Seminar

- Bound states and resonances in ERE (short range)

(◦ Complex compositeness and interpretation)

- near-threshold exotic hadrons
  - $X(3872) \sim D^0 \bar{D}^{*0}$ $\left(= \frac{1}{\sqrt{2}} (D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0}) \right)$
  - $T_{cc} \sim D^0 D^{*+}$

binding energy $\sim$ keV order $\ll$ typical scale (~ 200 MeV)

PDG $\quad M_{X(3872)} = 3871.64 \pm 0.06$ MeV

$$\Rightarrow \quad B_{X(3872)} = M_{D^0} + M_{\bar{D}^{*0}} - M_{X(3872)}$$

$$= 0.04 \pm 0.06 \text{ MeV}$$

$$\Gamma_{X(3872)} = 1.19 \pm 0.21 \text{ MeV} \quad (\to J/\psi\, \pi\pi, \cdots)$$

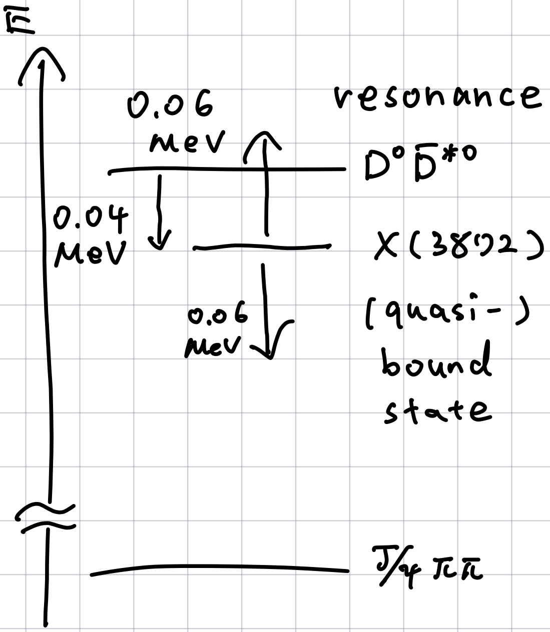
(quasi-) bound state

or resonance

uncertainty

$\leftarrow$ exp. resolution,

decay mode, ...



∘ Structure of $X(3872)$

$I(J^{PC}) = 0(1^{++})$

- $c\bar{c}$ in p-wave (quark core)
- $D^0\bar{D}^{*0} + \bar{D}^0 D^{*0}$ in s-wave (hadronic molecule)

$$|X(3872)\rangle = C^{c\bar{c}}|c\bar{c}\rangle + C^{00}|D^0\bar{D}^{*0}\rangle + C^{+-}|D^+D^{-*}\rangle + \cdots$$

$$\simeq \sqrt{X}\,|D^0\bar{D}^{*0}\rangle + \sqrt{Z}\,|\text{others}\rangle$$

compositeness $X = (C^{00})^2$

elementarity $Z = \sum_{i \neq 00} (C^i)^2$

normalization of $|X(3872)\rangle \Rightarrow X + Z = 1$

X is model- (renormalization-) dependent quantity

∘ weak-binding relation

a_0 scattering length

r_e effective range

effective range expansion (ERE)

$$f(k) = \frac{1}{-\frac{1}{a_0} + \frac{r_e}{2}k^2 + \mathcal{O}(k^4) - ik}$$

$R = 1/\sqrt{2\mu B} = 1/\sqrt{-2\mu E}$ length scale of eigenstate

R_{typ} length scale of interaction ($\sim 1\,\mathrm{fm}$, $1/m_\pi$, ---)

$$a_0 = R\left[\frac{2X}{X+1} + \mathcal{O}\left(\frac{R_{typ}}{R}\right)\right] \quad (1)$$

$$r_e = R\left[\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{typ}}{R}\right)\right] \quad (2)$$

(1) can be derived from EFT (with finite cutoff)

To obtain (2), we assume ERE convergence

neglecting $\mathcal{O}(k^4)$ terms, pole condition reads

$$-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik = 0$$

$$k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1} \quad (3)$$

$\Rightarrow$ relation between a_0, r_e, R, (1) $\Rightarrow$ (2)

For weakly bound (s-wave) states,

X can be determined by observables (a_0, R, r_e)

- completely composite $X = 1$, $Z = 0$

(1) $\Rightarrow$ $a_0 = R + \mathcal{O}(R_{typ})$

natural

(2) $\Rightarrow$ $r_e = \mathcal{O}(R_{typ})$

- completely elementary $X = 0$, $Z = 1$

(1) $\Rightarrow$ $a_0 = \mathcal{O}(R_{typ})$

unnatural

fine tuning required

(2) $\Rightarrow$ $r_e = -\infty$

- classification of poles in ERE

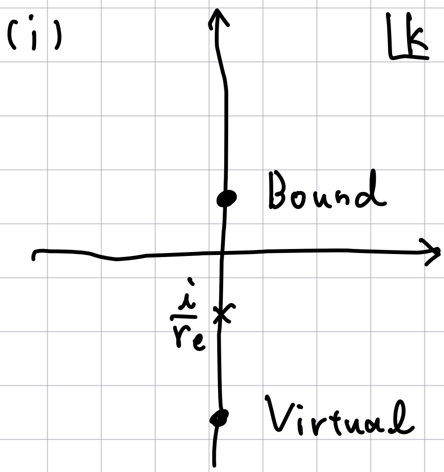
eigenmomenta (3)

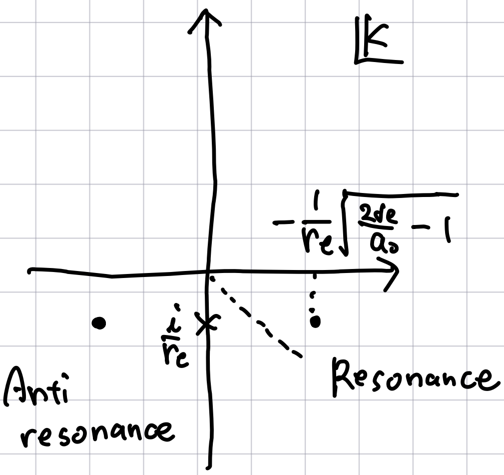
(i) $\frac{2r_e}{a_0} - 1 < 0$: two pure imaginary roots

(ii) $\frac{2r_e}{a_0} - 1 > 0$: two complex roots $k^- = -(k^+)^*$

symmetric w.r.t. imaginary axis

to obtain resonance solution, $r_e < 0$





to obtain $|\mathrm{Re}\, k^-| > |\mathrm{Im}\, k^-|$,

$\sqrt{\frac{2r_e}{a_0} - 1} > 1 \Rightarrow r_e < a_0 < 0$

a_0, r_e are both negative and $|a_0| < |r_e|$

near-threshold resonance : $|a_0|, |r_e| \gg R_{typ}$

fine tuning

← no centrifugal barrier in s wave

difficult to generate resonance above threshold

○ near-threshold mass scaling

bare state $|\Psi_0\rangle$ couples to two-body continuum

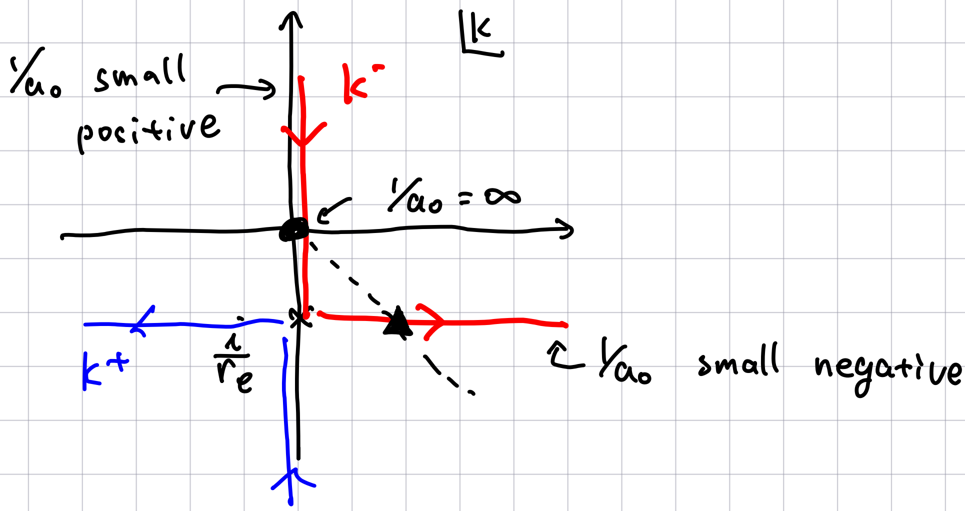
$\sim$ $c\bar{c}$ core couples to $D^0\bar{D}^{*0}$ continuum

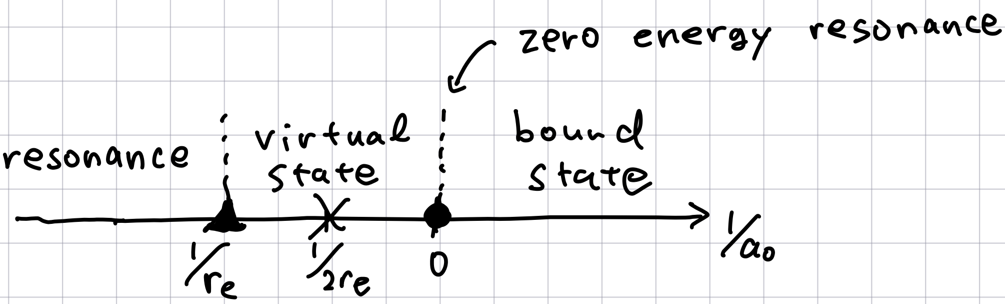
$\sim$ Feshbach resonance

closed-ch. bound state couples to open-ch. continuum

variation of $1/a_0$ for a fixed r_e

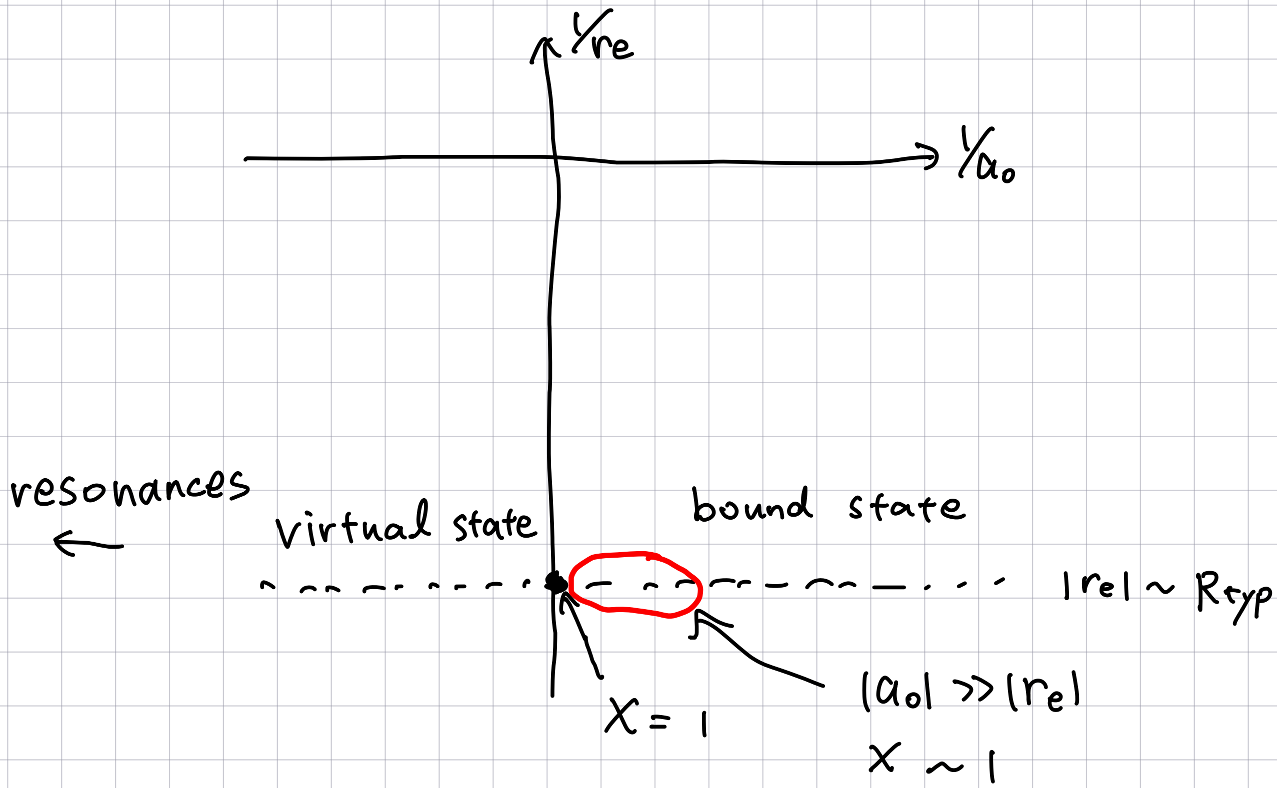
from (3) we obtain pole trajectory



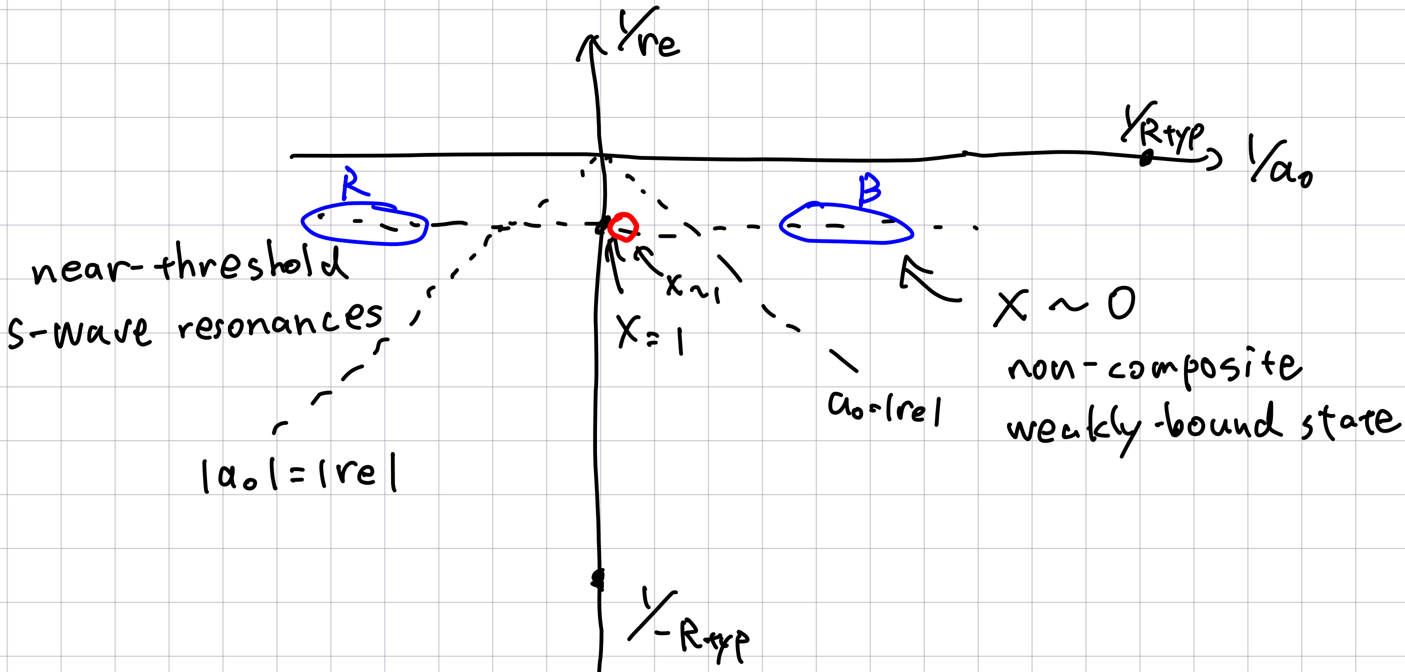


∘ Structure

natural size $|r_e| \sim R_{typ}$ (broad Feshbach resonance)



unnatural $|r_e| \gg R_{typ}$ (narrow Feshbach resonance)



B : $|r_e| \gg a_0 \gg R_{typ} \Rightarrow X \sim 0$

R : near-threshold s-wave resonance

- conclusion
  - natural case $|r_e| \sim R_{typ}$

    Near-threshold bound states are composite

    $X \sim 1$, $Z \sim 0$
  - unnatural case $|r_e| \gg R_{typ}$

    Near-threshold bound states can be non-composite ($X \sim 0$, $Z \sim 1$), though unlikely

    Near-threshold narrow s-wave resonance can appear only in this case. (unlikely)

    If such a resonance exists, it should be non-composite

    $X \sim 0$, $Z \sim 1$