

Femtoscscopy for exotic hadrons and nuclei



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Contents



Introduction — Femtoscopy primer



Femtoscopy for exotic hadrons

- K^-p correlations for $\Lambda(1405)$

[Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 \(2020\)](#)



Femtoscopy for hypernuclei

- $\Lambda\alpha$ correlations for Λ in medium

[A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 \[nucl-th\]](#)



Summary and future prospects

In memory of Akira Ohnishi



Akira Ohnishi

Yuki Kamiya

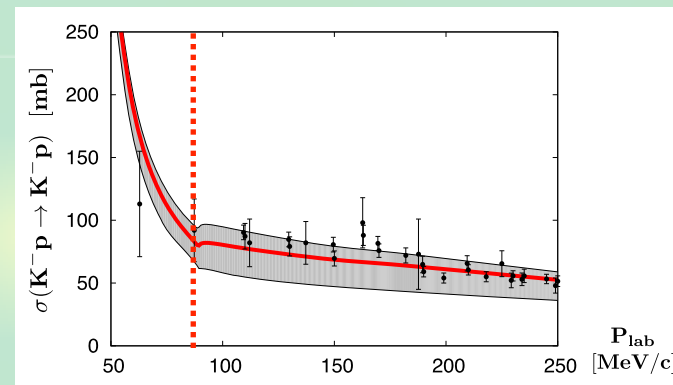
Sep. 13, 2019, after FemTUM19 workshop @ München

Scattering experiments and femtoscopy

Traditional methods: scattering experiments

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

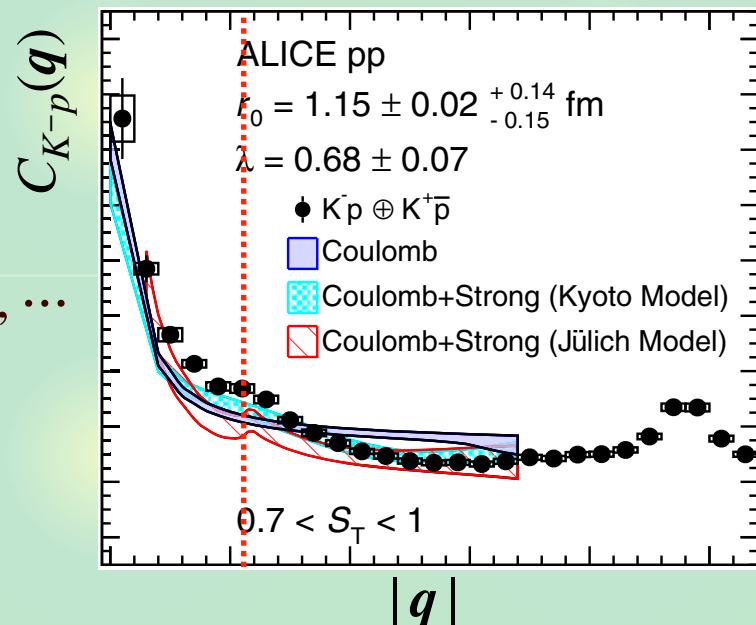
- **Limited channels:** $NN, YN, \pi N, KN, \bar{K}N, \dots$
- **Limited statistics (low-energy)**
- **Heavy (c, b) hadrons: impossible**



Femtoscopy: correlation function

ALICE collaboration, PRL 124, 092301 (2020)

- **Various systems:** $\Lambda\Lambda, N\Omega, \phi N, \bar{K}\Lambda, \textcolor{red}{DN}, \dots$
- Excellent **precision** ($\bar{K}^0_n$ cusp)
- Heavy hadrons: **possible!**

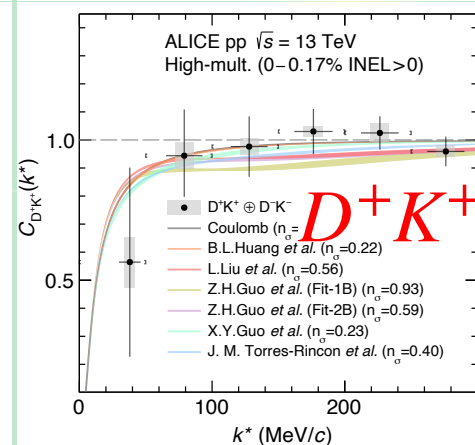
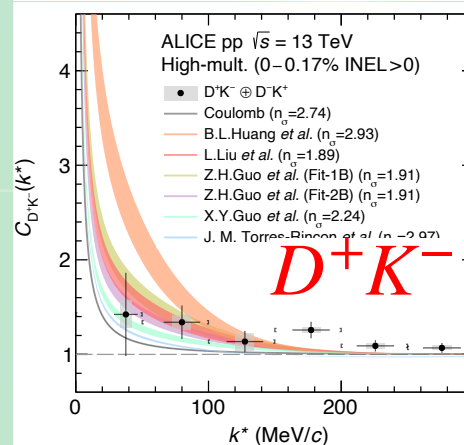
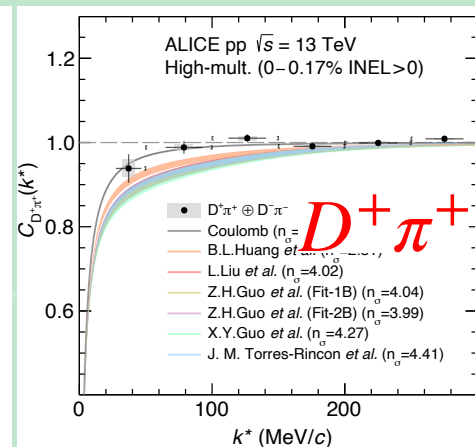
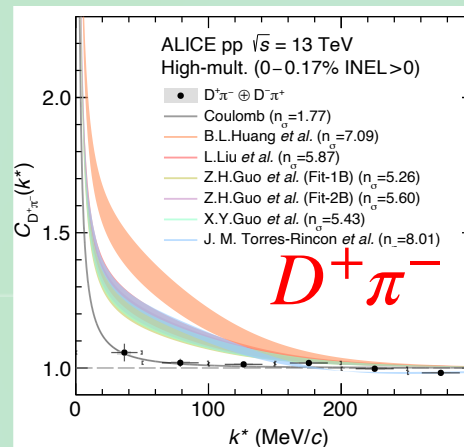
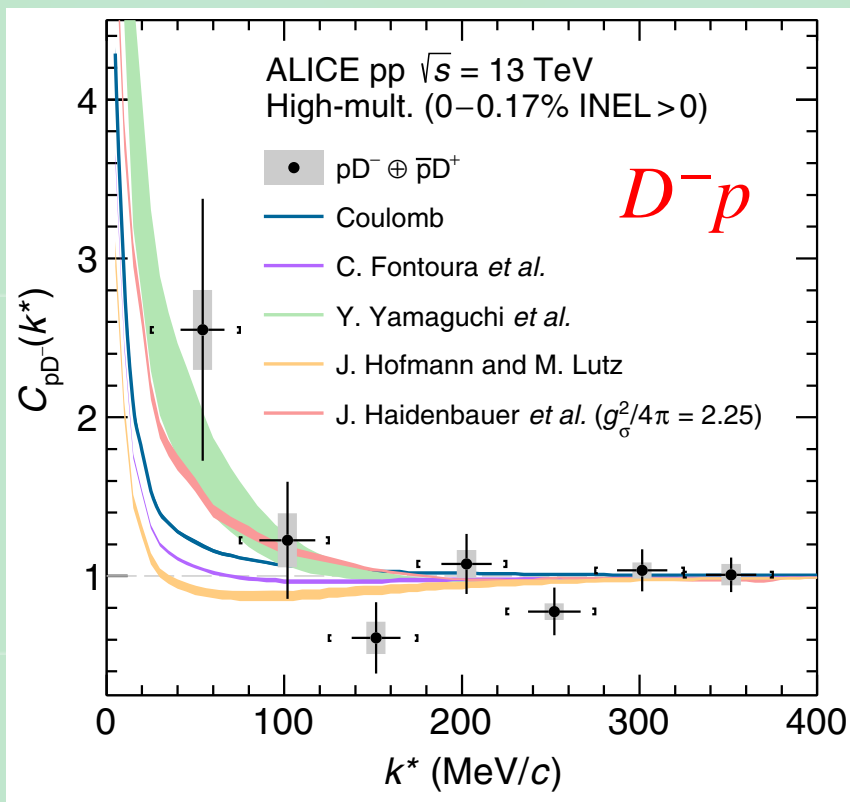


Experimental data in charm sector

Observed correlation functions with charm: $DN, D\pi, DK$

ALICE collaboration, PRD 106, 052010 (2022);

ALICE collaboration, arXiv:2401.13541 [nucl-ex]

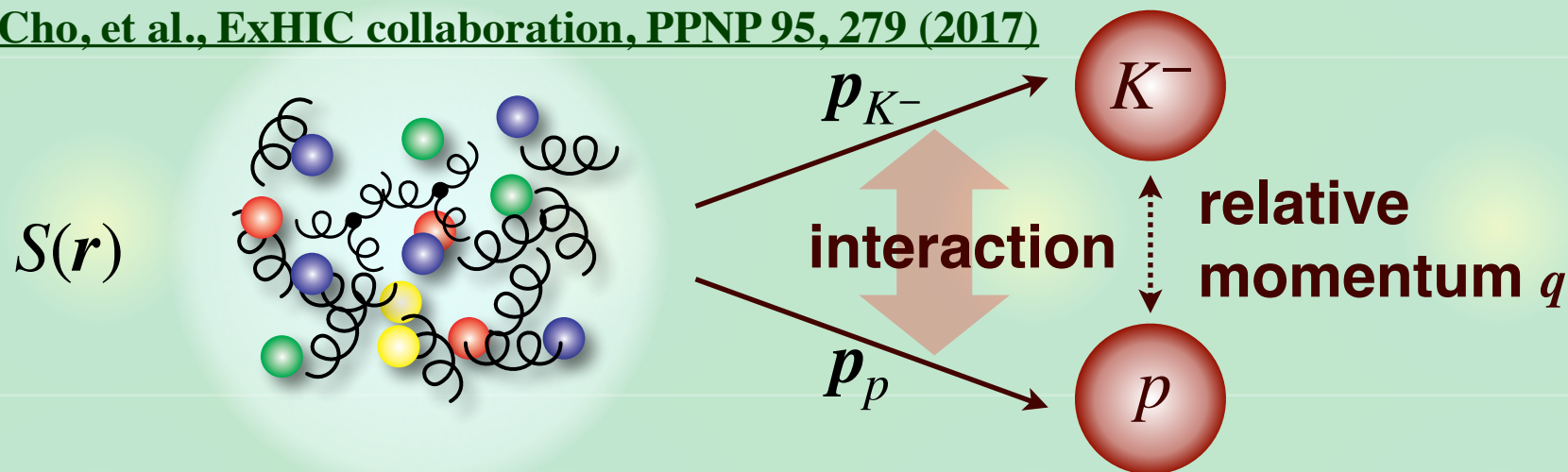


Unique way to obtain data in charm sector (yet low statistics)

Correlation function and KP formula

High-energy collision: chaotic source $S(r)$ of hadron emission

S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)



- Definition

$$C(q) = \frac{N_{K^-p}(p_{K^-}, p_p)}{N_{K^-}(p_{K^-})N_p(p_p)} \quad (= 1 \text{ in the absence of FSI/QS})$$

- Theory (Koonin-Pratt formula)

incoming + outgoing

S.E. Koonin, PLB 70, 43 (1977); S. Pratt, PRD 33, 1314 (1986)

$$C(q) \simeq \int d^3r \, S(r) \, |\Psi_q^{(-)}(r)|^2, \quad \Psi_q^{(-)}(r) \propto S^\dagger e^{-iqr} - e^{+iqr} \quad (r \rightarrow \infty)$$

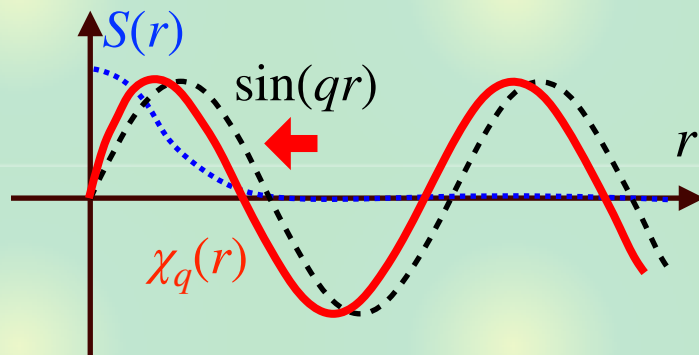
Source function $S(r) \longleftrightarrow$ wave function $\Psi_q^{(-)}(r)$ (interaction)

Wave functions and correlations

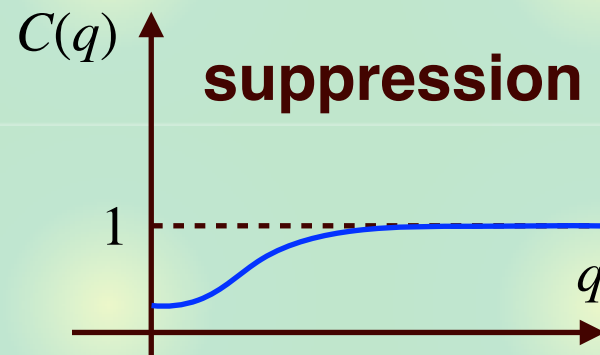
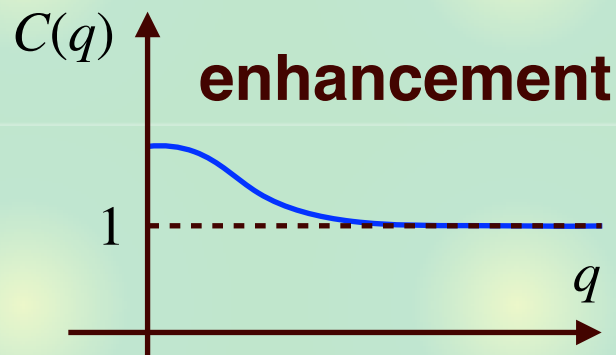
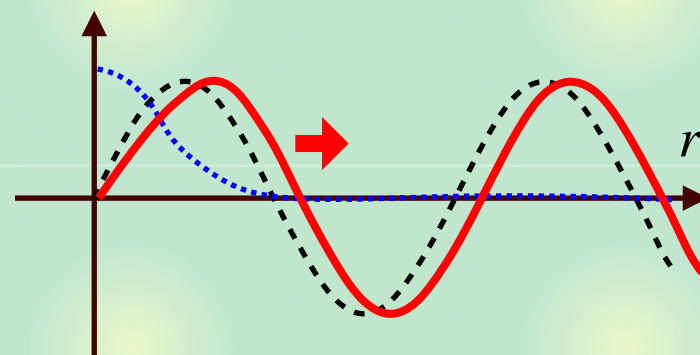
Spherical source with s-wave interaction dominance

$$C(q) \simeq 1 + \int_0^\infty dr S(r) \{ |\chi_q(r)|^2 - \sin^2(qr) \}$$

attraction



repulsion



Correlation function $\longleftrightarrow$ nature of interaction

LL formula

Correlation function $\longleftrightarrow$ **observables** ($a_0, r_e, f(q)$)

R. Lednicky, V.L. Lyuboshits, Yad. Fiz. **35**, 1316 (1981)

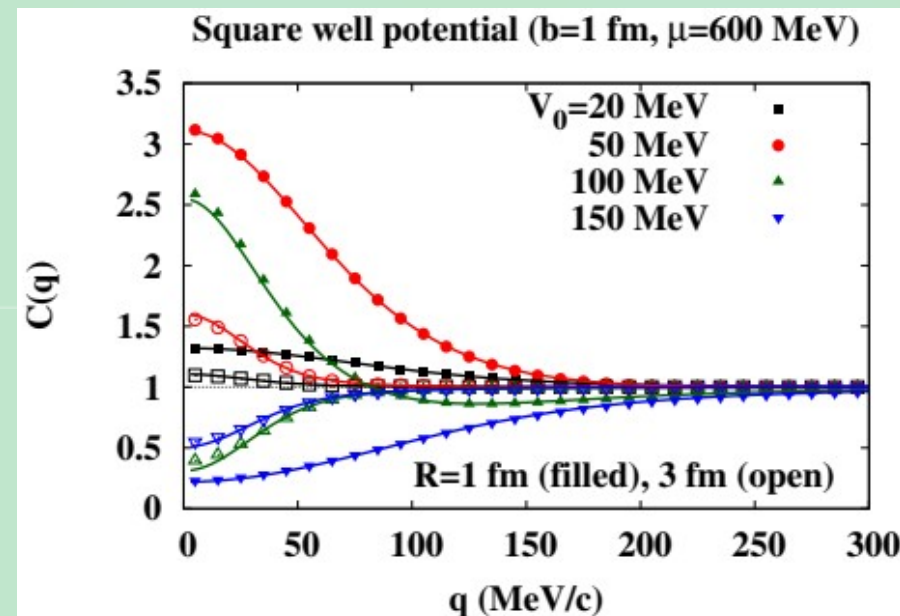
- **Gaussian source** $S(r) = \exp(-r^2/4R^2)/(4\pi R^2)^{3/2}$
- **zero-range interaction** $R \gg R_{\text{int}}$ (**asymptotic w.f.**)

$$C(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3(r_e/R) + \frac{2\text{Re } f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\text{Im } f(q)}{R} F_2(2qR)$$

Known to work even for $R \sim R_{\text{int}}$

- **square well potential**
- **symbols: KP, lines: LL**

A. Ohnishi, Talk@Bormio, 23-27 Jan. 2023

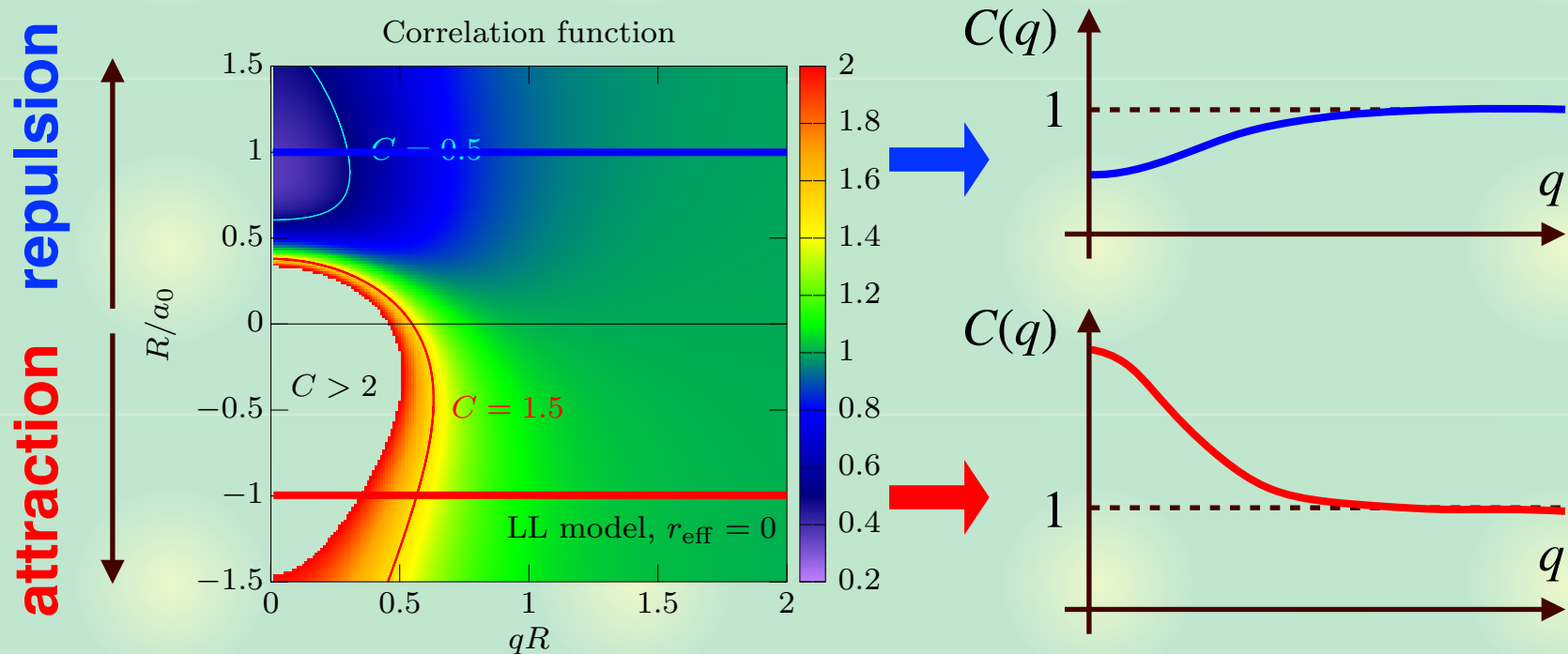


LL formula and correlations

LL formula with $r_e = 0$

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, T. Hatsuda, PRC 105, 014915 (2022)

- fixed $R > 0$
- **repulsion**: $R/a_0 > 0$, **attraction**: $R/a_0 < 0$



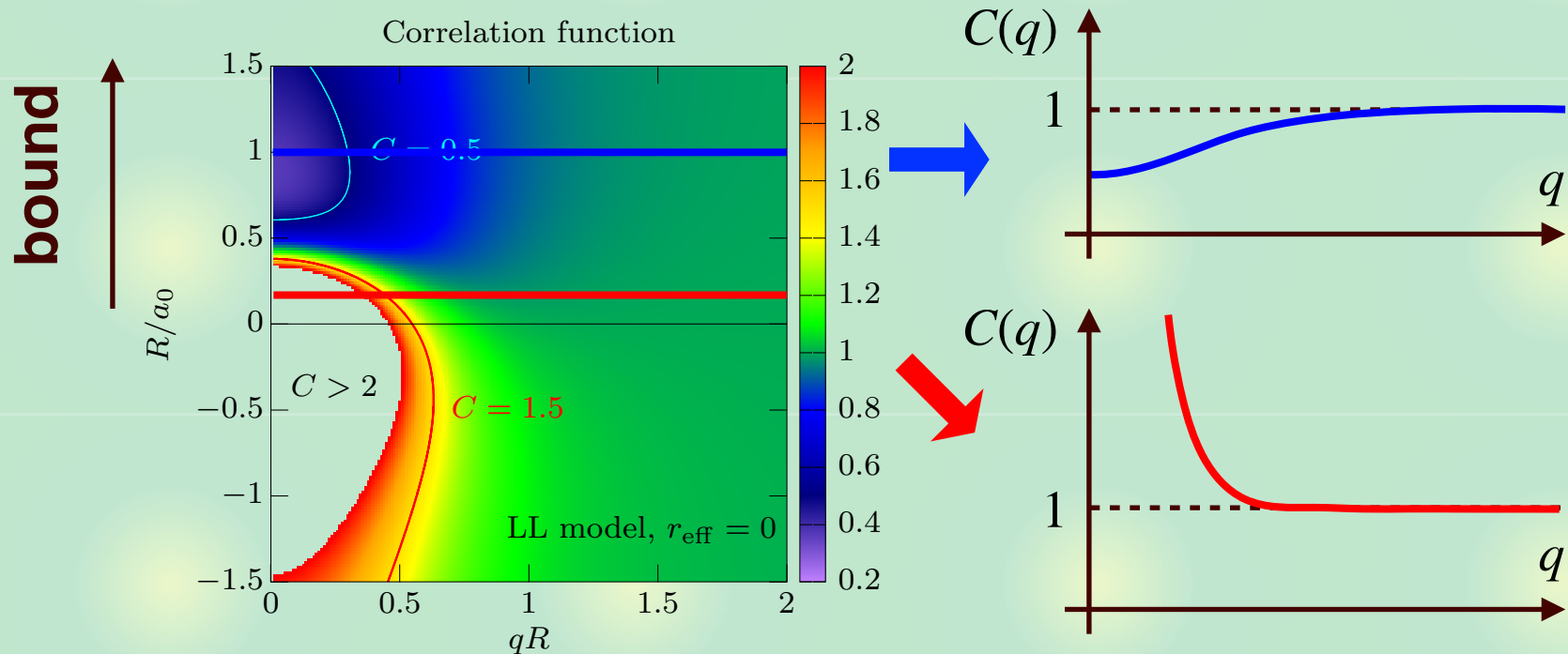
Consistent with KP formula

Shallow bound state case

LL formula with $r_e = 0$

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, T. Hatsuda, PRC 105, 014915 (2022)

- **shallow bound state: fixed** $a_0 > 0$, $|a_0| \gg R_{\text{int}}$
- **large source:** $R/a_0 \sim 1$, **small source:** $R/a_0 \ll 1$



—> **qualitative** difference in large/small source

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[Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 \(2020\)](#)



Femtoscopy for hypernuclei

- $\Lambda\alpha$ correlations for Λ in medium

[A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 \[nucl-th\]](#)

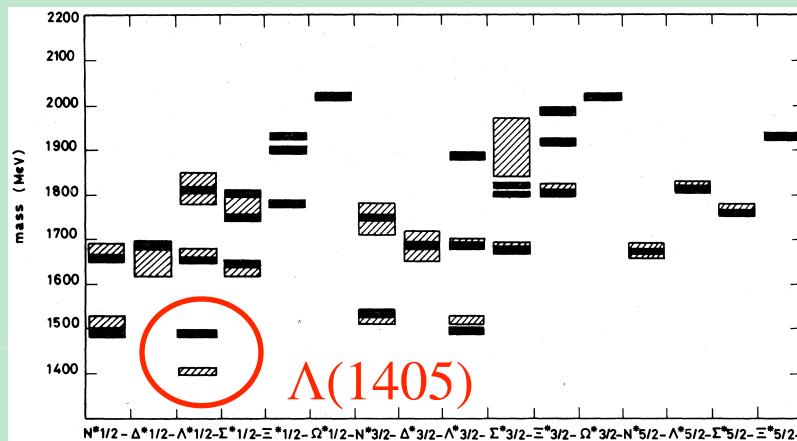
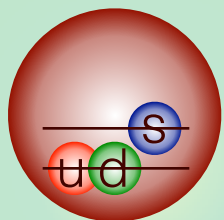


Summary and future prospects

$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture $\rightarrow$ exotic candidate

N. Isgur and G. Karl, PRD18, 4187 (1978)

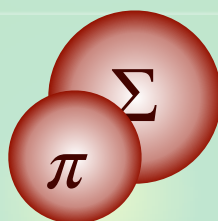
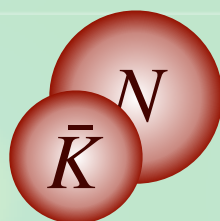


— : theory

▨ : experiment

Resonance in **coupled-channel** scattering

- Coupling to MB: chiral SU(3) dynamics



energy ↑

— $\bar{K}N$ threshold

▨ $\Lambda(1405)$

— $\pi\Sigma$ threshold

Coupled-channel effects

Schrödinger equation (s-wave)

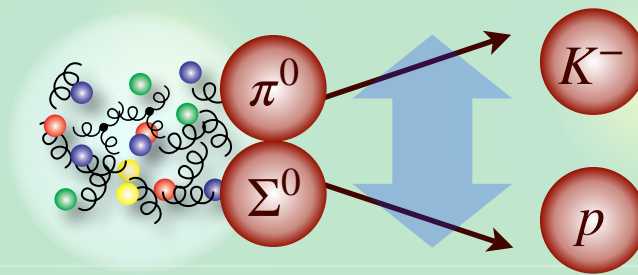
$$\begin{pmatrix} \frac{-1}{2\mu_1} \frac{d^2}{dr^2} + V_{11}(r) + \boxed{V_C(r)} & V_{12}(r) & \cdots \\ V_{21}(r) & \frac{-1}{2\mu_2} \frac{d^2}{dr^2} + V_{22}(r) + \boxed{\Delta_2} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0 n}(r) \\ \vdots \end{pmatrix} = E \begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0 n}(r) \\ \vdots \end{pmatrix}$$

Coulomb **threshold energy difference**

Asymptotic ($r \rightarrow \infty$) wave function (**incoming** + **outgoing**)

$$\begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0 n}(r) \\ \vdots \end{pmatrix} \propto \begin{pmatrix} \boxed{S_{11}^\dagger e^{-iqr}} - \boxed{e^{iqr}} \\ \boxed{S_{12}^\dagger e^{-iq_2 r}} - \boxed{0 \times e^{iq_2 r}} \\ \vdots \end{pmatrix} \quad (r \rightarrow \infty)$$

- **Transition** from $\bar{K}^0 n, \pi^+ \Sigma^-, \pi^0 \Sigma^0, \pi^- \Sigma^+, \pi^0 \Lambda$ is in $\psi_i(r)$ with $i \neq K^-p$



Coupled-channel correlation function

Coupled-channel Koonin-Pratt formula

R. Lednicky, V.V. Lyuboshitz, V.L. Lyuboshitz, Phys. Atom. Nucl. 61, 2950 (1998);

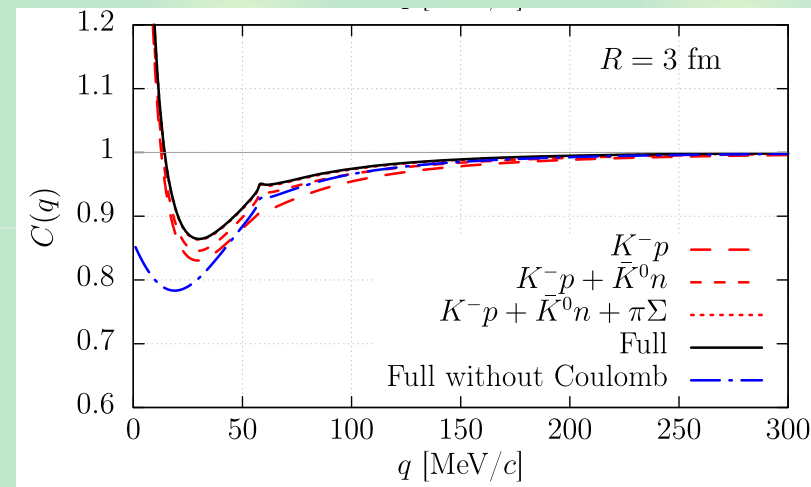
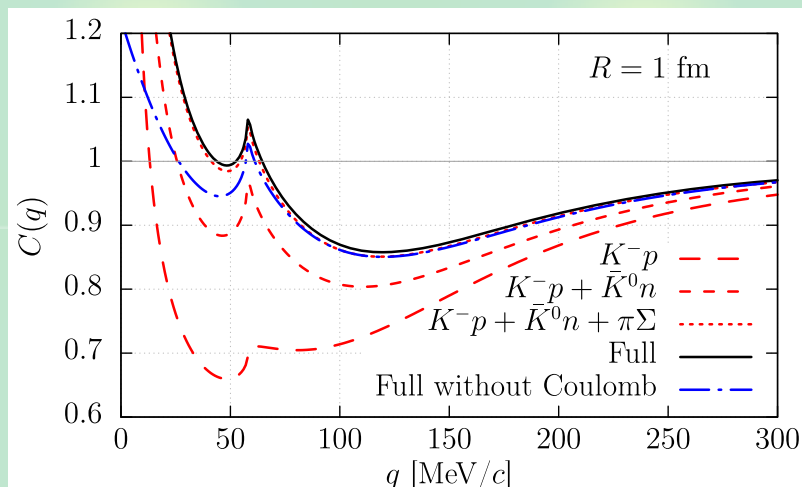
J. Haidenbauer, NPA 981, 1 (2019);

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)

$$C_{K^-p}(q) \simeq \int d^3r S_{K^-p}(r) |\Psi_{K^-p,q}^{(-)}(r)|^2 + \sum_{i \neq K^-p} \omega_i \int d^3r S_i(r) |\Psi_{i,q}^{(-)}(r)|^2$$

- Transition from $\bar{K}^0 n, \pi^+ \Sigma^-, \pi^0 \Sigma^0, \pi^- \Sigma^+, \pi^0 \Lambda$

- ω_i : weight of channel i source relative to K^-p



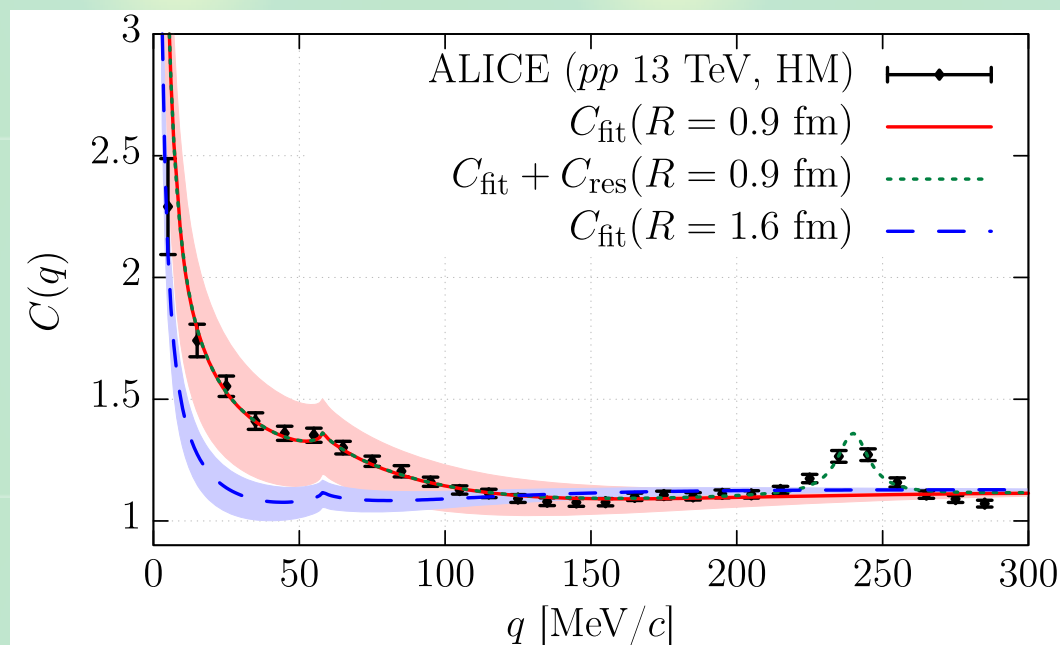
Coupled-channel effect is enhanced for **small sources**

Correlation from chiral SU(3) dynamics

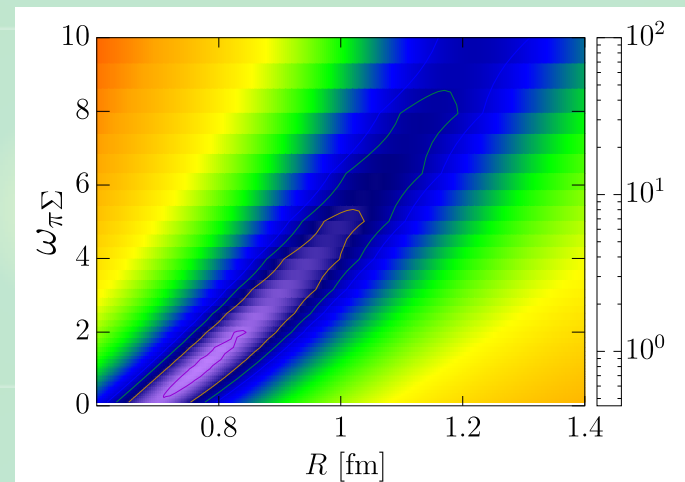
Wave function $\Psi_{i,q}^{(-)}(r)$: Kyoto $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential

K. Miyahara, T. Hyodo, W. Weise, PRC98, 025201 (2018)

- Source function $S(r)$: gaussian, $R \sim 1$ fm from K^+p data
- Source weight $\omega_{\pi\Sigma} \sim 2$ by simple statistical model estimate



reduced χ^2



Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)

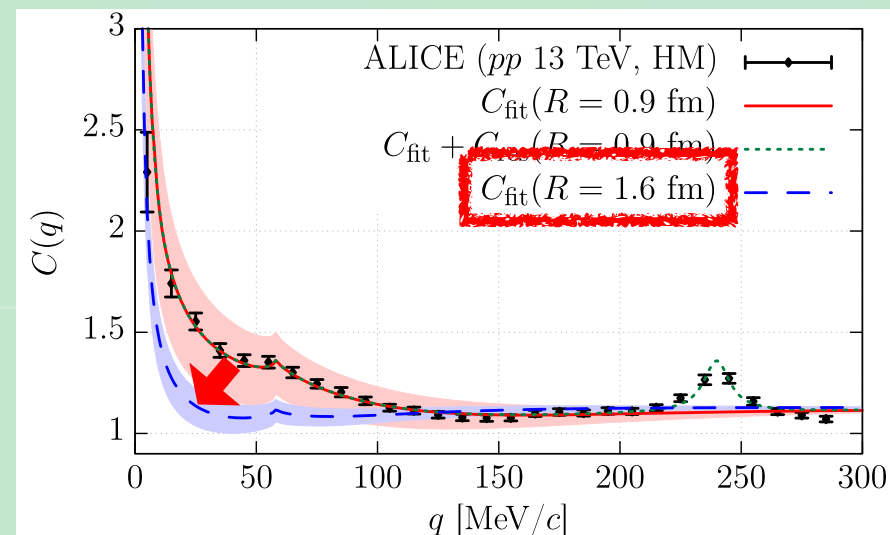
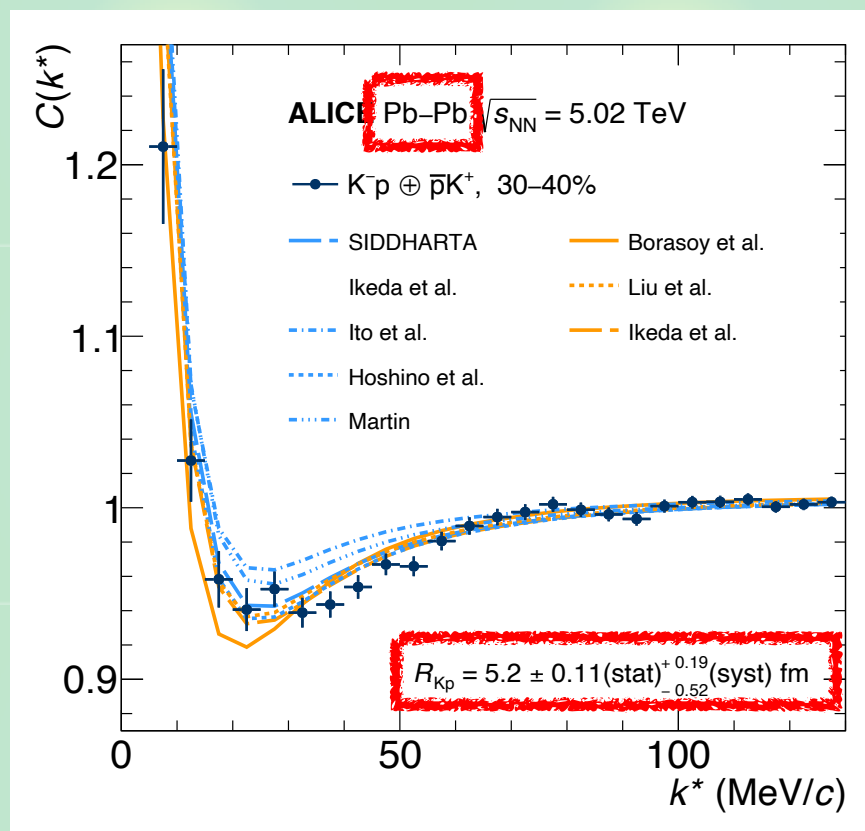
Correlation function by ALICE is **well reproduced**

Large source case

New data with Pb-Pb collisions at 5.02 TeV

ALICE collaboration, PLB 822, 136708 (2021)

- Scattering length $a_{K^-p} = -0.91 + 0.92i$ fm

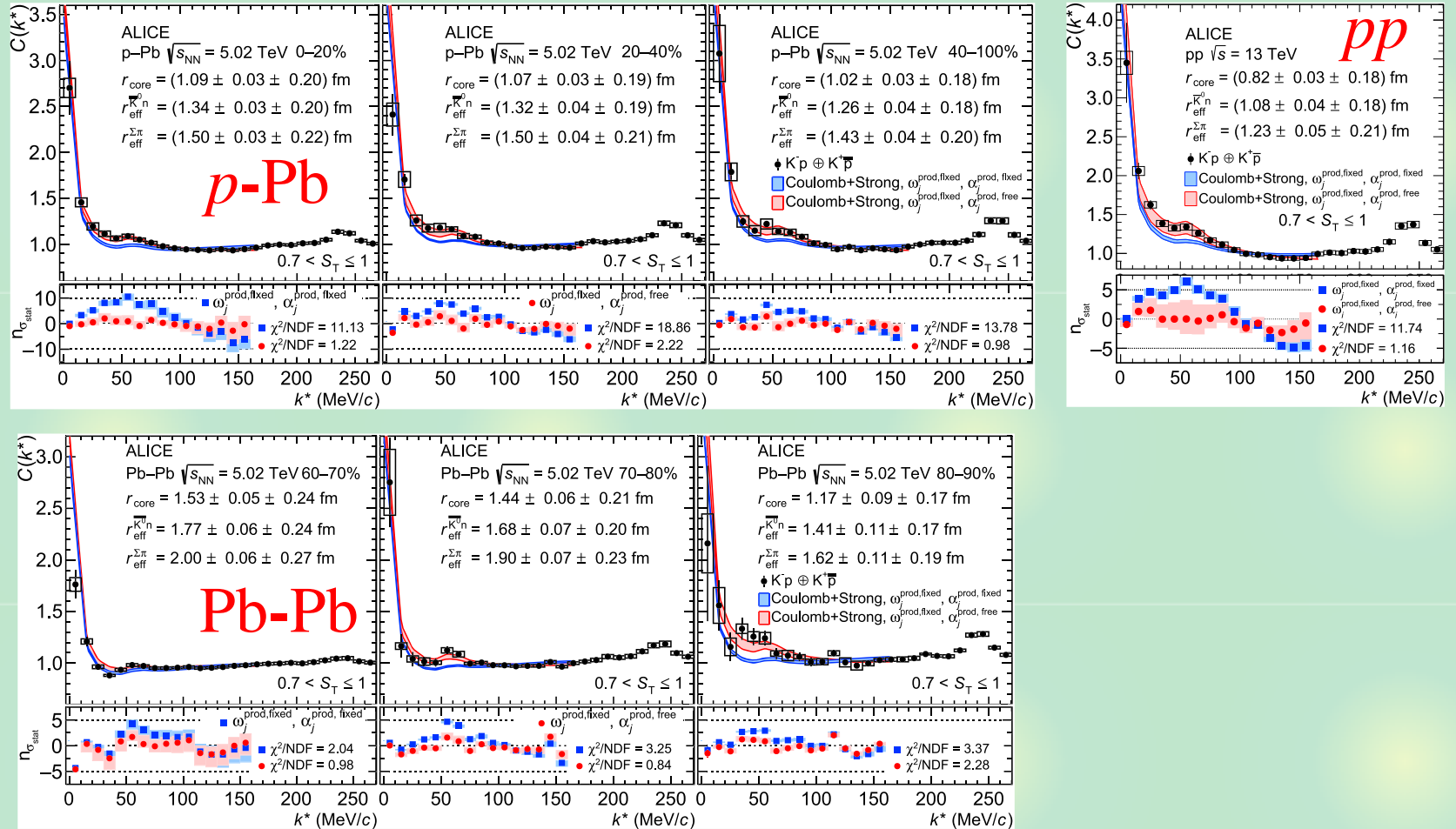


Correlation is suppressed at larger R , as predicted

Systematic study of source size dependence

Correlations in pp , p -Pb, Pb-Pb by Kyoto $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential

ALICE collaboration, EPJC 83, 340 (2023)



Correlations with different source size

Systematic study of source size dependence

Correlations in pp , p -Pb, Pb-Pb by Kyoto $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential

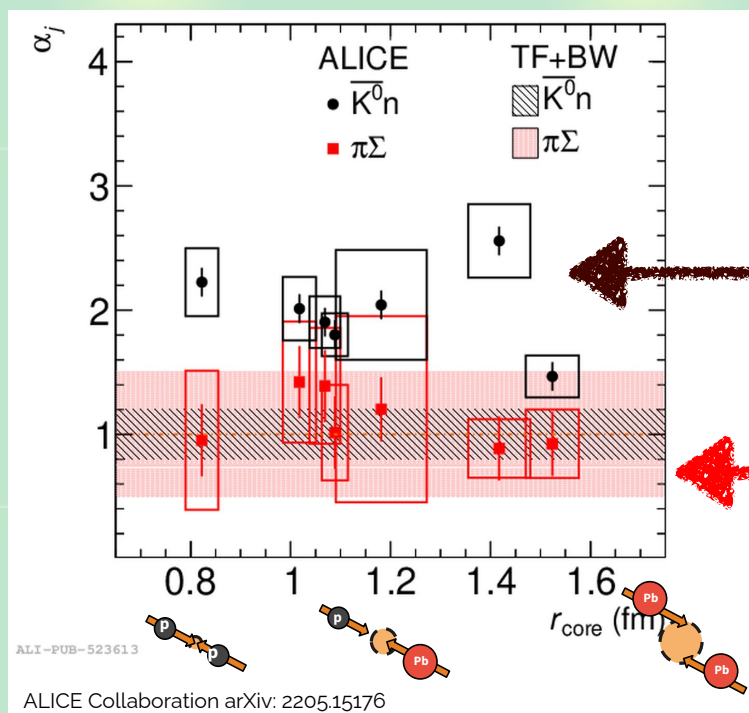
ALICE collaboration, EPJC 83, 340 (2023)

$$C_{K^-p}(q) \simeq \int d^3r S_{K^-p}(r) |\Psi_{K^-p,q}^{(-)}(r)|^2 + \sum_{i \neq K^-p} \alpha_i \omega_i \int d^3r S_i(r) |\Psi_{i,q}^{(-)}(r)|^2$$

ω_i : expected weight by
Thermal Fist + Blast Wave

enhancement needed to
explain data

expected weight is OK



More strength is needed in the $\bar{K}^0n$ channel

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[A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 \[nucl-th\]](#)



Summary and future prospects

Motivation

A solution to hyperon puzzle in neutron stars

- ΛNN **three-body force** for repulsion at high density

D. Gerstung, N. Kaiser, W. Weise, EPJA 55, 175 (2020)

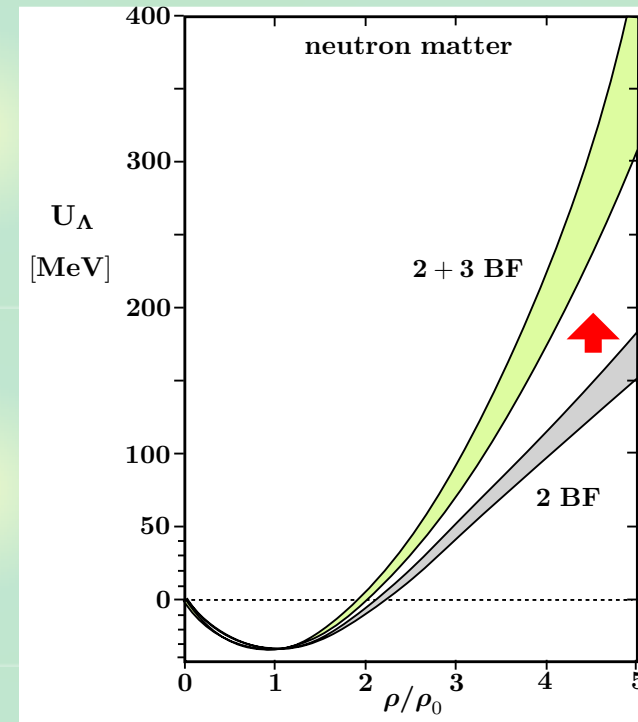
How to verify this in experiments?

- Λ **directed flow** in heavy ion collisions

Y. Nara, A. Jinno, K. Murase, A. Ohnishi,
PRC 106, 044902 (2022)

Λ -nucleus correlation function?

- Heavy nuclei are difficult to produce
- Strong binding of α : two-body treatment justified



$\Lambda\alpha$ correlation function $\rightarrow$ **nature of $\Lambda\alpha$ potential?**

$\Lambda\alpha$ potentials

Phenomenological $\Lambda\alpha$ potentials ($^5_\Lambda\text{He}$ binding energy)

I. Kumagai-Fuse, S. Okabe, Y. Akaishi, PLB 345, 386 (1997)

- **SG**: single gaussian
- **Isle**: two gaussians (with core)

Skyrme-Hartree Fock methods

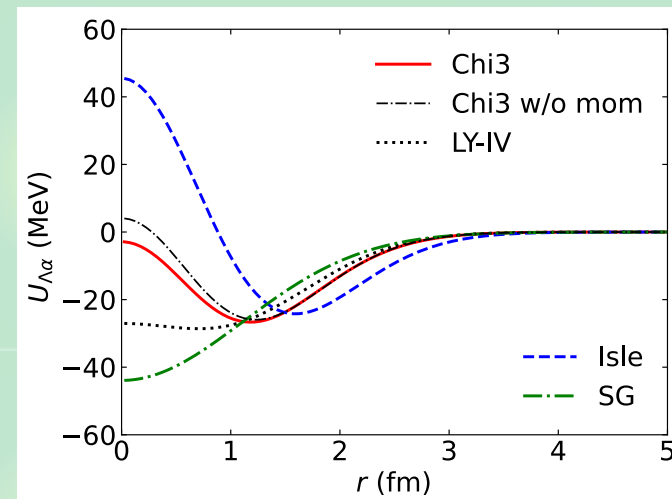
- **LY4**: phenomenorogical

D.E. Lanskoj, Y. Yamamoto, PRC 55, 2330 (1997)

- **Chi3**: based on chiral EFT with ΛNN force

A. Jinno, K. Murase, Y. Nara, A. Ohnishi, PRC 108, 065803 (2023)

- Both potentials reproduce hypernuclear data from C to Pb
- α density distribution $\rightarrow \Lambda\alpha$ potentials

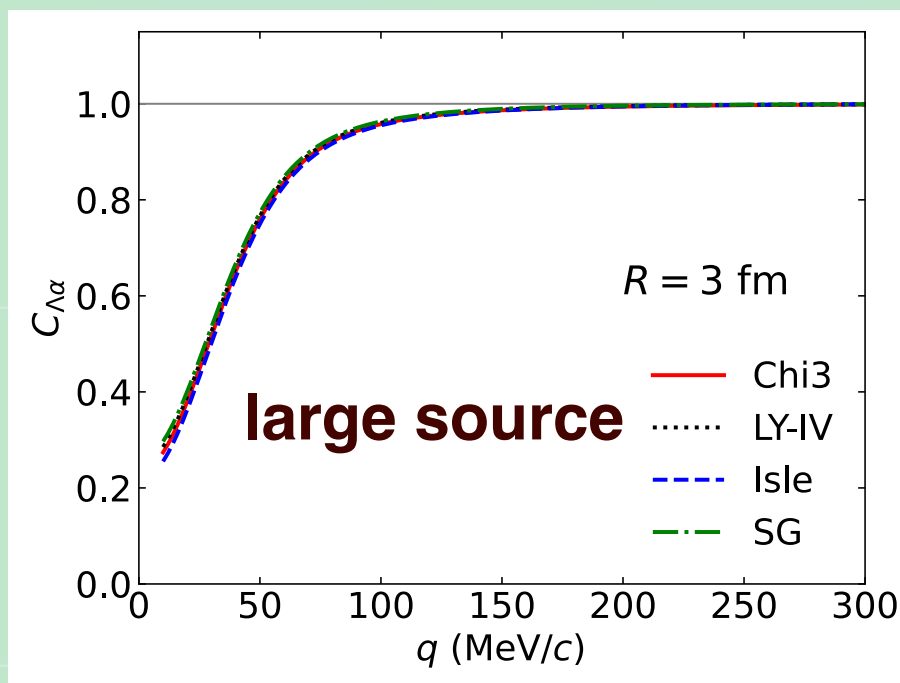
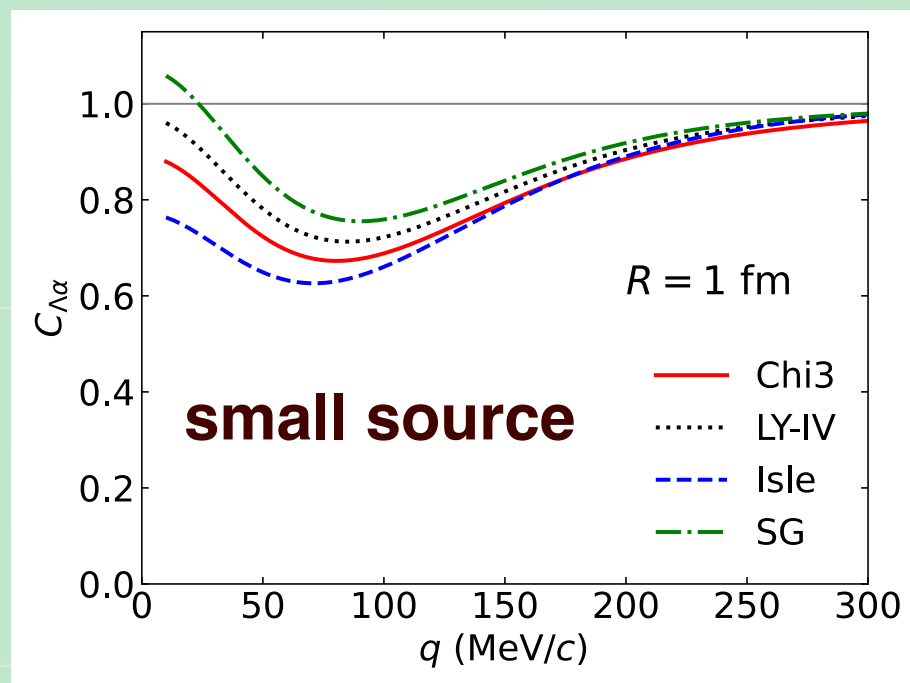


Effect of repulsive core $\rightarrow$ correlation function?

$\Lambda\alpha$ correlation functions: source size dependence

Correlation functions from small and large sources

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 [nucl-th]

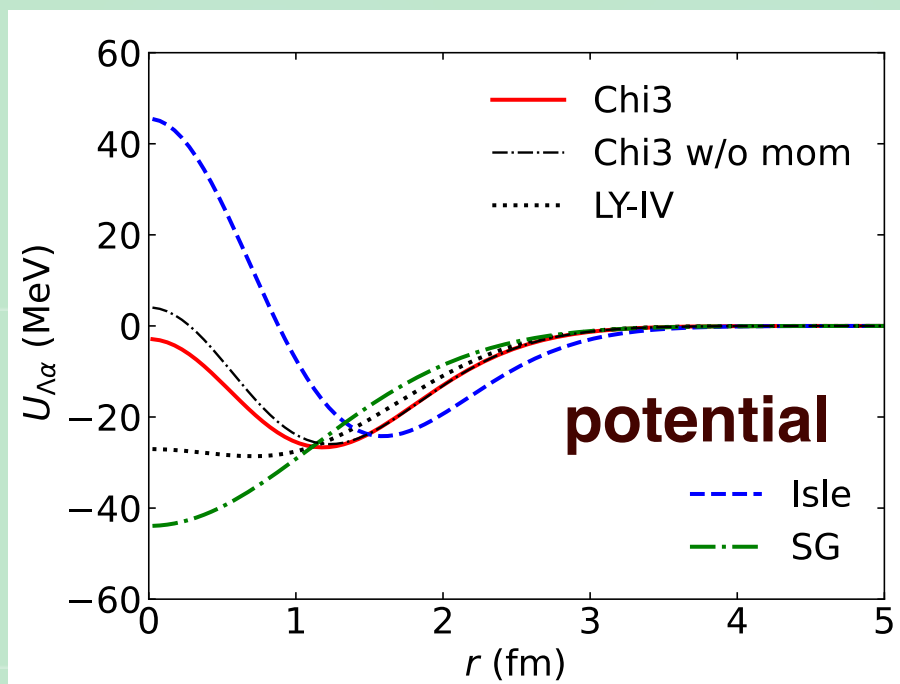
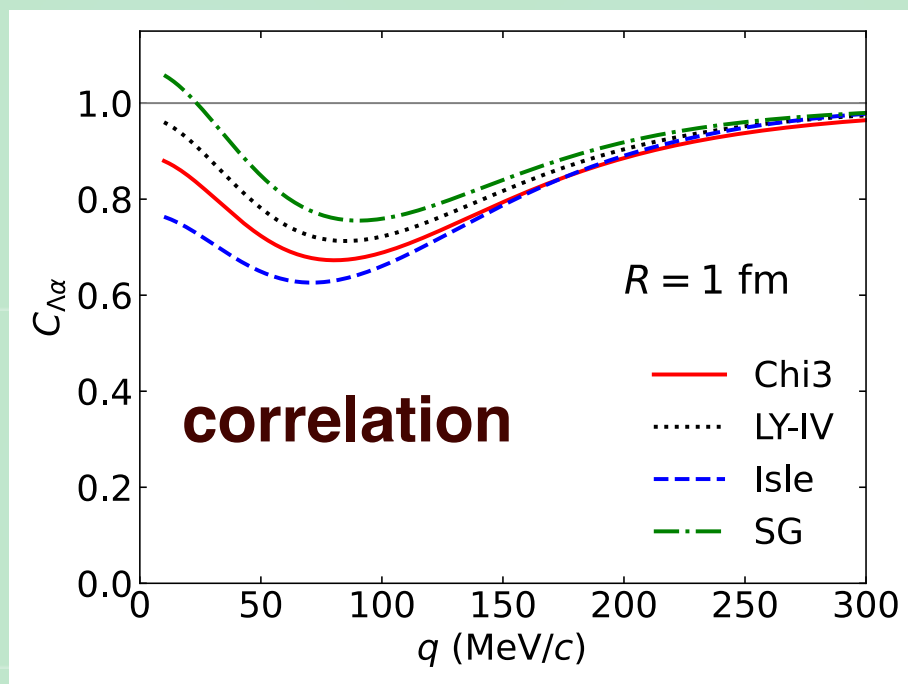


- Bound state signature (dip at low q in small source)
- No difference in large source ($R \sim 3$ fm)
- **Potential dependence** in small source ($R \sim 1$ fm)

$\Lambda\alpha$ correlation functions: potential dependence

Correlation functions and $\Lambda\alpha$ potentials

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 [nucl-th]

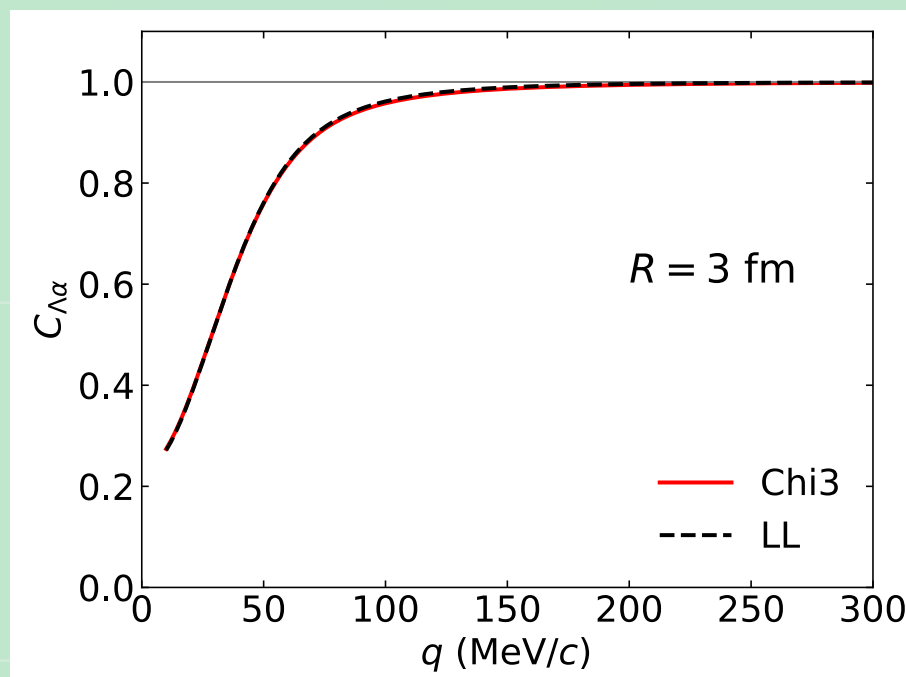
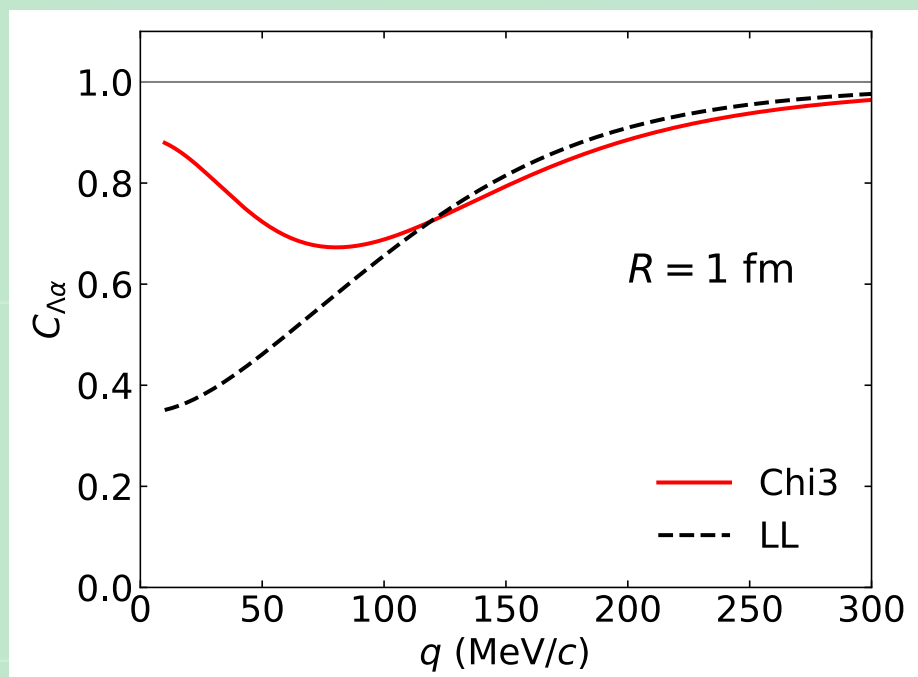


- $U_{\Lambda\alpha}(r = 0)$: **Isle** > **LY-IV** > **Chi3** > **SG**
- $C_{\Lambda\alpha}(q = 0)$: **Isle** < **LY-IV** < **Chi3** < **SG**
- Central repulsion **suppresses** correlation at low q

LL formula

Comparison with LL formula

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 [nucl-th]



- LL works for large source ($R \sim 3$ fm)
- Significant deviation from LL for small source
- ← large interaction range ($R_{\text{int}} \sim 3$ fm) of $\Lambda\alpha$ potential

Summary



Femtoscopy: novel and useful method to study interactions of exotic hadrons and nuclei

- unique tool to study **charm sector**



K^-p correlations

- precise test for $\Lambda(1405)$ and $\bar{K}N$ interactions

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)



$\Lambda\alpha$ correlations

- hint for repulsive core in $\Lambda\alpha$ interaction

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2403.09126 [nucl-th]

Future prospects



Formulation and basics

- derivation of KP/LL formulae
- higher partial wave contribution
- resonance contribution



Applications

- $\pi\Lambda$ correlation
- $\Xi\alpha$ correlation
- ...