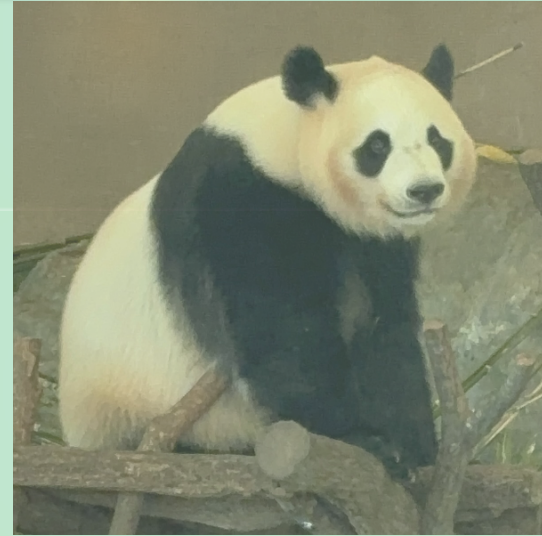


Symmetries in hadron scattering and quantum entanglement

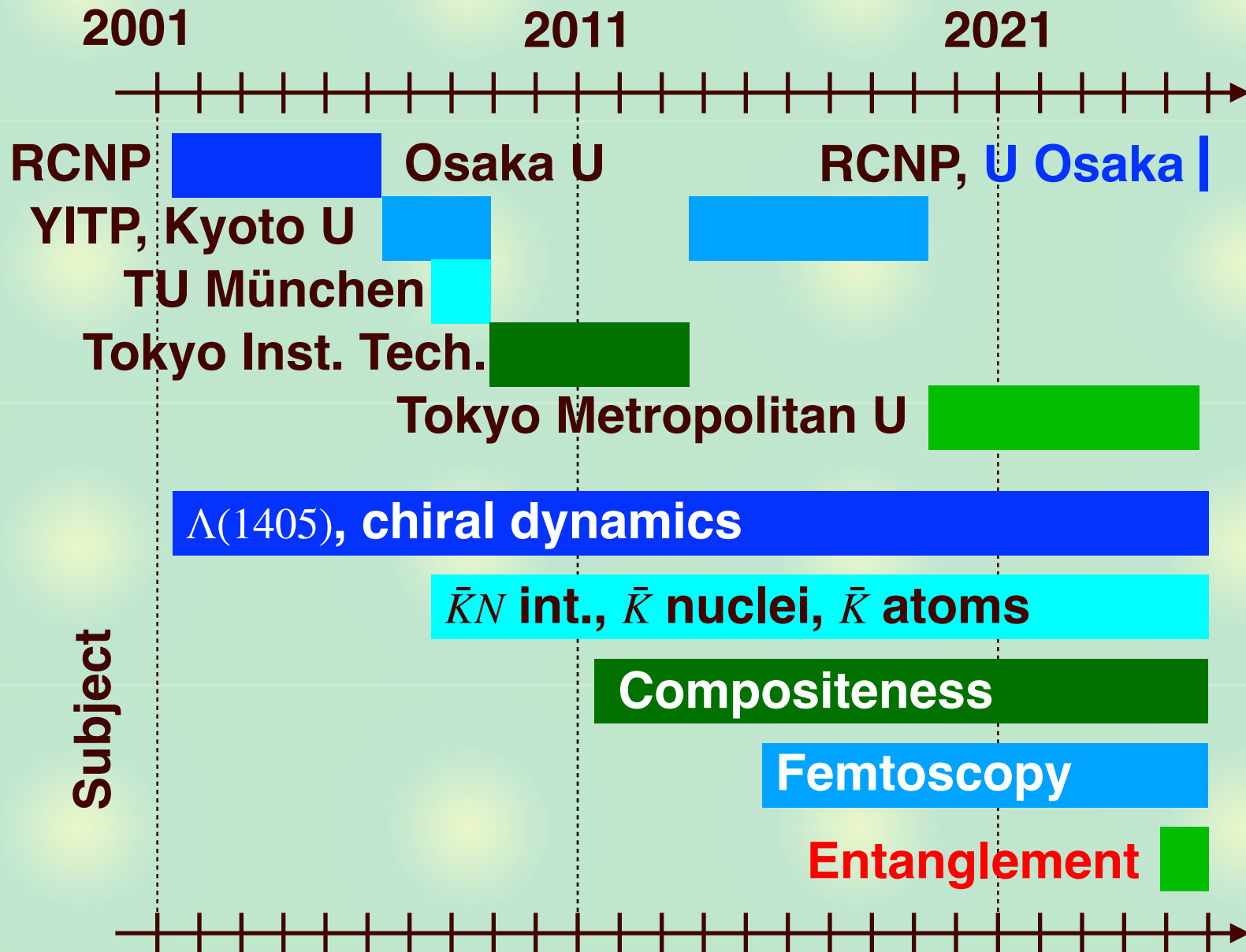


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2025, Dec. 16th ₁

Self-introduction





Overview

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

- Entanglement power
- Entanglement suppression for NN scattering



SU(3) baryon decuplet scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]
to appear in Phys. Rev. Res.

- Spin 3/2 scattering
- Application to $\Omega\Omega$ scattering



Summary

Basic concept of entanglement suppression

Entanglement power (EP) of S-matrix $E(\hat{S})$

- ability of generating entanglement



$$|\psi_{\text{out}}\rangle = \hat{S} |\psi_{\text{in}}\rangle$$



entangle?



Entanglement suppression

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019)

$E(\hat{S}) = 0 \rightarrow$ emergent **symmetries** of NN scattering

Generalization to other hadrons and nuclei, ...

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023);

T. Kirchner, W. Elkamhawy, H.-W. Hammer, Few-Body Syst. 65, 29 (2024);

T.R. Hu, S. Chen, F.K. Guo, PRD110, 014001 (2024); ...

Symmetries of NN scattering

Fundamental symmetries $SU(2)_{\text{spin}} \times SU(2)_{\text{isospin}} \leftarrow \text{QCD}$

- **spin** $SU(2)_{\text{spin}}$ $\{ |\uparrow\rangle, |\downarrow\rangle \}$
 - **isospin** $SU(2)_{\text{isospin}}$ $\{ |p\rangle, |n\rangle \}$
- $$\frac{1}{2} \otimes \frac{1}{2} = \underbrace{0}_A \oplus \underbrace{1}_S$$

s-wave NN scattering: spin $J \times$ isospin I should be A

$|J=0, I=1\rangle$ (1S_0) **and** $|J=1, I=0\rangle$ (3S_1), **independent**

Emergent symmetries $\leftarrow$ **entanglement suppression?**

- **large scattering length** $|a_0| \gg 1 \text{ fm}$: **NR conformal symmetry**

T. Mehen, I.W. Stewart, and M.B. Wise, PLB474, 145 (2000)

- **both** 1S_0 **and** 3S_1 **have large** $|a_0|$: **spin-flavor** $SU(4)$

$\{ |p\uparrow\rangle, |p\downarrow\rangle, |n\uparrow\rangle, |n\downarrow\rangle \}$

E. Wigner, PR51, 106 (1937), ..., D.B. Kaplan and M.J. Savage, PLB365, 244 (1996), ...

Entanglement measures

Entanglement **measure** for bipartite **state** $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

- **von Neumann entropy**

$$\mathcal{E}_{\text{vN}}(|\psi\rangle) = -\text{Tr}_A[\rho_A \ln \rho_A], \quad \rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$$

- **linear entropy (expansion $\ln \rho_A \sim (\rho_A - 1)$)**

$$\mathcal{E}(|\psi\rangle) = 1 - \text{Tr}_A[\rho_A^2]$$

- **product state: minimum (=0)**

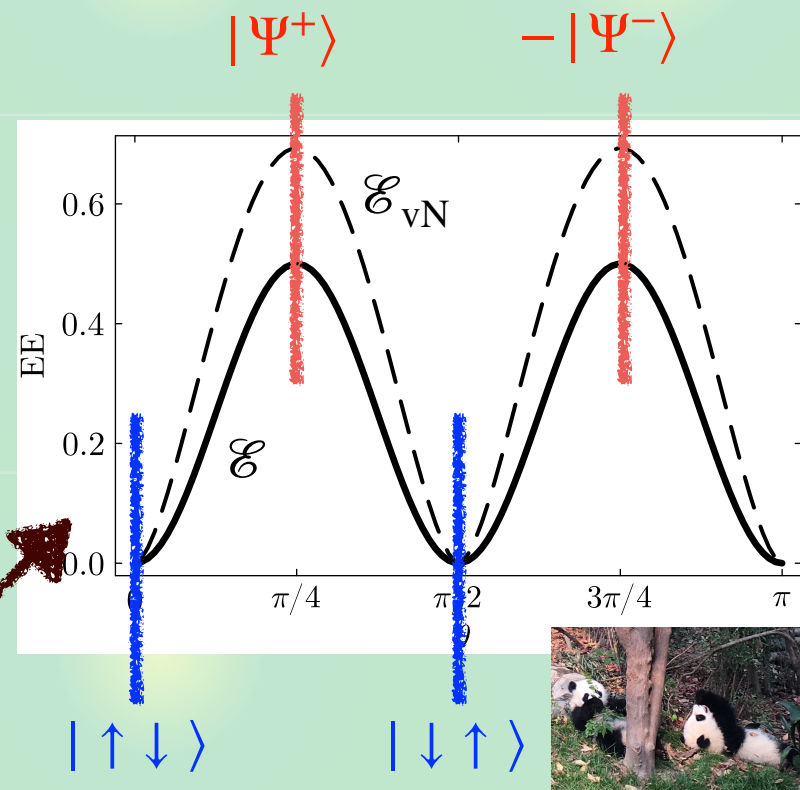
$$|\uparrow\downarrow\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle$$

- **Bell state: maximum (=1/2)**

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle$$

Linear combination:

$$|\psi\rangle = \cos \theta |\uparrow\downarrow\rangle + \sin \theta |\downarrow\uparrow\rangle$$



Entanglement power

Entanglement **power** (EP) of **operator** $\hat{U}$ acting on $|\psi\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

- average over entanglement measure of $\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle$

$$E(\hat{U}) = \overline{\mathcal{E}(\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle)}$$

$$\mathcal{E}(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$$

$$= \int d\omega_A d\omega_B \mathcal{E}(\hat{U}|\psi_A(\omega_A)\rangle \otimes |\psi_B(\omega_B)\rangle)$$

—> ability of $\hat{U}$ to **generate** entanglement (state independent)

- **spin 1/2 (qubit)**: rays in $\mathbb{C}^2 \simeq \mathbb{CP}^1$ —> Fubini-Study measure

$$|\psi(\omega_2)\rangle = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 e^{i\nu_1} \end{pmatrix}, \quad d\omega_2 = \frac{1}{\pi} d\theta_1 \cos \theta_1 \sin \theta_1 d\nu_1 \quad \begin{array}{l} \theta_1 \in [0, \pi/2) \\ \nu_1 \in [0, 2\pi) \end{array}$$

(equivalent to Bloch sphere with $\theta_1 = \theta/2$, $\nu_1 = \phi$, $d\omega_2 = d\Omega/4\pi$)

S-matrix for NN scattering

S-matrix for pn scattering: 2 phase shifts δ_0, δ_1

$$\hat{S} = \underbrace{\exp\{2i\delta_0\}}_{J=0} \frac{1 - \sigma_A \cdot \sigma_B}{4} + \underbrace{\exp\{2i\delta_1\}}_{J=1} \frac{3 + \sigma_A \cdot \sigma_B}{4}$$

(isospin is automatically determined by Pauli principle)

Entanglement power of NN scattering

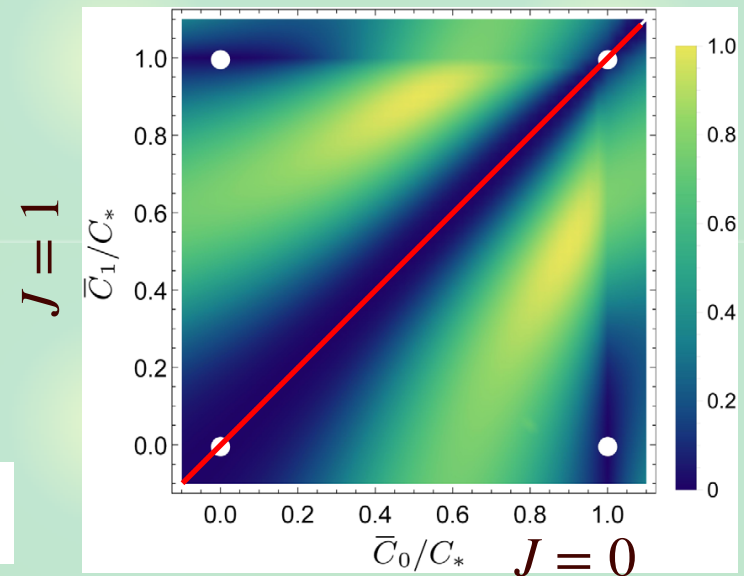
S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019)

$$E(\hat{S}) = \frac{1}{6} \sin^2[2(\delta_0 - \delta_1)] \geq 0$$

$E(\hat{S}) = 0$ is achieved if

$$\begin{cases} \delta_0 = \delta_1 & \text{spin-flavor SU(4)} \\ (\delta_0, \delta_1) = (0,0), (0,\pi/2), (\pi/2,0), (\pi/2,\pi/2) \end{cases}$$

$|a_0| \rightarrow \infty$: NR conformal symmetry



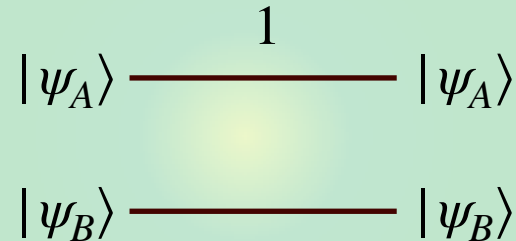
Interpretation by quantum information

S-matrix with $E(\hat{S}) = 0$: product state $\rightarrow$ product state

I. Low, T. Mehen, PRD104, 074014 (2021)

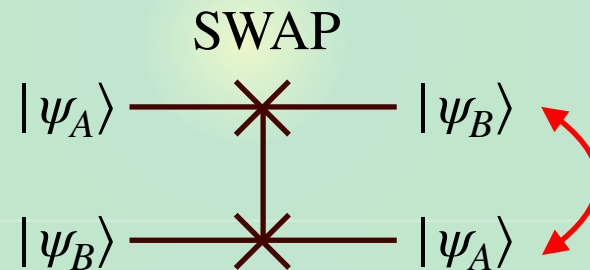
- **Identity gate:** $|\delta_0 - \delta_1| = 0$

$$\hat{S} \propto \frac{1 - \sigma_A \cdot \sigma_B}{4} + \frac{3 + \sigma_A \cdot \sigma_B}{4} = 1$$



- **SWAP gate:** $|\delta_0 - \delta_1| = \pi/2$ (spin exchange operator P_s)

$$\begin{aligned} \hat{S} &\propto -\frac{1 - \sigma_A \cdot \sigma_B}{4} + \frac{3 + \sigma_A \cdot \sigma_B}{4} \\ &= \frac{1 + \sigma_A \cdot \sigma_B}{2} = \text{SWAP} \end{aligned}$$



Identity and SWAP are the **only minimal entanglers**
 ($\leftarrow$ Cartan decomposition of $SU(4)/(SU(2) \times SU(2))$)

Further evidences of entanglement suppression

Spin-flavor symmetries with three flavors

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019)

- $SU(6) \sim (u, d, s) \otimes (\uparrow, \downarrow) \leftarrow$ large N_c
- $SU(16) \sim (n, p, \Lambda, \dots) \otimes (\uparrow, \downarrow) \leftarrow$ Entanglement suppression
- Lattice QCD evaluation of LEC favors $SU(16)$

M.L. Wagman, *et al.*, (NPLQCD), PRD96, 114510 (2017)

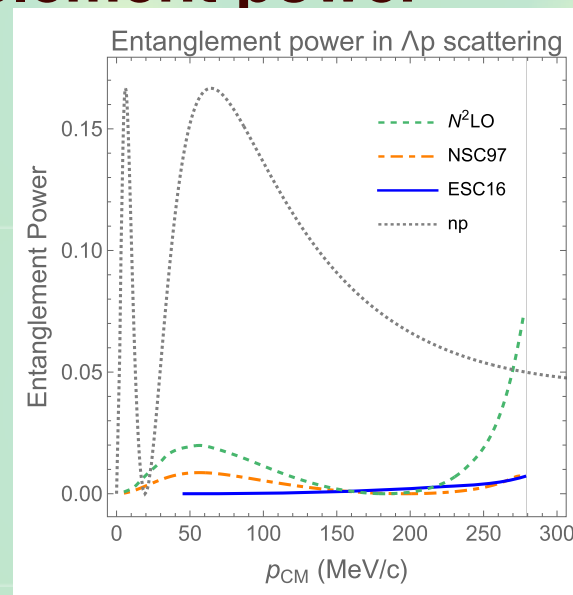
Momentum dependence of Entanglement power

Q. Liu, I. Low, PLB, 856, 138899 (2024)

- $np, \Lambda p$ scatterings

$$E(p_{\text{CM}}) = \frac{1}{6} \sin^2[2(\delta_0(p_{\text{CM}}) - \delta_1(p_{\text{CM}}))]$$

- EP suppressed at low energies



Summary of overview part



Entanglement power of S-matrix $\sim$ ability of generating entanglement

$$E(\hat{S}) = \overline{\mathcal{E}(\hat{S} |\psi_A\rangle \otimes |\psi_B\rangle)}$$



Entanglement suppression: **requiring** $E(\hat{S}) = 0$ for NN scattering, the following symmetries emerge

- **spin-flavor SU(4) symmetry** ($\hat{S} \propto 1$)
- **NR conformal symmetry** ($\hat{S} \propto \text{SWAP}$)

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)



Overview

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

- Entanglement power
- Entanglement suppression for NN scattering



SU(3) baryon decuplet scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]
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- Spin 3/2 scattering
- Application to $\Omega\Omega$ scattering



Summary

Motivation

Decuplet baryons

- spin 3/2
- ground states with Octet in quark model

Large scattering length a_0 ($\delta \sim \pi/2$)?

- $\Delta\Delta$ dibaryon, $d^*(2380)$?

F. Dyson, N.H. Xuong, PRL13, 815 (1964);

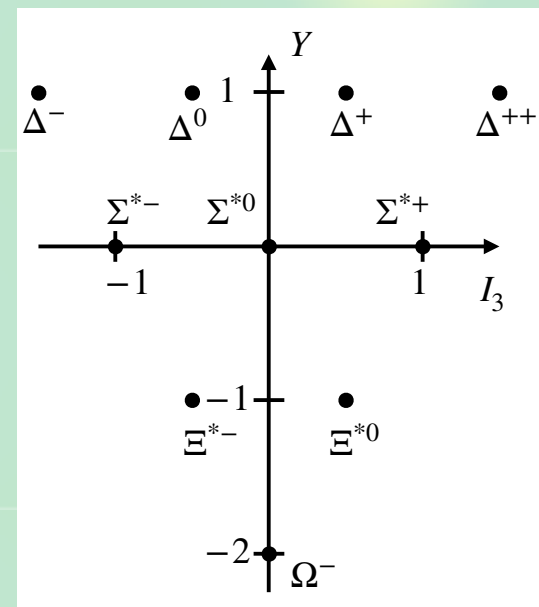
P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for $\Omega\Omega$ scattering: $a_0(J=0) = 4.6 \text{ fm} \gg 1 \text{ fm}$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Quantum information viewpoint

- spin 3/2 scattering = 4d-qudit quantum gates



Spin 3/2 state

Spin 3/2 (4d-qudit): rays in $\mathbb{C}^4 \simeq \mathbb{CP}^3$

$$|\psi_4\rangle = \begin{pmatrix} \cos \theta_1 \sin \theta_2 \sin \theta_3 \\ \sin \theta_1 \sin \theta_2 \sin \theta_3 e^{i\nu_1} \\ \cos \theta_2 \sin \theta_3 e^{i\nu_2} \\ \cos \theta_3 e^{i\nu_3} \end{pmatrix}, \quad d\omega_4 = \frac{3!}{\pi^3} \prod_{i=1}^3 d\theta_i d\nu_i \cos \theta_i \sin^{2i-1} \theta_i,$$

$$\theta_i \in [0, \pi/2), \quad \nu_i \in [0, 2\pi)$$

Spin decomposition (2 symmetric, 2 anti-symmetric)

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_A \oplus \underbrace{1 \oplus 3}_S$$

S-matrix: 4 components, 4 phase shifts $\delta_0, \delta_1, \delta_2, \delta_3$

$$\hat{S} = \mathcal{J}_0 \exp\{2i\delta_0\} + \mathcal{J}_1 \exp\{2i\delta_1\} + \mathcal{J}_2 \exp\{2i\delta_2\} + \mathcal{J}_3 \exp\{2i\delta_3\}$$

$$\mathcal{J}_0 = \frac{33}{128} + \frac{31}{96} (\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2}) - \frac{5}{72} (\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^2 - \frac{1}{18} (\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^3, \dots$$

Entanglement power

Entanglement power (~3 days on mathematica)

$$\begin{aligned}
 E(\hat{S}) = \frac{1}{200000} \{ & 77482 - 2100 \cos \left[2 (\delta_0 + \delta_1 - \delta_2 - \delta_3) \right] \\
 & - 2100 \cos \left[2 (\delta_0 - \delta_1 + \delta_2 - \delta_3) \right] - 2100 \cos \left[2 (\delta_0 - \delta_1 - \delta_2 + \delta_3) \right] \\
 & - 1200 \cos \left[2 (\delta_0 - 2\delta_1 + \delta_2) \right] - 4200 \cos \left[2 (\delta_0 + \delta_2 - 2\delta_3) \right] \\
 & - 8400 \cos \left[2 (\delta_1 - 2\delta_2 + \delta_3) \right] \\
 & - 375 \cos \left[4 (\delta_0 - \delta_1) \right] - 10800 \cos \left[2 (\delta_0 - \delta_2) \right] - 625 \cos \left[4 (\delta_0 - \delta_2) \right] \\
 & - 875 \cos \left[4 (\delta_0 - \delta_3) \right] - 2175 \cos \left[4 (\delta_1 - \delta_2) \right] - 26376 \cos \left[2 (\delta_1 - \delta_3) \right] \\
 & - 5481 \cos \left[4 (\delta_1 - \delta_3) \right] - 10675 \cos \left[2 (\delta_2 - \delta_3) \right] \}
 \end{aligned}$$

$E(\hat{S}) = 0$ is achieved if (all cos = +1)

$$\delta_0 = \delta_2 = \delta_{\text{even}}, \quad \delta_1 = \delta_3 = \delta_{\text{odd}}, \quad \begin{cases} |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 & : \text{Identity gate} \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 & : \text{SWAP gate} \end{cases}$$

Minimal entanglers

S-matrix with $E(\hat{S}) = 0$

- **Identity gate:** $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0$

$$\hat{S} \propto \mathcal{I}_0 + \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3 = 1$$

- **SWAP gate:** $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$

$$\hat{S} \propto -\mathcal{I}_0 + \mathcal{I}_1 - \mathcal{I}_2 + \mathcal{I}_3 = \sum_i \mathcal{S}_i - \sum_i \mathcal{A}_i$$

—> Minimal entanglement is achieved by Identity and SWAP, also for spin 3/2 scattering

Identity and SWAP are the **only minimal entanglers for any bipartite system with same dimension**

E. Alfsen, F. Shultz, JMP51, 052201 (2010);...

Decuplet-decuplet scattering

Spin-flavor decomposition: spin $\otimes$ flavor should be A

- **spin** $SU(2)_{\text{spin}}$ $\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_A \oplus \underbrace{1 \oplus 3}_S$

- **flavor** $SU(3)_{\text{flavor}}$ $10 \otimes 10 = \underbrace{27 \oplus 28}_S \oplus \underbrace{\overline{10} \oplus 35}_A$

8 phase shifts $\delta_{0,27}, \delta_{0,28}, \delta_{1,\overline{10}}, \delta_{1,35}, \delta_{2,27}, \delta_{2,28}, \delta_{3,\overline{10}}, \delta_{3,35}$

$$\begin{aligned} \hat{S} = & \mathcal{J}_0 \otimes (\mathcal{F}_{27} e^{2i\delta_{0,27}} + \mathcal{F}_{28} e^{2i\delta_{0,28}}) + \mathcal{J}_1 \otimes (\mathcal{F}_{\overline{10}} e^{2i\delta_{1,\overline{10}}} + \mathcal{F}_{35} e^{2i\delta_{1,35}}) \\ & + \mathcal{J}_2 \otimes (\mathcal{F}_{27} e^{2i\delta_{2,27}} + \mathcal{F}_{28} e^{2i\delta_{2,28}}) + \mathcal{J}_3 \otimes (\mathcal{F}_{\overline{10}} e^{2i\delta_{3,\overline{10}}} + \mathcal{F}_{35} e^{2i\delta_{3,35}}) \end{aligned}$$

$E(\hat{S}) = 0$ is achieved if

$$\delta_{0,F} = \delta_{2,F'} = \delta_{\text{even}}, \quad \delta_{1,F} = \delta_{3,F'} = \delta_{\text{odd}}, \quad \begin{cases} |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 \end{cases}$$

Emergent symmetries

Emergent symmetry ← effective Lagrangian

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023)

- **spin** $i = 1, 2, 3, 4$, **flavor** $T = (\Delta^{++}, \dots, \Omega^-)$

Effective Lagrangian for $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0$

$$\mathcal{L} = c_1 (T^{\dagger i} \cdot T^i)^2$$

- **rotation of 40 component spin-flavor vector**
 → SU(40) **spin-flavor symmetry**

Effective Lagrangian for $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$

$$\mathcal{L} = -\frac{2\pi}{\mu\Lambda} \left[-\frac{1}{4} (T^{\dagger i} \cdot T^i)^2 \pm \frac{1}{4} (T^{\dagger i} \cdot T^j) (T^{\dagger j} \cdot T^i) \right]$$

- **independent rotation of 4 spin and 10 flavor**
 → SU(4)_{spin} × SU(10)_{flavor} + NR conformal symmetry

Emergent symmetries

Emergent symmetry $\leftarrow$ degeneracy counting

	$\delta_{\text{even}}(J = 0, 2, F = 27, 28)$		$\delta_{\text{odd}}(J = 1, 3, F = \overline{10}, 35)$	
spin	$1 + 5 = 6$		$3 + 7 = 10$	
flavor	$27 + 28 = 55$		$10 + 35 = 45$	
total	330	+	450	= 780

- Identity gate: $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0 \rightarrow$ **spin-flavor** SU(40)

$$40 \otimes 40 = 820 \oplus 780$$

- SWAP gate: $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$

$\rightarrow$ $SU(4)_{\text{spin}} \times SU(10)_{\text{flavor}}$ + **NR conformal symmetry**

$$4 \otimes 4 = 10 \oplus 6, \quad 10 \otimes 10 = 55 \oplus 45$$

Justified by effective Lagrangian approach

Identical particles and sub-unitary S-matrix

Identical particles: symmetric spin states are forbidden

- nn scattering/ $\Omega\Omega$ scattering: only antisymmetric component

$$\hat{S}_{nn} = \mathcal{I}_0 \exp\{2i\delta_0\} \quad \text{spin 1 absent}$$

$$\hat{S}_{\Omega\Omega} = \mathcal{I}_0 \exp\{2i\delta_{0,28}\} + \mathcal{I}_2 \exp\{2i\delta_{2,28}\} \quad \text{spin 1 and 3 absent}$$

S-matrix is not unitary in full space of $|\psi_{\text{in}}\rangle : \hat{S}_{nn}\hat{S}_{nn}^\dagger = \mathcal{I}_0 \neq 1$

$$\hat{S} = \begin{pmatrix} \text{(Antisymmetric)} & 0 \\ 0 & \text{(Symmetric)} \end{pmatrix}, \quad \mathcal{I}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Density matrix of out state is not properly normalized

$$\rho = |\psi_{\text{out}}\rangle\langle\psi_{\text{out}}| = \hat{S}_{nn}|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|\hat{S}_{nn}^\dagger, \quad \text{Tr}[\rho] = \langle\psi_{\text{in}}|\mathcal{I}_0|\psi_{\text{in}}\rangle < 1$$

—> Properly **normalized** Entanglement power

Normalized entanglement power

Normalized density matrix

M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge

$$\tilde{\rho} = \frac{|\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|}{\langle\psi_{\text{out}}|\psi_{\text{out}}\rangle}$$

k-weighted normalized entanglement power

$$E_k(\hat{S}) = 1 - \frac{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^k \text{Tr}_A [\tilde{\rho}_A^2]}{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^k}$$

For nn scattering, result does not depend on k

$$E_k(\hat{S}_{nn}) = \frac{1}{2}$$

$k = 2$ is practically convenient

$$E_2(\hat{S}) = 1 - \frac{\int d\omega_A d\omega_B \text{Tr}_A [\rho_A^2]}{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^2}$$

$\Omega\Omega$ scattering **$\Omega\Omega$ S-matrix**

$$\hat{S}_{\Omega\Omega} = \mathcal{J}_0 \exp\{2i\delta_{0,28}\} + \mathcal{J}_2 \exp\{2i\delta_{2,28}\}$$

Properly normalized EP of $\Omega\Omega$ S-matrix

$$E_2(\hat{S}_{\Omega\Omega}) = \frac{1}{48} \left\{ 25 - \cos \left[4 (\delta_{0,28} - \delta_{2,28}) \right] \right\}$$

$E_2(\hat{S}_{\Omega\Omega})$ is minimized (1/2, not 0) if

$$\begin{cases} |\delta_{0,28} - \delta_{2,28}| = 0 & \hat{S} \propto \mathcal{J}_0 + \mathcal{J}_2 \equiv \mathcal{P}_A \\ |\delta_{0,28} - \delta_{2,28}| = \pi/2 & \hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_2 \equiv \text{SWAP}_A \end{cases}$$

$a_0(J=0) \gg 1$ fm **by lattice QCD suggests either**

- $|a_0(J=2)| \gg 1$ fm (**unitary limit**, $|\delta_{0,28} - \delta_{2,28}| = 0$) **or**
- $a_0(J=2) \sim 0$ (**noninteracting**, $|\delta_{0,28} - \delta_{2,28}| = \pi/2$)

Summary

- Entanglement suppression for baryon decuplet
- $E(\hat{S}) = 0$ is achieved only by $\hat{S} \propto 1$ or $\hat{S} \propto \text{SWAP}$ for spin 3/2 scattering
- Largest emergent symmetries
 - spin-flavor SU(40) symmetry ($\hat{S} \propto 1$)
 - $\text{SU}(4)_{\text{spin}} \times \text{SU}(10)_{\text{flavor}}$ + NR conformal ($\hat{S} \propto \text{SWAP}$)
- Sub-unitary S-matrix
 - Final state should be properly normalized
 - $\Omega\Omega$ scattering: implication to spin 2 channel

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph],
to appear in Phys. Rev. Res.