Compositeness, $\Lambda(1405)$, and kaonic nuclei

Clusters & Hierarchies

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2023, Sep. 23rd

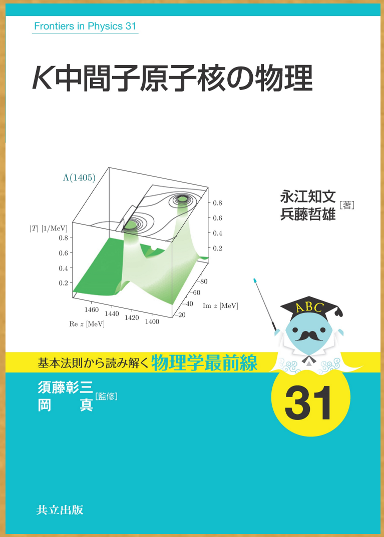
Contents

$\Lambda(1405)$ and $\bar{K}N$ interactions

T. Hyodo, M. Niiyama, Prog. Part. Nucl. Phys. 120, 103868 (2021);
Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL 124, 132501 (2020)

Compositeness

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Kinugawa, T. Hyodo, PRC106, 015205 (2022); in preparation



Kaonic nuclei

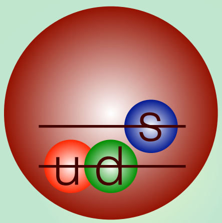
T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th] (Handbook of Nuclear Physics);
永江知文、兵藤哲雄「K中間子原子核の物理」（共立出版）

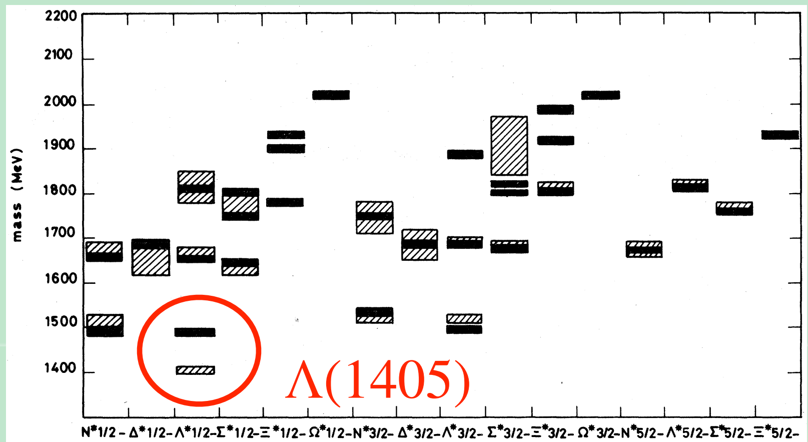
Summary

$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture —> exotic candidate

N. Isgur and G. Karl, PRD18, 4187 (1978)



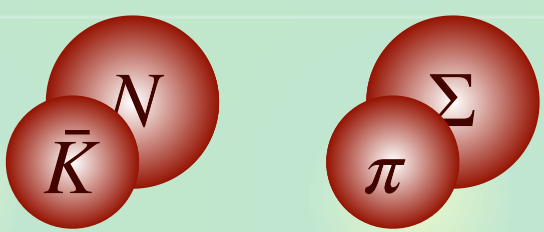


: theory

: experiment

Resonance in coupled-channel scattering

- Coupling to MB states



Detailed analysis of $\bar{K}N$-$\pi\Sigma$ scattering is necessary

Current PDG

Analysis by NLO chiral SU(3) dynamics

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012); ▲
Z.H. Guo, J.A. Oller, PRC 87, 035202 (2013); ×
M. Mai, U.G. Meißner, EPJA 51, 30 (2015) ■ ○

- **Particle Listing section:**

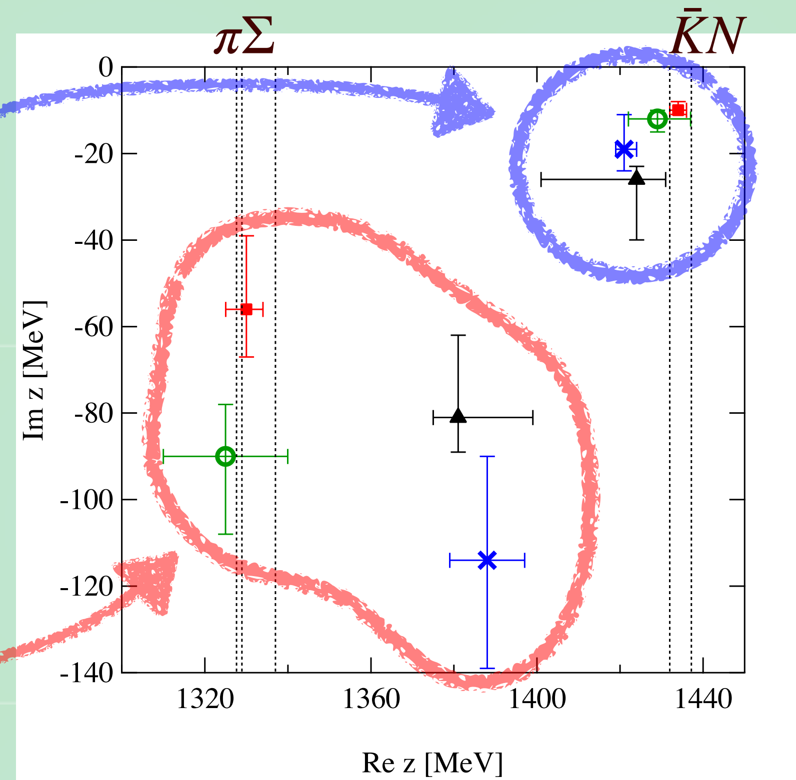
Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405)\ 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1380)\ 1/2^-$ $J^P = \frac{1}{2}^-$ Status: ∗∗

new!



T. Hyodo, M. Niiyama, PPNP 120, 103868 (2021)

- "$\Lambda(1405)$" is no longer at 1405 MeV but ~ 1420 MeV.

- Lower pole: two-star resonance $\Lambda(1380)$

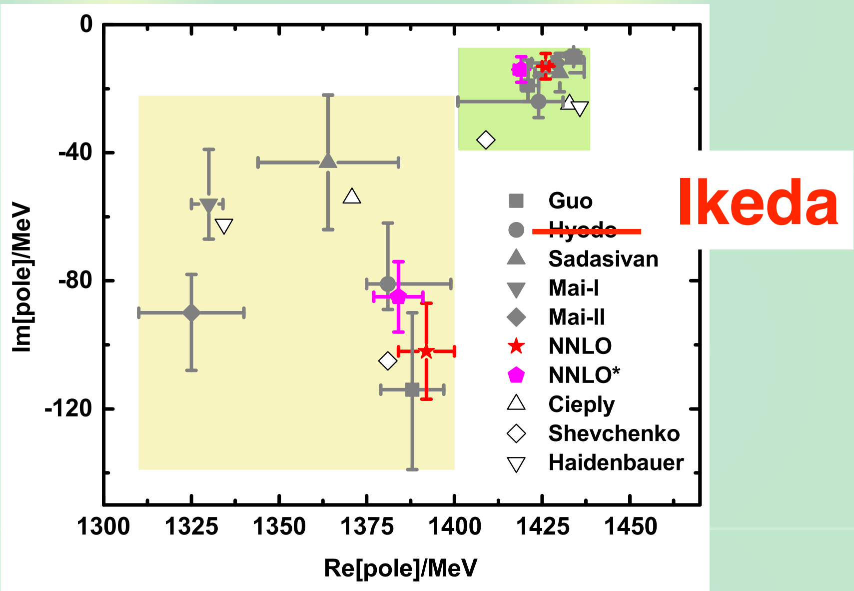
NNLO analysis and lattice QCD

Analysis at NNLO chiral SU(3) dynamics (KN and πN included)

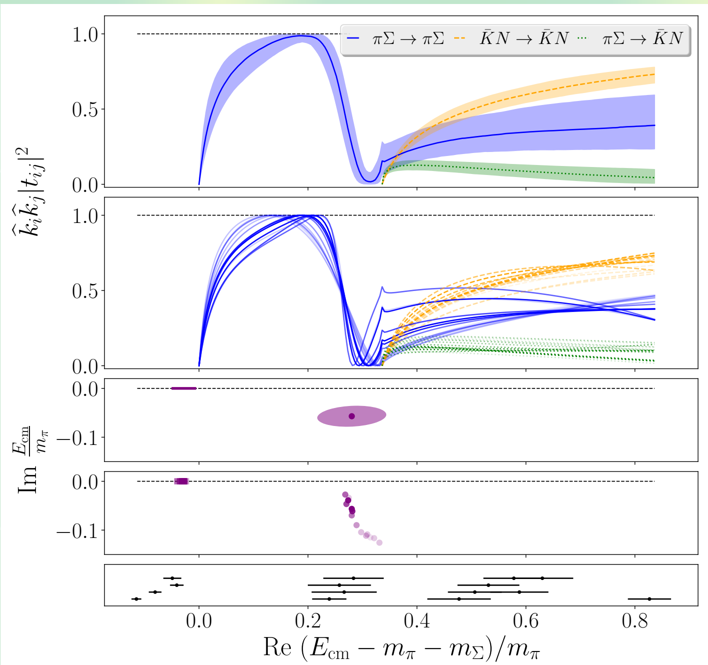
J.-X. Lu, L.S. Geng, M. Doering, M. Mai, PRL 130, 071902 (2023)

Lattice QCD calculation of $\bar{K}N$-$\pi\Sigma$ scattering ($m_\pi \sim 200$ MeV)

J. Bulava, *et al.* (BaSc), arXiv:2307.10413 [hep-lat]; arXiv:2307.13471 [hep-lat]



	Pole positions (MeV)
Λ(1380)	$1392 \pm 8 - i(102 \pm 15)$
Λ(1405)	$1425 \pm 1 - i(13 \pm 4)$



$$E_1 = 1392(9)(2)(16)\ \mathrm{MeV},$$

$$E_2 = [1455(13)(2)(17) - i11.5(4.4)(4)(0.1)]\ \mathrm{MeV},$$

Two states are confirmed at NNLO and lattice QCD

Construction of $\bar{K}N$ potentials

Local $\bar{K}N$ potential is useful for various applications

meson-baryon amplitude (chiral SU(3) at NLO)

T. Hyodo, W. Weise, PRC 77, 035204 (2008)

Kyoto $\bar{K}N$ potential (single-channel, complex)

K. Miyahara. T. Hyodo, PRC 93, 015201 (2016)

Kyoto $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential (coupled-channel, real)

K. Miyahara, T. Hyodo, W. Weise, PRC 98, 025201 (2018)

Kaonic nuclei

Kaonic deuterium

$K^{-}p$ correlation function

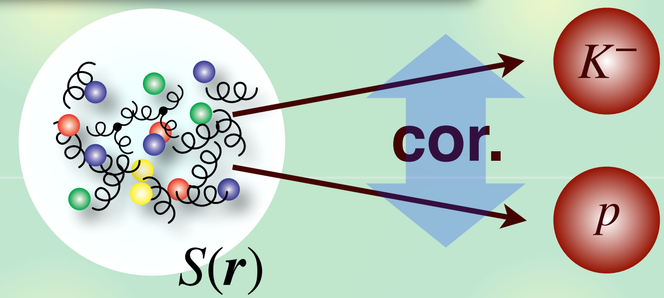
In memory of Akira Ohnishi

Sep. 13, 2019, after FemTUM19 workshop @ München

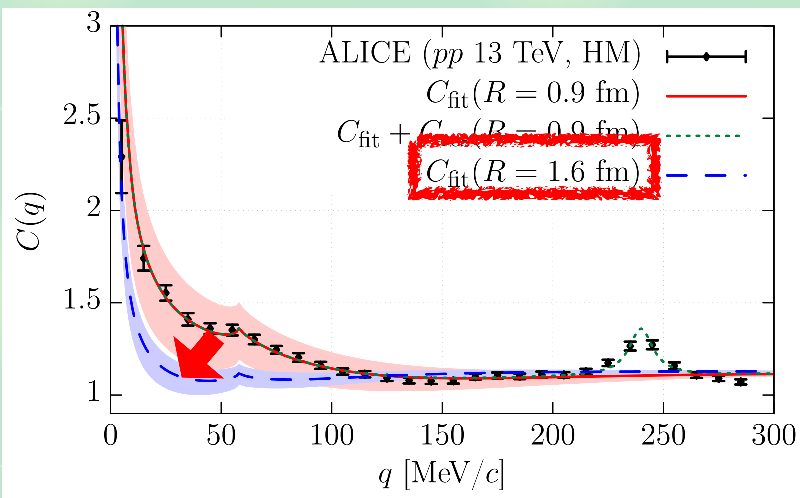
Correlation functions and femtoscopy

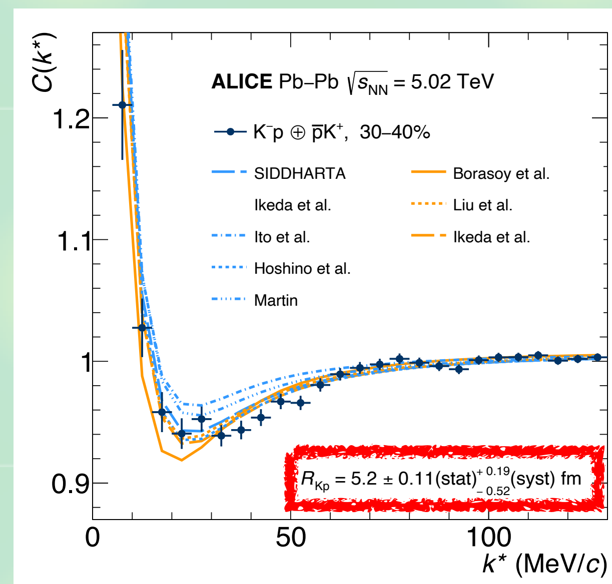
$K^{-}p$ correlation function $C(\boldsymbol{q})$

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-})N_p(\boldsymbol{p}_p)} \simeq \int d^3\boldsymbol{r}\, S(\boldsymbol{r})\, |\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2$$



- **Wave function $\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})$: Kyoto $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential**





S. Acharya *et al.* (ALICE), PRL 124, 092301 (2020)

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

S. Acharya *et al.* (ALICE), PLB 822, 136708 (2021)

Correlation functions are well reproduced and predicted

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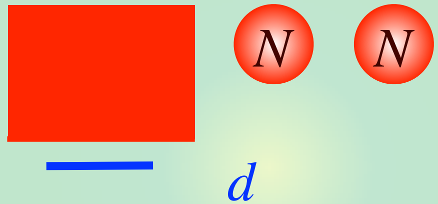
Summary

Weak-binding relation for stable states

Compositeness X of **stable** bound state

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{Z}\,|\text{others}\rangle, \quad X + Z = 1, \quad 0 \le X \le 1$$

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length (a_0) — radius of state (R)

- applicable only to stable bound states
- for shallow bound state $R \gg R_{\text{typ}}$, $X \leftarrow (a_0, B)$

(i) The particle must be stable; else Z is undefined. (However, it may be an adequate approximation to ignore the decay modes of a very narrow resonance.)
(ii) The particle must couple to a two-particle channel with threshold not too much above the particle mass.
(iii) It is crucial that this two-body channel have zero orbital angular momentum l, since for $l \neq 0$ the factor $(E)^{1/2}$ in the integrands of (24) and (32) would be $E^{l+(1/2)}$, and the integrals could not be approximated by their low-energy parts.

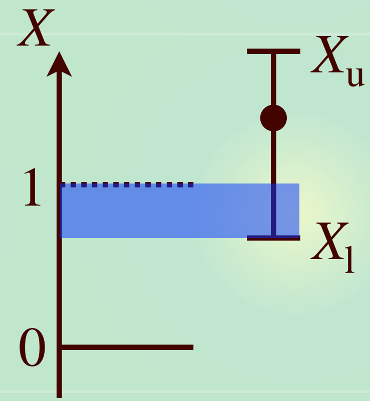
Problem: quantitative estimation —> $X = 1.68$?

Uncertainty and interpretation

Uncertainty estimation with $\mathcal{O}(R_{typ}/R)$ term

Y. Kamiya, T. Hyodo, PTEP2017, 023D02 (2017)

$$X_u = \frac{a_0/R + \xi}{2 - a_0/R - \xi}, \quad X_l = \frac{a_0/R - \xi}{2 - a_0/R + \xi}, \quad \xi = \frac{R_{typ}}{R}$$



Interpretation (with finite range correction)

T. Kinugawa, T. Hyodo, PRC 106, 015205 (2022)

- exclude region outside $0 \leq X \leq 1$

$$R_{typ} = \max\{R_{int}, R_{eff}\}$$

- X of hadrons, nuclei, and atoms
- X of deuteron is reasonable
- $X \geq 0.5$ in all cases studied

Bound state	Compositeness X
d	$0.74 \leqslant X \leqslant 1$
$X(3872)$	$0.53 \leqslant X \leqslant 1$
$D^*_{s0}(2317)$	$0.81 \leqslant X \leqslant 1$
$D_{s1}(2460)$	$0.55 \leqslant X \leqslant 1$
$N\Omega$ dibaryon	$0.80 \leqslant X \leqslant 1$
$\Omega\Omega$ dibaryon	$0.79 \leqslant X \leqslant 1$
${}^3_\Lambda$H	$0.74 \leqslant X \leqslant 1$
^{4}He dimer	$0.93 \leqslant X \leqslant 1$

Near-threshold bound states are mostly composite

Weak-binding relation for unstable states

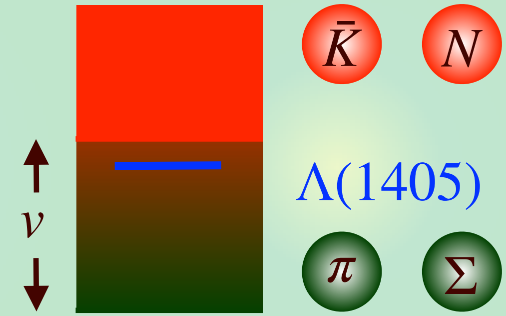
Compositeness X of unstable quasibound state

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

- complex eigenenergy: $-B \to E_h \in \mathbb{C}$

$$|\Lambda(1405)\rangle = \sqrt{X}\,|\bar{K}N\rangle + \sqrt{Z}\,|\text{others}\rangle, \quad X + Z = 1$$

- complex a_0, X



$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- correction from threshold energy difference

- for near-threshold quasibound state $|R| \gg (R_{\text{typ}}, \ell)$, $X \leftarrow (a_0, E_h)$

Interpretation of complex X

—> Poster by T. Kinugawa

$$\tilde{X} = \frac{1 - |Z| + |X|}{2}, \quad \tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} + \tilde{Z} = 1, \quad 0 \le \tilde{X} \le 1$$

Compositeness of $\Lambda(1405)$: central values

Generalized weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- Neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-\ \ 4 - i\ \ 8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-\ \ 3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, $X \sim 1$ and $\tilde{X} \sim 1$

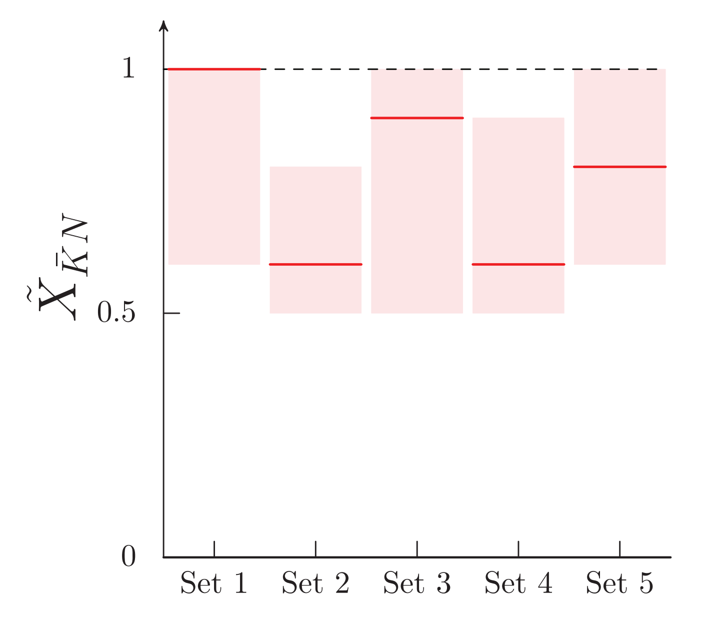
$\Lambda(1405)$: $\bar{K}N$ composite dominance <— observables

Compositeness of $\Lambda(1405)$: uncertainties

Estimation of correction terms: $|R| \sim 2$ fm

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- **ρ meson exchange picture:** $R_{\rm typ} \sim 0.25$ fm
- **Energy difference from $\pi\Sigma$:** $\ell \sim 1.08$ fm



$\bar{K}N$ composite dominance holds even with correction terms

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Summary

$\bar{K}NN$ system : simplest kaonic nucleus

Theoretical calculation with realistic $\bar{K}N$ interaction

- Fit to $K^{-}p$ cross sections and branching ratios
- SIDDHARTRA constraint of kaonic hydrogen

[1] J. Revai, N.V. Shevchenko, PRC 90, 034004 (2014)
[2] S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara. T. Hyodo, PRC95, 065202 (2017)

Potential	$\Lambda(1405)$ [MeV]	$\Lambda(1380)$ [MeV]	$B_{\bar{K}NN}$ [MeV]	$\Gamma_{\bar{K}NN\to\pi YN}$ [MeV]
$V^{1,\mathrm{SIDD}}_{\bar{K}N\text{-}\pi\Sigma}$	$1426-48i$ [3]	-	53.3 [1]	64.8 [1]
$V^{2,\mathrm{SIDD}}_{\bar{K}N\text{-}\pi\Sigma}$	$1414-58i$ [3]	$1386-104i$ [3]	47.4 [1]	49.8 [1]
$V^{\mathrm{chiral}}_{\bar{K}N\text{-}\pi\Sigma\text{-}\pi\Lambda}$	$1417-33i$ [4]	$1406-89i$ [4]	32.2 [1]	48.6 [1]
Kyoto $\bar{K}N$	$1424-26i$ [5]	$1381-81i$ [5]	25.3-27.9 [2]	30.9-59.4 [2]

[3] N.V. Shevchenko, NPA 890-891, 50 (2012)
[4] N.V. Shevchenko, J. Revai, PRC 90, 034003 (2014)
[5] K. Miyahara. T. Hyodo, PRC 93, 015201 (2016)

- Caution: $2N$ absorption (Γ_{YN}) is NOT included!!

Kaonic nuclei up to $A = 6$

Rigorous few-body approach up to $A = 6$ systems

S. Ohnishi, W. Horiuchi, T. Hoshino, K. Miyahara. T. Hyodo, PRC95, 065202 (2017)

- Stochastic variational method with correlated gaussians

$$\hat{V} = \hat{V}^{\bar{K}N}(\text{Kyoto } \bar{K}N) + \hat{V}^{NN}(AV4')$$ **(single channel)**

Results for kaonic nuclei with $A = 2, 3, 4, 6$

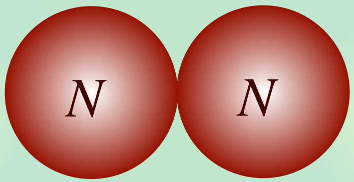
	$\bar{K}NN$	$\bar{K}NNN$	$\bar{K}NNNN$	$\bar{K}NNNNNN$
$I(J^P)$	$1/2(0^-)$	$0(1/2^-)$	$1/2(0^-)$	$1/2(0^-, 1^-)$
B [MeV]	25.3-27.9	45.3-49.7	67.9-75.5	69.8-80.7
$\Gamma_{\text{mes.}}$ [MeV]	30.9-59.4	25.5-69.4	28.0-74.5	23.7-75.6

- for $A = 6$ system, 0^- and 1^- are almost degenerated
- quasi-bound state below the lowest threshold
- decay width (without multi-N absorption) ~ binding energy

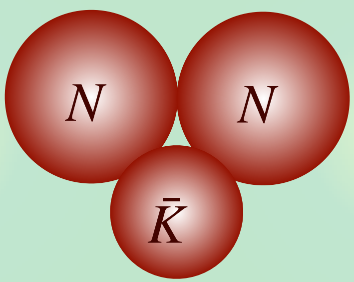
Interplay between NN and $\bar{K}N$ correlations 1

Two-nucleon system

$^{1}S_0(I_{NN} = 1)$



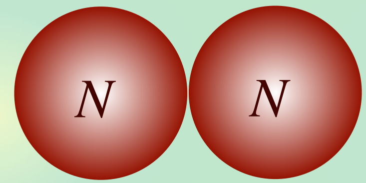
unbound



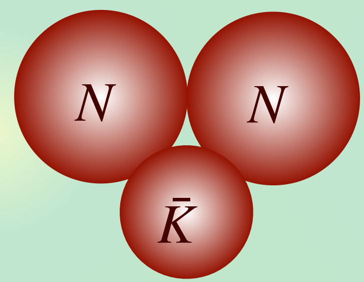
(quasi-)bound

$$\frac{\bar{K}N(I=0)}{\bar{K}N(I=1)} = 3$$

$^{3}S_1(I_{NN} = 0)$



bound (d)



$\Lambda(1405)$

unbound

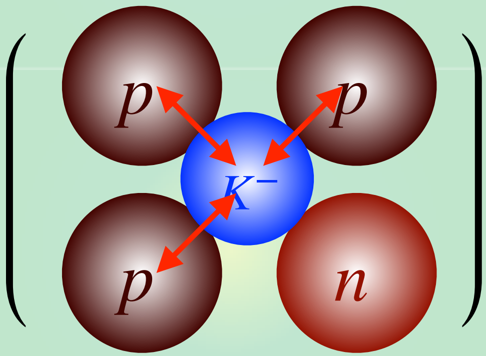
$$\frac{\bar{K}N(I=0)}{\bar{K}N(I=1)} = \frac{1}{3}$$

NN **correlation** $<$ $\bar{K}N$ **correlation**

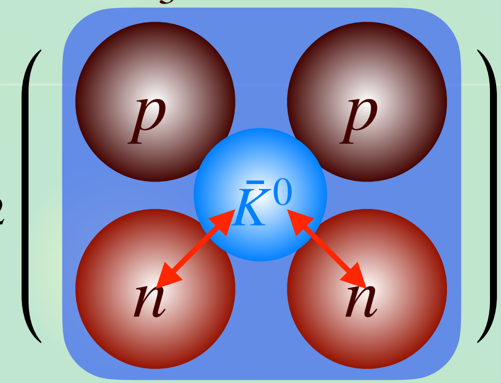
Interplay between NN and $\bar{K}N$ correlations 2

Four-nucleon system with $J^P = 0^-$, $I = 1/2$, $I_3 = +1/2$

$|\bar{K}NNNN\rangle = C_1$



$+ C_2$



- **$\bar{K}N$ correlation**

  $I = 0$ **pair in K^-p (3 pairs) or $\bar{K}^0 n$ (2 pairs) :** $|C_1|^2 > |C_2|^2$

- **NN correlation**

  $ppnn$ **forms** α **:** $|C_1|^2 < |C_2|^2$

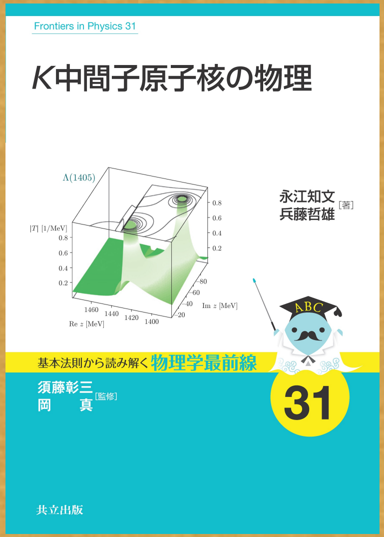
- **Numerical result**

  $|C_1|^2 = 0.08, \quad |C_2|^2 = 0.92$

NN correlation $>$ $\bar{K}N$ correlation

Summary

- $\Lambda(1405)$ **and** $\bar{K}N$ **interactions**
  - **- precise determination of** $\Lambda(1405)$ **and** $\Lambda(1380)$
  - **-** $K^{-}p$ **correlation function**
- **Compositeness**
  - **- applicable to nuclei, atoms, …**
  - **-** $\bar{K}N$ **molecule picture for** $\Lambda(1405)$
- **Kaonic nuclei**
  - **- realistic calculations**
  - **- interplay between** NN **and** $\bar{K}N$



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