Higher partial waves and resonance contributions in femtoscopy

Koichi Murase, Tetsuo Hyodo

Tokyo Metropolitan Univ.

2025, Sep. 29th

Contents

- Introduction — Femtoscopy
- Contribution from s-wave resonance

  S. Watanabe, T. Hyodo, in preparation

  - Origin of resonance peak
- Contribution from higher partial waves

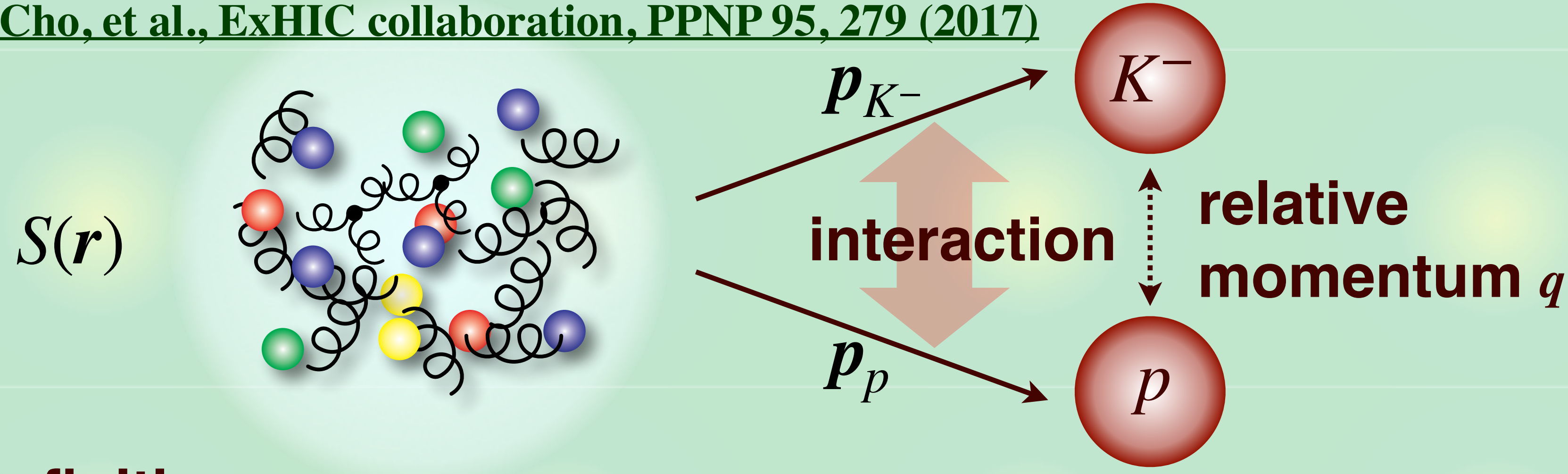
  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]

  - Correlation function with $l > 0$
  - Regularized LL formula for $l > 0$
- Summary

Correlation function and KP formula

High-energy collision: chaotic source $S(r)$ of hadron emission

S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)



- **Definition**

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-}) N_p(\boldsymbol{p}_p)}$$ **(= 1 in the absence of FSI/QS)**

- **Theory: KP (Koonin-Pratt) formula**

S.E. Koonin, PLB 70, 43 (1977); S. Pratt, PRD 33, 1314 (1986)

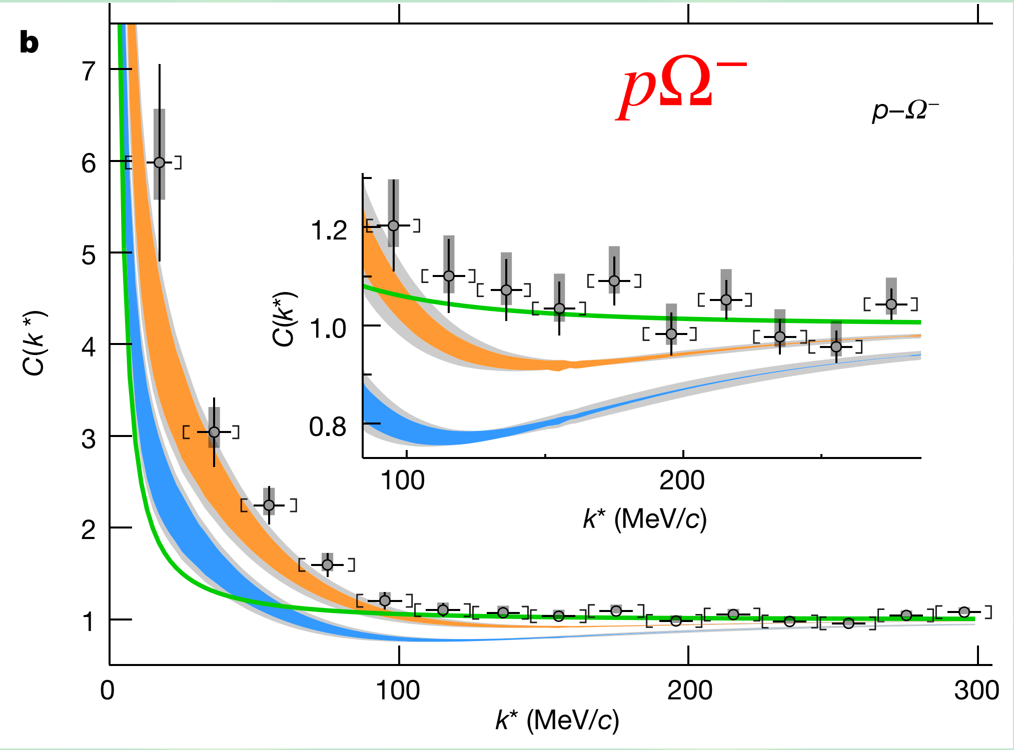
$$C(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\, S(\boldsymbol{r})\, |\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2$$

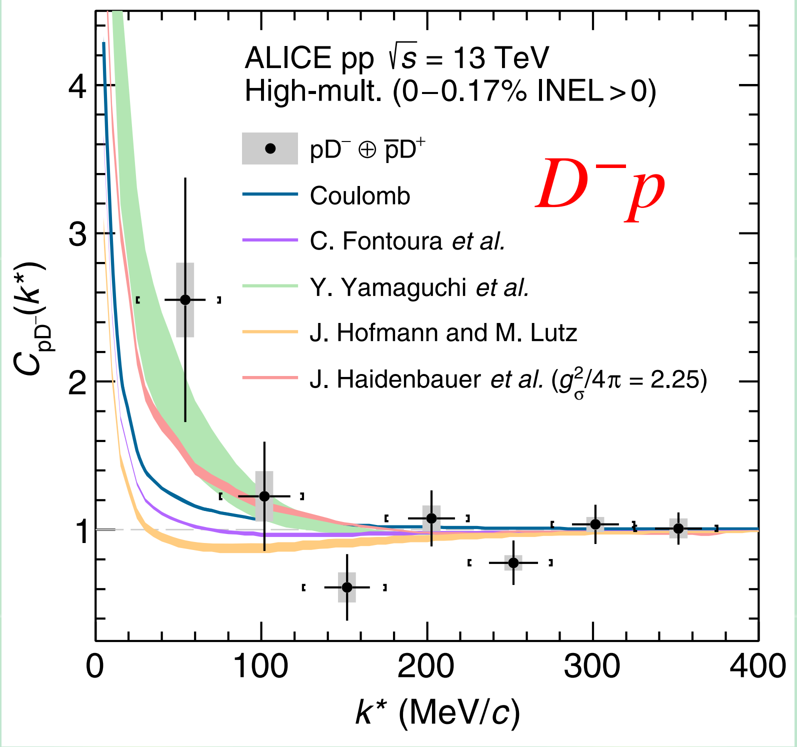
Source function $S(\boldsymbol{r})$ <—> wave function $\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})$ (interaction)

Experimental data with strangeness and charm

Correlation functions observed by ALICE@LHC

ALICE collaboration, Nature 588, 232 (2020);
ALICE collaboration, PRD 106, 052010 (2022)





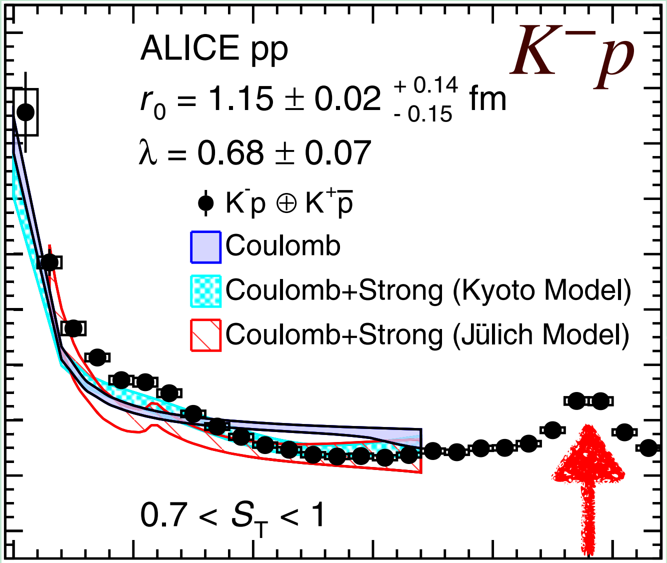
$\Omega^- \sim sss$: **strangeness** $S=-3$, $D^- \sim \bar{c}d$: **charm** $C=-1$

Almost impossible in scattering experiments

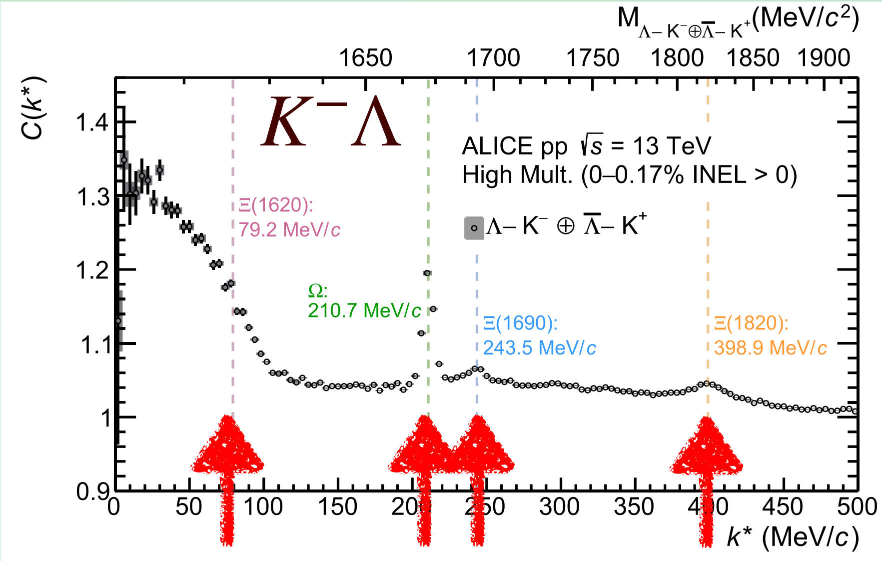
Resonance contributions

Resonance peaks in experimental data ($\ell = 0$ and $\ell \neq 0$)

ALICE collaboration, PRL 124, 092301 (2020); PLB845, 138145 (2023)



$\Lambda(1520)$: d wave



$\Xi(1620)$, $\Xi(1690)$: s wave,

Ω : p wave (weak decay),

$\Xi(1820)$: d wave

Questions:

- **Origin** of resonance peak?
- Contribution from **higher partial waves**?

Contents

- Introduction — Femtoscopy
- Contribution from s-wave resonance

  S. Watanabe, T. Hyodo, in preparation

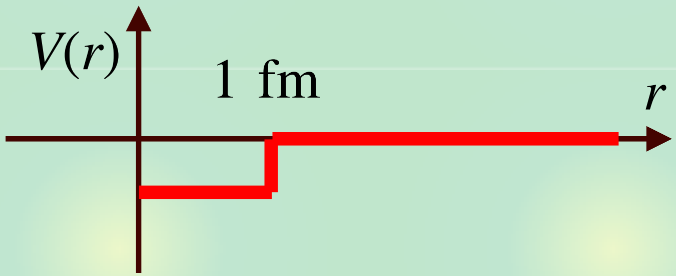
  - Origin of resonance peak
- Contribution from higher partial waves

  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]

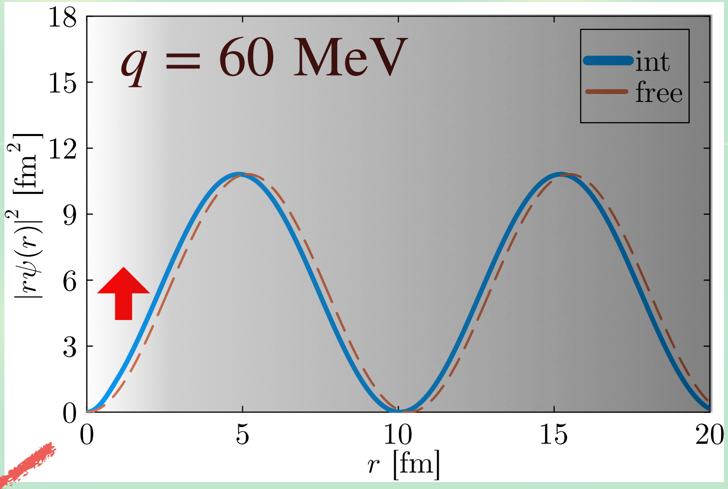
  - Correlation function with $l > 0$
  - Regularized LL formula for $l > 0$
- Summary

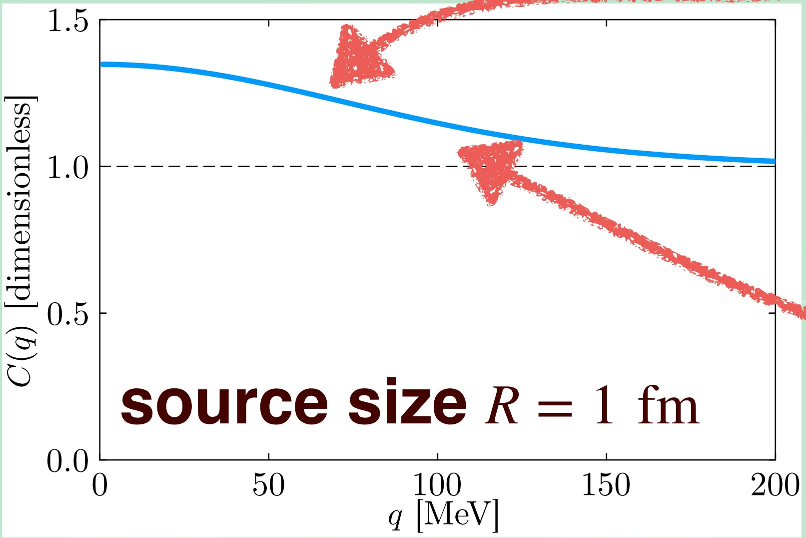
Wave function and correlation (attraction)

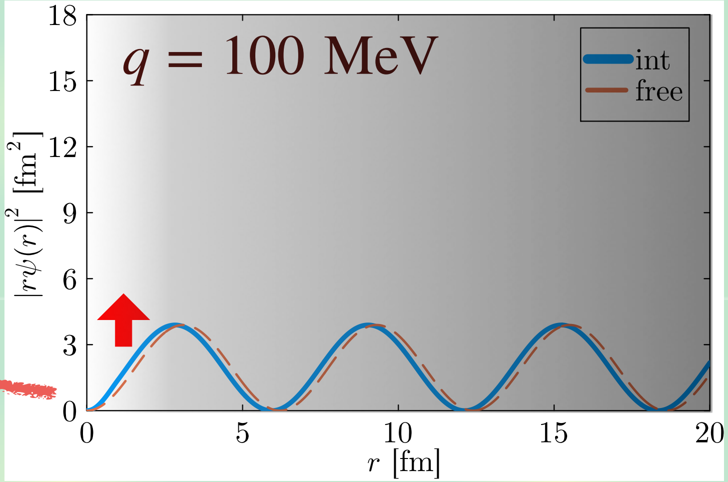
Attractive well (no bound state)



$$C(q) \simeq 1 + \int_0^\infty dr\ \frac{4\pi}{q^2} S(r)\{\,|\,|r q \psi_q(r)|^2 - \sin^2(qr)\}$$



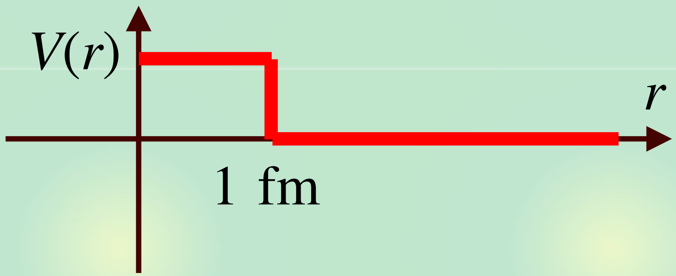




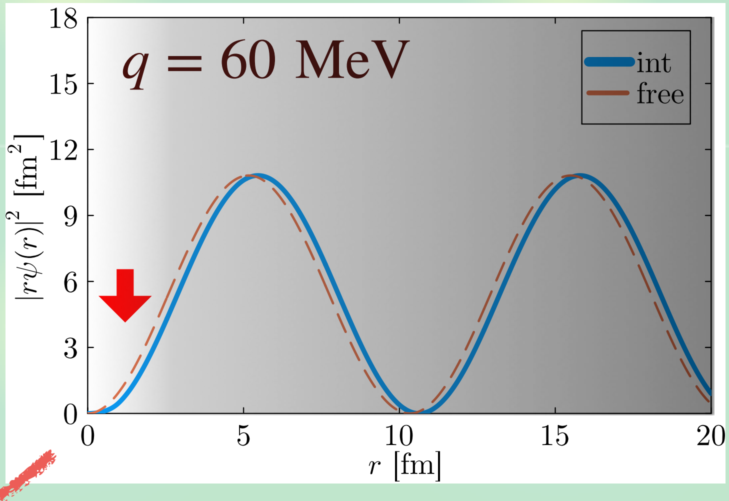
W.f. is pulled in, increased at $r \lesssim R$ —> $C(q)$ enhancement

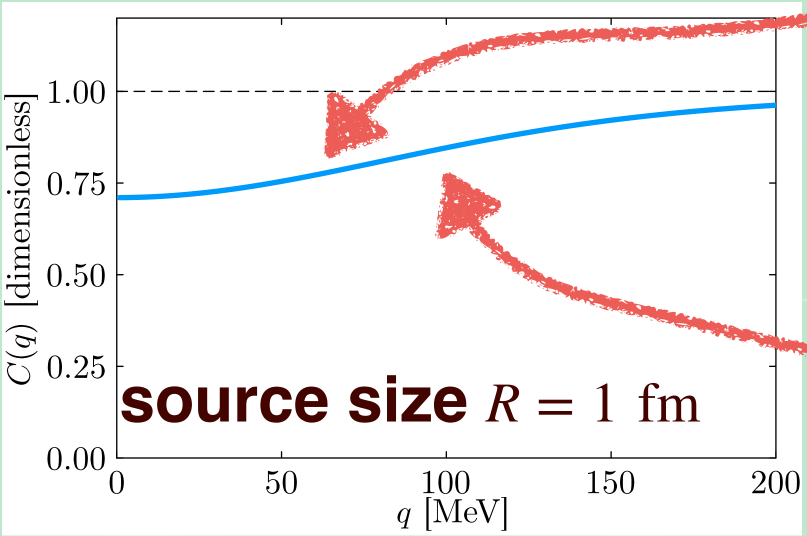
Wave function and correlation (repulsion)

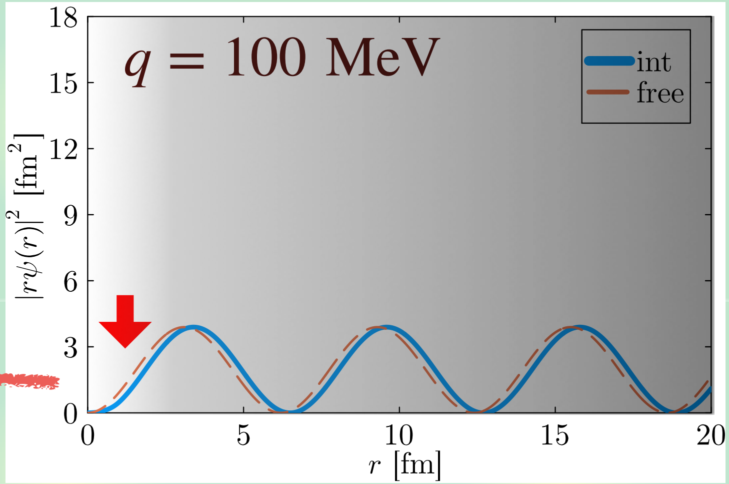
Repulsive rectangular potential



$$C(q) \simeq 1 + \int_0^\infty dr\ \frac{4\pi}{q^2} S(r)\{\,||rq\psi_q(r)|^2 - \sin^2(qr)\}$$



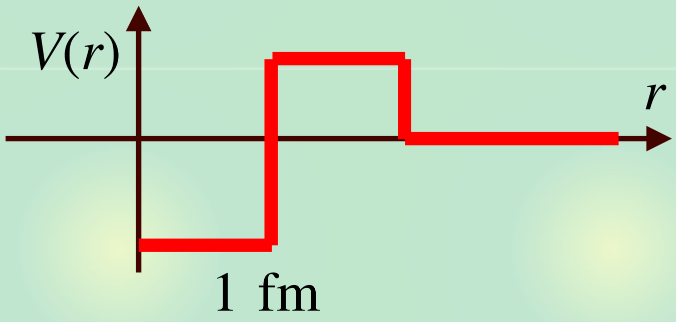


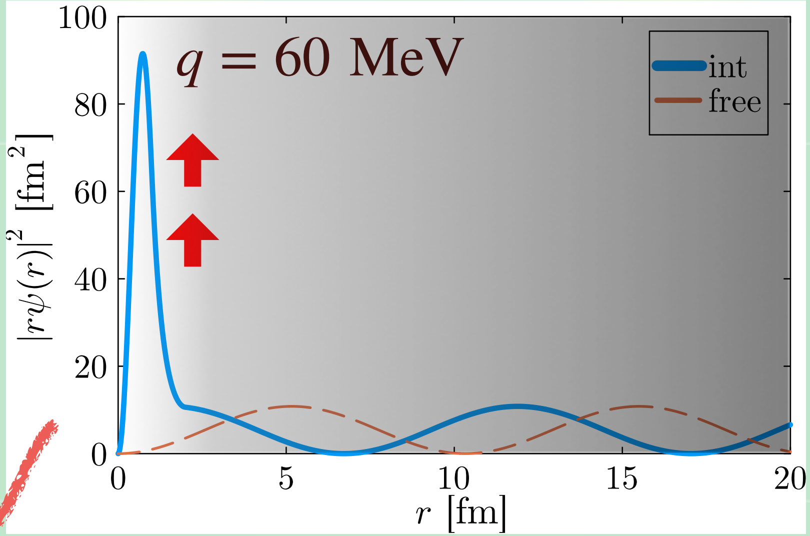


W.f. is pushed out, decreased at $r \lesssim R$ —> $C(q)$ suppression

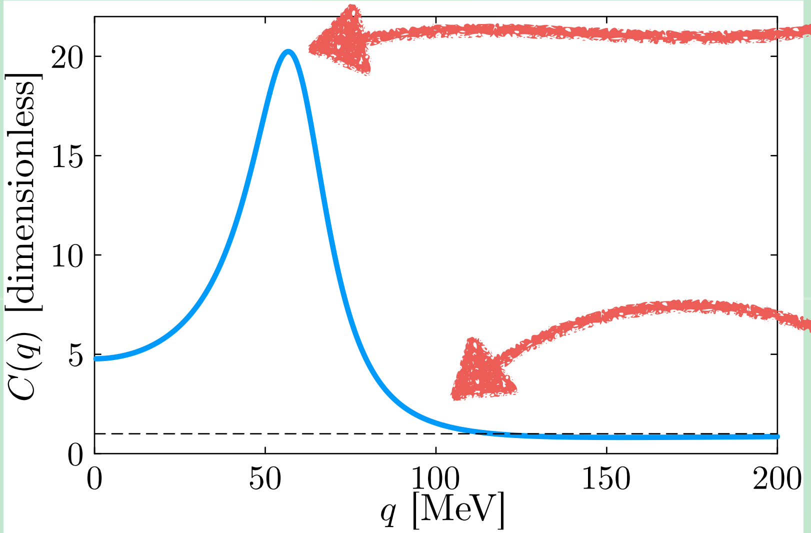
Wave function and correlation (resonance)

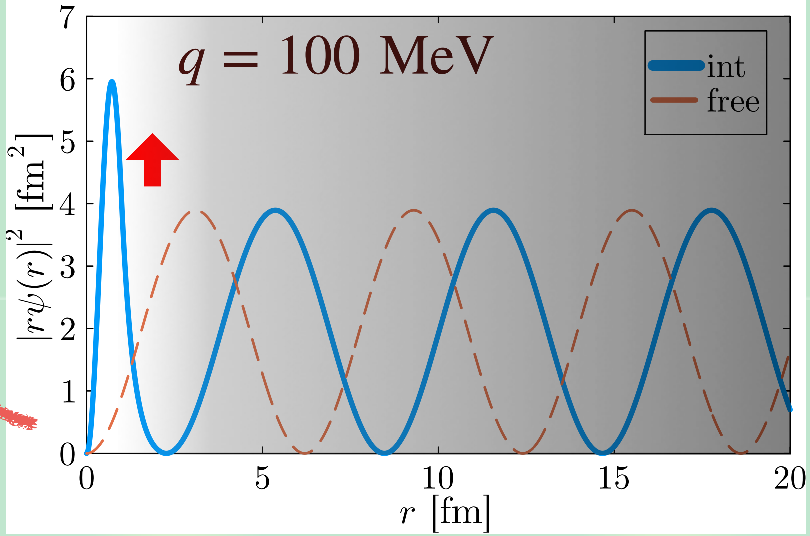
Well + barrier potential





- resonance @ $q^- = 59 - 14i$ MeV





W.f. is localized in $r \lesssim R$ at pole momentum —> peak in $C(q)$

Contents

Higher partial waves

KP formula with $l > 0$ (spherical source)

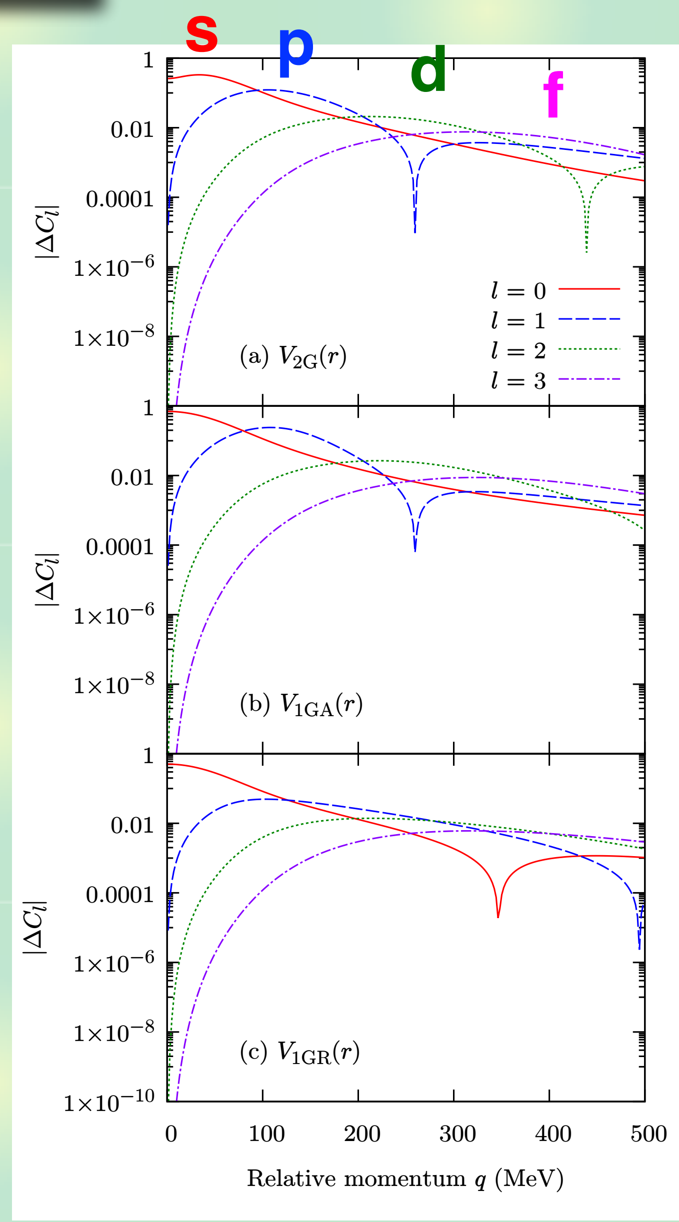
$$C(q) = 1 + \sum_{l=0}^{\infty} \Delta C_l(q)$$

$$\Delta C_l(q) = (2l+1) \times \int_0^{\infty} dr\, 4\pi r^2 S(r) [\, |R_l(r)|^2 - |j_l(qr)|^2]$$

- sum of partial wave contributions
- interacting w.f. $R_l(r)$ — free w.f. $j_l(qr)$

Gaussian potentials (range ~ 1.25 fm)

- $l > 0$ components at **larger** q
- l-th wave dominant at $q \sim l/r \sim 160 l$ MeV



Resonance contribution

With $q \to \infty$, $C(q)$ approaches unity

- How can resonances be seen?

Resonances in higher partial waves

- p-wave resonance at

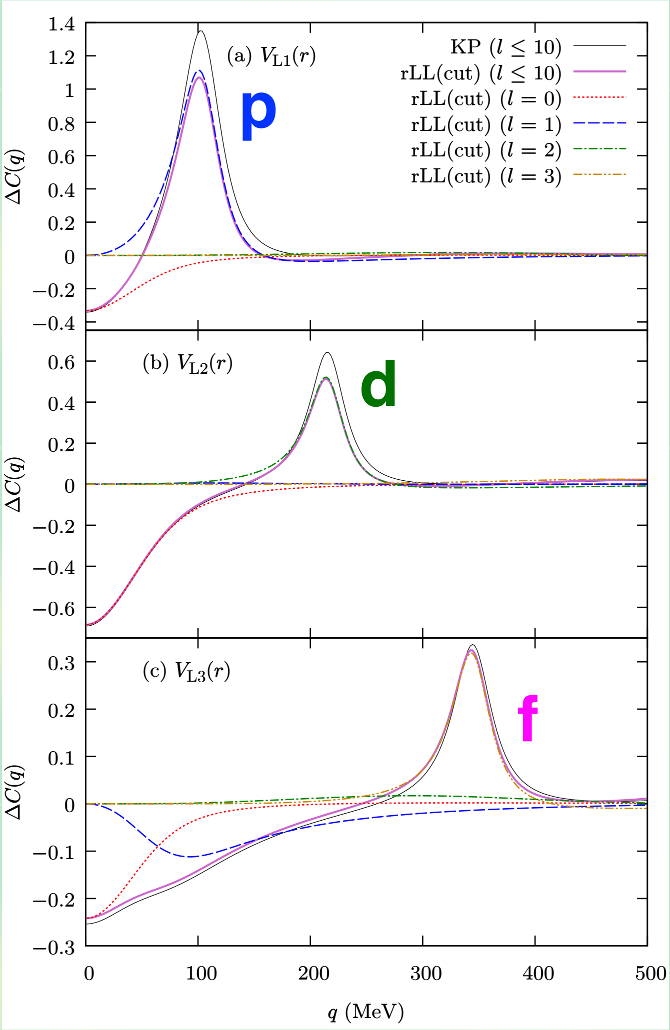
$$q^- \sim 105 - 23i \text{ MeV}$$

- d-wave resonance at

$$q^- \sim 216 - 20i \text{ MeV}$$

- f-wave resonance at

$$q^- \sim 345 - 21i \text{ MeV}$$



With resonances, $l > 0$ components is enahnced

LL formula for higher partial wave?

Lednicky-Lyuboshits (LL) formula for s-wave

R. Lednicky, V.L. Lyuboshits, Yad. Fiz. 35, 1316 (1981);
K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025)

- Asymptotic w.f. for entire r <— on-shell observable

$$\Delta C(q) = \int_0^\infty dr\, \frac{4\pi}{q^2} S(r)[\sin^2(qr - \delta(q)) - \sin^2(q)]$$

- corresponding to zero range limit (point-like interaction)

Naive generalization for $l > 0$ is not possible

$$\Delta C_l(q) = (2l+1)\int_0^\infty dr\, 4\pi r^2 S(r)[\,|R_l^{\mathrm{aym}}(r)|^2 - |j_l(qr)|^2\,]$$

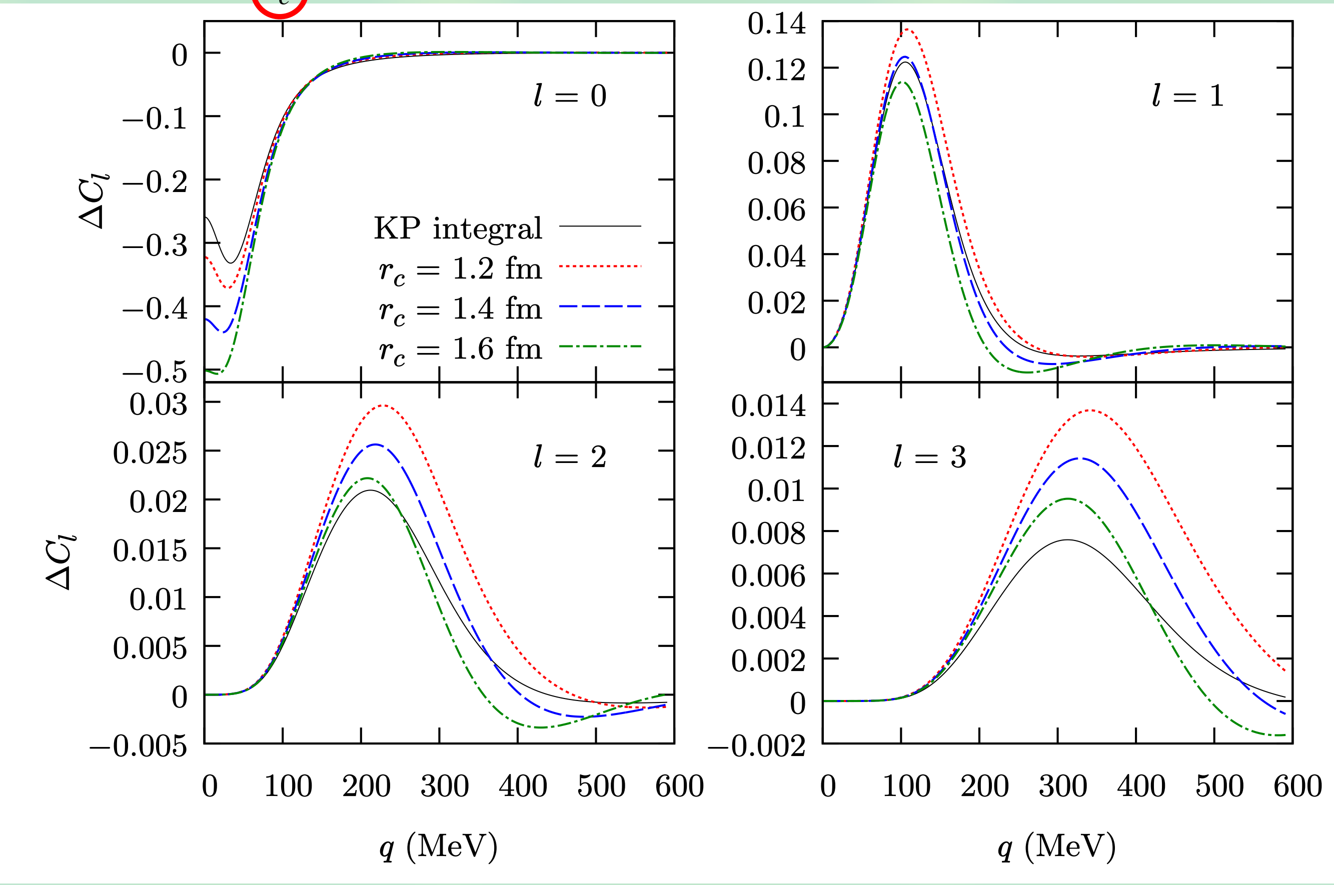
- $R_l^{\mathrm{aym}}(r)$ is too singular for $l > 0$ ($n_l(qr) \sim r^{-l-1}$) at $r \to 0$

Some regularization for $r \to 0$ is needed

Regularized LL formula

Simplest choice: introducing cutoff r_c

$$\Delta C_l(q) = (2l+1)\int_{r_c}^{\infty} dr\, 4\pi r^2 S(r)[\,|R_l^{\mathrm{aym}}(r)|^2 - |j_l(qr)|^2\,]$$



Works for $l > 0$ with r_c ~ interaction range (1.25 fm)

Summary

- Resonace peaks in correlation functions

- s-wave resonance peak <— localization of wave function at interacting region

  S. Watanabe, T. Hyodo, in preparation

- Higher partial wave ($l > 0$) contributions becomes important for larger q

- Regularized LL formula with suitable cutoff r_c works for $l > 0$

  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]