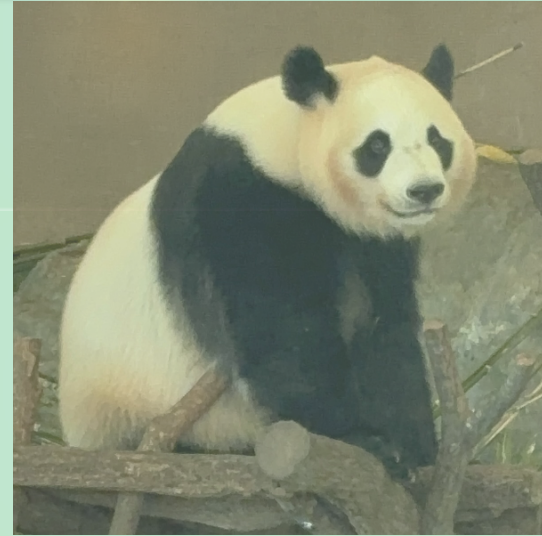


Entanglement suppression and hadron scatterings



**Tao-Ran Hu, Katsuyoshi Sone, Feng-Kun Guo,
Tetsuo Hyodo, Ian Low**

Tokyo Metropolitan Univ.

2025, Jun. 7th

24 years ago

Hosaka-san came to RCNP in 2001

(M0: Tamenaga, Nagata)

M1: Nam, Geng, Hyodo

M2: Nakamura, Hamamoto

D1: Yasui, Manabe

D2: Takahashi

**D3: Koma (Takayama),
Sugimoto**



2004?

保坂さん24年間おつかれさまでした。

退職おめでとうございます！

Introduction to entanglement suppression

Entanglement power (EP) of S-matrix $E(\hat{S})$

- ability of generating entanglement

$$|\text{out}\rangle = \hat{S} |\text{in}\rangle$$



entangle?

Entanglement suppression

S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$E(\hat{S}) = 0 \rightarrow$ emergent **symmetries** of NN scattering

Scatterings of octet baryons, pd , nd , dd , heavy mesons, ...

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023);

T. Kirchner, W. Elkamhawy, H.-W. Hammer, Few-Body Syst. 65, 29 (2024);

T.R. Hu, S. Chen, F.K. Guo, PRD110, 014001 (2024); ...

Symmetries of NN scattering

Fundamental symmetries ← QCD

- **spin** $SU(2)_{\text{spin}}$ $\{ |\uparrow\rangle, |\downarrow\rangle \}$

- **isospin** $SU(2)_{\text{isospin}}$ $\{ |p\rangle, |n\rangle \}$

Low-energy (s-wave) NN scattering under $SU(2)_{\text{spin}} \times SU(2)_{\text{isospin}}$

→ **two components**, 1S_0 and 3S_1

1S_0 and 3S_1 are **independent**, no constraint on their **strength**

Emergent symmetries ← entanglement suppression?

- **spin-flavor** $SU(4)$ $\{ |p\uparrow\rangle, |p\downarrow\rangle, |n\uparrow\rangle, |n\downarrow\rangle \}$ $|1/a_0(^1S_0)| \sim |1/a_0(^3S_1)|$

E. Wigner, PR51, 106 (1937), ..., D.B. Kaplan, M.J. Savage, PLB365, 244 (1996), ...

- **nonrelativistic conformal symmetry** $|a_0| \gg 1/m_\pi$

T. Mehen, I.W. Stewart, M.B. Wise, PLB474, 145 (2000)

Entanglement measures

Entanglement **measure** for bipartite **state** $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

- von Neumann entropy

$$\mathcal{E}_{\text{vN}}(|\psi\rangle) = -\text{Tr}_A[\rho_A \ln \rho_A], \quad \rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$$

- linear entropy (expansion $\ln \rho_A \sim (\rho_A - 1)$)

$$\mathcal{E}(|\psi\rangle) = 1 - \text{Tr}_A[\rho_A^2]$$

- product state: minimum $\mathcal{E}(|\psi\rangle) = 0$

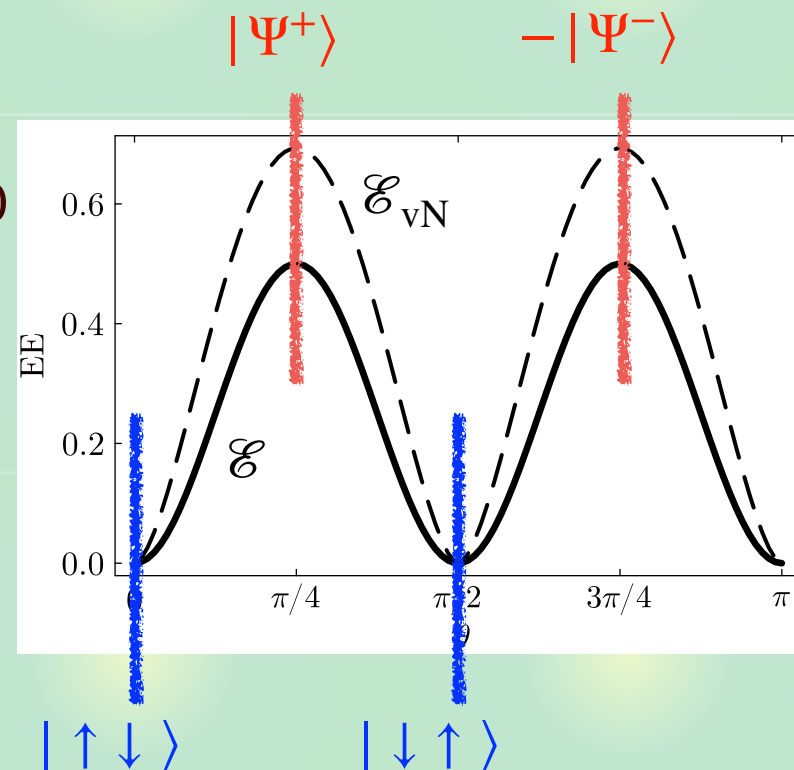
$$|\psi\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle = |\uparrow\downarrow\rangle$$

- Bell state: maximum $\mathcal{E}(|\Psi^\pm\rangle) = 1/2$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle$$

Example:

$$|\psi\rangle = \cos \theta |\uparrow\downarrow\rangle + \sin \theta |\downarrow\uparrow\rangle$$



Entanglement power

Entanglement **power** (EP) of **operator** U acting on $|\psi\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

- average over entanglement measure of $U|\psi_A\rangle \otimes |\psi_B\rangle$

$$E(U) = \overline{\mathcal{E}(U|\psi_A\rangle \otimes |\psi_B\rangle)}$$

$$\mathcal{E}(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$$

$$= \int d\omega_A d\omega_B \mathcal{E}(U|\psi_A(\omega_A)\rangle \otimes |\psi_B(\omega_B)\rangle)$$

—> ability of U to **generate** entanglement

- **spin 1/2 (qubit)**: rays in $\mathbb{C}^2 \simeq \mathbb{CP}^1$ —> **Fubini-Study measure**

$$|\psi(\omega_2)\rangle = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 e^{i\nu_1} \end{pmatrix}, \quad d\omega_2 = \frac{1}{\pi} d\theta_1 \cos \theta_1 \sin \theta_1 d\nu_1 \quad \begin{array}{l} \theta_1 \in [0, \pi/2) \\ \nu_1 \in [0, 2\pi) \end{array}$$

(equivalent to Bloch sphere with $\theta_1 = \theta/2$, $\nu_1 = \phi$, $d\omega_2 = d\Omega/4\pi$)

S-matrix for NN scattering

S-matrix for pn scattering

$$\hat{S} = \underbrace{\exp\{2i\delta_0\}}_{^1S_0} \frac{1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} + \underbrace{\exp\{2i\delta_1\}}_{^3S_1} \frac{3 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4}$$

(isospin is automatically determined by Pauli principle)

Entanglement power of NN scattering

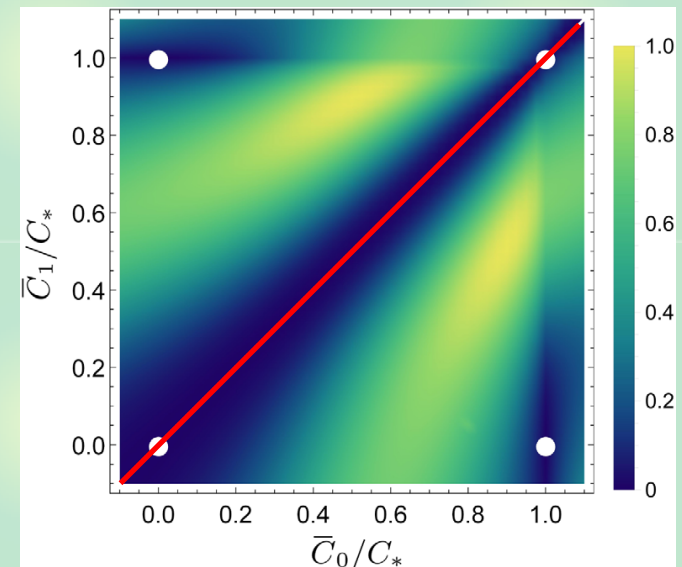
S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$$E(\hat{S}) = \frac{1}{6} \sin^2[2(\delta_0 - \delta_1)] \geq 0$$

$E(\hat{S}) = 0$ is achieved if

$$\begin{cases} \delta_0 = \delta_1 & \text{SU(4) symmetry} \\ (\delta_0, \delta_1) = (0,0), (0,\pi/2), (\pi/2,0), (\pi/2,\pi/2) \end{cases}$$

NR conformal symmetry



Interpretation by quantum information

S-matrix with $E(\hat{S}) = 0$: product state $\rightarrow$ product state

I. Low, T. Mehen, PRD104, 074014 (2021)

- **Identity gate:** $|\delta_0 - \delta_1| = 0$

$$\hat{S} \propto \frac{1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} + \frac{3 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} = 1, \quad 1 |\psi_A\rangle \otimes |\psi_B\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

- **SWAP gate:** $|\delta_0 - \delta_1| = \pi/2$ (spin exchange operator P_s)

$$\hat{S} \propto -\frac{1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} + \frac{3 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} = \frac{1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{2} = \text{SWAP}$$

$$\text{SWAP} |\psi_A\rangle \otimes |\psi_B\rangle = |\psi_B\rangle \otimes |\psi_A\rangle$$


Identity and SWAP are the **only minimal entanglers**

$\leftarrow$ parametrization of $SU(4)/(SU(2) \times SU(2))$

Motivation

Decuplet baryons

- spin 3/2
- Octet + decuplet: ground state of SU(6)

Possibly large scattering length

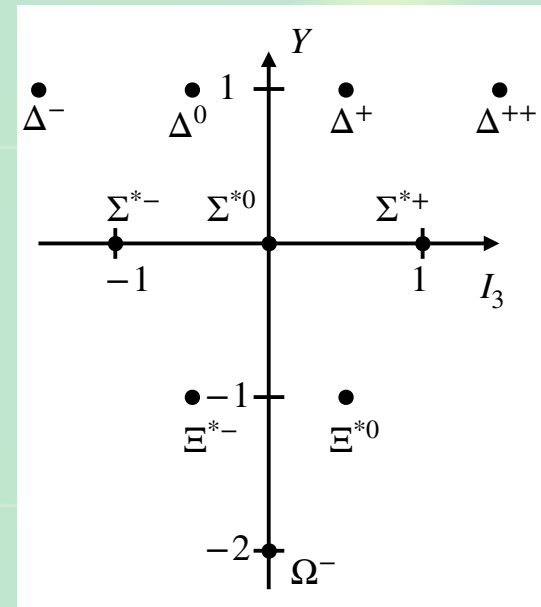
- $\Delta\Delta$ dibaryon, $d^*(2380)$?

F. Dyson, N.H. Xuong, PRL13, 815 (1964);

P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for $\Omega\Omega$ scattering: $a_0(^1S_0) = 4.6$ fm

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)



Quantum information viewpoint

- scattering of spin 3/2 particles = two-qudit quantum gate

Spin 3/2 state

Spin 3/2 (qudit): rays in $\mathbb{C}^4 \simeq \mathbb{CP}^3$

$$|\psi_4\rangle = \begin{pmatrix} \cos \theta_1 \sin \theta_2 \sin \theta_3 \\ \sin \theta_1 \sin \theta_2 \sin \theta_3 e^{i\nu_1} \\ \cos \theta_2 \sin \theta_3 e^{i\nu_2} \\ \cos \theta_3 e^{i\nu_3} \end{pmatrix}, \quad d\omega_4 = \frac{3!}{\pi^3} \prod_{i=1}^3 d\theta_i d\nu_i \cos \theta_i \sin^{2i-1} \theta_i,$$

$$\theta_i \in [0, \pi/2), \quad \nu_i \in [0, 2\pi)$$

Spin decomposition (2 symmetric, 2 anti-symmetric)

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_A \oplus \underbrace{1 \oplus 3}_S$$

S-matrix: 4 components, 4 phase shifts

$$\hat{S} = \mathcal{J}_0 \exp\{2i\delta_0\} + \mathcal{J}_1 \exp\{2i\delta_1\} + \mathcal{J}_2 \exp\{2i\delta_2\} + \mathcal{J}_3 \exp\{2i\delta_3\}$$

$$\mathcal{J}_0 = \frac{33}{128} + \frac{31}{96} (\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2}) - \frac{5}{72} (\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^2 - \frac{1}{18} (\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^3, \dots$$

Entanglement power

Entanglement power (~3 days on mathematica)

$$\begin{aligned}
 E(\hat{S}) = \frac{1}{200000} \{ & 77482 - 2100 \cos \left[2 (\delta_0 + \delta_1 - \delta_2 - \delta_3) \right] \\
 & - 2100 \cos \left[2 (\delta_0 - \delta_1 + \delta_2 - \delta_3) \right] - 2100 \cos \left[2 (\delta_0 - \delta_1 - \delta_2 + \delta_3) \right] \\
 & - 1200 \cos \left[2 (\delta_0 - 2\delta_1 + \delta_2) \right] - 4200 \cos \left[2 (\delta_0 + \delta_2 - 2\delta_3) \right] \\
 & - 8400 \cos \left[2 (\delta_1 - 2\delta_2 + \delta_3) \right] \\
 & - 375 \cos \left[4 (\delta_0 - \delta_1) \right] - 10800 \cos \left[2 (\delta_0 - \delta_2) \right] - 625 \cos \left[4 (\delta_0 - \delta_2) \right] \\
 & - 875 \cos \left[4 (\delta_0 - \delta_3) \right] - 2175 \cos \left[4 (\delta_1 - \delta_2) \right] - 26376 \cos \left[2 (\delta_1 - \delta_3) \right] \\
 & - 5481 \cos \left[2 (\delta_1 - \delta_3) \right] - 10675 \cos \left[2 (\delta_2 - \delta_3) \right] \}
 \end{aligned}$$

$E(\hat{S}) = 0$ is achieved if (all cos = +1)

$$\delta_0 = \delta_2 = \delta_{\text{even}}, \quad \delta_1 = \delta_3 = \delta_{\text{odd}}, \quad \begin{cases} |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 & \mathbf{1} \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 & \mathbf{SWAP} \end{cases}$$

$\Omega\Omega$ scattering**(properly normalized) Entanglement power of $\Omega\Omega$ S-matrix**

$$E_2(\hat{S}_{\Omega\Omega}) = \frac{1}{48} \left\{ 25 - \cos \left[4 \left(\delta_{0,28} - \delta_{2,28} \right) \right] \right\}$$

 $E_2(\hat{S}) = 0$ is achieved if

$$\begin{cases} |\delta_{0,28} - \delta_{2,28}| = 0 \\ |\delta_{0,28} - \delta_{2,28}| = \pi/2 \end{cases}$$

Lattice QCD shows $a_0(^1S_0) = 4.6 \text{ fm} \gg 1/m_\pi \rightarrow \delta_{0,28} \sim \pi/2$ **S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)****Entanglement suppression suggests either**

- **spin 2 channel is near the unitary limit ($|\delta_{0,28} - \delta_{2,28}| = 0$), or**
- **spin 2 channel is almost noninteracting ($|\delta_{0,28} - \delta_{2,28}| = \pi/2$)**

Summary of overview part



Entanglement power of S-matrix $\sim$ ability of generating entanglement

$$E(\hat{S}) = \overline{\mathcal{E}(\hat{S} |\psi_A\rangle \otimes |\psi_B\rangle)}$$



Entanglement suppression: requiring $E(\hat{S}) = 0$ for NN scattering, the following symmetries emerge

- SU(4) **spin-flavor symmetry** ($\hat{S} \propto 1$)
- NR **conformal symmetry** ($\hat{S} \propto \text{SWAP}$)

S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

Summary

- Entanglement suppression for baryon decuplet
- $E(\hat{S}) = 0$ is achieved only by $\hat{S} \propto 1$ or $\hat{S} \propto \text{SWAP}$ for spin 3/2 scattering
- Largest emergent symmetries
 - SU(40) spin-flavor symmetry ($\hat{S} \propto 1$)
 - $\text{SU}(4)_{\text{spin}} \times \text{SU}(10)_{\text{flavor}}$ + NR conformal ($\hat{S} \propto \text{SWAP}$)
- Sub-unitary S-matrix
 - final state should be normalized
 - $\Omega\Omega$ scattering: implication to spin 2 channel

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]