E|} > 0$$

- **w.f. at $r \to \infty$**

$$\chi(r) \to \cancel{A^{-}(i\kappa)e^{+\kappa r}} + A^{+}(i\kappa)e^{-\kappa r}$$





- **w.f. is square integrable : b.c. $\chi(r \to \infty) = 0$**

—> $A^{-}(i\kappa) = 0$: vanishing incoming wave

Discrete eingenstate satisfies outgoing b.c. at $r \to \infty$

Resonance solution

Pure imaginary momentum $p = i\kappa$ for bound state

- **Physical scattering momentum p is real**

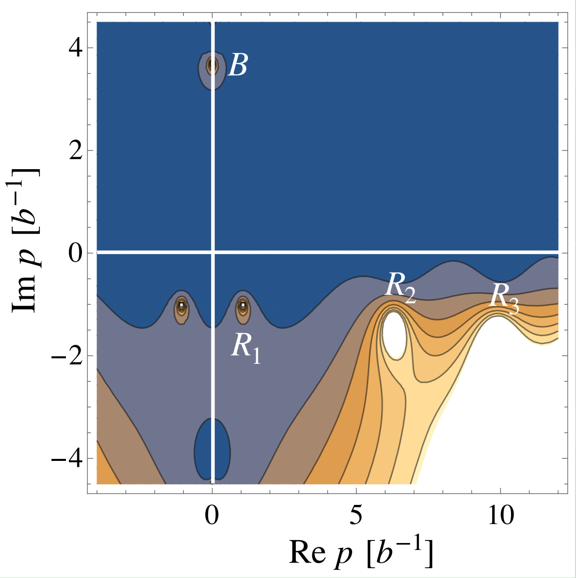
—> Bound state : analytic continuation of momentum

Resonance : momentum continued to complex p

- **definition of resonance 3)**
- **outgoing b.c. $A^{-}(p) = 0, \quad p \in \mathbb{C}$**
- **numerical solution with $V_0 = 10b^{-2}$**

	$p\ [b^{-1}]$	$E = p^2/2\ [b^{-2}]$
Bound state B	$+3.68i$	$-\ 6.78$
1st resonance R_1	$1.06 - 1.02i$	$0.05 - 1.08i$
2nd resonance R_2	$6.29 - 1.41i$	$18.8 - 8.86i$
3rd resonance R_3	$9.90 - 1.69i$	$47.6 - 16.8i$
$\vdots$		

$1/|A^{-}(p)|$ in complex p plane



W.f. of resonance

$1/|A^{-}(p)|$ **in complex** p **plane**

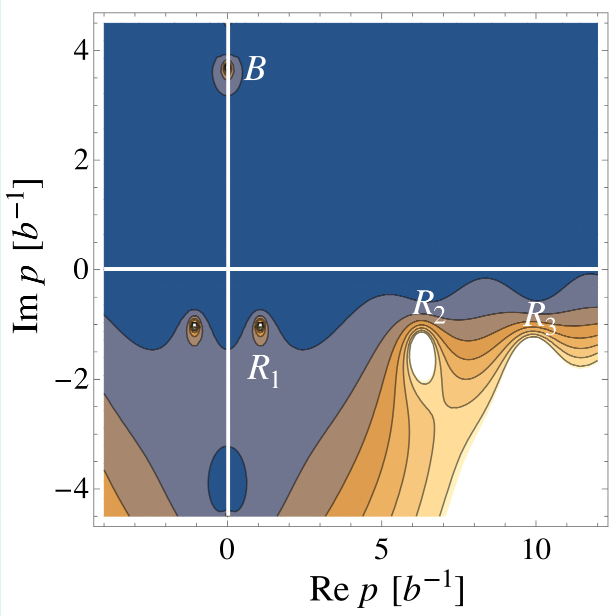
- **Resonance solutions in lower half** ($\mathrm{Im}\ p < 0$) **of** p **plane**

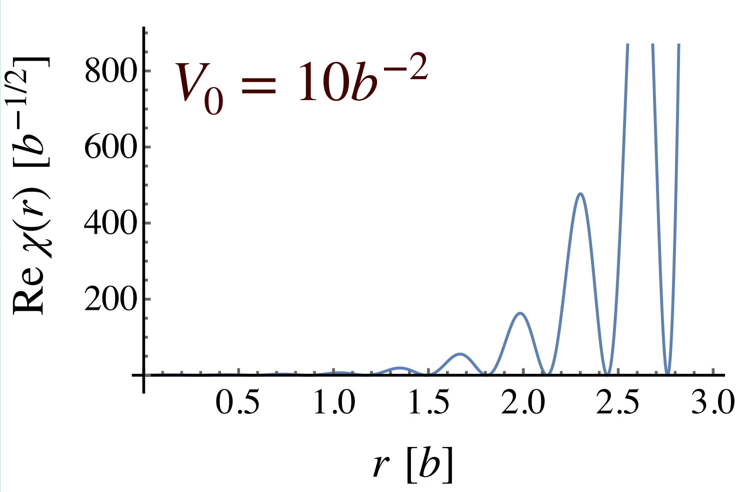
$$p = p_R - ip_I, \quad p_R, p_I > 0$$

- **Behavior of w.f.**

$$\chi(r) \to A^{+}(p)e^{ipr} \propto \underbrace{e^{ip_R r}}_{\text{oscillating}}\ \underbrace{e^{+p_I r}}_{\text{increasing}}$$

—> diverges at $r \to \infty$
need modified normalization





Complex eigenvalue <— non-square-integrable functions

Scattering theory : setup

Basic setup

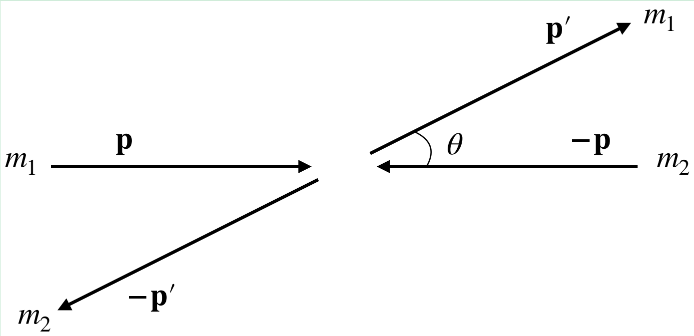
- nonlerativistic two-body scattering of m_1, m_2
- elastic scattering without internal d.o.f. (spin, etc.)
- rotational symmetry (central potential, $[H, L] = 0$)
- short range force

Two parameters to determine kinematics

- Energy E or momentum p (1-to-2 correspondence)

$$E = \frac{p^2}{2\mu}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

- scattering angle θ
- θ <—> angular momentum ℓ



Scattering theory : physical quantities

Relation between scattering quantities

J.R. Taylor, *Scattering theory* (Wiley, New York, 1972)

- **S matrix $s_\ell(p) \in \mathbb{C}$ in ℓ-th partial wave**

$$\langle E', \ell', m' | \mathsf{S} | E, \ell, m \rangle = \delta(E' - E)\delta_{\ell'\ell}\delta_{m'm}s_\ell(E)$$

- **phase shift $\delta_\ell(p) \in \mathbb{R}$ in ℓ-th partial wave**

$$s_\ell(p) = \exp\{2i\delta_\ell(p)\}$$

- **scattering amplitude $f_\ell(p) \in \mathbb{C}$ in ℓ-th partial wave**

$$f_\ell(p) = \frac{s_\ell(p) - 1}{2ip}$$

- **Total cross section**

$$\sigma(p) = \sum_\ell 4\pi(2\ell + 1)|f_\ell(p)|^2 = \sum_\ell \sigma_\ell$$ **<— ℓ-th wave cross section**

Relation to wave function

Asymptotic wave function (below we focus on $\ell = 0$)

$$\psi_p(r) \to \frac{i}{2}[J(p)e^{-ipr} - J(-p)e^{+ipr}] \quad (r \to \infty)$$

- **Jost function** $J(p)$: amplitude of incoming wave e^{-ipr}
- amplitude of outgoing wave is $-J(-p)$
- normalization called "regular solution"

S matrix, scattering amplitude and Jost function

$$s(p) = \frac{J(-p)}{J(p)}$$ ~ outgoing / incoming ratio

$$f(p) = \frac{s(p) - 1}{2ip} = \frac{J(-p) - J(p)}{2ipJ(p)}$$

Condition for discrete eigenstate

Eigenstate condition : outgoing b.c.

$$\psi_p(r) \to \frac{i}{2}[\cancel{J(p)e^{-ipr}} - J(-p)e^{+ipr}] \quad (r \to \infty)$$

- **Zero of Jost function** $J(p_R) = 0$ **with complex** p_R

- p_R **is pole of S matrix**

$$s(p_R) = \frac{J(-p_R)}{J(p_R)} \to \infty$$

- p_R **is pole of scattering amplitude —> definition 4)**

$$f(p_R) = \frac{J(-p_R) - J(p_R)}{2ip_R J(p_R)} \to \infty$$

Property $J(-p^*) = [J(p)]^*$ **—> if** p **is a solution, so is** $-p^*$

Complex energy and Riemann sheet

Relation between energy E and momentum p

$$E = \frac{p^2}{2\mu} = \frac{|p|^2}{2\mu} e^{2i\theta_p} = |E| e^{i\theta_E}$$

- **Complex momentum :** $p = |p| e^{i\theta_p}$
- **Phase of energy** $\theta_E = 2\theta_p$
- **If** θ_p **moves** $0 \to 2\pi$**, then** θ_E **moves** $0 \to 4\pi$
- p **and** $-p$ **(**θ_p **and** $\theta_p + \pi$ **) are mapped onto the same** E

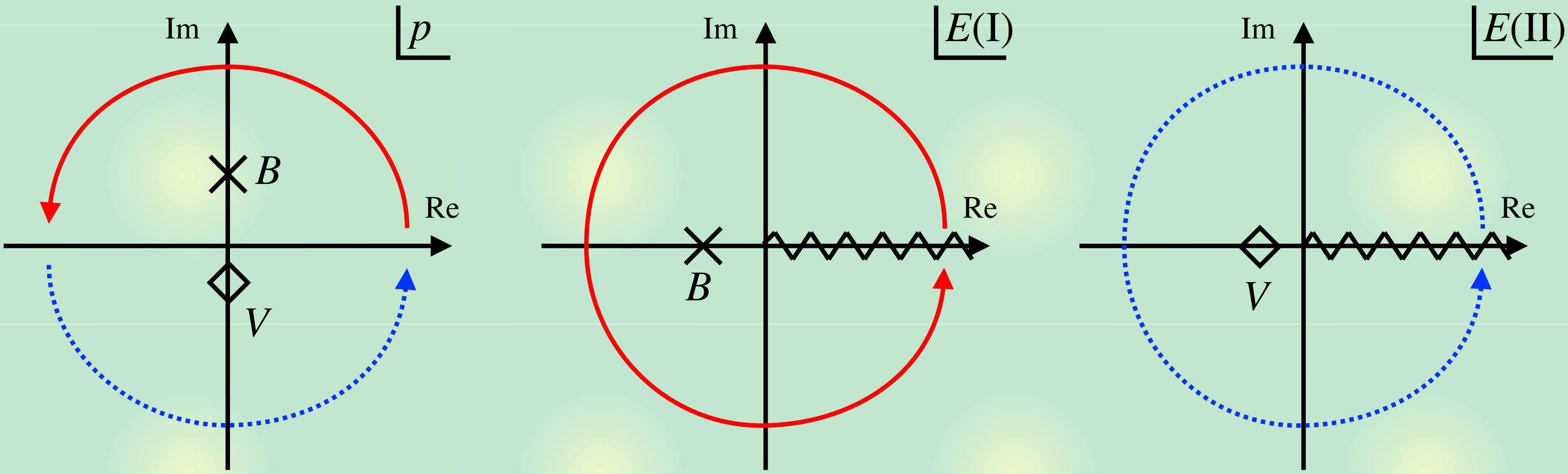
Meromorphic function of p ($s(p), f(p)$)

—> defined on two-sheeted Riemann surface of E

- $0 \leq \theta_E < 2\pi$ **: 1st Riemann sheet of E (upper half of p)**
- $2\pi \leq \theta_E < 4\pi$ **: 2nd Riemann sheet of E (lower half of p)**

Discrete eigenstates (on imaginary axis)

Pure imaginary solution in complex p plane ($p = -p^*$)



- bound state (B) : $E_B < 0$ on 1st Riemann sheet

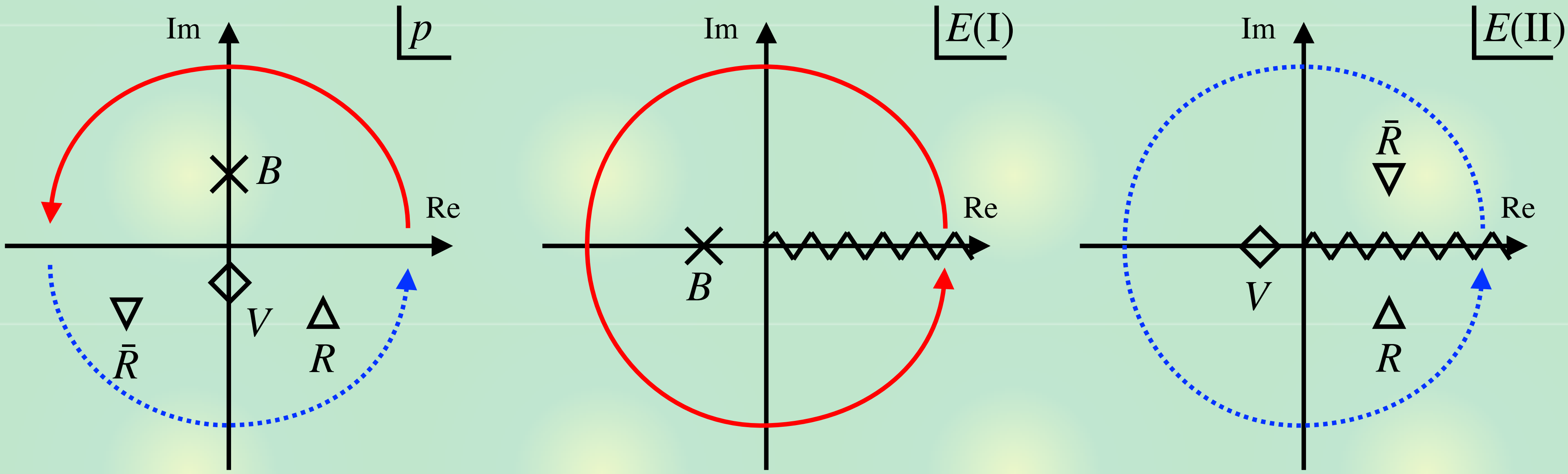
$$\mathrm{Re}\,[p_B] = 0, \quad \mathrm{Im}\,[p_B] > 0$$

- virtual state (V) : $E_V < 0$ on 2nd Riemann sheet

$$\mathrm{Re}\,[p_V] = 0, \quad \mathrm{Im}\,[p_V] < 0$$

Discrete eigenstates (in complex plane)

Complex solution, $p \neq -p^*$ **, always in pair**



- resonance (R**) :** $\mathrm{Re}\,[E_R] > 0,\ \mathrm{Im}\,[E_R] < 0$ **on 2nd Riemann sheet**

$\mathrm{Re}\,[p_R] > 0, \quad \mathrm{Im}\,[p_R] < 0$

- anti-resonance ($\bar{R}$**) :** $\mathrm{Re}\,[E_{\bar{R}}] > 0,\ \mathrm{Im}\,[E_{\bar{R}}] > 0$ **on 2nd sheet**

$\mathrm{Re}\,[p_{\bar{R}}] < 0, \quad \mathrm{Im}\,[p_{\bar{R}}] < 0$

Scattering amplitude on pole and on real axis

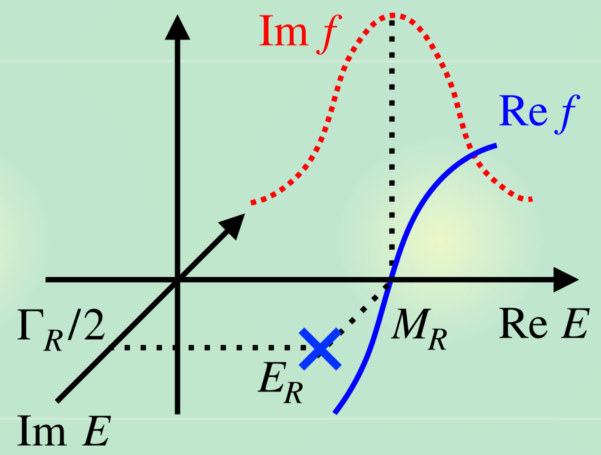
Laurent expansion around pole at $E = E_R$

$$f(E) = \underbrace{\frac{C_{-1}}{E - E_R}}_{\text{Breit-Wigner}} + \underbrace{\sum_{n=0}^{\infty} C_n (E - E_R)^n}_{\text{non-resonant terms}} = f_{BW}(E) + f_{NR}(E)$$

With $E_R = M_R - i\Gamma_R/2$**, amplitude is**

$$f(E) = -\frac{\Gamma_R}{2p}\frac{1}{E - M_R + i\Gamma_R/2} + f_{NR}(E)$$

$$\simeq -\frac{\Gamma_R}{2p}\frac{E - M_R - i\Gamma_R/2}{(E - M_R)^2 + \Gamma_R^2/4}$$



$f(E)$ **diverges at** $E = E_R$ **(pole) if with** f_{NR}

- If we assume that f_{NR} **is negligible in comparison with** f_{BW}**,**

—> real part vanishes and imaginary part peaks at $E = M_R$

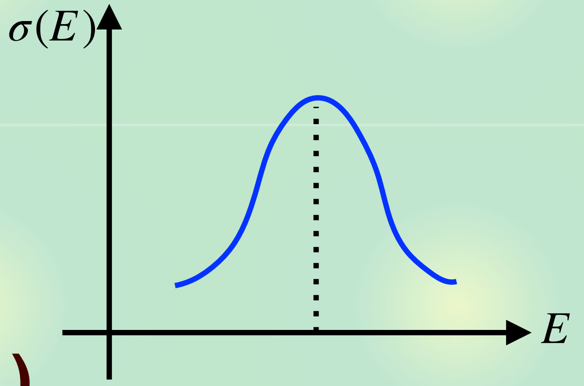
Effect on cross section and phase shift

$\mathrm{Im}\, f$ **peaks at** $E = M_R$

- **optical theorem**

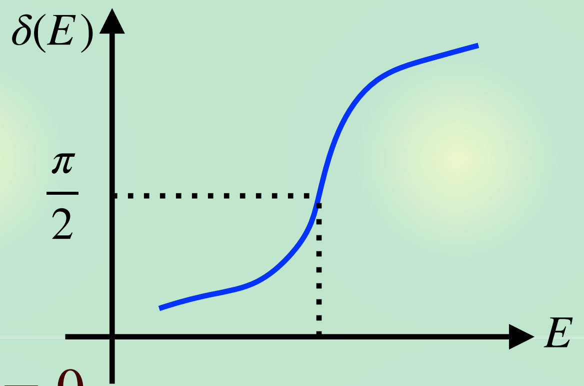
$$\mathrm{Im}\ f(E) = \frac{p}{4\pi}\sigma(E)$$

- **peak of cross section** σ **—> definition 1)**



$\mathrm{Re}\, f = 0$ **at** $E = M_R$

$$0 = \mathrm{Re}\ [f(M_R)] = \mathrm{Re}\ \left[\frac{s(M_R) - 1}{2i\sqrt{2\mu M_R}}\right]$$



- **numerator should be real <—** $\mathrm{Im}\ [s(M_R)] = 0$
- $s = e^{2i\delta}$ **: phase shift** $\delta = \pi/2$ **at** $E = M_R$ **—> definition 2)**

Both definitions valid only when we assume that f_{NR} **is small**

Summary of Part I

- As a consequence of **chiral SU(3) symmetry**, Weinberg-Tomozawa theorem suggests strongly attractive $\bar{K}N(I=0)$ interaction.

- Resonance : **pole** of amplitude = eigenstate

T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

Schrödinger eq. + outgoing b.c.
at energy E $(p=\sqrt{2\mu E})$

- bound states $(E<0)$

$$p = i\kappa \quad (\kappa > 0)$$

- resonances $(E \in \mathbb{C})$

$$p \in \mathbb{C} \quad (\mathrm{Im}\ p < 0)$$

$\Leftrightarrow$

zero of Jost function

$$\mathscr{f}_{\ell}(p) = 0$$

$\Leftrightarrow$

pole of s-matrix/
scattering amplitude

$$|f_{\ell}(p)| \to \infty$$

$$|s_{\ell}(p)| \to \infty$$