Femtoscopy for exotic hadrons and nuclei

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- Introduction: femtoscopy
- Resonance contribution

  S. Watanabe, T. Hyodo, in preparation

  - s-wave resonance in correlation function
  - Validity of LL formula
- Other recent topics (not covered in this talk)
  - higher partial waves —> Murase san
  - $\Lambda\alpha, \Xi\alpha$ correlations —> Jinno san
- Summary

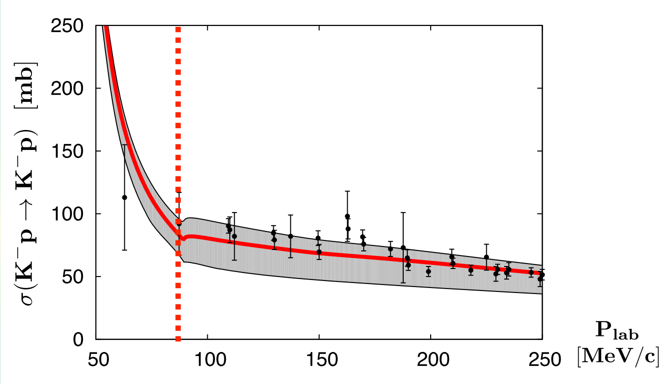
In memory of Akira Ohnishi

Sep. 13, 2019, after FemTUM19 workshop @ München

Scattering experiments and femtoscopy

Traditional methods: scattering experiments

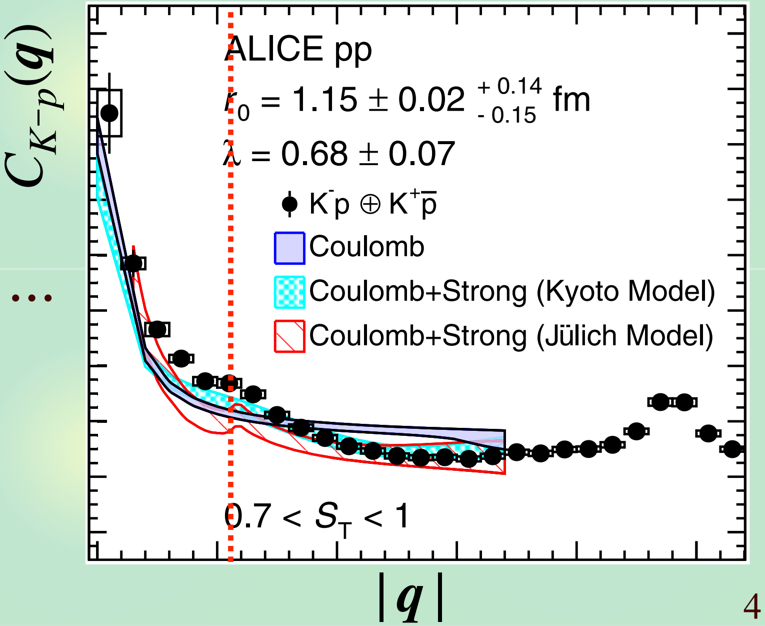
Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)



- Limited channels: $NN, YN, \pi N, KN, \bar{K}N, \cdots$
- Limited statistics (low-energy)
- Heavy (c, b) hadrons: impossible

Femtoscopy: correlation function

ALICE collaboration, PRL 124, 092301 (2020)



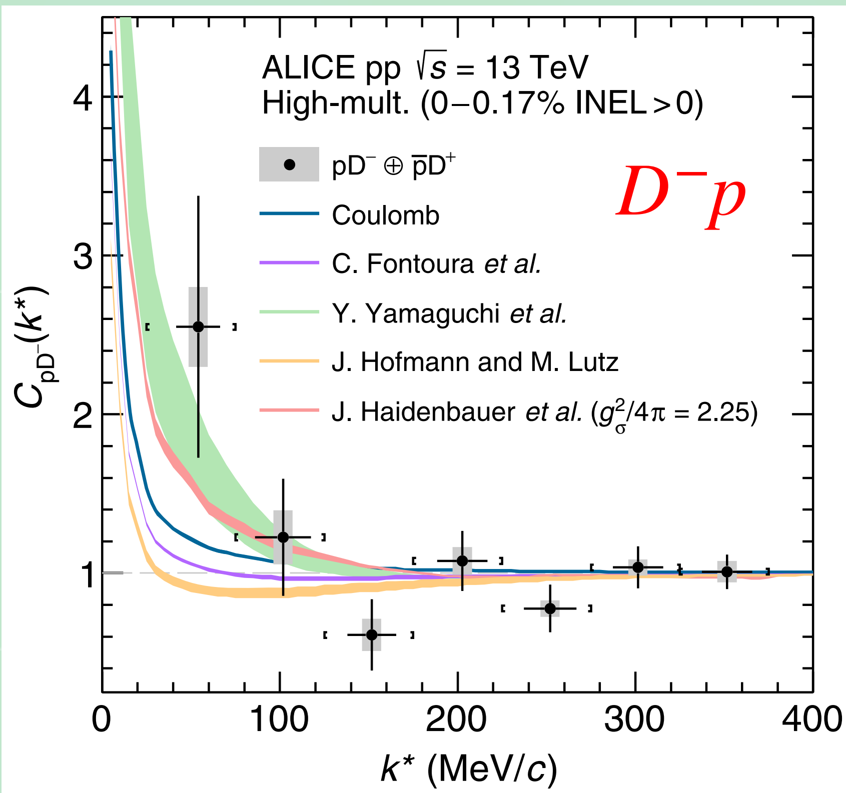
- Various systems: $\Lambda\Lambda, N\Omega, \phi N, \bar{K}\Lambda, DN, \cdots$
- Excellent precision ($\bar{K}^0 n$ cusp)
- Heavy hadrons: possible!

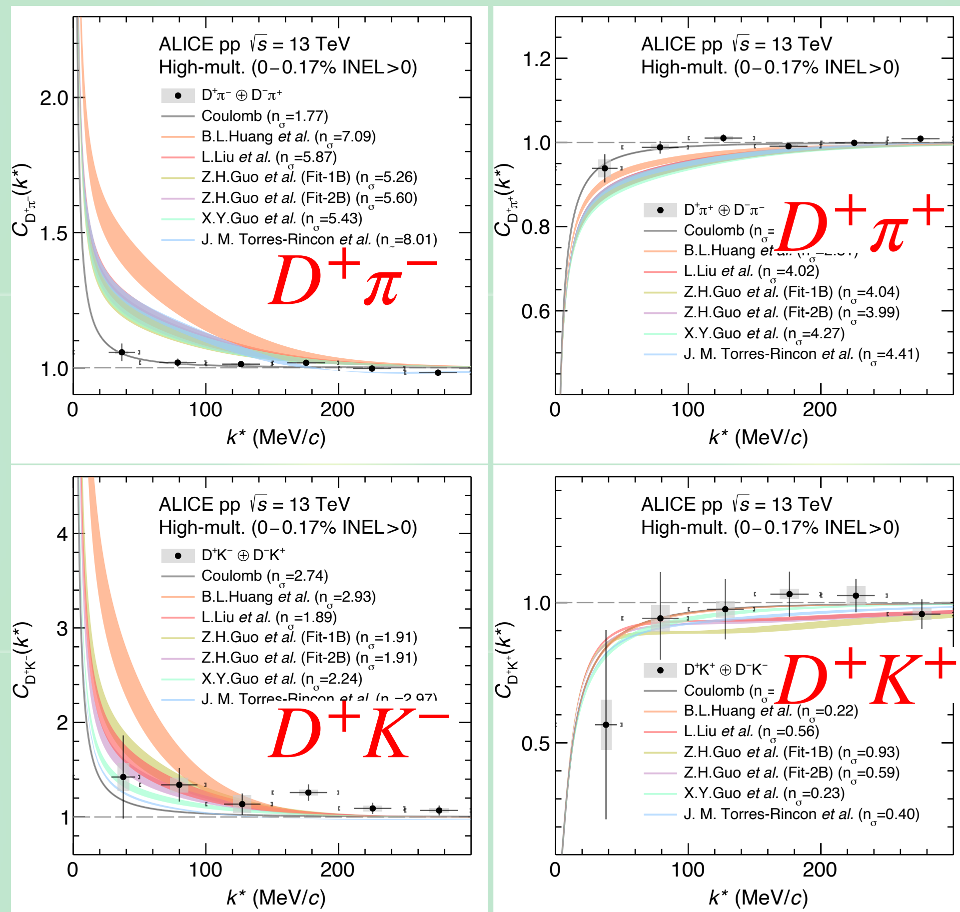
Experimental data in charm sector

Observed correlation functions with charm: $DN, D\pi, DK$

ALICE collaboration, PRD 106, 052010 (2022);

ALICE collaboration, arXiv:2401.13541 [nucl-ex]



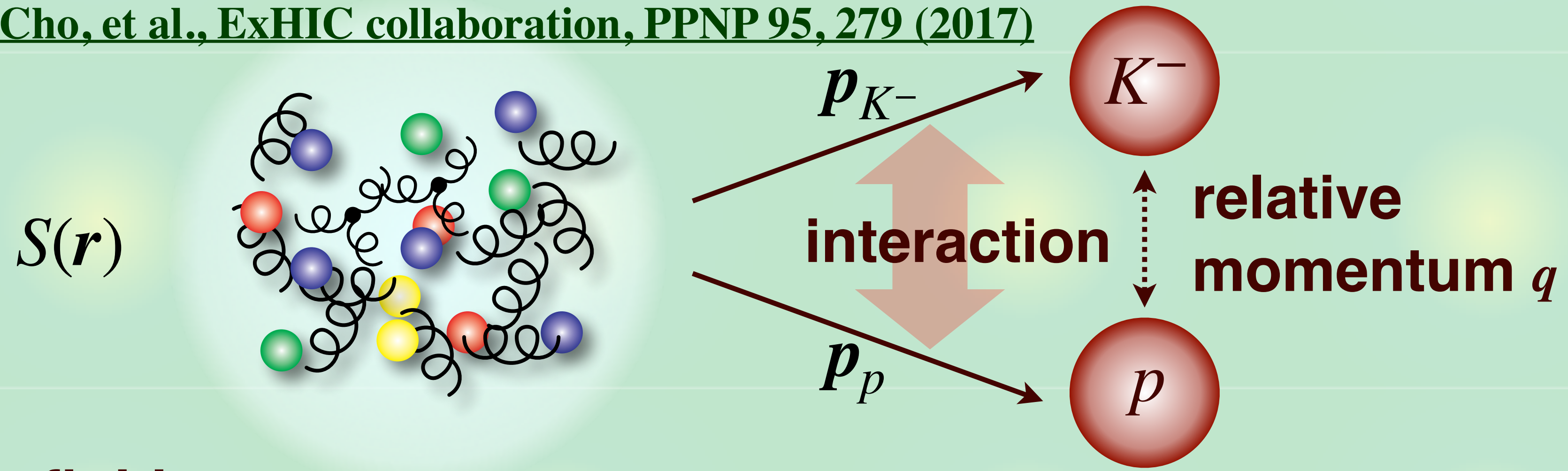


Unique way to obtain data in charm sector (yet low statistics)

Correlation function and KP formula

High-energy collision: chaotic source $S(r)$ of hadron emission

S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)



- Definition

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-})N_p(\boldsymbol{p}_p)}$$ (= 1 in the absence of FSI/QS)

- Theory (Koonin-Pratt formula) incoming + outgoing

S.E. Koonin, PLB 70, 43 (1977); S. Pratt, PRD 33, 1314 (1986)

$$C(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\, S(\boldsymbol{r})\, |\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2, \quad \Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r}) \propto S^\dagger e^{-iqr} - e^{+iqr} \quad (r \to \infty)$$

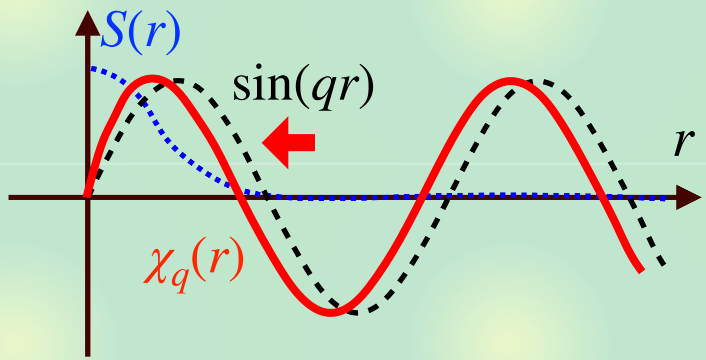
Source function $S(\boldsymbol{r})$ <—> wave function $\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})$ (interaction)

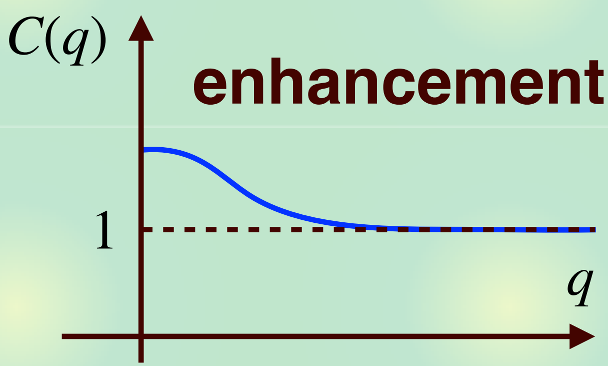
Wave functions and correlations

Spherical source with s-wave interaction dominance

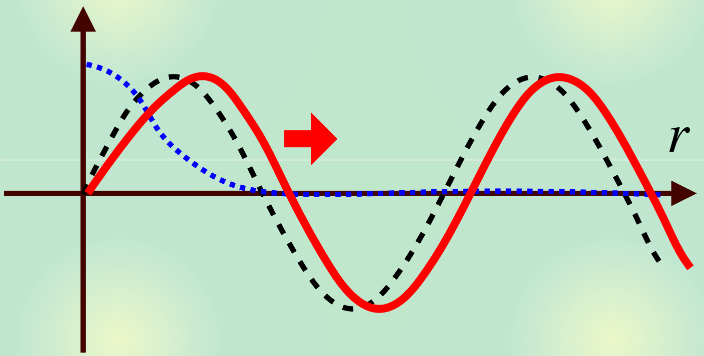
$$C(q) \simeq 1 + \int_0^\infty dr\, S(r) \{ |\chi_q(r)|^2 - \sin^2(qr) \}$$

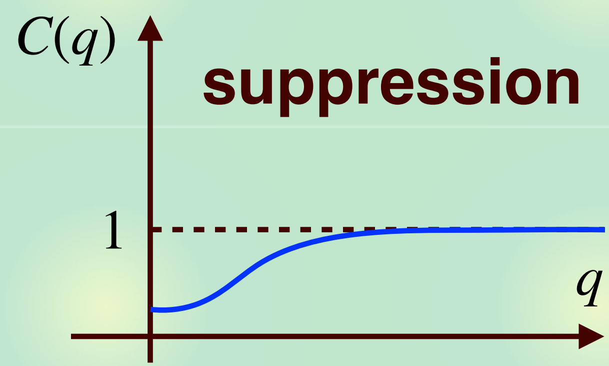
attraction





repulsion





Correlation function <—> nature of interaction

LL formula

Correlation function <—> observables ($a_0, r_e, f(q)$)

R. Lednicky, V.L. Lyuboshits, Yad. Fiz. 35, 1316 (1981)

- Gaussian (relative) source $S(r) = \exp(-r^2/4R^2)/(4\pi R^2)^{3/2}$
- R : source size (gaussian width is $\sqrt{2}R$)
- s-wave interaction only
- zero-range interaction : $R \gg R_{\mathrm{int}}$ (use asymptotic w.f.)

$$C(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3(r_e/R) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R} F_2(2qR)$$

- $F_i(x)$: known functions, $f(q)$: s-wave scattering amplitude

S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)

$$f(q) = \frac{1}{q\cot\delta - iq} \simeq \frac{1}{-\frac{1}{a_0} + \frac{r_e}{2}q^2 - iq}$$

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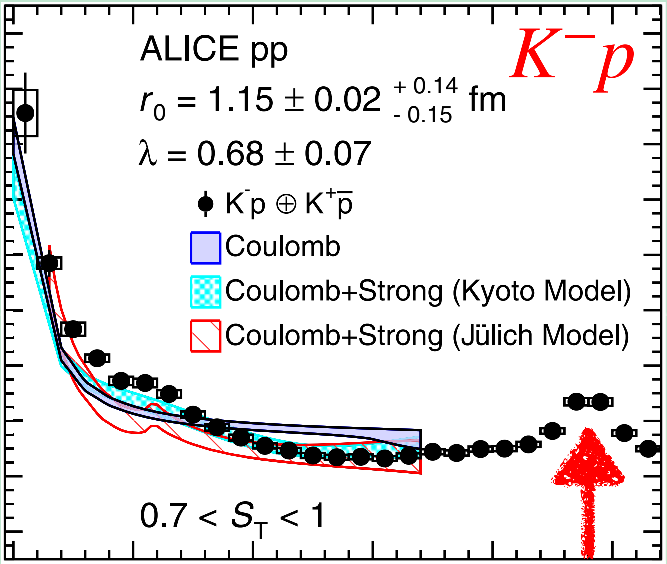
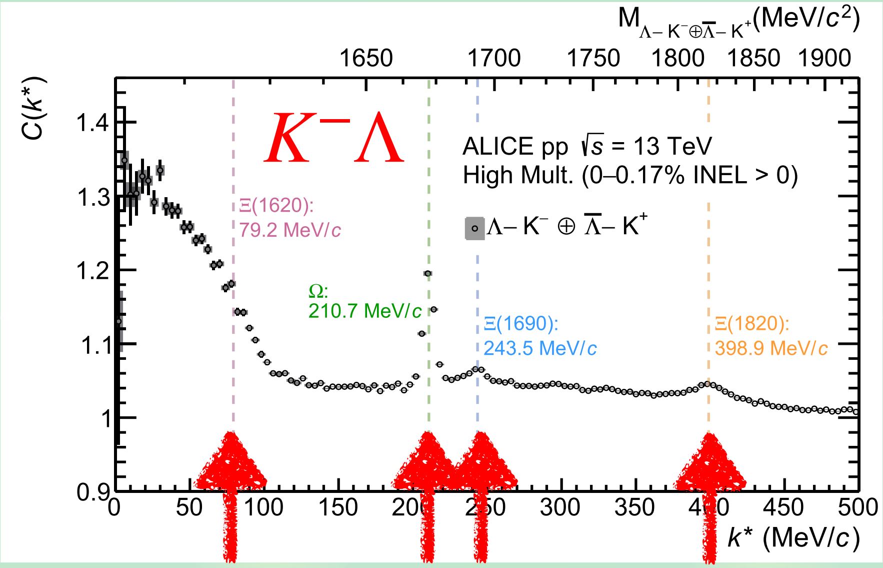
- Summary and future prospects

Higher partial waves and resonance contributions

Resonances in $\ell = 0$ and in $\ell \neq 0$ are seen

- Simple Breit-Wigner function has been used





$\Lambda(1520)$ **: d-wave**

$\Xi(1620)$, $\Xi(1690)$**: s-wave**

Ω **: p-wave (weak decay)**

$\Xi(1820)$**: d-wave**

Questions

- Contribution from higher partial waves?

- Is Breit-Wigner function fine for resonance?

Resonance with effective range expansion

Pole in the effective range expansion (s-wave)

T. Hyodo, PRL 111, 132002 (2013),

T. Kinugawa, T. Hyodo, arXiv:2403.12635 [nucl-th]

$$f(q) = \frac{1}{-\frac{1}{a_0} + \frac{r_e}{2}q^2 - iq}, \quad q^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1 + i0^+}$$

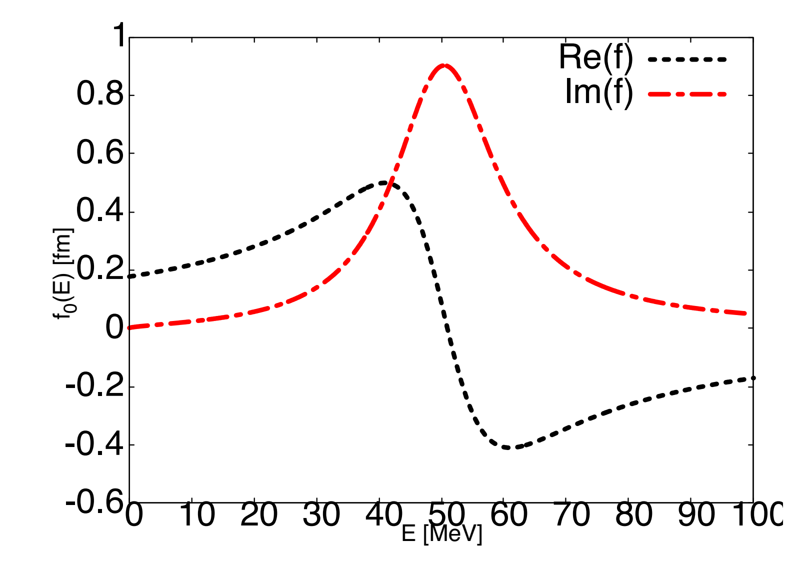
- condition to have resonance ($-\pi/4 \leq \arg(k^-) < 0$)

$$r_e \leq a_0 < 0$$

Choosing $a_0 = -0.18\ \text{fm},\ r_e = -9.1\ \text{fm}$

$$q^- = 217 - 21i\ \text{MeV}$$

$$E^- = 50 - 10i\ \text{MeV}$$



$f(E)$ **shows BW resonance shape**

—> cross section $\sigma \propto \text{Im}\, f(E)$ **: peak**

Resonance with effective range expansion

LL formula can be simplified by optical theorem

K. Murase, T. Hyodo, in preparation

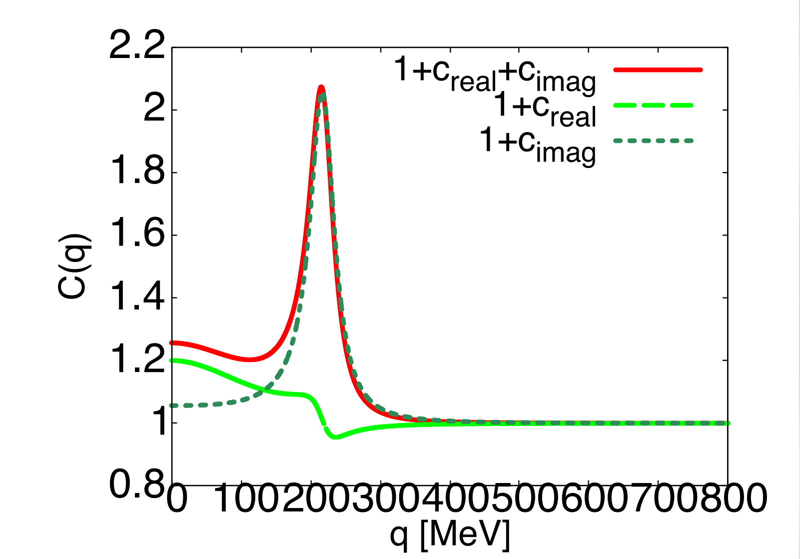
$$C(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3(r_e/R) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R} F_2(2qR)$$

$$= 1 + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) + \frac{\mathrm{Im}\, f(q)}{2qR^2} \left(e^{-(2qR)^2} - \frac{r_e/R}{2\sqrt{\pi}} \right)$$

With $a_0 = -0.18\ \mathrm{fm},\ r_e = -9.1\ \mathrm{fm}$

$q^- = 217 - 21i\ \mathrm{MeV}$

S. Watanabe, T. Hyodo, in preparation



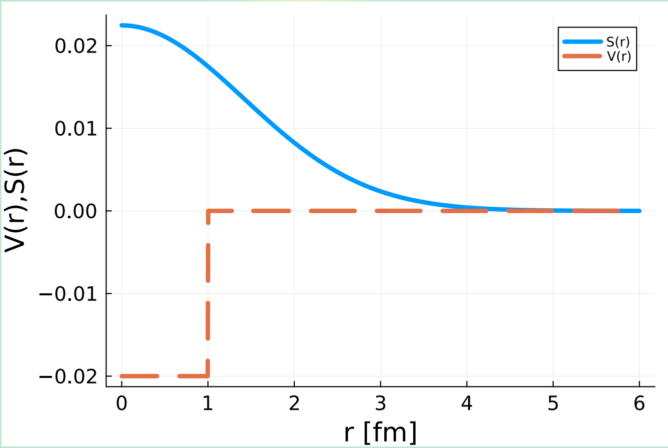
Correlation function: Peak (Im) + Background (Re)

Validity of LL formula

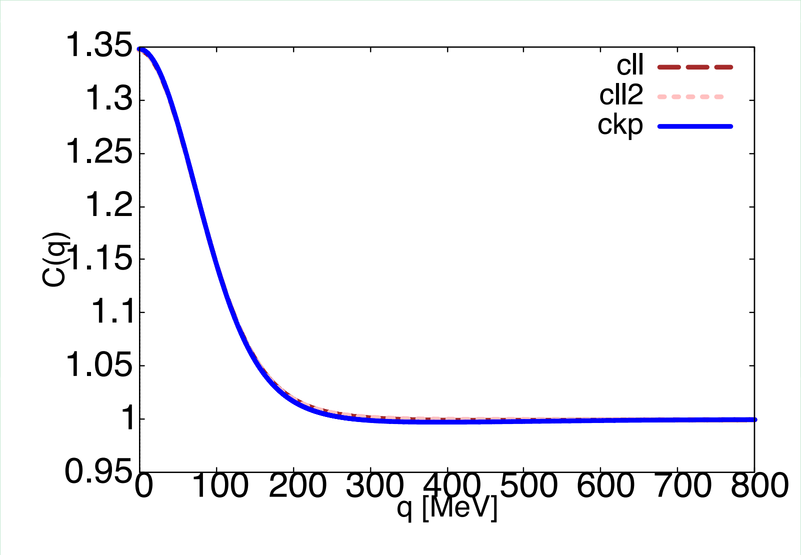
Square-well potential

$$V(r) = \begin{cases} V_0 & (0 \le r \le R_{\rm int}) \\ 0 & (R_{\rm int} < r) \end{cases}$$

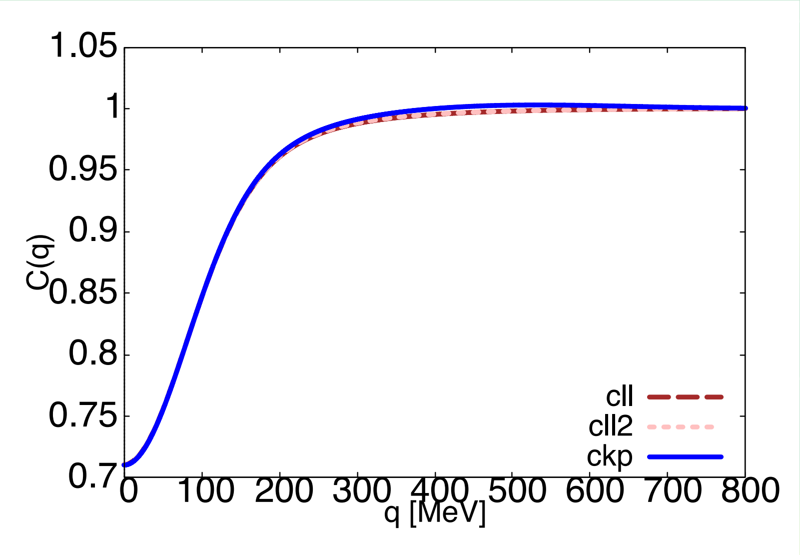
$\mu = 470$ MeV, $R_{\rm int} = 1$ fm



- source size $R = 1$ fm



attraction $V_0 = -27$ MeV



repulsion $V_0 = 58$ MeV

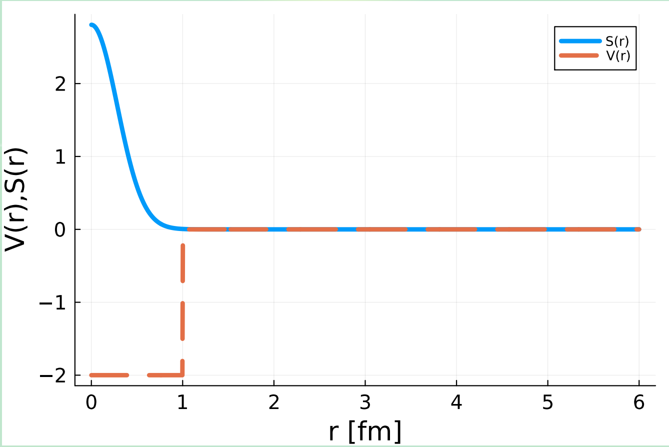
LL formula works well even for $R = R_{\rm int}$

Too small source size

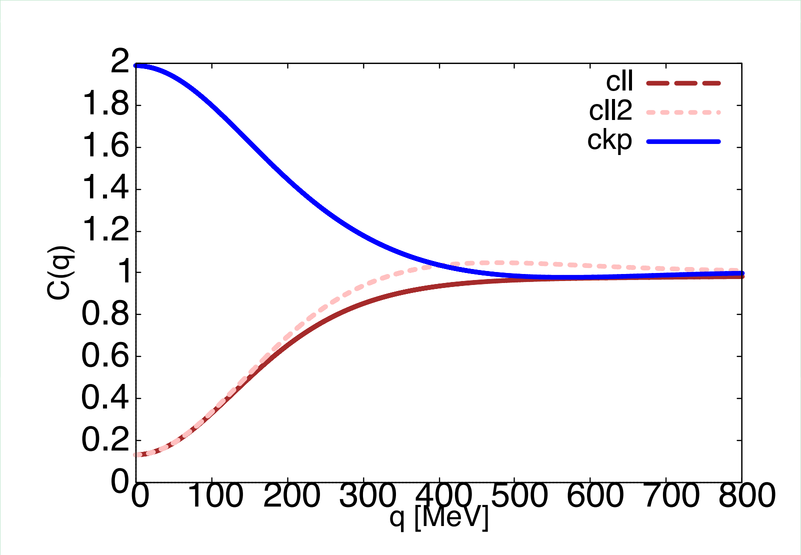
Square-well potential

$$V(r) = \begin{cases} V_0 & (0 \le r \le R_{\rm int}) \\ 0 & (R_{\rm int} < r) \end{cases}$$

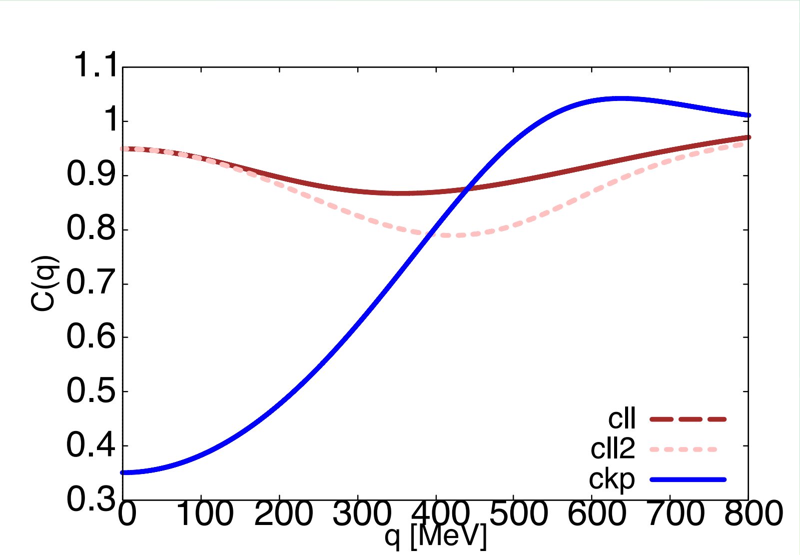
$$\mu = 470 \ \mathrm{MeV}, \ R_{\rm int} = 1 \ \mathrm{fm}$$



- source size $R = 0.2 \ \mathrm{fm}$



attraction $V_0 = -27 \ \mathrm{MeV}$



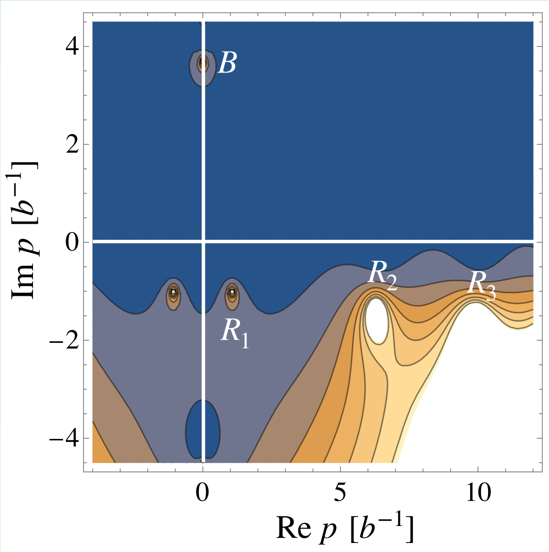
repulsion $V_0 = 58 \ \mathrm{MeV}$

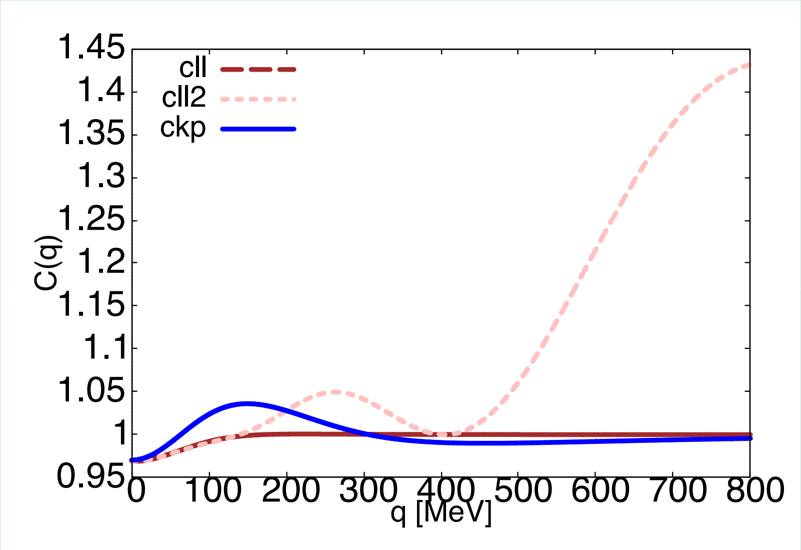
LL formula breaks down for $R = 0.2 R_{\rm int}$

When a near-threshold resonance exists

Square-well potential has resonance solutions

T.A. Weber, C.L. Hammer, V.S. Zidell, Am. J. Phys. 50, 839 (1982)





attraction $V_0 = -830\ \mathrm{MeV}$

—> resonance $q = -206 - 201i\ \mathrm{MeV}$

- Correlation with $R = R_{\mathrm{int}}$

LL may not work when a near-threshold resonance exists

Summary

- **Femtoscopy: novel and useful method to study interactions of exotic hadrons and nuclei**

  - unique tool to study **charm sector**

- **Resonance contributions**

  - Resonance contribution in LL : peak (Im) plus **background** (Re)

  - LL formula **works well even for** $R = R_{\mathrm{int}}$, but breaks down for $R \simeq 0.2 R_{\mathrm{int}}$

  - Near-threshold resonance —> LL fails?

S. Watanabe, T. Hyodo, in preparation