

# Theory of few-body kaon-nuclear systems



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# Contents



## **$\Lambda(1405)$ and $\bar{K}N$ potentials**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012);  
K. Miyahara, T. Hyodo, PRC 93, 015201 (2016);  
K. Miyahara, T. Hyodo, W. Weise, PRC 98, 025201 (2018);  
Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)



## **Applications to few-body systems**

### **- Kaonic nuclei**

S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara. T. Hyodo, PRC95, 065202 (2017)

### **- Kaonic deuterium**

T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)



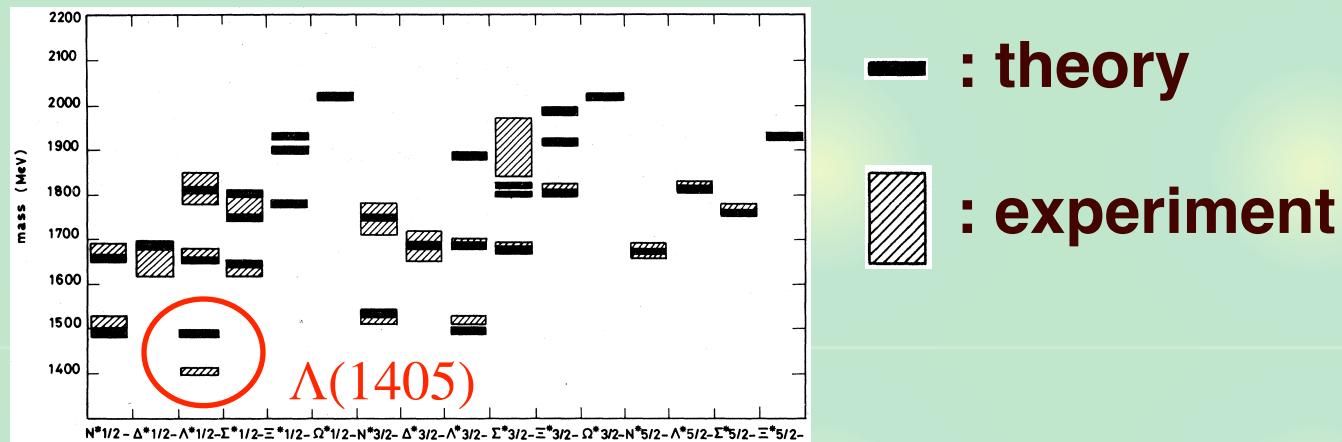
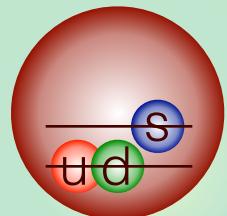
## **Summary**

T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021);  
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

## $\Lambda(1405)$ and $\bar{K}N$ scattering

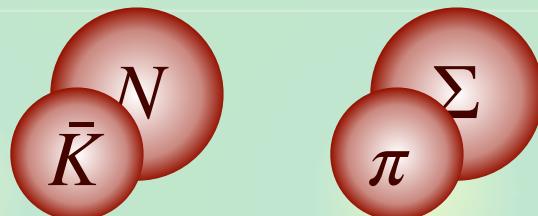
$\Lambda(1405)$  does not fit in standard picture  $\rightarrow$  exotic candidate

N. Isgur and G. Karl, PRD18, 4187 (1978)



Resonance in coupled-channel scattering

- Coupling to MB states



Detailed analysis of  $\bar{K}N-\pi\Sigma$  scattering is necessary

Strategy for  $\bar{K}N$  interaction

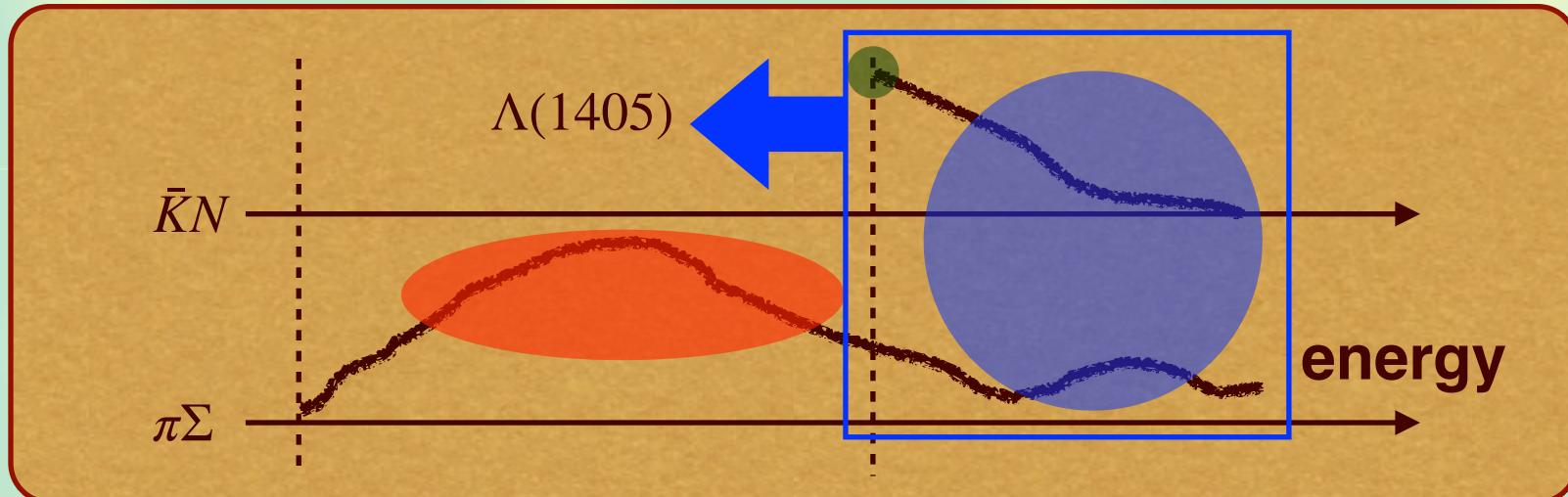
Above the  $\bar{K}N$  threshold : direct constraints

- $K^- p$  total cross sections (old data)
- $\bar{K}N$  threshold branching ratios (old data)
- $K^- p$  scattering length (new data : SIDDHARTA)

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

Below the  $\bar{K}N$  threshold: indirect (reaction model needed)

- $\pi\Sigma$  mass spectra (LEPS, CLAS, HADES, J-PARC, ...)



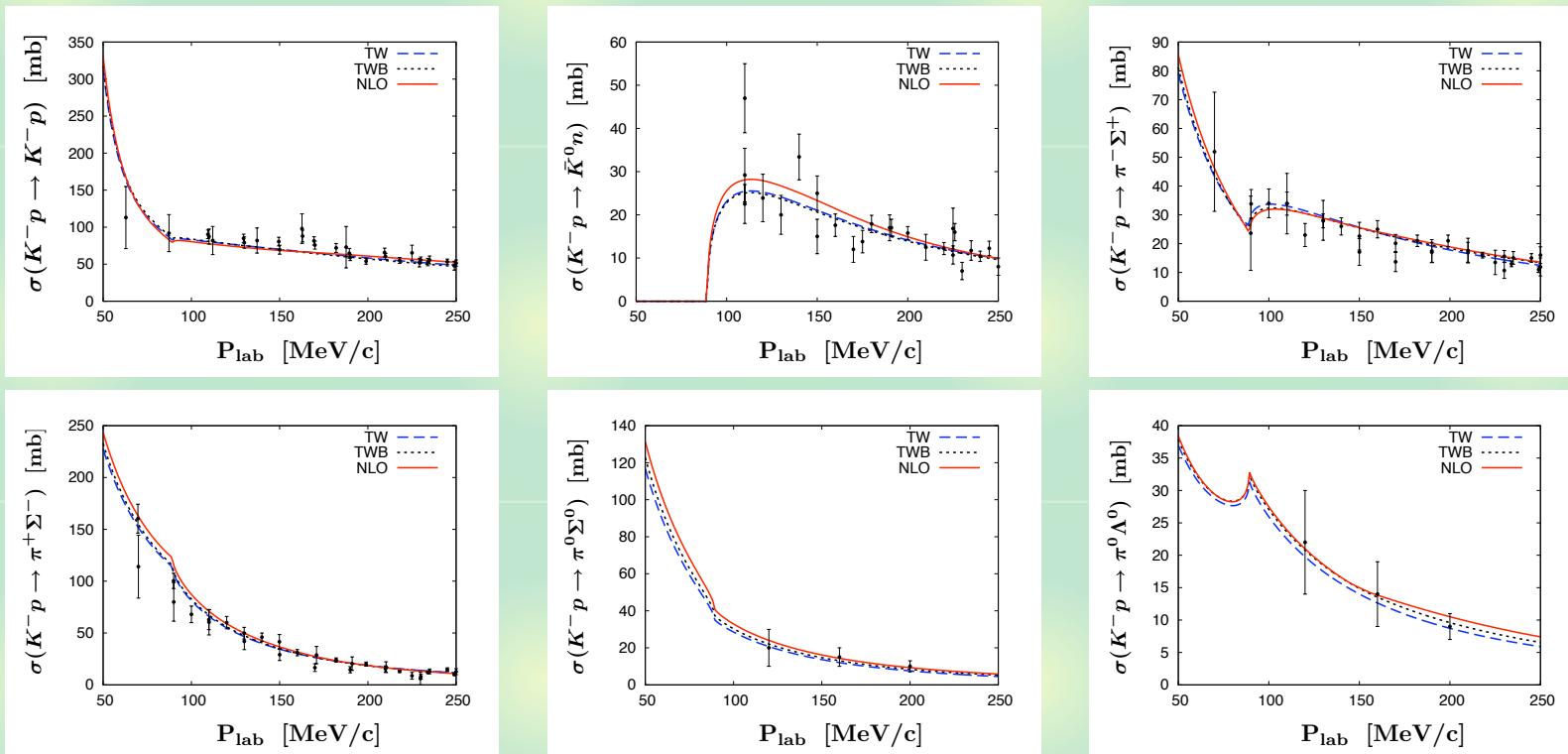
## Best-fit results

$K$  at rest

	TW	TWB	NLO	Experiment	
$\Delta E$ [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
$\Gamma$ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
$\gamma$	2.36	2.36	2.37	$2.36 \pm 0.04$	[11]
$R_n$	0.20	0.19	0.19	$0.189 \pm 0.015$	[11]
$R_c$	0.66	0.66	0.66	$0.664 \pm 0.011$	[11]
$\chi^2/\text{d.o.f}$	1.12	1.15	0.96		

} SIDDHARTA  
} Branching ratios

$K^- p$  cross sections



Accurate description of all existing data ( $\chi^2/\text{d.o.f} \sim 1$ )

# PDG has changed

## 2020 update of PDG

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012); ▲

Z.H. Guo, J.A. Oller, PRC87, 035202 (2013);  $\times$

M. Mai, U.G. Meißner, EPJA51, 30 (2015) ■ ○

### - Particle Listing section:

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405) \frac{1}{2}^-$

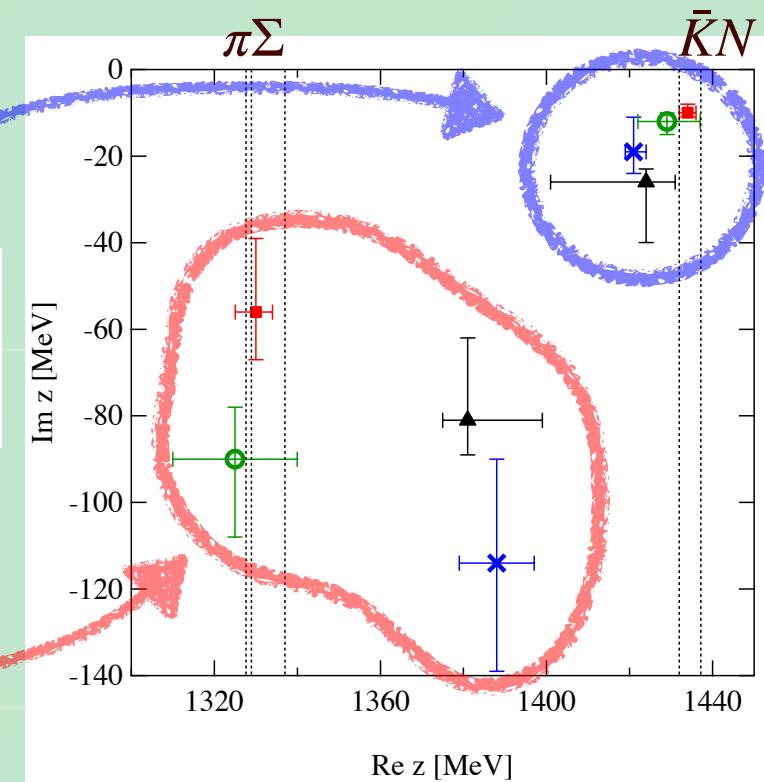
$I(J^P) = 0(\frac{1}{2}^-)$  Status: \* \* \* \*

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1380) \frac{1}{2}^-$

$J^P = \frac{1}{2}^-$  Status: \* \*

**new!**



T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021)

- “ $\Lambda(1405)$ ” is no longer at 1405 MeV but  $\sim 1420$  MeV.
- Lower pole : two-star resonance  $\Lambda(1380)$

## Construction of $\bar{K}N$ potentials

Local  $\bar{K}N$  potential is useful for various applications

meson-baryon amplitude  
(chiral SU(3) EFT)



T. Hyodo, W. Weise, PRC 77, 035204 (2008)

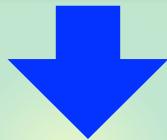


Kyoto  $\bar{K}N$  potential  
(single-channel, complex)

K. Miyahara, T. Hyodo,  
PRC 93, 015201 (2016)

Kyoto  $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$  potential  
(coupled-channel, real)

K. Miyahara, T. Hyodo, W. Weise,  
PRC 98, 025201 (2018)



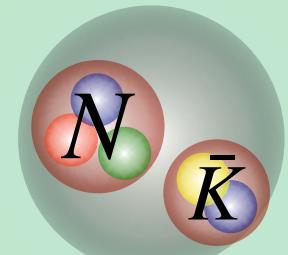
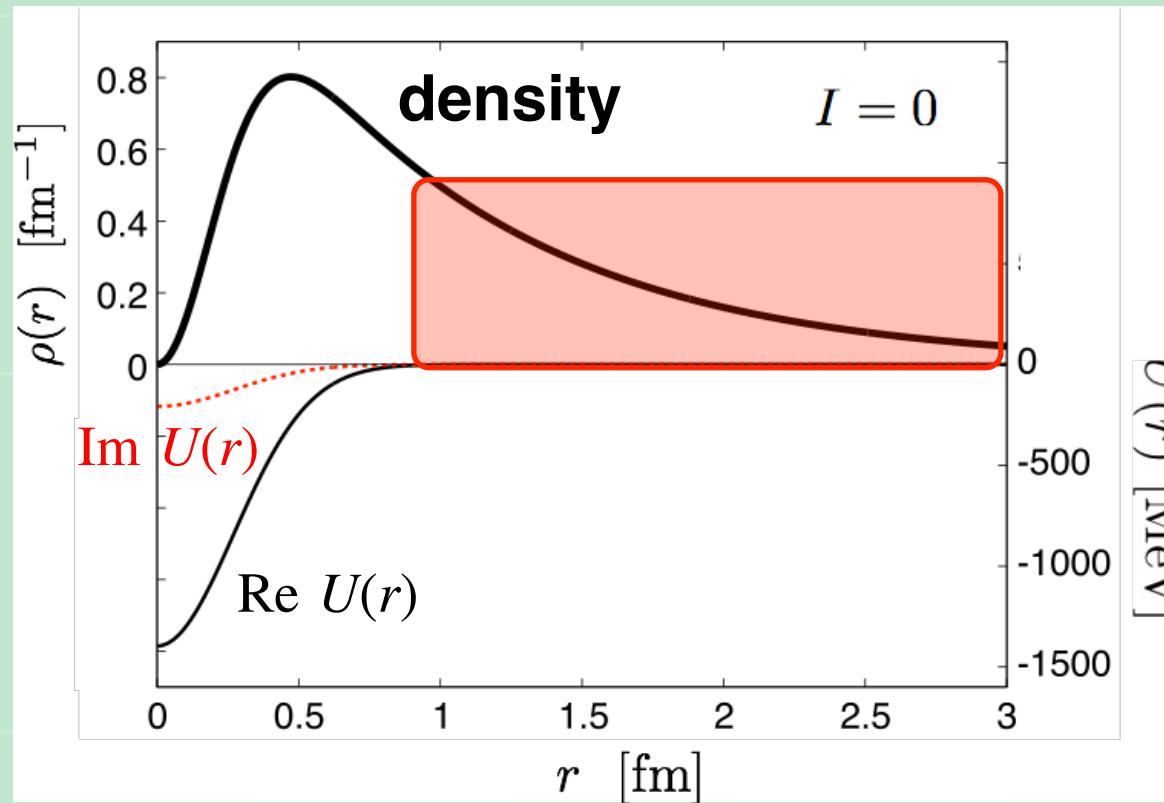
Kaonic nuclei

Kaonic deuterium

$K^-p$  correlation function

Spatial structure of  $\Lambda(1405)$  $\bar{K}N$  wave function at  $\Lambda(1405)$  pole

K. Miyahara, T. Hyodo, PRC93, 015201 (2016)



- substantial distribution at  $r > 1$  fm

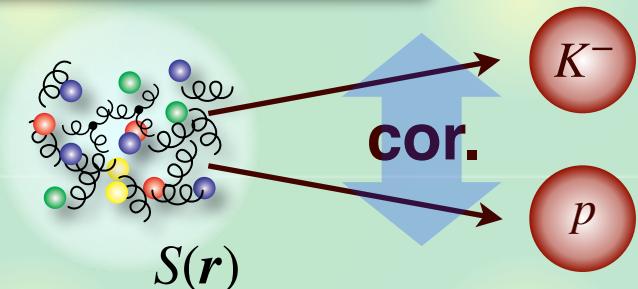
- root mean squared radius  $\sqrt{\langle r^2 \rangle} = 1.44$  fm

The size of  $\Lambda(1405)$  is much larger than ordinary hadrons

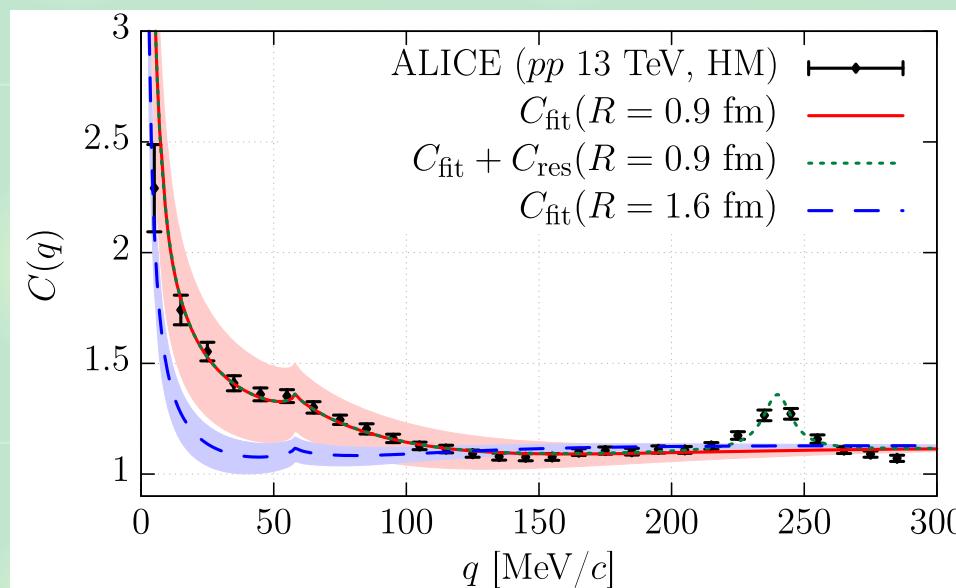
# Correlation function and femtoscopy

$K^-p$  correlation function  $C(q)$

$$C(q) = \frac{N_{K^-p}(\mathbf{p}_{K^-}, \mathbf{p}_p)}{N_{K^-}(\mathbf{p}_{K^-})N_p(\mathbf{p}_p)} \simeq \int d^3r S(r) |\Psi_q^{(-)}(\mathbf{r})|^2$$



- Wave function  $\Psi_q^{(-)}(\mathbf{r})$  : coupled-channel  $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$  potential



→ Talks by Y. Kamiya  
(Wed.), R. Lea (Thu.)

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

Correlation function is well reproduced

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Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)



## **Applications to few-body systems**

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## **Summary**

T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021);  
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th]

# $\bar{K}NN$ system : simplest $\bar{K}$ -nucleus

## Theoretical calculation with realistic $\bar{K}N$ interaction

- Fit to  $K^- p$  cross sections and branching ratios
- SIDDHARTRA constraint of kaonic hydrogen

[1] J. Revai, N.V. Shevchenko, PRC 90, 034004 (2014)

[2] S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara, T. Hyodo, PRC95, 065202 (2017)

Potential	$\Lambda(1405)$ [MeV]	$\Lambda(1380)$ [MeV]	$B_{\bar{K}NN}$ [MeV]	$\Gamma_{\bar{K}NN \rightarrow \pi YN}$ [MeV]
$V_{\bar{K}N-\pi\Sigma}^{1,\text{SIDD}}$	$1426 - 48i$ [3]	-	53.3 [1]	64.8 [1]
$V_{\bar{K}N-\pi\Sigma}^{2,\text{SIDD}}$	$1414 - 58i$ [3]	$1386 - 104i$ [3]	47.4 [1]	49.8 [1]
$V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{\text{chiral}}$	$1417 - 33i$ [4]	$1406 - 89i$ [4]	32.2 [1]	48.6 [1]
Kyoto $\bar{K}N$	$1424 - 26i$ [5]	$1381 - 81i$ [5]	25.3-27.9 [2]	30.9-59.4 [2]

[3] N.V. Shevchenko, NPA 890-891, 50 (2012)

[4] N.V. Shevchenko, J. Revai, PRC 90, 034003 (2014)

[5] K. Miyahara, T. Hyodo, PRC 93, 015201 (2016)

- Caution:  $2N$  absorption ( $\Gamma_{YN}$ ) is NOT included!!

# Kaonic nuclei

## Rigorous few-body approach up to $A = 6$ systems

S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara, T. Hyodo, PRC95, 065202 (2017)

- Stochastic variational method with correlated gaussians

$$\hat{V} = \hat{V}^{\bar{K}N}(\text{Kyoto } \bar{K}N) + \hat{V}^{NN}(AV4') \quad (\text{single channel})$$

## Results for kaonic nuclei with $A = 2, 3, 4, 6$

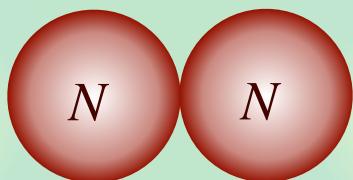
	$\bar{K}NN$	$\bar{K}NNN$	$\bar{K}NNNN$	$\bar{K}NNNNNN$
$I(J^P)$	$1/2(0^-)$	$0(1/2^-)$	$1/2(0^-)$	$1/2(0^-, 1^-)$
$B$ [MeV]	25.3-27.9	45.3-49.7	67.9-75.5	69.8-80.7
$\Gamma_{\text{mes.}}$ [MeV]	30.9-59.4	25.5-69.4	28.0-74.5	23.7-75.6

- for  $A = 6$  system,  $0^-$  and  $1^-$  are almost degenerated
- quasi-bound state below the lowest threshold
- decay width (without multi- $N$  absorption)  $\sim$  binding energy

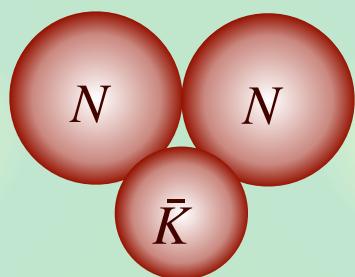
# Interplay between $NN$ and $\bar{K}N$ correlations 1

## Two-nucleon system

$^1S_0(I_{NN} = 1)$



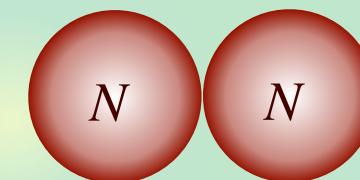
**unbound**



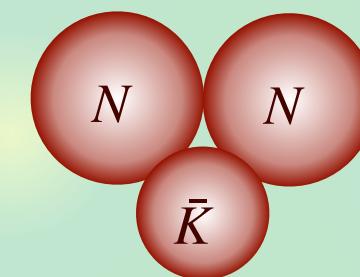
**(quasi-)bound**

$$\frac{\bar{K}N(I=0)}{\bar{K}N(I=1)} = \frac{1}{3}$$

$^3S_1(I_{NN} = 0)$



**bound ( $d$ )**



$\Lambda(1405)$

**unbound**

$$\frac{\bar{K}N(I=0)}{\bar{K}N(I=1)} = \frac{1}{3}$$

$NN$  correlation <  $\bar{K}N$  correlation

# Interplay between $NN$ and $\bar{K}N$ correlations 2

Four-nucleon system with  $J^P = 0^-, I = 1/2, I_3 = +1/2$

$$|\bar{K}NNNN\rangle = C_1 \left( \begin{array}{c} \text{Diagram of } \bar{K}^- N N N \\ \text{Three pions } \bar{K}^- \text{ in the center, } 3 \text{ protons } p \text{ and } 1 \text{ neutron } n \end{array} \right) + C_2 \left( \begin{array}{c} \text{Diagram of } \bar{K}^0 N N N \\ \text{Two pions } \bar{K}^0 \text{ in the center, } 2 \text{ protons } p \text{ and } 2 \text{ neutrons } n \end{array} \right)$$

## - $\bar{K}N$ correlation

$I = 0$  pair in  $\bar{K}^- p$  (3 pairs) or  $\bar{K}^0 n$  (2 pairs) :  $|C_1|^2 > |C_2|^2$

## - $NN$ correlation

$ppnn$  forms  $\alpha$  :  $|C_1|^2 < |C_2|^2$

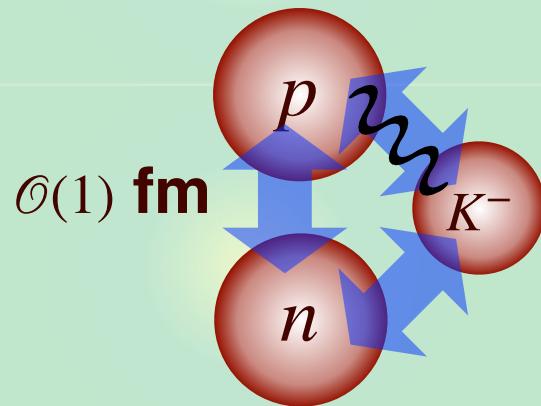
## - Numerical result

$$|C_1|^2 = 0.08, \quad |C_2|^2 = 0.92$$

$NN$  correlation  $>$   $\bar{K}N$  correlation

# Kaonic deuterium

$K^-pn$  system with strong + Coulomb interaction



Potential	$\Delta E - i\Gamma/2$ [eV]
$V_{\bar{K}N-\pi\Sigma}^{1,\text{SIDD}}$	$767 - 464i$ [1]
$V_{\bar{K}N-\pi\Sigma}^{2,\text{SIDD}}$	$782 - 469i$ [1]
$V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{\text{chiral}}$	$835 - 502i$ [1]
Kyoto $\bar{K}N$	$670 - 508i$ [2]

Theoretical requirements :

- Rigorous three-body treatment of strong + Coulomb
- Inclusion of SIDDHARTRA constraint (**realistic  $\bar{K}N$** )

[1] J. Revai, PRC 94, 054001 (2016)

[2] T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, PRC96, 045204 (2017)

- Experiments : J-PARC E57, SIDDHARTA-2

# Summary



**Experimental data constrain pole structure of the  $\Lambda(1405)$  region : “ $\Lambda(1405)$ ”  $\rightarrow \Lambda(1405) + \Lambda(1380)$**

$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****



**$\bar{K}N$  potentials are useful to calculate kaonic nuclei and kaonic deuterium**

Potential	$\Lambda(1405)$ [MeV]	$\Lambda(1380)$ [MeV]	$B_{\bar{K}NN}$ [MeV]	$\Gamma_{\bar{K}NN \rightarrow \pi YN}$ [MeV]	$\Delta E - i\Gamma/2$ [eV]
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