Hadronic molecules and their structure (part II)

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Clusters & Hierarchies

2022, Jun. 28th

Contents

Part I : Basics

- Chiral symmetry

S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory* (Springer, Berlin, 2012)

- Resonances

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972);
T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

Part II : Application

- $\bar{K}N$ scattering and $\Lambda(1405)$ resonance

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

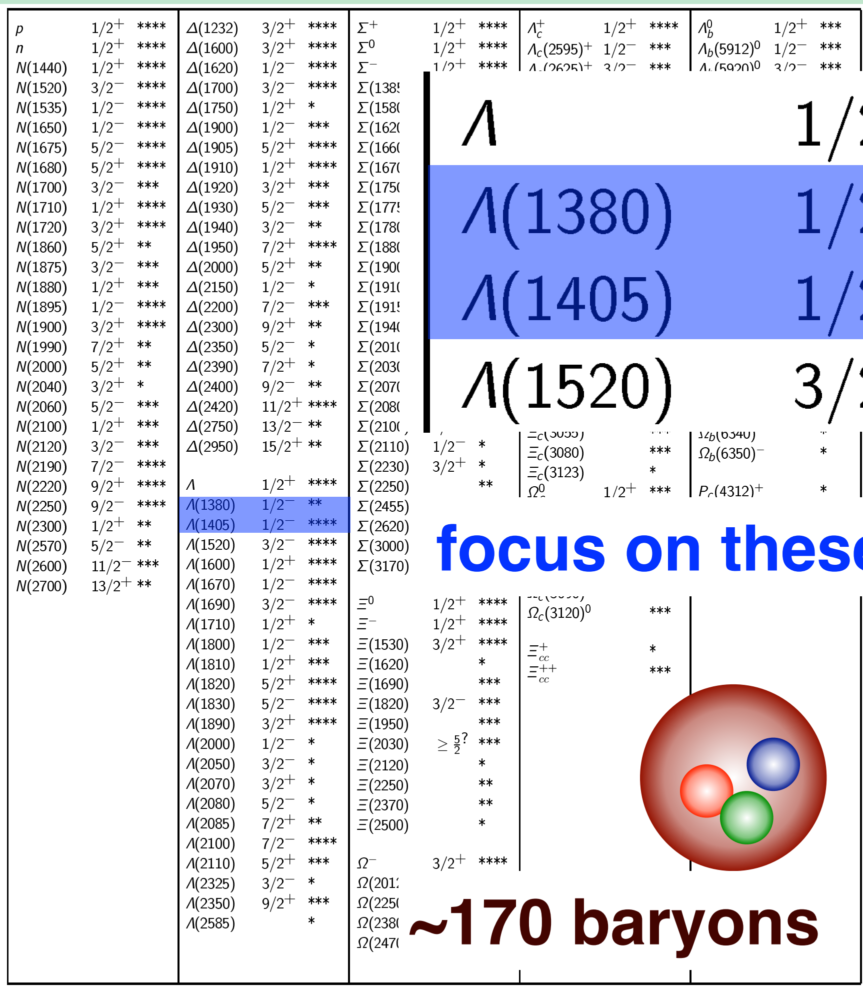
- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Observed hadrons (2022)

Particle Data Group (PDG) 2022 eddition

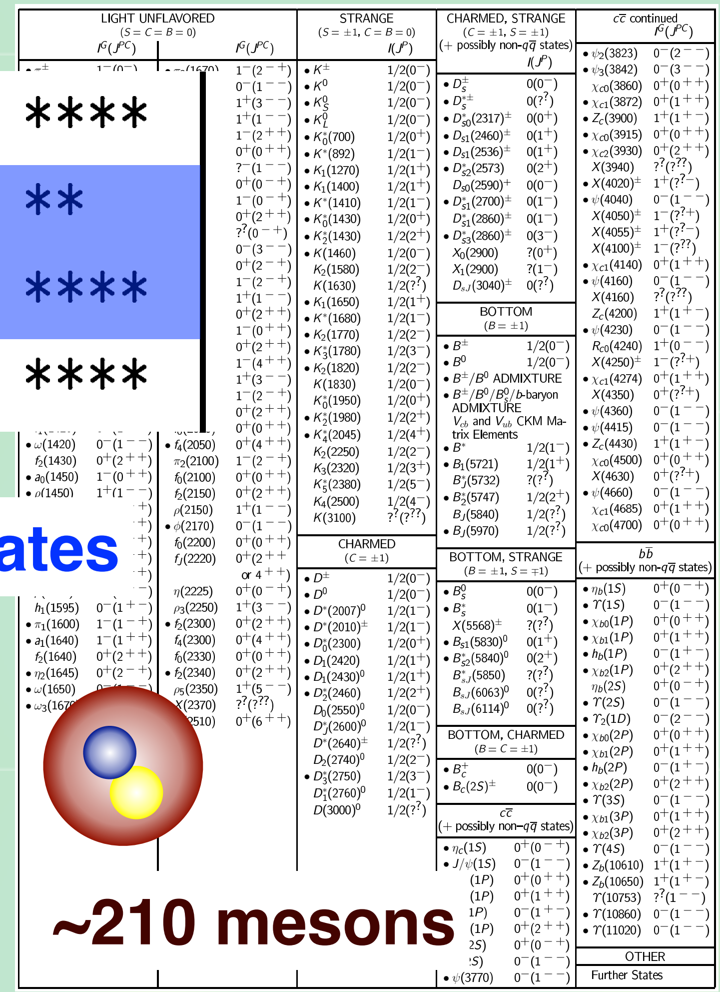
http://pdg.lbl.gov/



Λ	$1/2^+$	****
$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****

focus on these states

~170 baryons



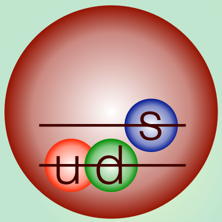
~210 mesons

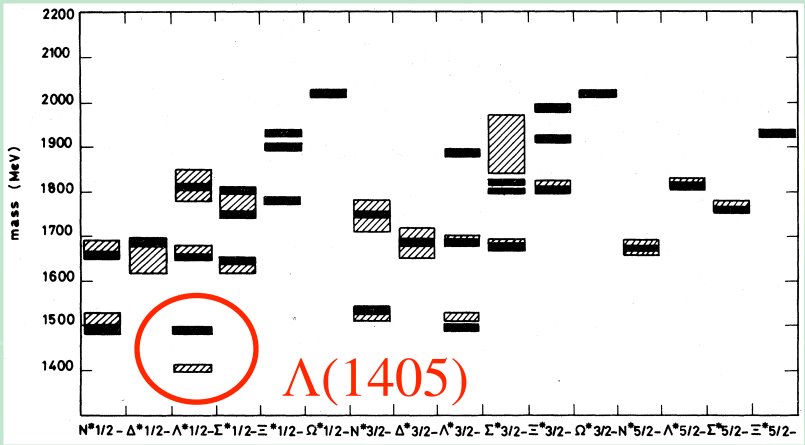
All ~ 380 hadrons emerge from single QCD Lagrangian

$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture —> exotic candidate

N. Isgur and G. Karl, PRD18, 4187 (1978)



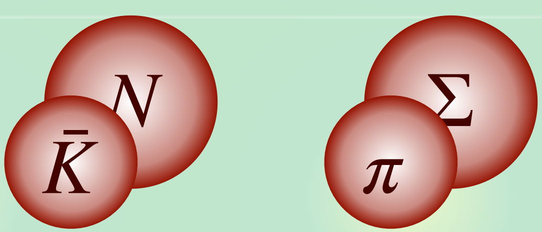


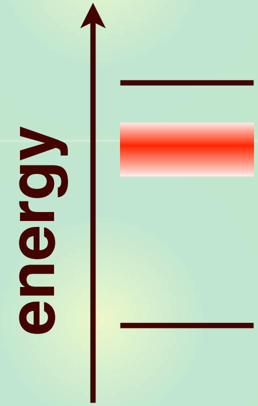
▬ : theory

▨ : experiment

Resonance in coupled-channel scattering

- Coupling to MB states





Detailed analysis of $\bar{K}N$-$\pi\Sigma$ scattering is necessary

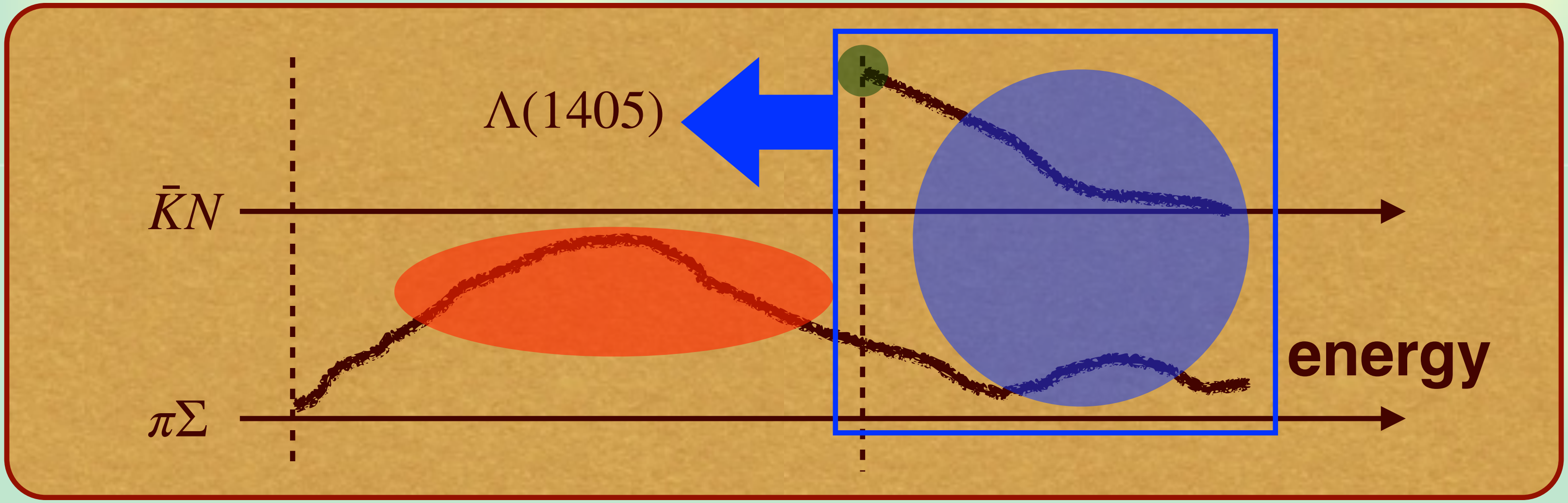
Strategy for $\bar{K}N$ interaction

Above the $\bar{K}N$ threshold : direct constraints

- K^-p **total cross sections** (old data)
- $\bar{K}N$ **threshold branching ratios** (old data)
- K^-p **scattering length** (new data : SIDDHARTA)

Below the $\bar{K}N$ threshold: indirect constraints

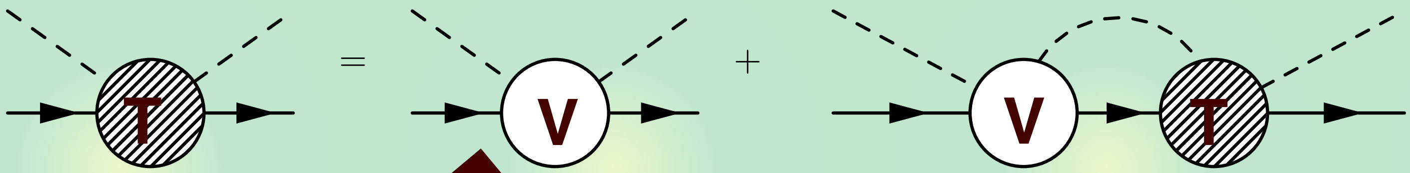
- $\pi\Sigma$ **mass spectra** (new data : LEPS, CLAS, HADES, ...)



Construction of the realistic amplitude

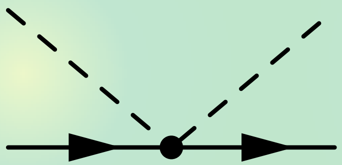
Chiral SU(3) coupled-channels $(\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, K\Xi)$ approach

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)



Chiral perturbation theory

1) TW term

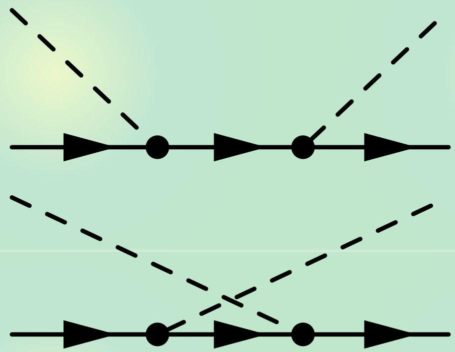


$\mathcal{O}(p)$

6 cutoffs

TW model

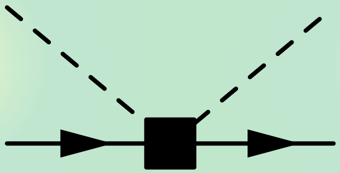
2) Born terms



$\mathcal{O}(p)$

TWB model

3) NLO terms



$\mathcal{O}(p^2)$

7 LECs

NLO model

Best-fit results

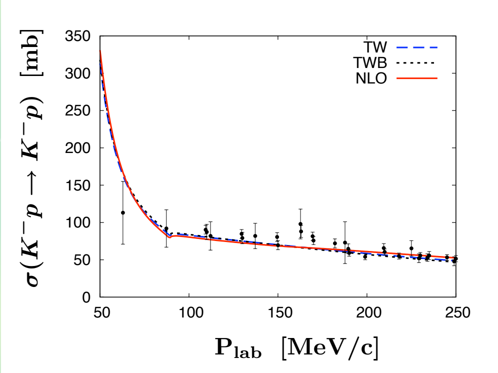
K at rest

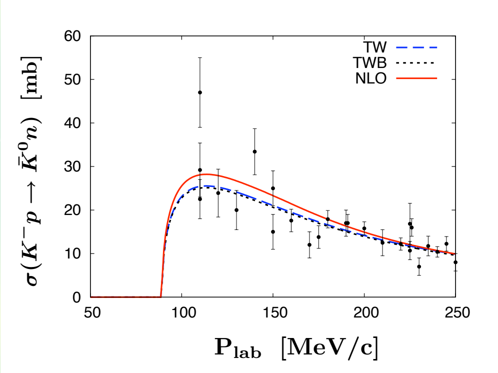
	TW	TWB	NLO	Experiment	
ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
Γ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
γ	2.36	2.36	2.37	2.36 ± 0.04	[11]
R_n	0.20	0.19	0.19	0.189 ± 0.015	[11]
R_c	0.66	0.66	0.66	0.664 ± 0.011	[11]
χ^2/d.o.f	1.12	1.15	0.96		

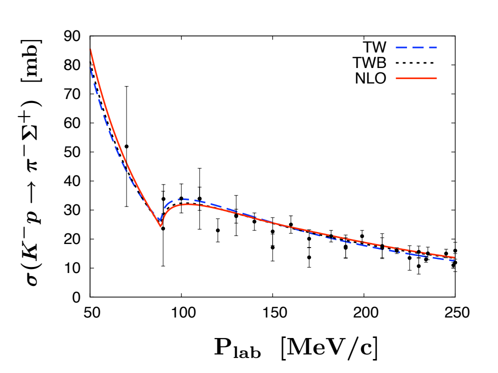
} **SIDDHARTA** (ΔE, Γ)

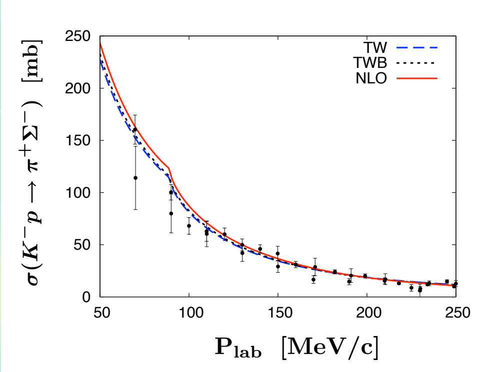
} **Branching ratios** (γ, R_n, R_c)

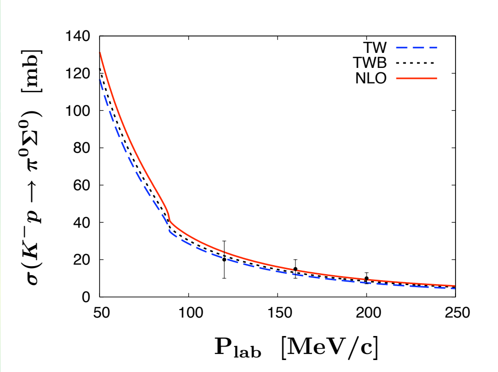
$K^- p$ cross sections

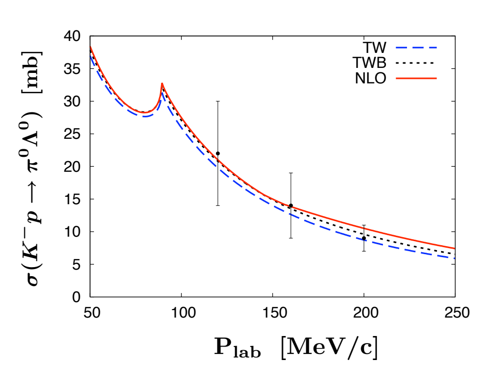








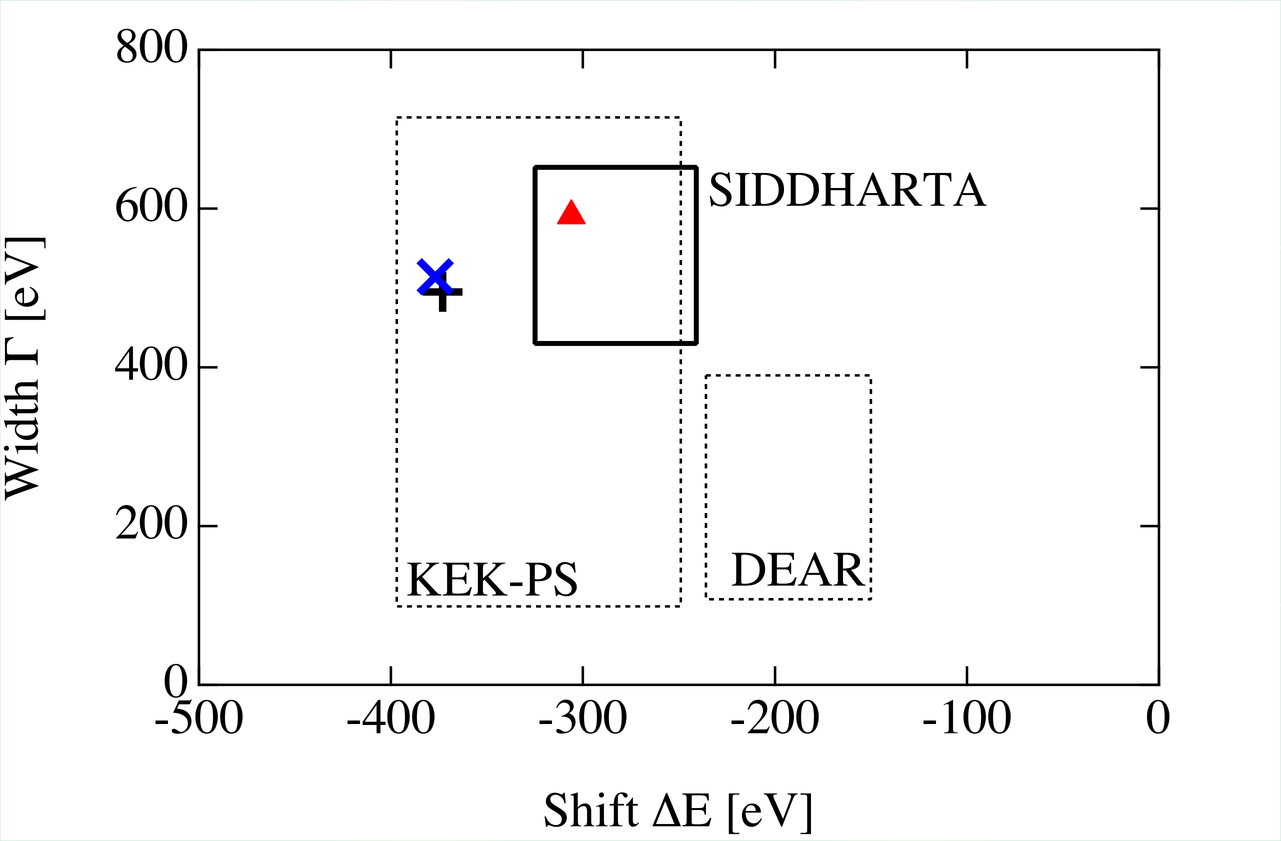




Accurate description of all existing data (χ^2/d.o.f ~ 1)

Comparison with SIDDHARTA

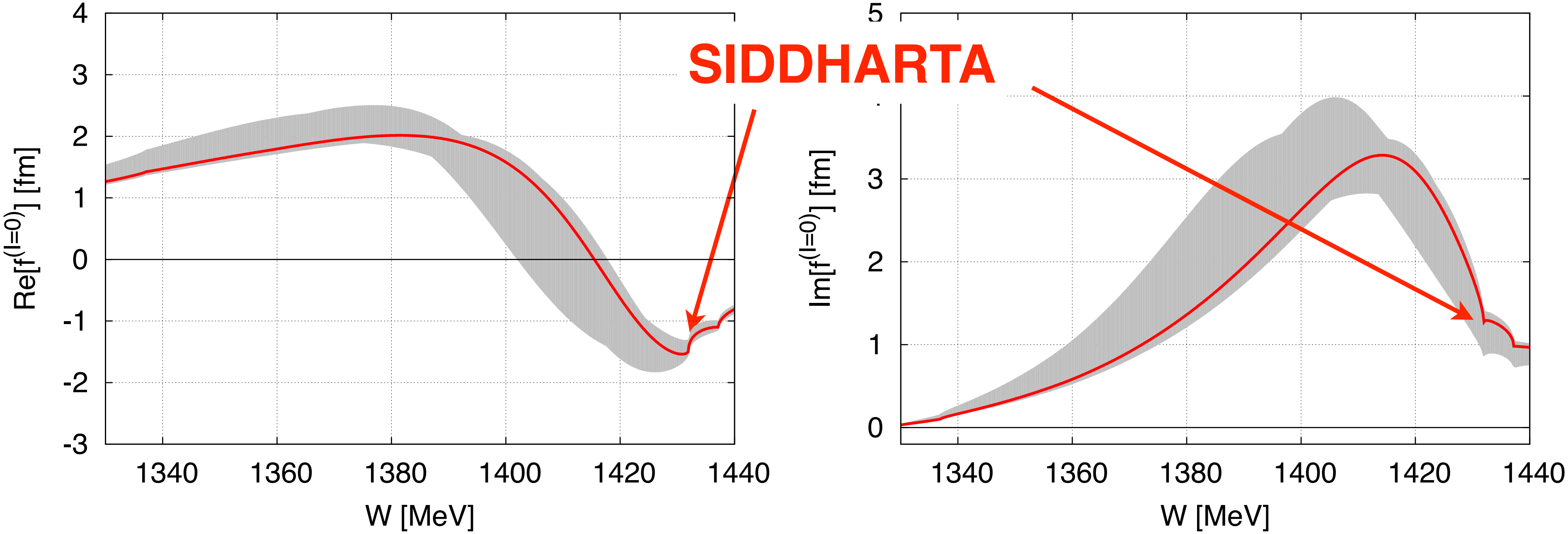
	TW	TWB	NLO
χ^2/d.o.f.	1.12	1.15	0.957



TW and TWB are reasonable, while best-fit requires NLO

Subthreshold extrapolation

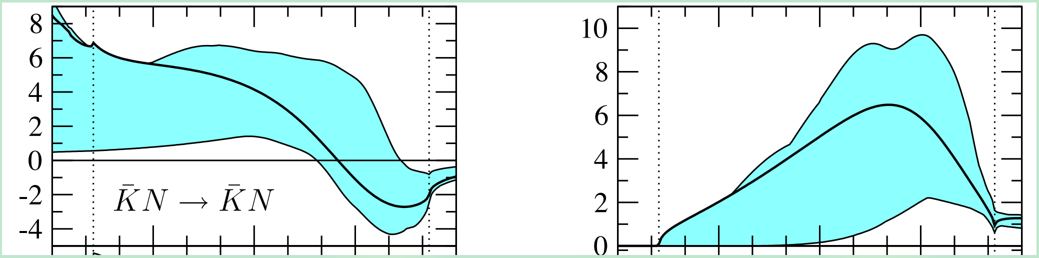
Uncertainty of $\bar{K}N \to \bar{K}N(I=0)$ amplitude below threshold



Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise, NPA 954, 41 (2016)

- c.f. without SIDDHARTA

R. Nissler, Doctoral Thesis (2007)

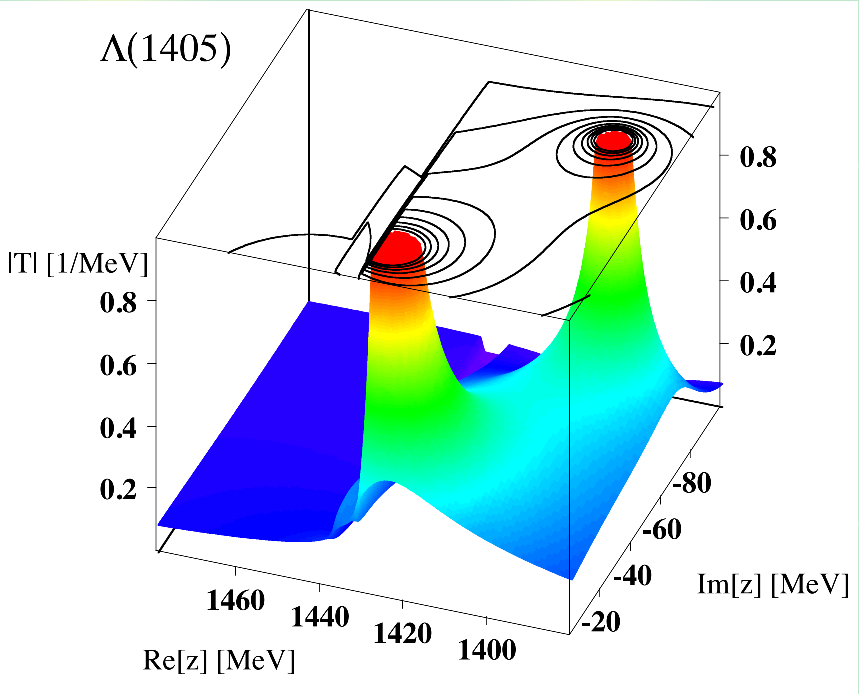


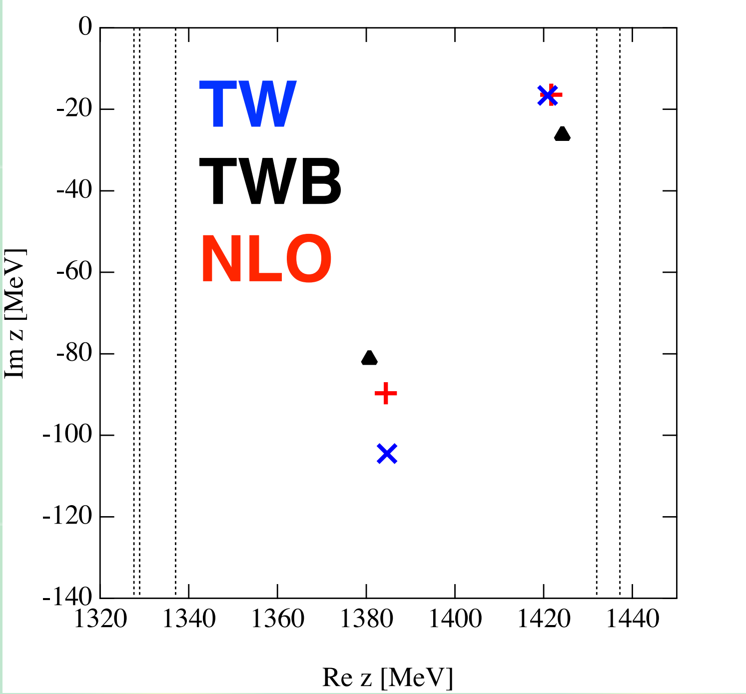
SIDDHARTA is essential for subthreshold extrapolation

Extrapolation to complex energy: two poles

Two poles : superposition of two eigenstates

J.A. Oller, U.G. Meißner, PLB 500, 263 (2001);
D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meißner, NPA 723, 205 (2003);
U.G. Meißner, Symmetry 12, 981 (2020); M. Mai, Eur. Phys. J. ST 230 6, 1593 (2021);
T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)





T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

NLO analysis confirms the two-pole structure

PDG has changed

2020 update of PDG

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012); ▲
Z.H. Guo, J.A. Oller, PRC87, 035202 (2013); ×
M. Mai, U.G. Meißner, EPJA51, 30 (2015) ■ ○

- Particle Listing section:

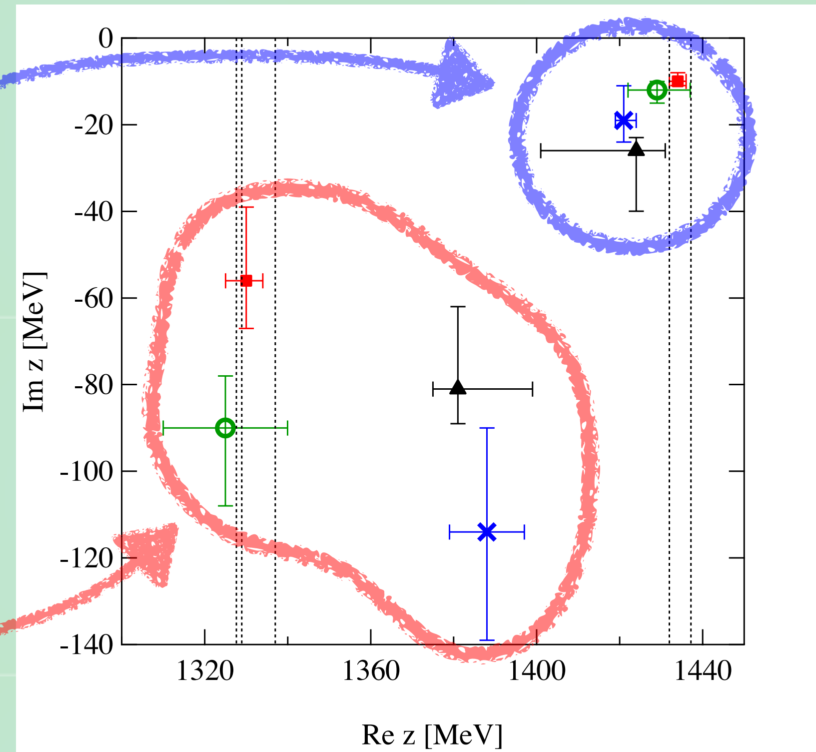
Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405)\ 1/2^-$ $\qquad I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1380)\ 1/2^-$ $\qquad J^P = \frac{1}{2}^-$ Status: ∗∗

new!



T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

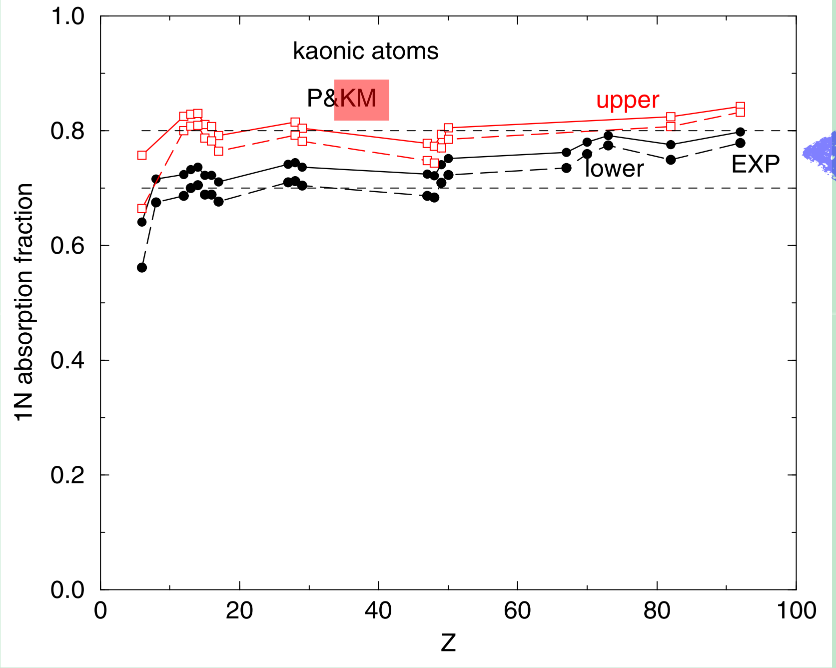
- "$\Lambda(1405)$" is no longer at 1405 MeV but ~ 1420 MeV.

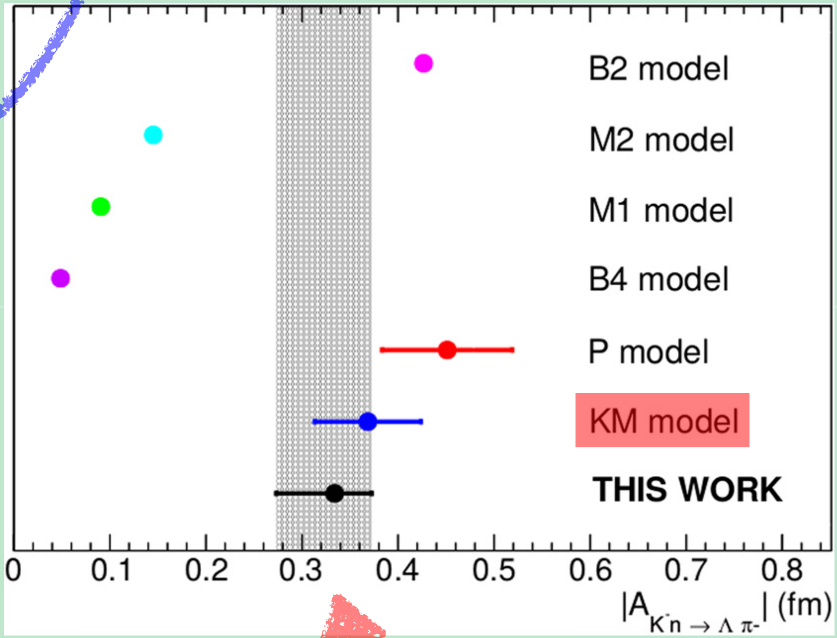
- Lower pole : two-star resonance $\Lambda(1380)$

Further check of amplitude

Single-nucleon absorption on kaonic atoms

E. Friedman, A. Gal, NPA959, 66 (2017)





$|f_{K^-n\to\pi^-\Lambda}|$ from K^- absorption on $^4\mathrm{He}$ at DAΦNE

K. Piscicchia, *et al.*, PLB782, 339 (2018)

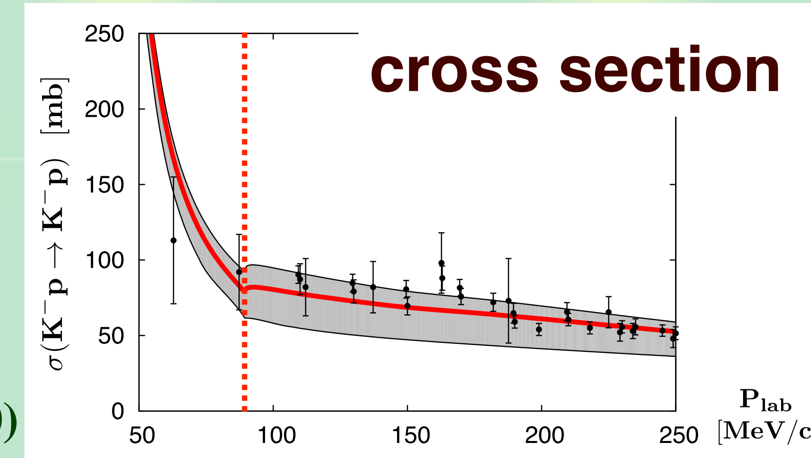
Our amplitude (KM model) is compatible with these analyses

New data : K^-p correlation function

K^-p total cross sections

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

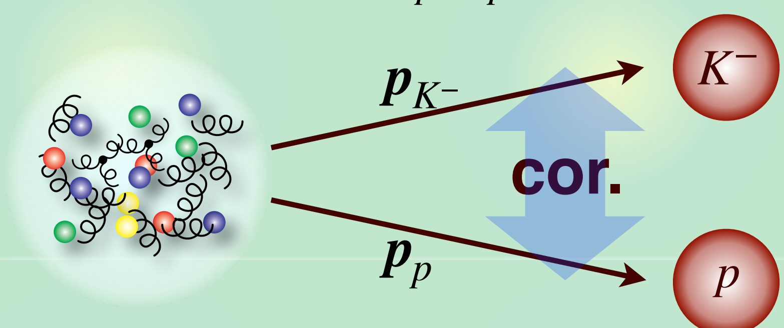
- Old bubble chamber data

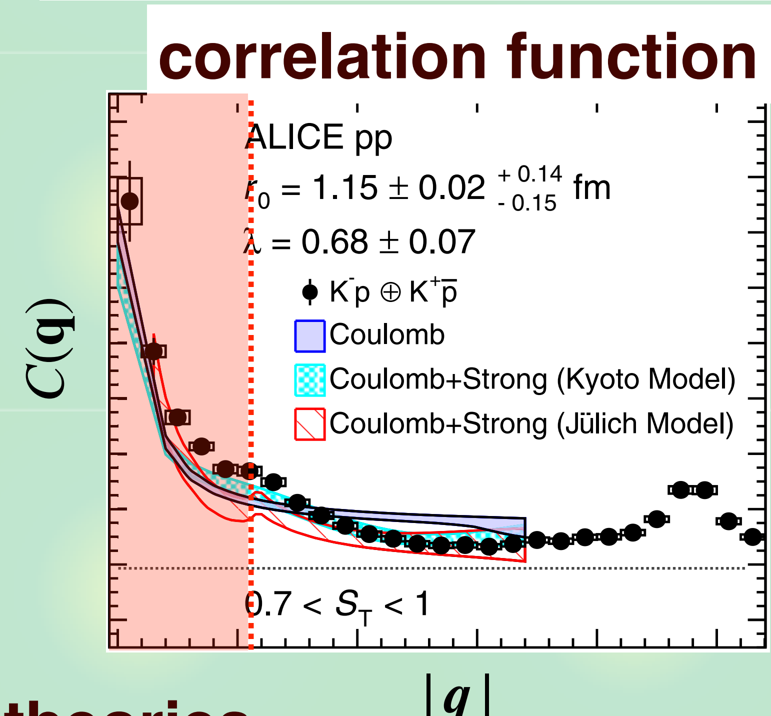


K^-p correlation function

S. Acharya *et al.* (ALICE), PRL 124, 092301 (2020)

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-})N_p(\boldsymbol{p}_p)}$$





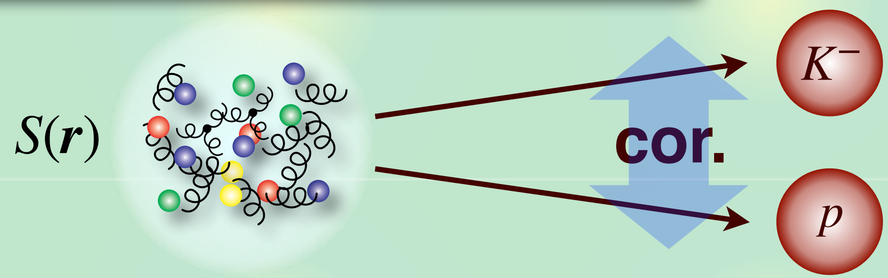
- Excellent precision ($\bar{K}^0n$ cusp)
- Low-energy data below $\bar{K}^0n$

—> Important constraint on $\Lambda(1405)$ theories

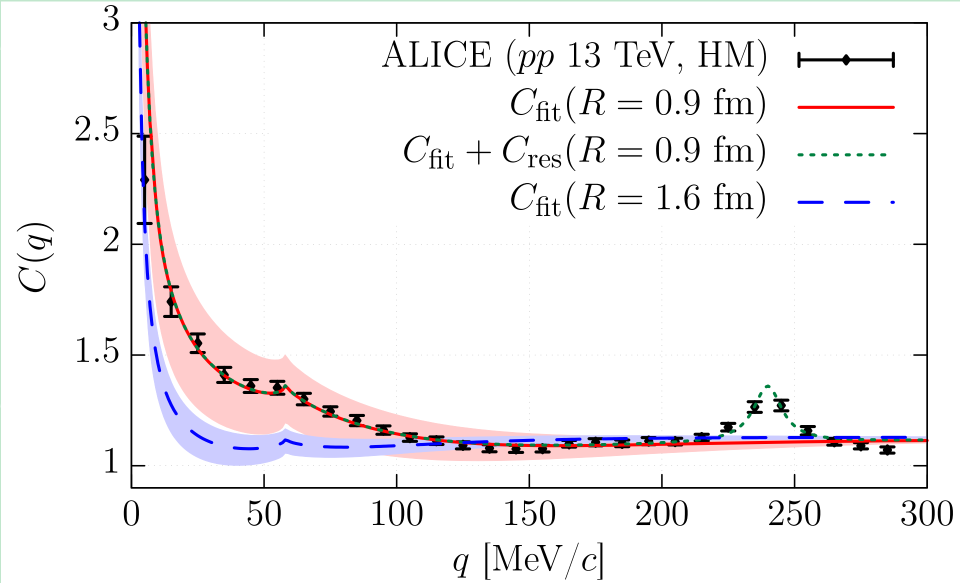
Prediction from chiral SU(3) dynamics

Theoretical calculation of $C(q)$

$$C(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\, S(\boldsymbol{r})\, |\Psi^{(-)}_{\boldsymbol{q}}(\boldsymbol{r})|^2$$



- Wave function $\Psi^{(-)}_{\boldsymbol{q}}(\boldsymbol{r})$: coupled-channel $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential
- Source function $S(r)$: estimated by K^+p data



Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

Correlation function is well reproduced

Contents

Lecture 1 : Basics

- Chiral symmetry

S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory* (Springer, Berlin, 2012)

- Resonances

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972);
T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

Lecture 2 : Application

- $\bar{K}N$ scattering and $\Lambda(1405)$ resonance

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Compositeness of hadrons

- Structure of a given resonance (pole)?
- Weak binding relation for stable bound states

  S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness X threshold channel	or	"Elementariness" Z other contributions

↑

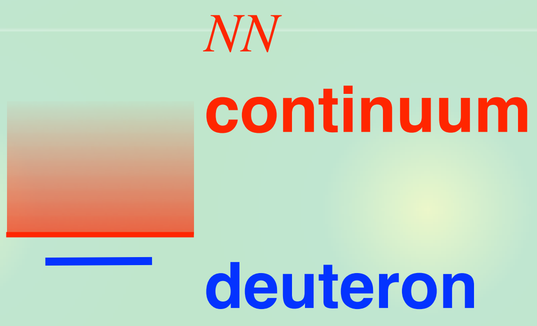
observables (a_0, B)

- Effective field theory —> description of low-energy scattering amplitude, generalization to unstable resonances

Weak-binding relation for stable states

Compositeness X of s-wave weakly bound state $(R \gg R_{\mathrm{typ}})$

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{1-X}\,|\mathrm{others}\rangle$$

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length (a_0) — **radius of state** (R)

- **Deuteron is NN composite :** $a_0 \sim R \Rightarrow X \sim 1$
- **Internal structure from observables** (a_0, B)

(i) The particle must be stable; else Z is undefined. (However, it may be an adequate approximation to ignore the decay modes of a very narrow resonance.)
(ii) The particle must couple to a two-particle channel with threshold not too much above the particle mass.
(iii) It is crucial that this two-body channel have zero orbital angular momentum l, since for $l \neq 0$ the factor $(E)^{1/2}$ in the integrands of (24) and (32) would be $E^{l+(1/2)}$, and the integrals could not be approximated by their low-energy parts.

Problem: applicable only to stable states

Effective field theory

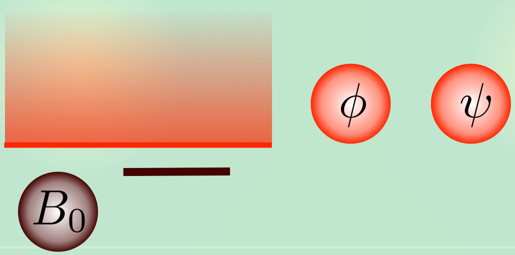
Low-energy scattering with near-threshold bound state

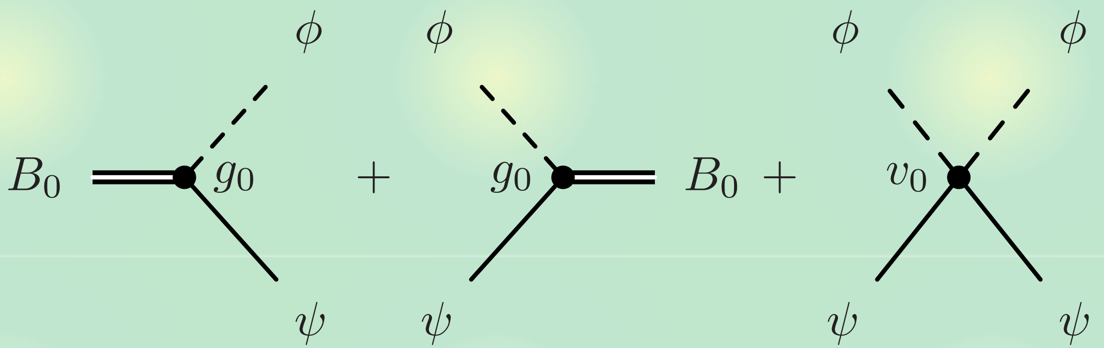
- **Nonrelativistic EFT with contact interaction**

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M} \nabla\psi^\dagger \cdot \nabla\psi + \frac{1}{2m} \nabla\phi^\dagger \cdot \nabla\phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \omega_0 B_0^\dagger B_0 \right]$$

$$H_{\text{int}} = \int d\mathbf{r} \left[g_0 \left(B_0^\dagger \phi\psi + \psi^\dagger \phi^\dagger B_0 \right) + v_0 \psi^\dagger \phi^\dagger \phi\psi \right]$$





- **Cutoff** : $\Lambda \sim 1/R_{\text{typ}}$ **(interaction range of microscopic theory)**
- **At low momentum** $p \ll \Lambda$**, interaction ~ contact**

Compositeness and "elementariness"

Eigenstates

$$H_{\rm free}|B_0\rangle = \omega_0|B_0\rangle, \quad H_{\rm free}|\boldsymbol{p}\rangle = \frac{\boldsymbol{p}^2}{2\mu}|\boldsymbol{p}\rangle$$

free (discrete + continuum)

$$(H_{\rm free} + H_{\rm int})|B\rangle = -B|B\rangle$$

full (bound state)

- Normalization of $|B\rangle$ + completeness relation

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\boldsymbol{p}\rangle\langle \boldsymbol{p}|$$

- Projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle \boldsymbol{p}|B\rangle|^2$$

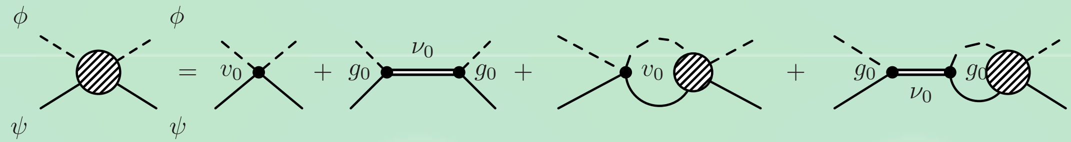
"elementarity" (Z) **compositeness** (X)

Z, X **: real and nonnegative —> interpreted as probability**

Weak binding relation

$\psi\phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E - \omega_0}, \quad G(E) = \frac{1}{2\pi^2}\int_0^{\Lambda} dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness $X \leftarrow v(E), G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R = \sqrt{2\mu B}$ expansion of scattering length a_0

$$a_0 = -f(E=0) = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

renormalization dependent

renormalization independent

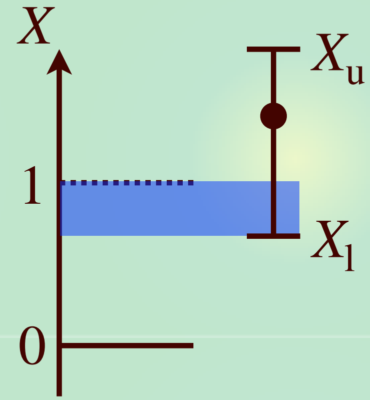
If $R \gg R_{\rm typ}$, correction terms neglected: $X \leftarrow (a_0, B)$

Application to bound states

Uncertainty estimation with $\mathcal{O}(R_{\rm typ}/R)$ term

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$X_{\rm u} = \frac{a_0/R + \xi}{2 - a_0/R - \xi}, \quad X_{\rm l} = \frac{a_0/R - \xi}{2 - a_0/R + \xi}, \quad \xi = \frac{R_{\rm typ}}{R}$$



- exclude region outside $0 \leq X \leq 1$

Application and finite range correction

$$R_{\rm typ} = \max\{R_{\rm int}, R_{\rm eff}\}$$

T. Kinugawa, T. Hyodo, arXiv:2205.08470 [hep-ph]

bound state	compositeness X
d	$0.74 \leq X \leq 1$
$X(3872)$	$0.53 \leq X \leq 1$
$D^*_{s0}(2317)$	$0.81 \leq X \leq 1$
$D_{s1}(2460)$	$0.55 \leq X \leq 1$
$N\Omega$ dibaryon	$0.80 \leq X \leq 1$
$\Omega\Omega$ dibaryon	$0.79 \leq X \leq 1$
${}^3_\Lambda\mathrm{H}$	$0.74 \leq X \leq 1$
${}^4\mathrm{He}$ dimer	$0.93 \leq X \leq 1$

Inclusion of decay channel

Introduce decay channel

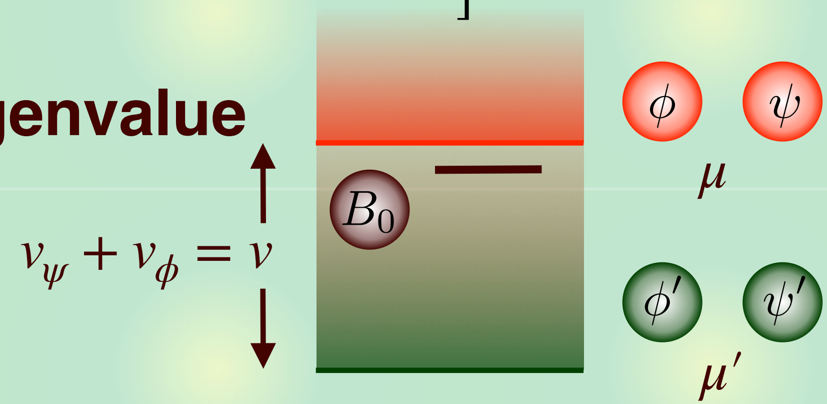
$$H'_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right]$$

$$H'_{\text{int}} = \int d\mathbf{r} \left[g'_0 \left(B_0^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} B_0 \right) + v'_0 \psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + v_0^t (\psi^{\dagger} \phi^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} \phi \psi) \right]$$

Quasi-bound state : complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H \,|\, h \rangle = E_h \,|\, h \rangle, \quad E_h \in \mathbb{C}$$



Generalized relation : correction from threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O}\left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

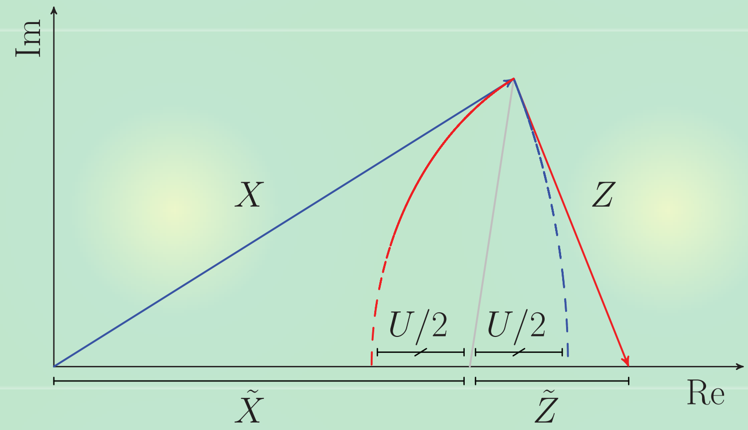
If $|R| \gg (R_{\text{typ}}, \ell)$, correction terms neglected: $X \leftarrow (a_0, E_h)$

Complex compositeness

Unstable states —> complex Z and X

$$Z+X=1, \quad Z, X \in \mathbb{C}$$

- Probabilistic interpretation?



New definition

$$\tilde{Z}=\frac{1-|X|+|Z|}{2}, \quad \tilde{X}=\frac{1-|Z|+|X|}{2}$$

- Interpreted as probabilities $\tilde{Z}+\tilde{X}=1, \quad \tilde{Z}, \tilde{X} \in [0,1]$
- reduces to Z and X in the bound state limit

$U/2$: uncertainty of interpretation

$$U=|Z|+|X|-1$$

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

- Sensible interpretation only for small $U/2$ case

Evaluation of compositeness

Generalized weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- Neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-\ \ 4 - i\ \ 8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-\ \ 3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, $X \sim 1$ with small $U/2$ (complex nature)

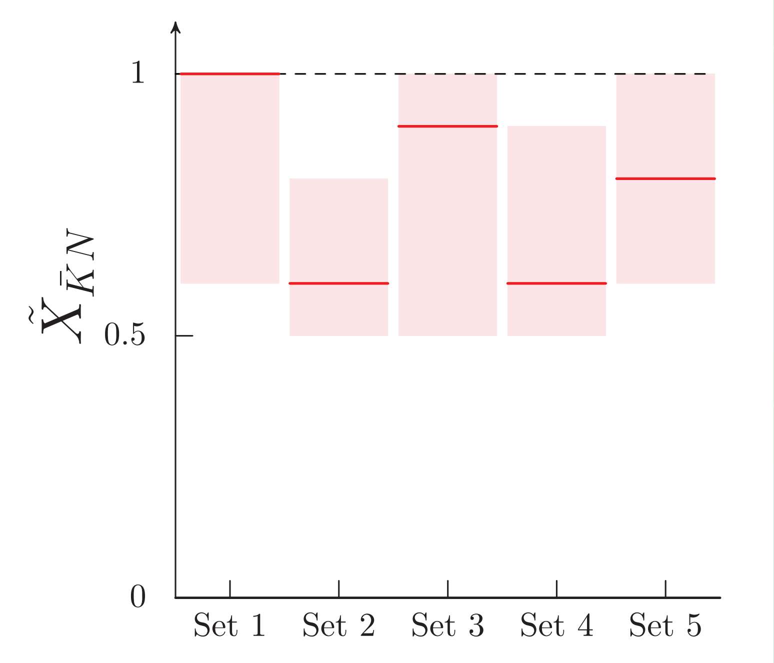
$\Lambda(1405)$: $\bar{K}N$ composite dominance <— observables

Uncertainty estimation

Estimation of correction terms: $|R| \sim 2$ fm

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **ρ meson exchange picture:** $R_{\rm typ} \sim 0.25$ fm
- **Energy difference from $\pi\Sigma$:** $\ell \sim 1.08$ fm



$\bar{K}N$ composite dominance holds even with correction terms

Summary of Part II

- **Pole structure in the $\Lambda(1405)$ region is now well constrained by experimental data.**

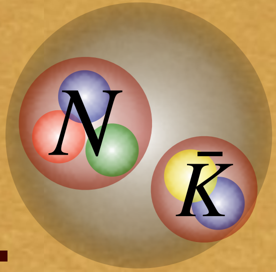
  "$\Lambda(1405)$" —> $\Lambda(1405)$ **and** $\Lambda(1380)$

  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****

- **Compositeness of hadrons can be determined from observables by the weak-binding relation.**

  Generalized weak-binding relation shows that the structure of $\Lambda(1405)$ is dominated by $\bar{K}N$ molecular component.



Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)