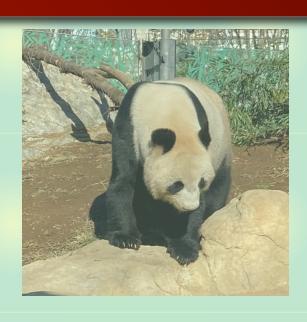
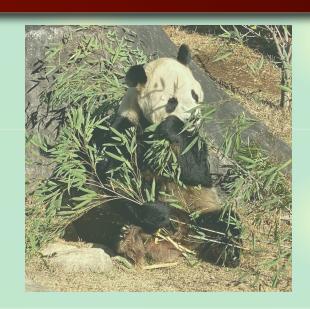
Study of hadron interactions and compositeness





Tetsuo Hyodo

Tokyo Metropolitan Univ.

Contents



Introduction — Exotic hadrons



Hadron interactions — Femtoscopy



Applications

- K^-p correlations for $\Lambda(1405)$

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

- Compositeness of $\Lambda(1405)$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)



Summary and future prospects

Observed hadrons (2022)

Particle Data Group (PDG) 2022 edition

http://pdg.lbl.gov/

р	1/2+ ****	∆(1232)	3/2+ ****	Σ^{+}	1/2+ ****	Λ_c^+ 1	1/2+ ****	$A_b^0 = 1/2$	2+ ***
n	1/2+ ****	$\Delta(1600)$	3/2+ ****	Σ^0	1/2+ ****		1/2 ***	$\Lambda_b(5912)^0$ 1/3	
N(1440)	1/2+ ****	$\Delta(1620)$	1/2- ****	Σ-	1/2+ ****		3/2- ***	$A_b(5920)^0$ 3/3	_
N(1520)	3/2- ****	Δ (1700)	3/2- ****	Σ(1385)	3/2+ ****	$\Lambda_c(2765)^+$	*		2+ ***
N(1535)	1/2- ****	Δ (1750)	1/2+ *	$\Sigma(1580)$	3/2- *		3/2+ ***	$A_b(6152)^0$ 5/3	
N(1650)	1/2- ****	Δ (1900)	1/2- ***	$\Sigma(1620)$	1/2- *		5/2 ⁺ ***		2+ ***
N(1675)	5/2- ****	$\Delta(1905)$	5/2 ⁺ ****	$\Sigma(1660)$	1/2+ ***		3/2 ⁻ ***	Σ_b^* 3/2	2+ ***
N(1680)	5/2 ⁺ ****	$\Delta(1910)$	1/2+ ****	$\Sigma(1670)$	3/2- ****		L/2 ⁺ ****	$\Sigma_b(6097)^+$	***
N(1700)	3/2- ***	$\Delta(1920)$	3/2+ ***	$\Sigma(1750)$	1/2- ***		3/2 ⁺ ***	$\Sigma_b(6097)^-$	***
N(1710)	1/2+ ****	Δ (1930)	5/2 ⁻ ***	$\Sigma(1775)$	5/2- ****	$\Sigma_c(2800)$	***		2+ ***
N(1720)	3/2+ ****	$\Delta(1940)$	3/2- **	$\Sigma(1780)$	3/2+ *		1/2+ ***		2+ ***
N(1860)	5/2 ⁺ **	$\Delta(1950)$	7/2+ ****	$\Sigma(1880)$	1/2+ **		L/2 ⁺ ****	=b 1/.	2+ ***
N(1875)	3/2- ***	$\Delta(2000)$	5/2 ⁺ **	$\Sigma(1900)$	1/2- **	-c 1 ='+ 1	1/2 ⁺ ***		
N(1880)	1/2+ ***	$\Delta(2150)$	1/2- *	$\Sigma(1910)$	3/2- ***	=c 1			-
N(1895)	1/2- ****	$\Delta(2200)$	7/2 ⁻ ***	$\Sigma(1915)$	5/2 ⁺ ****] = č	-/-	J /	- 1
N(1900)	3/2+ ****	$\Delta(2300)$	9/2+ **	$\Sigma(1940)$	3/2+ *		J/ -	$\Xi_b(6100)^-$ 3/2	2- ***
N(1990)	7/2 ⁺ **	$\Delta(2350)$	5/2 ⁻ *	$\Sigma(2010)$	3/2- *	٠, ,	1/2- ***	$\Xi_b(6227)^-$	***
N(2000)	5/2 ⁺ **	$\Delta(2390)$	7/2 ⁺ *	$\Sigma(2030)$	7/2 ⁺ ****	~ /	3/2- ***	$\Xi_b(6227)^0$	
N(2040)	3/2+ *	$\Delta(2400)$	9/2 ⁻ **	$\Sigma(2070)$	5/2+ *	$\Xi_c(2923)$	**		2+ ***
N(2060)	5/2 ⁻ ***	$\Delta(2420)$	11/2+ ****	$\Sigma(2080)$	3/2+ *	$\Xi_c(2930)$		Ω_b (6316)-	
N(2100)	1/2+ ***	$\Delta(2750)$	13/2- **	$\Sigma(2100)$	7/2- *	. ,	1/2+ ***	Ω_b (6330)-	*
N(2120)	3/2- ***	$\Delta(2950)$	15/2+ **	$\Sigma(2110)$	1/2- *	$\Xi_c(3055)$	***	Ω_b (6340)-	*
N(2120)	7/2- ****	<u> </u> (2330)	13/2	$\Sigma(2230)$	3/2+ *	$\Xi_c(3080)$	***	Ω_b (6350) $^-$	*
N(2220)	9/2+ ****	Λ	1/2+ ****	$\Sigma(2250)$	**	$\Xi_c(3123)$		D (4010)±	*
N(2250)	9/2- ****	/(1380)	1/2- **	$\Sigma(2455)$	*		1/2+ ***	$P_c(4312)^+$	
N(2300)	1/2+ **	Λ(1405)	1/2- ****	$\Sigma(2620)$	*		3/2+ ***	$P_c(4380)^+$	*
N(2570)	5/2- **	Λ(1520)	3/2- ***	$\Sigma(3000)$	*	$\Omega_c(3000)^0$	***	$P_c(4440)^+$	*
N(2510)	11/2- ***	Λ(1600)	1/2+ ****	$\Sigma(3170)$	*	$\Omega_{c}(3050)^{0}$	***	$P_c(4457)^+$	*
	13/2+ **	Λ(1670)	1/2- ****	2(3170)		$\Omega_c(3065)^0$	***		
N(2700)	13/2	Λ(1690)	3/2- ****	<u>=</u> 0	1/2+ ****	$\Omega_c(3090)^0$	***		
		Λ(1710)	1/2+ *	I <u>=</u> −	1/2+ ****	$\Omega_c(3120)^0$	***		
		Λ(1710) Λ(1800)	1/2 ***	<i>=</i> (1530)	3/2+ ****				
		Λ(1810)	1/2+ ***	$\Xi(1620)$	3/2	Ξ_{cc}^{+}	*		
		Λ(1820)	5/2+ ****	$\Xi(1690)$	***	\equiv_{cc}^{++}	***		
		Λ(1830)	5/2 ⁻ ****	$\Xi(1820)$	3/2- ***				
		Λ(1890)	3/2+ ****	$\Xi(1950)$	3/2 ***				
		Λ(1890) Λ(2000)	1/2- *	$\Xi(2030)$	≥ ½? ***				
		Λ(2000) Λ(2050)	3/2- *	$\Xi(2120)$	≤ <u>2</u> **				
		Λ(2070)	3/2+ *	$\Xi(2120)$ $\Xi(2250)$	**				
		Λ(2070) Λ(2080)	5/2 ⁻ *	. ,	**				
		Λ(2085)	7/2 ⁺ **	$\Xi(2370)$ $\Xi(2500)$	*				
1		Λ(2100)	7/2- ****	[_(2000)	**				
		Λ(2100) Λ(2110)	5/2 ⁺ ***	Ω^{-}	3/2+ ****				
		/ (ZIIO)	3/2	1 34	3/2	1			
		1 سا	I QN	hs	aryc	ne			
		~	I UU	UC	11 Y C	<i>/</i> 113			_

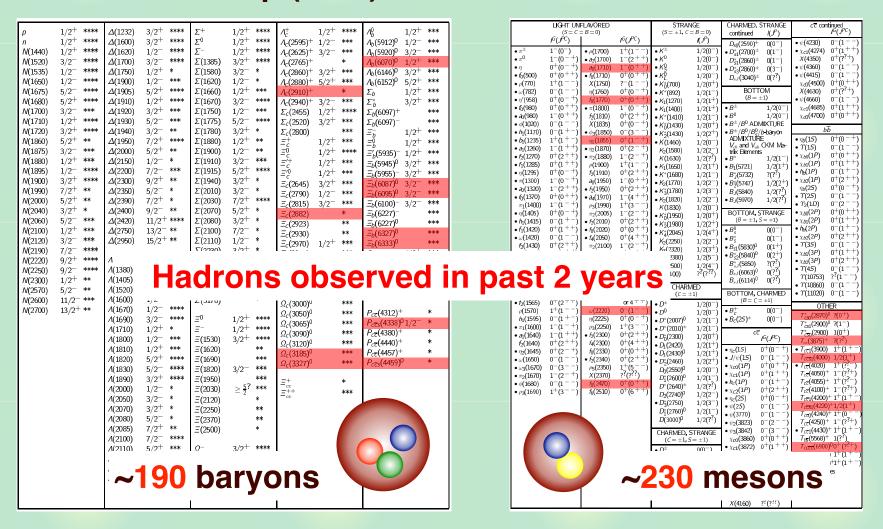
LIGHT UNFLA		STRANGE		CHARMED, STRANGE		c continued	
(S = C = B) (S = C = B)	(S = ±1, C =	:B=0) I(J ^P)	$(C = \pm 1, S = \pm 1)$ $(+ \text{ possibly non-}q\overline{q})$			$I^{G}(\tilde{J}^{PC})$	
	I ^G (J ^{PC})	• K [±]		1()		 ψ₂(3823) ψ₂(3823) 	0-(2)
	$\pi_2(1670)$ 1 ⁻ (2 ⁻⁺) $\phi(1680)$ 0 ⁻ (1)	• K ⁰	1/2(0 ⁻) 1/2(0 ⁻)	• D _s 0(0	<u>-)</u>	 ψ₃(3842) χ_{c0}(3860) 	0-(3)
	ρ _β (1690) 1 ⁺ (3 ⁻ ⁻)	• K&	1/2(0)	• D _s ^{*±} 0(?		• χ _{c1} (3872)	0+(1++)
	ρ(1700) 1+(1)	• K9	1/2(0-)	• D _{\$0} (2317)± 0(0		• Z _c (3900)	1+(1+-)
72 / 13 1	a ₂ (1700) 1 ⁻ (2 ⁺ +)	• K*(700)	1/2(0+)	• D _{s1} (2460) [±] 0(1		• χ _{c0} (3915)	0+(0++)
• ω(782) 0 ⁻ (1 ⁻ -) •	f ₀ (1710) 0+(0++)	• K*(892)	1/2(1-)	 D_{S1}(2536)[±] 0(1 		 χ_{C2}(3930) 	0+(2++)
	X(1750) ?-(1)	• K ₁ (1270)	1/2(1+)	• D ₅₂ (2573) 0(2	+)	X(3940)	??(???)
• f ₀ (980) 0 ⁺ (0 ⁺ +)	$\eta(1760) 0^{+}(0^{-})$	 K₁(1400) 	1/2(1+)	$D_{s0}(2590)^{+}$ 0(0		• X(4020) [±]	1+(??-)
	π(1800) 1 ⁻ (0 ^{- +})	• K*(1410)	1/2(1-)	• D _{S1} (2700)± 0(1		• ψ(4040)	0-(1) 1-(?-+)
• φ(1020) 0 ⁻ (1 ⁻ ⁻) • h ₁ (1170) 0 ⁻ (1 ⁺ ⁻)	f ₂ (1810) 0 ⁺ (2 ⁺ +) X(1835) ? ² (0 ⁻ +)	• K ₀ *(1430)	1/2(0+)	$D_{s1}^*(2860)^{\pm}$ 0(1		X(4050) [±] X(4055) [±]	1+(??-)
	X(1835) ? ^f (0 - +) • \(\phi_3(1850) \) 0 - (3)	• K ₂ (1430)	1/2(2+)	• D ₅₃ (2860) [±] 0(3		X(4100)±	1-(???)
	η ₂ (1870) 0 ⁺ (2 ⁻ ⁺)	• K(1460) K₂(1580)	1/2(0 ⁻) 1/2(2 ⁻)	$X_0(2900)$?(0 $X_1(2900)$?(1		• χ _{C1} (4140)	0+(1++)
	π ₂ (1880) 1 ⁻ (2 ⁻⁺)	K(1630)	1/2(2)	$D_{sJ}(3040)^{\pm}$ 0(?		• ψ(4160)	0 - (1)
• f ₁ (1285) 0 ⁺ (1 ⁺ +)	ρ(1900) 1 ⁺ (1)	• K ₁ (1650)	1/2(1+)		,	X(4160)	??(???)
• η(1295) 0 ⁺ (0 ⁻⁺)	f ₂ (1910) 0 ⁺ (2 ^{+ +})	• K*(1680)	1/2(1-)	BOTTOM		$Z_{C}(4200)$	1+(1+-)
• \pi(1300) 1^-(0 - +)	$a_0(1950)$ $1^-(0^{++})$	• K ₂ (1770)	1/2(2-)	(B = ±1)		 ψ(4230) 	0-(1)
• a ₂ (1320) 1 ⁻ (2 + +) •	f ₂ (1950) 0 ⁺ (2 ⁺ ⁺)	• K ₃ (1780)	1/2(3-)	• B [±] 1/2	2(0-)	R _{c0} (4240)	1 ⁺ (0) 1 ⁻ (? ^{?+})
	a ₄ (1970) 1 ⁻ (4 ⁺⁺)	 K₂(1820) 	1/2(2-)	• B ⁰ 1/2 • B [±] /B ⁰ ADMIXTU	(0_)	X(4250)±	0+(1++)
• π ₁ (1400) 1 ⁻ (1 ⁻ +) • η(1405) 0 ⁺ (0 ⁻ +)	$\rho_3(1990)$ 1 ⁺ (3 ⁻) $\pi_2(2005)$ 1 ⁻ (2 ⁻ +)	K(1830)	1/2(0-)	• B [±] /B ⁰ /B ⁰ /b-ban		• χ _{C1} (4274) Χ(4350)	0+(??+)
	f ₂ (2010) 0 ⁺ (2 ⁺ +)	K ₀ (1950)	1/2(0+)	ADMIXTURE	yon	• ψ(4360)	0-(1)
• f ₁ (1420) 0+(1++)	f ₀ (2020) 0+(0++)	• K ₂ (1980)	1/2(2+)	V_{cb} and V_{ub} CKM	Ma-	• ψ(4415)	0-(1)
	f ₄ (2050) 0 ⁺ (4 ⁺ ⁺)	• K ₄ (2045)	1/2(4+)	trix Elements • B* 1/2	χ ₁ -)	• Z _c (4430)	1+(1+-)
$f_2(1430)$ $0^+(2^{++})$	π ₂ (2100) 1 ⁻ (2 ⁻⁺)	K ₂ (2250) K ₃ (2320)	1/2(2 ⁻) 1/2(3 ⁺)		X(1 ⁺)	χ _{c0} (4500)	0+(0++)
• a ₀ (1450) 1 ⁻ (0 ^{+ +})	$f_0(2100)$ $0^+(0^{++})$	K ₅ (2380)	1/2(5-)	B*(5732) ?(?		X(4630)	0+(??+)
 ρ(1450) 1⁺(1 [−] [−]) 	$f_2(2150)$ $0^+(2^{++})$	K ₄ (2500)	1/2(4-)	• B ₂ *(5747) 1/2	(2 ⁺)	 ψ(4660) 	0-(1)
• η(1475) 0 ⁺ (0 ⁻ +)	ρ(2150) 1 ⁺ (1)	K(3100)	,(,;;),	B _J (5840) 1/2	2(??)	χ _{c1} (4685)	0+(1++)
• f ₀ (1500) 0 ⁺ (0 ⁺ +) f ₁ (1510) 0 ⁺ (1 ⁺ +)	φ(2170) 0 ⁻ (1 ⁻) f ₀ (2200) 0 ⁺ (0 ⁺ +)			• B _J (5970) 1/2	2(??)	$\chi_{C0}(4700)$	0+(0++)
• f' ₂ (1525) 0 ⁺ (2 ⁺ +)	f _J (2220) 0 ⁺ (2 ⁺⁺	CHARMED $(C = \pm 1)$		BOTTOM, STRANGE		ЬЪ	
f ₂ (1565) 0 ⁺ (2 ⁺ +)	or 4 ++)	• D [±]	1/2(0-)	$(B = \pm 1, S = \mp 1)$		(+ possibly no	
ρ(1570) 1+(1)	$\eta(2225)$ 0+(0 - +)	• D ⁰	1/2(0-)	• B _S ⁰ 0(0	·-)	 η_b(1S) 	0+(0-+)
$h_1(1595) 0^-(1^{+-})$	$\rho_3(2250)$ 1 ⁺ (3)	• D*(2007)0	1/2(1-)	• B _s * 0(1	-)	 Υ(15) 	0-(1)
	f ₂ (2300) 0 ⁺ (2 ⁺⁺)	• D*(2010)±	1/2(1-)	X(5568)± ?(?		 χ_{b0}(1P) 	0+(0++)
• a ₁ (1640) 1 ⁻ (1 + +)	f ₄ (2300) 0 ⁺ (4 ^{+ +})	• D ₀ *(2300)	1/2(0+)	• B _{S1} (5830) ⁰ 0(1		• χ _{b1} (1P)	0+(1++)
f ₂ (1640) 0 ⁺ (2 + +)	f ₀ (2330) 0 ⁺ (0 ⁺ +)	• D ₁ (2420)	1/2(1+)	$\bullet B_{S2}^* (5840)^0 = 0(2$	(+)	 h_b(1P) χ_{b2}(1P) 	0 ⁻ (1 ⁺ -) 0 ⁺ (2 ⁺ +)
	ρ ₅ (2340) 0 ⁺ (2 ⁺ ⁺) ρ ₅ (2350) 1 ⁺ (5 ⁻ ⁻)	 D₁(2430)⁰ 	1/2(1+)	$B_{s,l}^{*}(5850)$?(?	;)	$\eta_b(2S)$	0+(0-+)
• ω(1650) 0 ⁻ (1 ⁻ ⁻) • ω ₃ (1670) 0 ⁻ (3 ⁻ ⁻)	ρ ₅ (2350) 1 ⁺ (5) X(2370) ? [?] (? ^{??})	• D ₂ (2460)	1/2(2+)	B _{sJ} (6063) ⁰ 0(?	?)	• Υ(2S)	0-(1)
123(1010) 0 (3)	f ₆ (2510) 0 ⁺ (6 ⁺ ⁺)	D ₀ (2550) ⁰	1/2(0 ⁻) 1/2(1 ⁻)	B _{s.J} (6114) ⁰ 0(?	')	 Υ₂(1D) 	0-(2)
1	5/ - (- /	D*(2600) ⁰ D*(2640)±	1/2(1)	BOTTOM, CHARN	ΛED	 χ_{b0}(2P) 	0+(0++)
1		$D_2(2740)^0$	1/2(!*)	$(B = C = \pm 1)$		 √b1(2P) 	0+(1++)
1		• D ₃ (2750)	1/2(3-)	• B _c ⁺ 0(0		• h _b (2P)	0-(1+-)
		D ₁ (2760) ⁰	1/2(1-)	• $B_c(2S)^{\pm}$ 0(0	_)	• χ _{b2} (2P)	0+(2++)
		D(3000)0	1/2(??)	cc	-	 Υ(3S) χ_{b1}(3P) 	0 ⁻ (1 ⁻ -) 0 ⁺ (1 ⁺ +)
			,	(+ possibly non–q7]s	states)	 χ_{b1}(3P) χ_{b2}(3P) 	0+(2++)
				• η _C (1S) 0 ⁺ (0	- + ₎	 Υ(45) 	0-(1)
					j		1+(1+-)
							ι+(1+-)
							??(1)
		าวก	n	1esc		ne)-(1)
	, ~ <u></u>	LZU	' ▮ ▮	1626	JI	19)-(1)
	_						R
1							
		i		 ψ(3770) 0[−](1)	Further Sta	tes

All ~ 400 hadrons basically emerge as qqq or $q\bar{q}$

Observed hadrons (2024)

Particle Data Group (PDG) 2024 edition

http://pdg.lbl.gov/



New states are continuously discovered

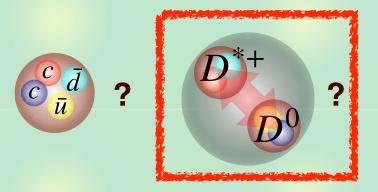
Exotic hadrons

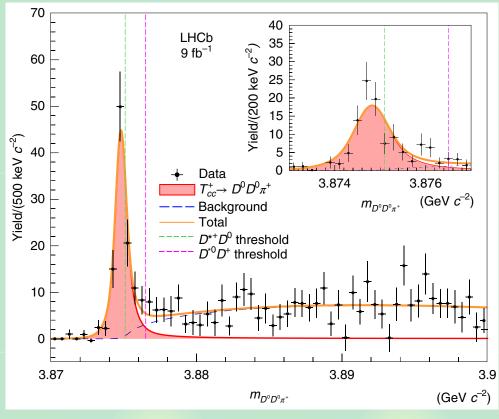
Observation of tetraquark T_{cc}

LHCb collaboration, Nature Phys. 18, 7, 751 (2022); Nature Commun. 13, 1, 3351 (2022)



- Quark content ~ ccūd̄
- Internal structure?





Hadronic molecules <— hadron interactions

F. K. Guo, et al., Rev. Mod. Phys. 90, 072501 (2018)

Contents



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Applications

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Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

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Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)



Summary and future prospects

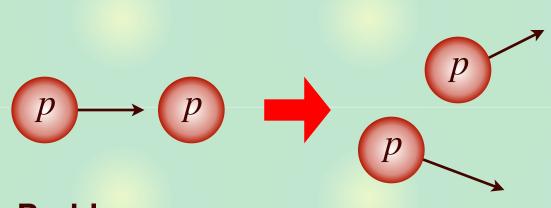
Hadron interactions — Femtoscopy

Study of hadron interactions

Traditional methods: scattering experiments

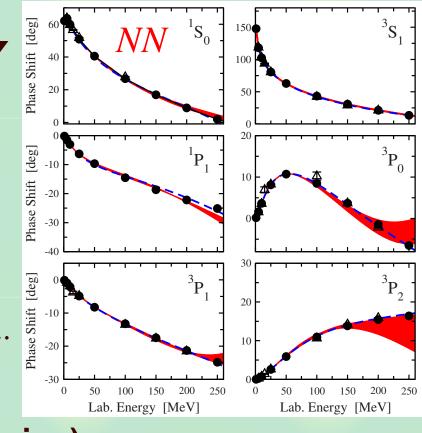
- differential cross sections -> phase shift

E. Epelbaum, H.W. Hammer, U.-G Meißner RMP 81, 1773 (2009)



Problems

- Stable beam/target particles
- Limited channels: NN, πN , KN, $\bar{K}N$, \cdots
- Heavy (c,b) hadrons: impossible
- Limited statistics (low-energy region)



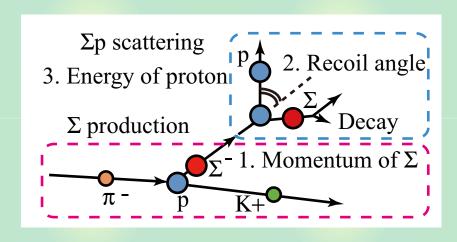
Hadron interactions — Femtoscopy

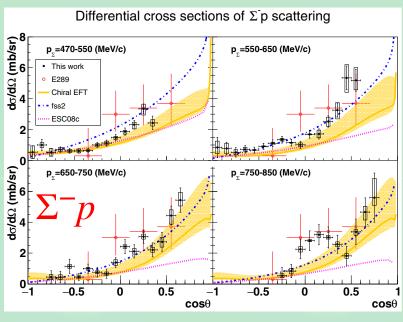
New developments

$\Sigma^{-}p$ scattering (J-PARC E40)

K. Miwa, et al., J-PARC E40, PRC 104, 045204 (2021); PRL 128, 072501 (2022)

- high intencity π beam

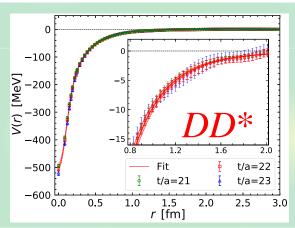




Lattice QCD (HAL QCD)

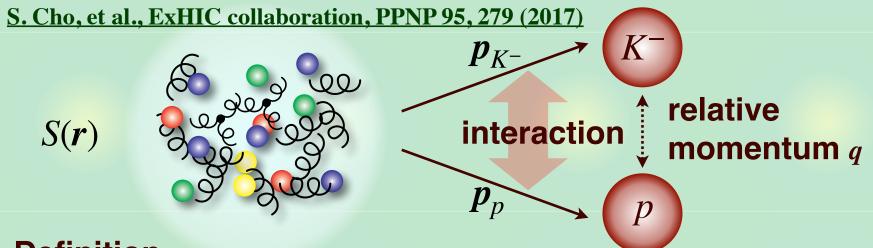
Y. Liu, et al., HAL QCD, PRL 131, 161901 (2023)

- First principle QCD calculation
- Hadron-hadron potential for T_{cc}



Femtoscopy: Correlation function

High-energy collision: chaotic source S(r) of hadron emission



- Definition

$$C(q) = \frac{N_{K^-p}(p_{K^-}, p_p)}{N_{K^-}(p_{K^-})N_p(p_p)}$$
 (= 1 in the absence of FSI/QS)

- Theory (Koonin-Pratt formula)

S.E. Koonin, PLB 70, 43 (1977); S. Pratt, PRD 33, 1314 (1986)

$$C(\boldsymbol{q}) \simeq \int d^3 \boldsymbol{r} \, S(\boldsymbol{r}) |\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2$$

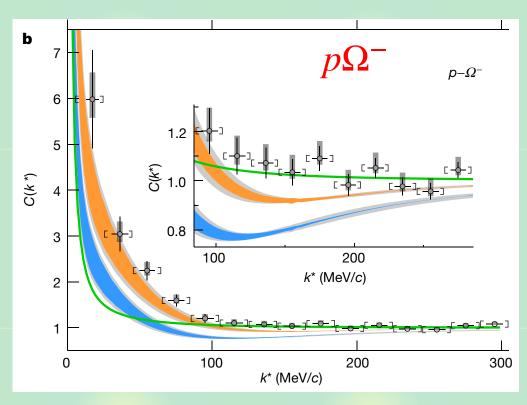
Source function S(r) < -> wave function $\Psi_q^{(-)}(r)$ (interaction)

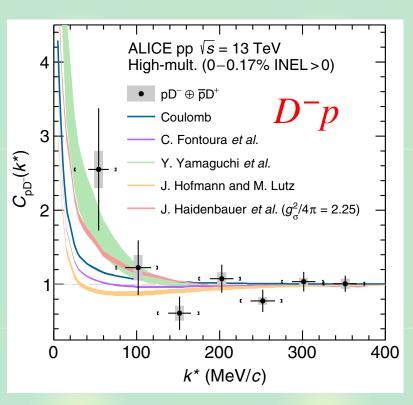
Experimental data in charm sector

Correlation functions observed by ALICE@LHC

ALICE collaboration, Nature 588, 232 (2020);

ALICE collaboration, PRD 106, 052010 (2022)





 $\Omega^- \sim sss$: strangeness S=-3, $D^- \sim \bar{c}d$: charm C=-1

Almost impossible in scattering experiments

Contents



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Applications

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- Compositeness of $\Lambda(1405)$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

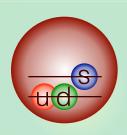


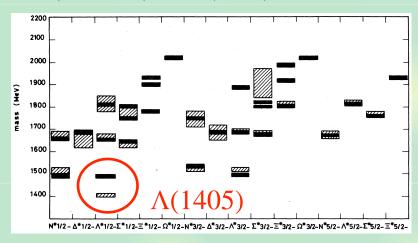
Summary and future prospects

$\Lambda(1405)$ and $\bar{K}N$ scattering

$\Lambda(1405)$ does not fit in standard picture —> exotic candidate

N. Isgur and G. Karl, PRD18, 4187 (1978)



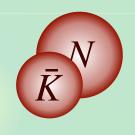


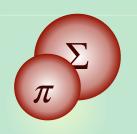
: theory

: experiment

Resonance in coupled-channel scattering

- Coupling to MB: chiral SU(3) dynamics







T. Hyodo, W. Weise, arXiv:2202.06181 [nucl-th] (Handbook of Nuclear Physics)

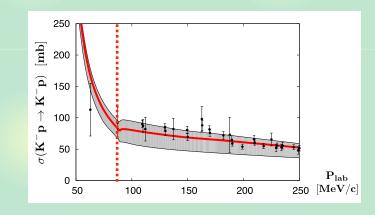
Applications: K^-p correlations for $\Lambda(1405)$

Scattering experiments and femtoscopy

K[−]*p* scattering data

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

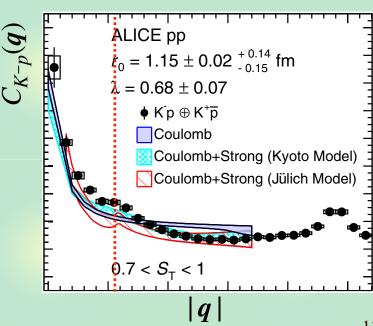
- Old bubble chamber experiments
- Limited statistics (low-energy)



Femtoscopy: correlation function

ALICE collaboration, PRL 124, 092301 (2020)

- Excellent precision (\bar{K}^0n cusp)
- Data below \bar{K}^0n threshold



Important constraints on $\Lambda(1405)$

Applications: K^-p correlations for $\Lambda(1405)$

Coupled-channel effects

Coupled-channel Schrödinger equation (s-wave)

$$\begin{pmatrix} \frac{-1}{2\mu_1}\frac{d^2}{dr^2} + V_{11}(r) + V_{C}(r) & V_{12}(r) & \cdots \\ V_{21}(r) & \frac{-1}{2\mu_2}\frac{d^2}{dr^2} + V_{22}(r) + \Delta_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0n}(r) \\ \vdots \\ \vdots \end{pmatrix} = E \begin{pmatrix} \psi_{K^-p}(r) \\ \psi_{\bar{K}^0n}(r) \\ \vdots \\ \vdots \end{pmatrix}$$
 Coulomb threshold energy difference

Coupled-channel Koonin-Pratt formula

R. Lednicky, V.V. Lyuboshitz, V.L. Lyuboshitz, Phys. Atom. Nucl. 61, 2950 (1998);

J. Haidenbauer, NPA 981, 1 (2019);

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)

$$C_{K^{-}p}(\boldsymbol{q}) \simeq \int d^3 \boldsymbol{r} \, S_{K^{-}p}(\boldsymbol{r}) \, |\Psi_{K^{-}p,\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2 + \sum_{i \neq K^{-}p} \boldsymbol{\omega_i} \int d^3 \boldsymbol{r} \, S_i(\boldsymbol{r}) \, |\Psi_{i,\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2$$

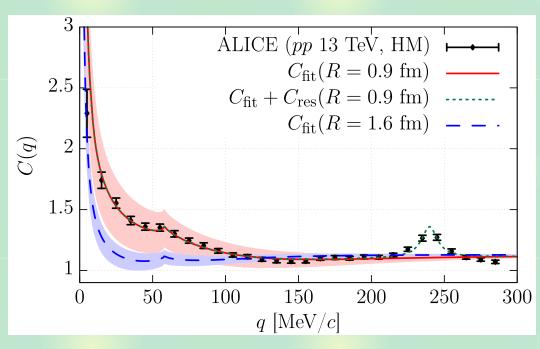
- Transition from $\bar{K}^0 n, \pi^+ \Sigma^-, \pi^0 \Sigma^0, \pi^- \Sigma^+, \pi^0 \Lambda$
- ω_i : weight of channel *i* source relative to K^-p

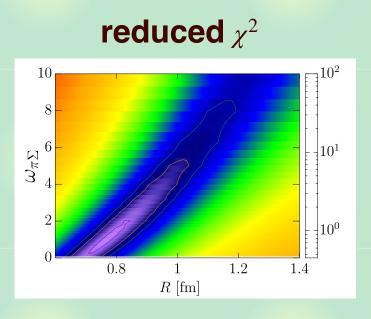
Correlation from chiral SU(3) dynamics

Wave function $\Psi_{i,q}^{(-)}(r)$: Kyoto $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential

K. Miyahara, T. Hyodo, W. Weise, PRC98, 025201 (2018)

- Source function S(r): gaussian, $R \sim 1$ fm from K^+p data
- Source weight $\omega_{\pi\Sigma} \sim 2$ by simple statistical model estimate





Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise, PRL124, 132501 (2020)

Correlation is well reproduced by chiral SU(3) potential

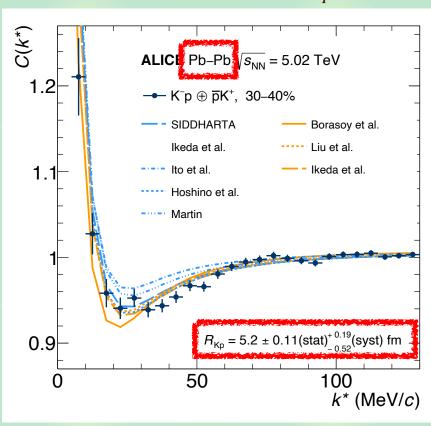
Applications: K^-p correlations for $\Lambda(1405)$

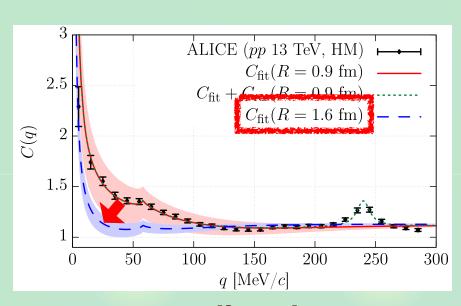
Large source case

New data with Pb-Pb collisions at 5.02 TeV

ALICE collaboration, PLB 822, 136708 (2021)

- Scattering length $a_{K^-p} = -0.91 + 0.92i$ fm





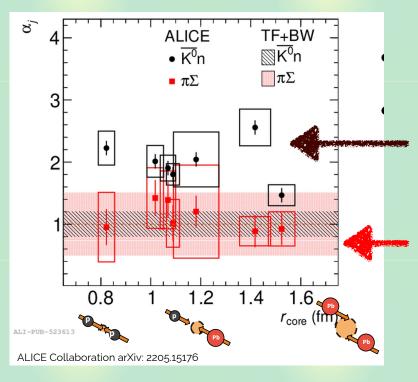
Correlation is suppressed at larger R, as predicted

Systematic study of source size dependence

Correlations in pp, p-Pb, Pb-Pb by Kyoto $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential

ALICE collaboration, EPJC 83, 340 (2023)

$$C_{K^{-}p}(\boldsymbol{q}) \simeq \int d^3 \boldsymbol{r} \, S_{K^{-}p}(\boldsymbol{r}) \, |\Psi_{K^{-}p,\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2 + \sum_{i \neq K^{-}p} \alpha_i \omega_i \int d^3 \boldsymbol{r} \, S_i(\boldsymbol{r}) \, |\Psi_{i,\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2$$



 ω_i : expected weight by Thermal Fist + Blast Wave

enhancement needed to explain data

expected weight is OK

More strength is needed in the \bar{K}^0n channel

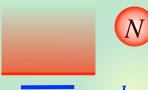
Weak-binding relation for stable states

Compositeness *X* of stable bound state

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$|d\rangle = \sqrt{X} |NN\rangle + \sqrt{Z} |\text{others}\rangle, \quad X + Z = 1, \quad 0 \le X \le 1$$





range of interaction

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$
scattering length radius of

radius of bound state

- for shallow bound state $R \gg R_{\rm typ}$, $X \leftarrow (a_0, B)$

- (i) The particle must be stable; else Z is undefined. (However, it may be an adequate approximation to ignore the decay modes of a very narrow resonance.)
- (ii) The particle must couple to a two-particle channel with threshold not too much above the particle mass.
- (iii) It is crucial that this two-body channel have zero orbital angular momentum l, since for $l\neq 0$ the factor $(E)^{1/2}$ in the integrands of (24) and (32) would be $E^{l+(1/2)}$, and the integrals could not be approximated by their low-energy parts.

Problem: applicable only to stable states

Weak-binding relation for unstable states

Compositeness X of unstable quasibound state

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

- complex eigenenergy: $-B \rightarrow E_h \in \mathbb{C}$ $|\Lambda(1405)\rangle = \sqrt{X} |\bar{K}N\rangle + \sqrt{Z} |\text{ others}\rangle, \quad X + Z = 1$

$$X + Z = 1$$

- complex a_0 , X

$$\mathcal{L} = \frac{1}{\sqrt{2\mu F}}, \quad \mathcal{E} \equiv \frac{1}{\sqrt{2\mu F}}$$

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O}\left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- correction from threshold energy difference
- for near-threshold quasibound state $|R| \gg (R_{\text{typ}}, \ell)$, $X \leftarrow (a_0, E_h)$

Interpretation of complex X

$$\tilde{X} = \frac{1 - |Z| + |X|}{2}, \quad \tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} + \tilde{Z} = 1, \quad 0 \le \tilde{X} \le 1$$

Compositeness of $\Lambda(1405)$: central values

Generalized weak-binding relation

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O}\left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- Neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{ar{K}N}$	$ ilde{X}_{ar{K}N}$	U/2
Set 1 [35]	-10 - i26	1.39 - i0.85	1.2 + i0.1	1.0	0.3
Set 2 [36]	-4-i8	1.81 - i0.92	0.6 + i0.1	0.6	0.0
Set 3 [37]	-13 - i20	1.30 - i0.85	0.9 - i0.2	0.9	0.1
Set 4 [38]	2 - i10	1.21 - i1.47	0.6 + i0.0	0.6	0.0
Set 5 [38]	-3-i12	1.52 - i1.85	1.0 + i0.5	0.8	0.3

- In all cases, $X \sim 1$ and $\tilde{X} \sim 1$

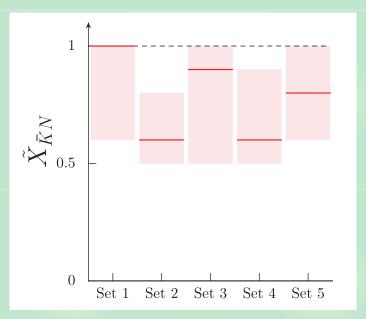
 $\Lambda(1405)$: $\bar{K}N$ composite dominance <— observables

Compositeness of $\Lambda(1405)$: uncertainties

Estimation of correction terms: $|R| \sim 2 \text{ fm}$

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left| \frac{R_{\text{typ}}}{R} \right| \right) + \mathcal{O}\left(\left| \frac{\ell}{R} \right|^3 \right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- ρ meson exchange picture: $R_{\rm typ} \sim 0.25~{\rm fm}$
- Energy difference from $\pi\Sigma$: $\ell \sim 1.08~\mathrm{fm}$



 $\bar{K}N$ composite dominance holds even with correction terms





Hadron interactions —> hadronic molecules



Femtoscopy: novel method to study hadron interactions



Femtoscopy for K^-p correlations

- precise test for $\Lambda(1405)$ and $\bar{K}N$ interactions

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)



Compositeness of $\Lambda(1405)$

- $\bar{K}N$ molecule picture from observables

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Future prospects



Hadron interactions

- Combination of scattering experiment, femtoscopy, and lattice QCD
- scattering length a_0 and binding energy B
 - -> compositeness X



New direction: femtoscopy with nuclei

- $\Lambda \alpha$, $\Xi \alpha$ correlations: complementary to ΛN , ΞN
- hopefully at J-PARC HI

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC110, 014001 (2024); Y. Kamiya, A. Jinno, T. Hyodo, A. Ohnishi, arXiv:2409.13207 [nucl-th]