Compositeness of near-threshold s-wave resonances

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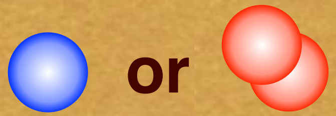
2024, Sep. 24th

Contents

- Introduction: structure of unstale hadrons

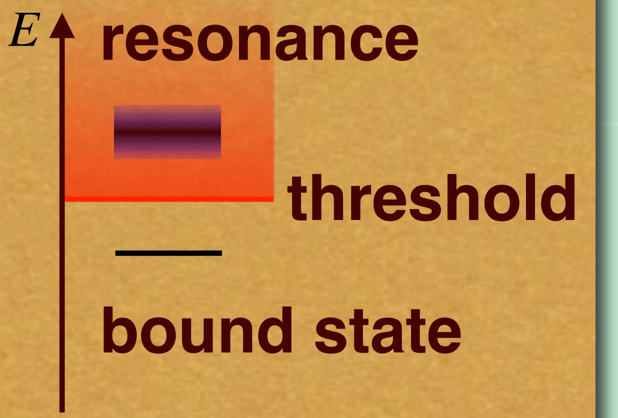
- Compositeness of near-threshold bound states

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Hyodo, PRC90, 055208 (2014);
T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)


or

- Near-threshold resonances

T. Hyodo, PRL111, 132002 (2013);
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]


E

resonance

threshold

bound state

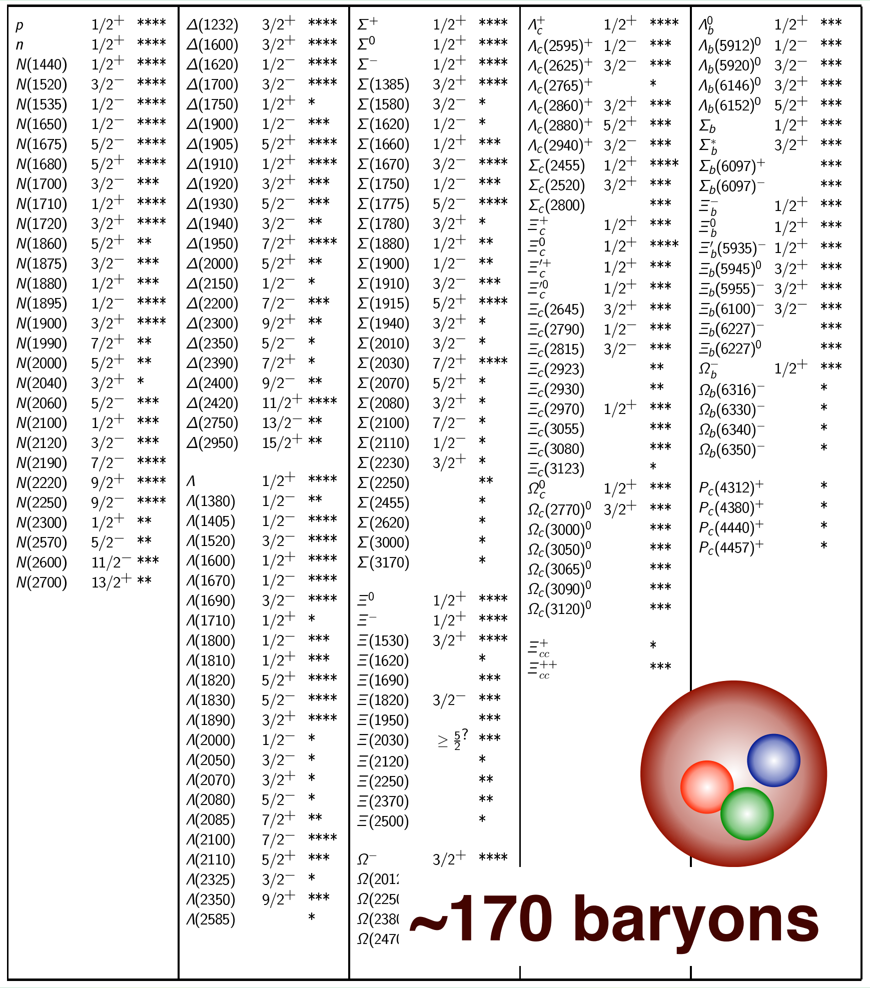
- Summary

Introduction: structure of unstable hadrons

Observed hadrons

Particle Data Group (PDG)

http://pdg.lbl.gov/



p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

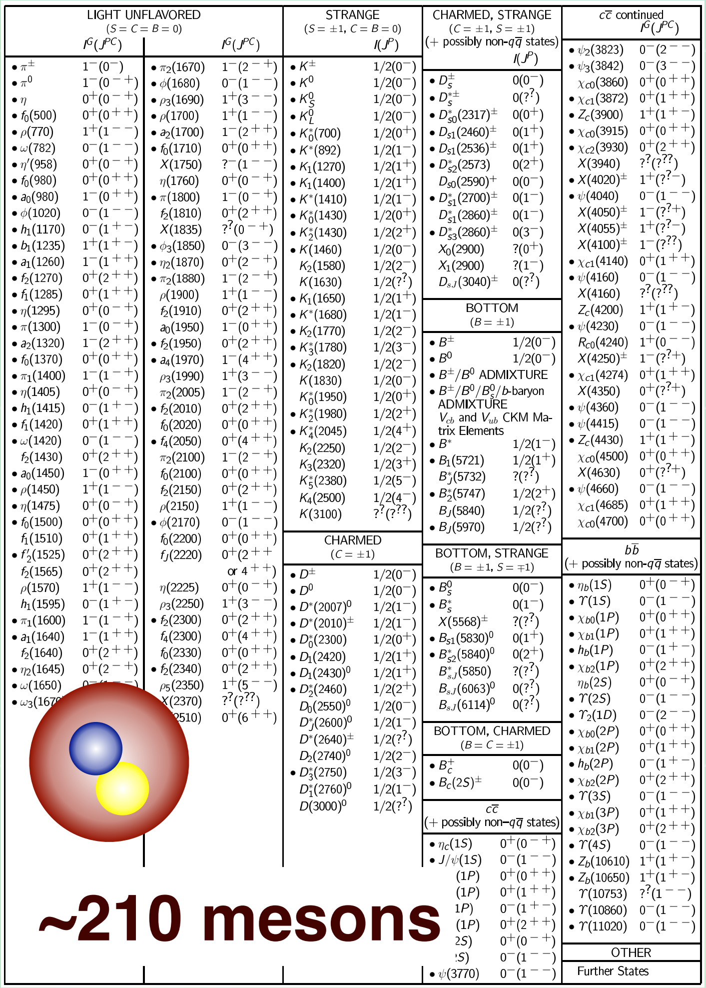
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		*

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2110)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		**
$\Sigma(2455)$		*
$\Sigma(2620)$		*
$\Sigma(3000)$		*
$\Sigma(3170)$		*
Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(201$		
$\Omega(225$		
$\Omega(238$		
$\Omega(247$		

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2923)$		**
$\Xi_c(2930)$		**
$\Xi_c(2970)$	$1/2^+$	***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3000)^0$		***
$\Omega_c(3050)^0$		***
$\Omega_c(3065)^0$		***
$\Omega_c(3090)^0$		***
$\Omega_c(3120)^0$		***
Ξ_{cc}^+		*
Ξ_{cc}^{++}		***

Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^-	$1/2^+$	***
Ξ_b^0	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6100)^-$	$3/2^-$	***
$\Xi_b(6227)^-$		***
$\Xi_b(6227)^0$		***
Ω_b^-	$1/2^+$	***
$\Omega_b(6316)^-$		*
$\Omega_b(6330)^-$		*
$\Omega_b(6340)^-$		*
$\Omega_b(6350)^-$		*
$P_c(4312)^+$		*
$P_c(4380)^+$		*
$P_c(4440)^+$		*
$P_c(4457)^+$		*

~170 baryons



LIGHT UNFLAVORED
$(S = C = B = 0)$

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(500)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	• $a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$X(1750)$	$?^-(1^{--})$
• $f_0(980)$	$0^+(0^{++})$	$\eta(1760)$	$0^+(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $\phi(1020)$	$0^-(1^{--})$	$f_2(1810)$	$0^+(2^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	$X(1835)$	$?^?(0^{-+})$
• $b_1(1235)$	$1^+(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $a_1(1260)$	$1^-(1^{++})$	• $\eta_2(1870)$	$0^+(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_1(1285)$	$0^+(1^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $\eta(1295)$	$0^+(0^{-+})$	$f_2(1910)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $a_4(1970)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2005)$	$1^-(2^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2020)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	• $f_4(2050)$	$0^+(4^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $a_0(1450)$	$1^-(0^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
• $\eta(1475)$	$0^+(0^{-+})$	$\rho(2150)$	$1^+(1^{--})$
• $f_0(1500)$	$0^+(0^{++})$	• $\phi(2170)$	$0^-(1^{--})$
$f_1(1510)$	$0^+(1^{++})$	$f_0(2200)$	$0^+(0^{++})$
• $f_2'(1525)$	$0^+(2^{++})$	$f_J(2220)$	$0^+(2^{++}$
$f_2(1565)$	$0^+(2^{++})$		or $4^{++})$
$\rho(1570)$	$1^+(1^{--})$	$\eta(2225)$	$0^+(0^{-+})$
$h_1(1595)$	$0^-(1^{+-})$	$\rho_3(2250)$	$1^+(3^{--})$
• $\pi_1(1600)$	$1^-(1^{-+})$	• $f_2(2300)$	$0^+(2^{++})$
• $a_1(1640)$	$1^-(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
$f_2(1640)$	$0^+(2^{++})$	$f_0(2330)$	$0^+(0^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$	• $f_2(2340)$	$0^+(2^{++})$
• $\omega(1650)$	$0^-(1^{--})$	$\rho_5(2350)$	$1^+(5^{--})$
• $\omega_3(167$		$X(2370)$	$?^?(?^{??})$
		$2510)$	$0^+(6^{++})$

STRANGE
$(S = \pm 1, C = B = 0)$

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
• $K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
• $K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
• $K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED
$(C = \pm 1)$

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)$	$1/2(0^+)$
• $D_1(2420)$	$1/2(1^+)$
• $D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)$	$1/2(2^+)$
$D_0(2550)^0$	$1/2(0^-)$
$D_J^*(2600)^0$	$1/2(1^-)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D_2(2740)^0$	$1/2(2^-)$
• $D_3^*(2750)$	$1/2(3^-)$
$D_1^*(2760)^0$	$1/2(1^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE
$(C = \pm 1, S = \pm 1)$
$(+$ possibly non-$q\overline{q}$ states$)$

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
$D_{s0}(2590)^+$	$0(0^-)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
• $D_{s3}^*(2860)^\pm$	$0(3^-)$
$X_0(2900)$	$?(0^+)$
$X_1(2900)$	$?(1^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM
$(B = \pm 1)$

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)$	$1/2(2^+)$
$B_J(5840)$	$1/2(?^?)$
• $B_J(5970)$	$1/2(?^?)$

BOTTOM, STRANGE
$(B = \pm 1, S = \mp 1)$

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$
$B_{sJ}(6063)^0$	$0(?^?)$
$B_{sJ}(6114)^0$	$0(?^?)$

BOTTOM, CHARMED
$(B = C = \pm 1)$

• B_c^+	$0(0^-)$
• $B_c(2S)^\pm$	$0(0^-)$

$c\overline{c}$
$(+$ possibly non$-q\overline{q}$ states$)$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
$(1P)$	$0^+(0^{++})$
$(1P)$	$0^+(1^{++})$
$1P)$	$0^-(1^{+-})$
$(1P)$	$0^+(2^{++})$
$2S)$	$0^+(0^{-+})$
$S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$

$c\overline{c}$ continued

	$I^G(J^{PC})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
$X(4630)$	$0^+(?^{?+})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c1}(4685)$	$0^+(1^{++})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\overline{b}$
$(+$ possibly non-$q\overline{q}$ states$)$

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

OTHER

Further States

~210 mesons

All ~ 380 hadrons basically emerge as qqq or $q\bar{q}$

Exotic hadrons

Observation of tetraquark T_{cc}

LHCb collaboration, Nature Phys. 18, 7, 751 (2022); Nature Commun. 13, 1, 3351 (2022)

nature physics

LETTERS

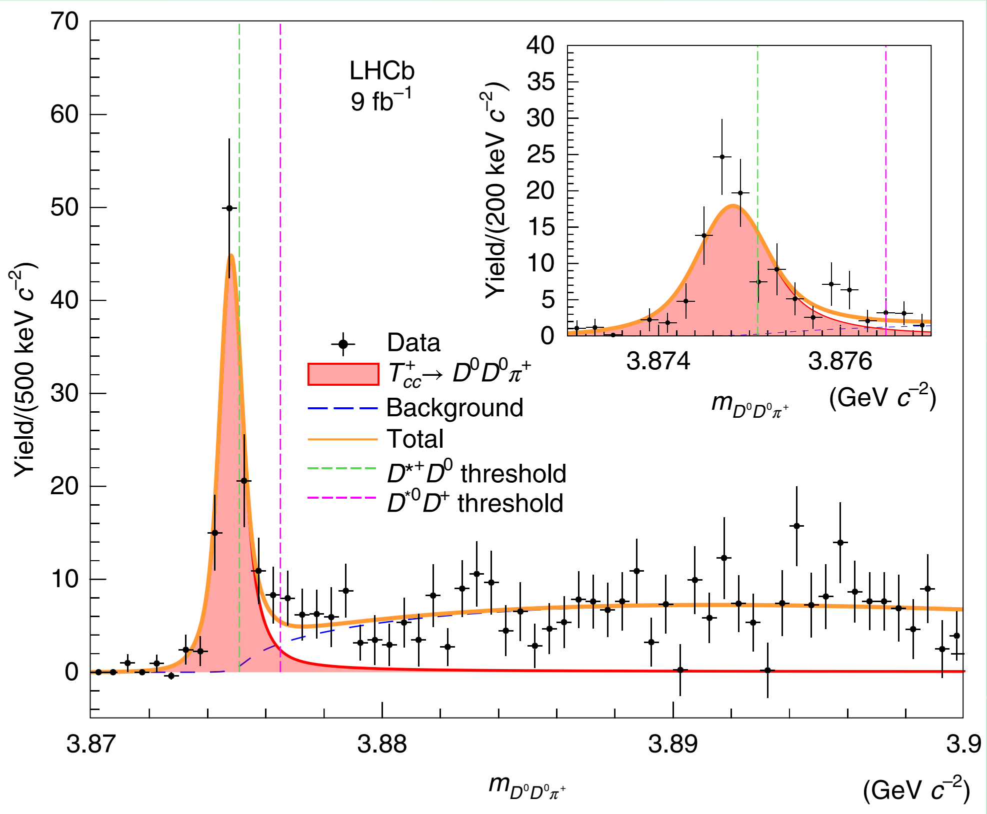
https://doi.org/10.1038/s41567-022-01614-y

Check for updates

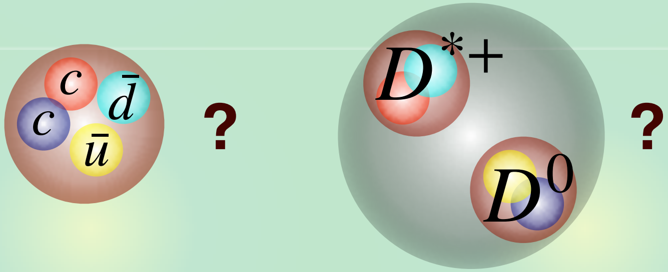
OPEN

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*



- **Quark content** $\sim cc\bar{u}\bar{d}$
- **Near** $D^{*+}D^0$ **threshold**
- **Internal structure?**

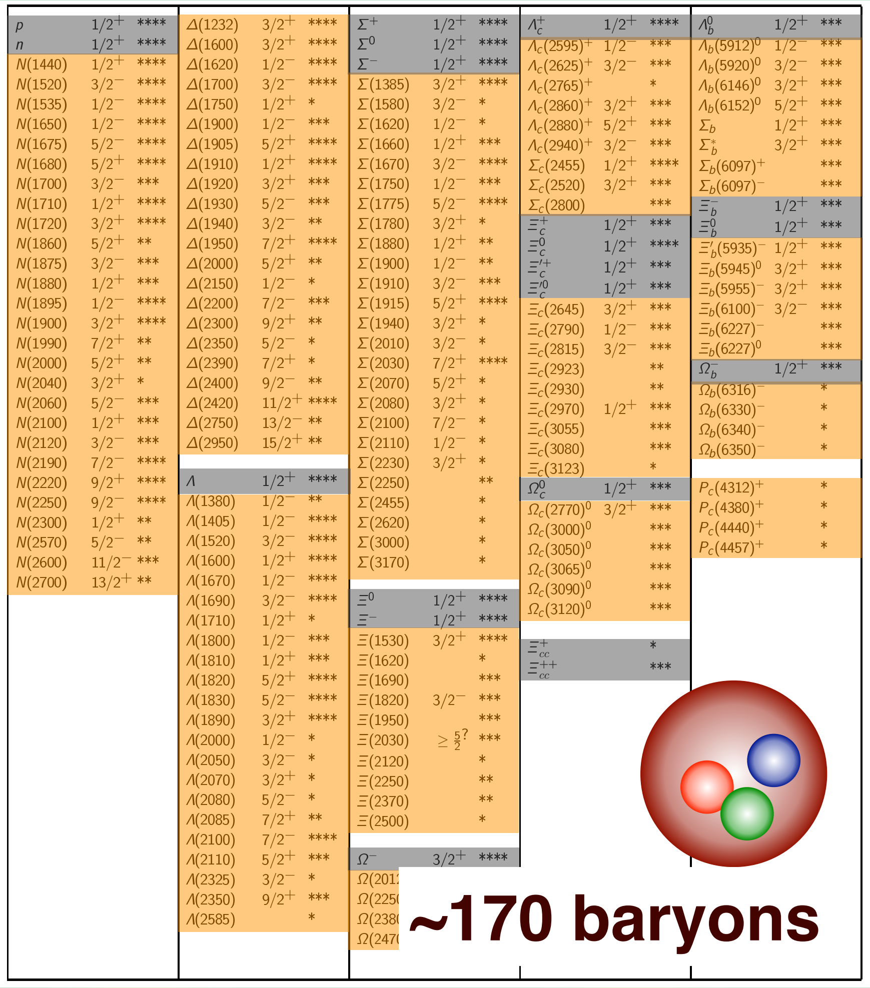


Decay into $D^0D^0\pi^+$: **structure of unstable states**

Unstable states via strong interaction

Stable/unstable hadrons

http://pdg.lbl.gov/



p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**

Λ	$1/2^+$	****
$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		*

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2110)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		**
$\Sigma(2455)$		*
$\Sigma(2620)$		*
$\Sigma(3000)$		*
$\Sigma(3170)$		*

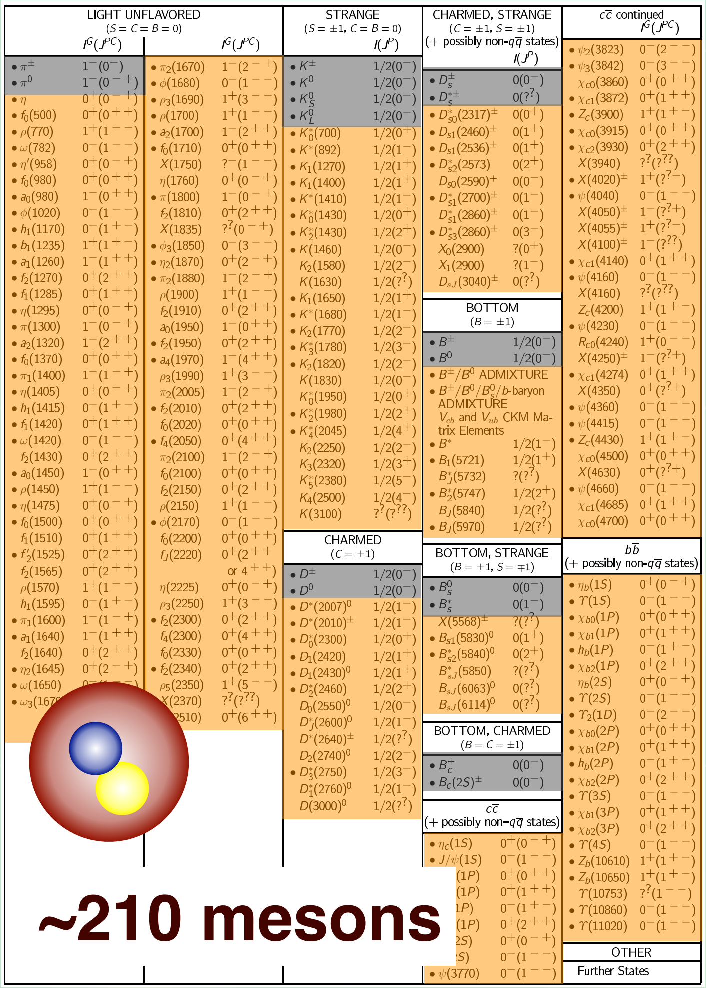
Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*

Ω^-	$3/2^+$	****
$\Omega(2012$		
$\Omega(2250$		
$\Omega(2380$		
$\Omega(247$		

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2923)$		**
$\Xi_c(2930)$		**
$\Xi_c(2970)$	$1/2^+$	***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3000)^0$		***
$\Omega_c(3050)^0$		***
$\Omega_c(3065)^0$		***
$\Omega_c(3090)^0$		***
$\Omega_c(3120)^0$		***
Ξ_{cc}^+		*
Ξ_{cc}^{++}		***

Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^-	$1/2^+$	***
Ξ_b^0	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6100)^-$	$3/2^-$	***
$\Xi_b(6227)^-$		***
$\Xi_b(6227)^0$		***
Ω_b^-	$1/2^+$	***
$\Omega_b(6316)^-$		*
$\Omega_b(6330)^-$		*
$\Omega_b(6340)^-$		*
$\Omega_b(6350)^-$		*
$P_c(4312)^+$		*
$P_c(4380)^+$		*
$P_c(4440)^+$		*
$P_c(4457)^+$		*

~170 baryons



LIGHT UNFLAVORED
($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
• $a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	
• $\omega_3(167$	

	$I^G(J^{PC})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$X(1750)$	$?^-(1^{--})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(0^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
• $\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
$a_0(1950)$	$1^-(0^{++})$
• $f_2(1950)$	$0^+(2^{++})$
• $a_4(1970)$	$1^-(4^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
$\pi_2(2005)$	$1^-(2^{-+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$X(2370)$	$?^?(?^{??})$
$2510)$	$0^+(6^{++})$

STRANGE
($S = \pm 1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
• $K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
• $K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
• $K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED
($C = \pm 1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)$	$1/2(0^+)$
• $D_1(2420)$	$1/2(1^+)$
• $D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)$	$1/2(2^+)$
$D_0(2550)^0$	$1/2(0^-)$
$D_J^*(2600)^0$	$1/2(1^-)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D_2(2740)^0$	$1/2(2^-)$
• $D_3^*(2750)$	$1/2(3^-)$
$D_1^*(2760)^0$	$1/2(1^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE
($C = \pm 1$, $S = \pm 1$)
(+ possibly non-$q\overline{q}$ states)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
$D_{s0}(2590)^+$	$0(0^-)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
• $D_{s3}^*(2860)^\pm$	$0(3^-)$
$X_0(2900)$	$?(0^+)$
$X_1(2900)$	$?(1^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM
($B = \pm 1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)$	$1/2(2^+)$
$B_J(5840)$	$1/2(?^?)$
• $B_J(5970)$	$1/2(?^?)$

BOTTOM, STRANGE
($B = \pm 1$, $S = \mp 1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$
$B_{sJ}(6063)^0$	$0(?^?)$
$B_{sJ}(6114)^0$	$0(?^?)$

BOTTOM, CHARMED
($B = C = \pm 1$)

• B_c^+	$0(0^-)$
• $B_c(2S)^\pm$	$0(0^-)$

$c\overline{c}$
(+ possibly non-$q\overline{q}$ states)

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
$(1P)$	$0^+(0^{++})$
$(1P)$	$0^+(1^{++})$
$1P)$	$0^-(1^{+-})$
$(1P)$	$0^+(2^{++})$
$2S)$	$0^+(0^{-+})$
$S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$

$c\overline{c}$ continued

	$I^G(J^{PC})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
$X(4630)$	$0^+(?^{?+})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c1}(4685)$	$0^+(1^{++})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\overline{b}$
(+ possibly non-$q\overline{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

OTHER

Further States

~210 mesons

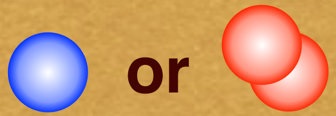
Most hadrons are unstable (above two-hadron threshold)

Contents

- Introduction: structure of unstale hadrons

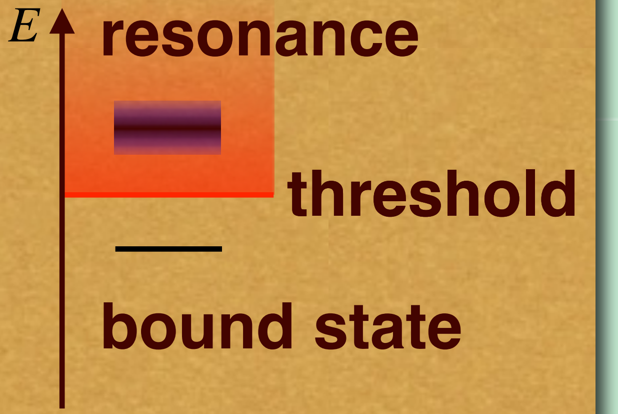
- Compositeness of near-threshold bound states

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Hyodo, PRC90, 055208 (2014);
T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



- Near-threshold resonances

T. Hyodo, PRL111, 132002 (2013);
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

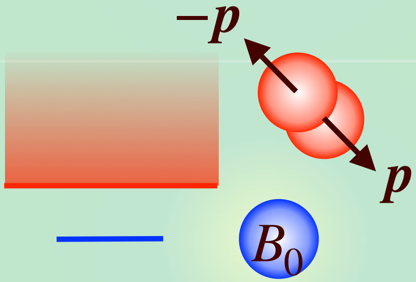


- Summary

Compositeness and elementarity

Compositeness: quantitative measure of internal structure

- Eigenstates of Hamiltonian $H = H_0 + V$



$$H_0|B_0\rangle = \nu_0|B_0\rangle, \quad H_0|\boldsymbol{p}\rangle = \frac{\boldsymbol{p}^2}{2\mu}|\boldsymbol{p}\rangle, \quad H|B\rangle = -B|B\rangle$$

- Normalization of $|B\rangle$ + completeness of free eigenstates

$$\langle B|B\rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\boldsymbol{p}\rangle\langle\boldsymbol{p}| = Q + P$$

- Definition

$$1 = Z + X, \quad Z \equiv |\langle B_0|B\rangle|^2, \quad X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\boldsymbol{p}|B\rangle|^2 = \langle B|P|B\rangle$$

“elementarity” compositeness

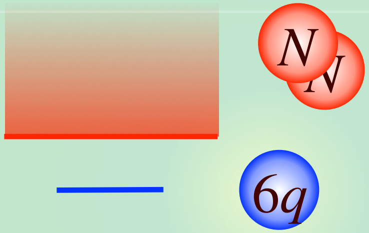
- Z, X: real and nonnegative —> interpreted as probability

Weak-binding relation

Compositeness X of **stable** bound state: deuteron

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



$$|d\rangle = \sqrt{X}\,|NN\rangle + \sqrt{Z}\,|6q\rangle, \quad X + Z = 1, \quad 0 \leq X \leq 1$$

range of interaction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

scattering length

radius of bound state

- **Shallow bound state ($R \gg R_{\text{typ}}$): $X \leftarrow$ observables (a_0, B)**
- $X = 1$ **for** $B = 0$ **<— low-energy universality** $a_0 = R$

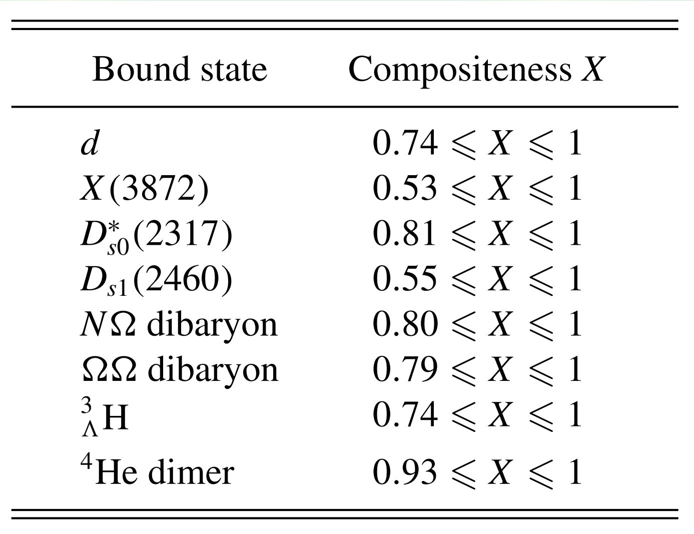
  T. Hyodo, PRC90, 055208 (2014)
- $X < 1$ **gives violation of universality by coupling to** $|6q\rangle$

Near-threshold bound states

Application to physical systems

T. Kinugawa, T. Hyodo, PRC 106, 015205 (2022)

- X of hadrons, nuclei, and atoms
- Uncertainty from $\mathcal{O}(R_{\mathrm{typ}}/R)$ term
- $X \geq 0.5$ in all cases studied



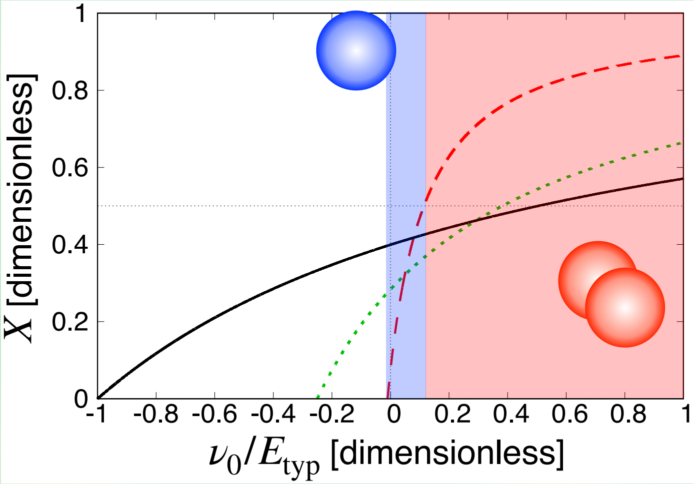
Bound state	Compositeness X
d	$0.74 \leqslant X \leqslant 1$
$X(3872)$	$0.53 \leqslant X \leqslant 1$
$D^{*}_{s0}(2317)$	$0.81 \leqslant X \leqslant 1$
$D_{s1}(2460)$	$0.55 \leqslant X \leqslant 1$
$N\Omega$ dibaryon	$0.80 \leqslant X \leqslant 1$
$\Omega\Omega$ dibaryon	$0.79 \leqslant X \leqslant 1$
${}^{3}_{\Lambda}\mathrm{H}$	$0.74 \leqslant X \leqslant 1$
${}^{4}\mathrm{He}$ dimer	$0.93 \leqslant X \leqslant 1$

Analysis with effective field theory

T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)

- Shallow bound state with $X \sim 0$

  <— Only with fine tuning = unlikely



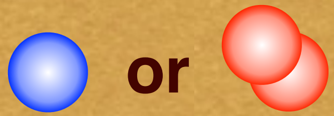
Near-threshold bound states are mostly composite

Contents

- Introduction: structure of unstale hadrons

- Compositeness of near-threshold bound states

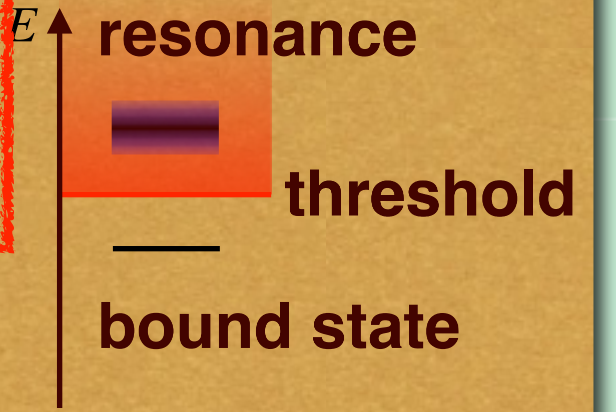
  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
  T. Hyodo, PRC90, 055208 (2014);
  T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)



- Near-threshold resonances

  T. Hyodo, PRL111, 132002 (2013);
  T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

- Summary



Resonances in effective range expansion

Effective range expansion (ERE): valid for small k

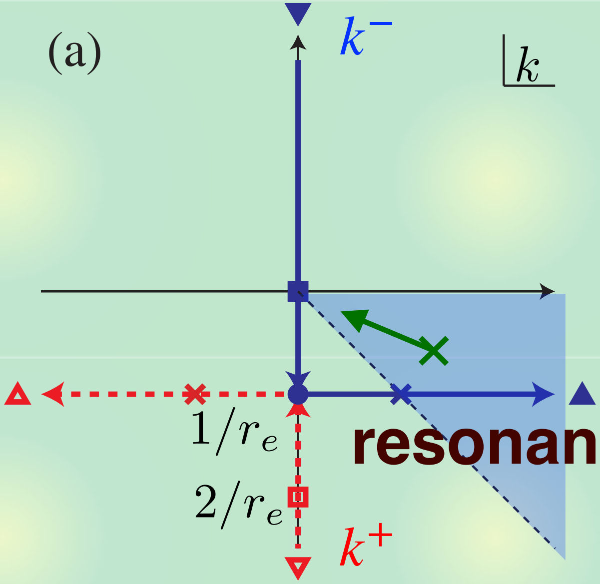
$$f(k)=\left[-\frac{1}{a_0}+\frac{r_e}{2}k^2-ik\right]^{-1}$$

- **Pole positions $k^{\pm}$ <—> (a_0, r_e)**

T. Hyodo, PRL111, 132002 (2013);
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

$$k^{\pm}=\frac{i}{r_e}\pm\frac{1}{r_e}\sqrt{\frac{2r_e}{a_0}-1+i0^+}$$

bound state



virtual state

Resonance solution ($r_e<0$)

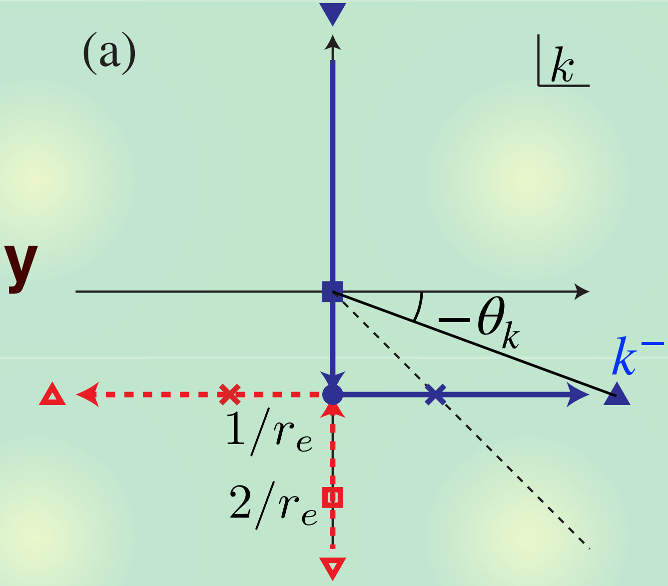
$$\frac{1}{|r_e|}\sqrt{\frac{2r_e}{a_0}-1}\geq\frac{1}{|r_e|},\quad\Rightarrow\quad\frac{r_e}{a_0}\geq 1,\quad\Rightarrow\quad|a_0|\leq|r_e|$$

- **Resonance with $|k^-|\to 0$: not only $|a_0|\to\infty$ but also $|r_e|\to\infty$**
- **Energy $E_R=M_R-i\frac{\Gamma_R}{2}$ <—> (a_0, r_e)**

Compositeness of resonances

Compositeness: pure imaginary <— weak-binding relation

$$X=\sqrt{\frac{1}{1-\frac{2r_e}{a_0}}}=-i\tan(\theta_k),\quad k^-=|k^-|e^{i\theta_k}$$



Resonance state: complex eigenenergy

$$H|R\rangle=E_R|R\rangle,\quad E_R=M_R-i\frac{\Gamma_R}{2}\in\mathbb{C}$$

- **Normalization by Gamow vector**

$$\langle R|H=\langle R|E_R^*,\quad \langle\tilde{R}|H=\langle\tilde{R}|E_R$$

$$\langle R|R\rangle\to\infty,\quad \langle\tilde{R}|R\rangle=1$$

- **Complex X: probability?**

$$X\equiv\langle\tilde{R}|P|R\rangle=\int\frac{d\boldsymbol{p}}{(2\pi)^3}\langle\tilde{R}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|R\rangle\in\mathbb{C}$$

Uncertain nature of resonance

Resonance contribution in a prepared state $1 = \langle \psi | \psi \rangle$

T. Berggren, PLB33, 547 (1970)

- Completeness relation with contour deformation

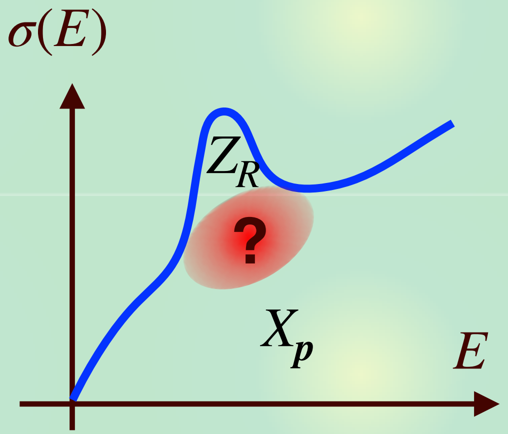
T. Berggren, NPA109, 547 (1968)

$$1 = |R\rangle\langle \tilde{R}| + \int_C \frac{d\boldsymbol{p}}{(2\pi)^3} |\boldsymbol{p} \text{ full}\rangle\langle \boldsymbol{p} \text{ full}|$$

$$1 = Z_R + X_{\boldsymbol{p}}, \quad Z_R = \langle \psi | R \rangle \langle \tilde{R} | \psi \rangle \in \mathbb{C}$$

Introduce three probabilities

- certainly find $|R\rangle$: a
- certainly find not $|R\rangle$: b
- uncertain: c



$$a + b + c = 1, \quad a + c = |Z_R|, \quad b + c = |1 - Z_R|$$

New interpretation scheme

Decomposition of resonance wave function $\langle \tilde{R} | R \rangle = 1$

T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

$$1 = |B_0\rangle\langle B_0| + \int \frac{d\boldsymbol{p}}{(2\pi)^3} |\boldsymbol{p}\rangle\langle \boldsymbol{p}|$$

$$1 = Z + X, \quad X = \int \frac{d\boldsymbol{p}}{(2\pi)^3} \langle \tilde{R} | \boldsymbol{p}\rangle\langle \boldsymbol{p} | R\rangle \in \mathbb{C}$$

Introduce three probabilities

- certainly find composite : $\mathcal{X}$
- certainly find elementary : $\mathcal{Z}$
- uncertain: $\mathcal{Y}$

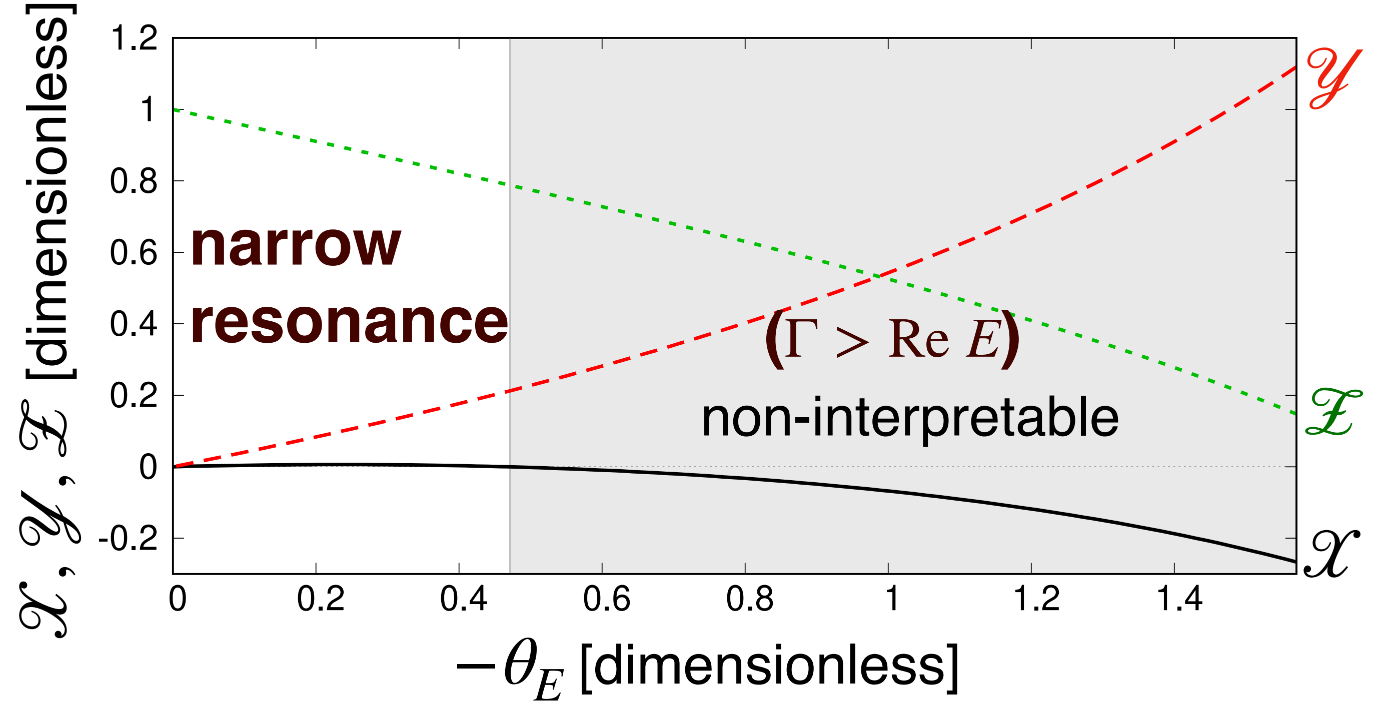
$$\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1, \quad \mathcal{X} + \alpha\mathcal{Y} = |X|, \quad \mathcal{Z} + \alpha\mathcal{Y} = |1 - X| = |Z|$$

- α: parameter to control degree of uncertainty

Compositeness of resonances

$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ as functions of argument of eigenenergy

$$E_R = |E_R| e^{i\theta_E}$$



- Large elementarity: $\mathscr{Z} \gtrsim 0.8$

<— Narrow s-wave state is Feshbach resonance

Near-threshold resonances are not composite dominant

Summary

- **Compositeness X: probability of finding**

- **Bound state exactly at threshold**

  T. Hyodo, PRC90, 055208 (2014);

  \- completely composite $X = 1$

- **Near threshold bound states**

  T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024)

  \- in general, composite $X \sim 1$

- **Near-threshold resonances**

  T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

  \- non-composite, $\mathcal{X} \lesssim 0.2$