Higher partial waves and resonance contributions in femtoscopy

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Contents

- Introduction — Femtoscopy
- Contribution from s-wave resonance

  S. Watanabe, T. Hyodo, in preparation

  - Origin of resonance peak
- Contribution from higher partial waves

  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]

  - Correlation function with $l > 0$
  - Regularized LL formula for $l > 0$
- Summary

Exotic hadrons and hadron interactions

Observation of tetraquark T_{cc}

LHCb collaboration, Nature Phys. 18, 7, 751 (2022); Nature Commun. 13, 1, 3351 (2022)


nature physics

LETTERS

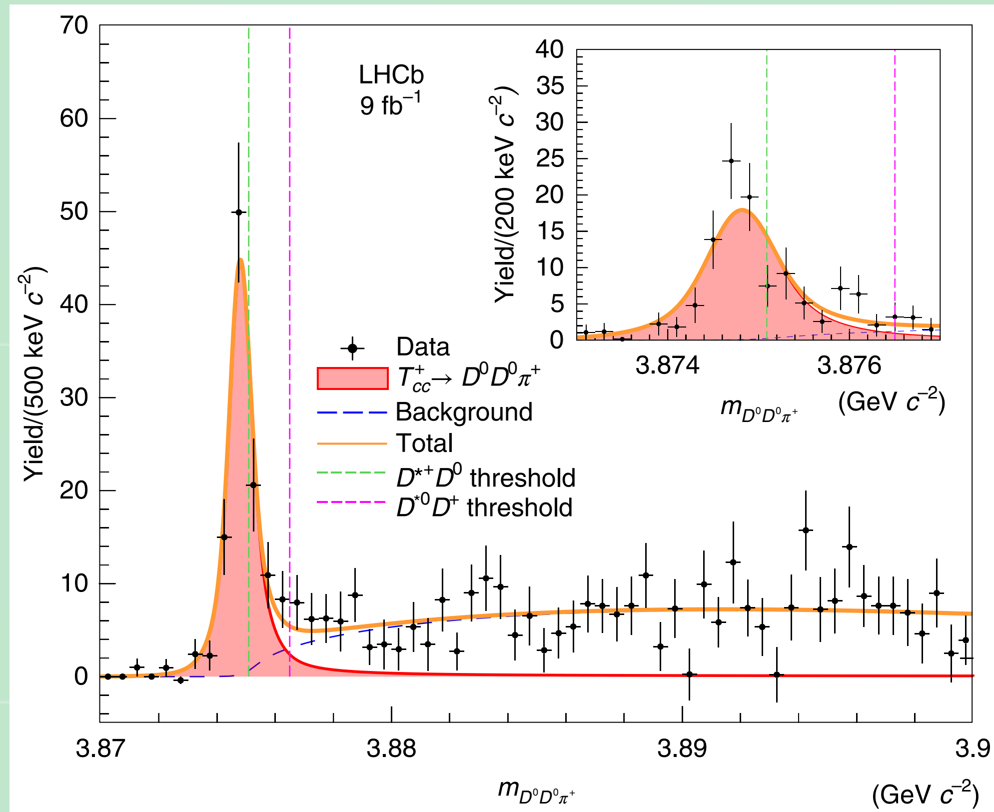
https://doi.org/10.1038/s41567-022-01614-y

Check for updates

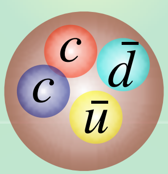
OPEN

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*



- **Quark content** $\sim cc\bar{u}\bar{d}$
- **Internal structure?**



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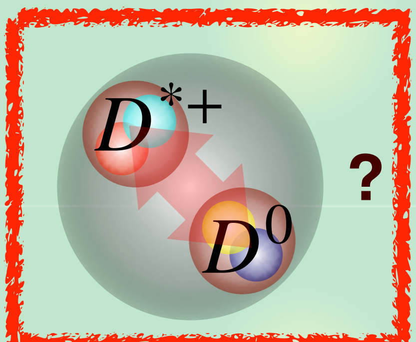


?

Hadronic molecules <— hadron interactions

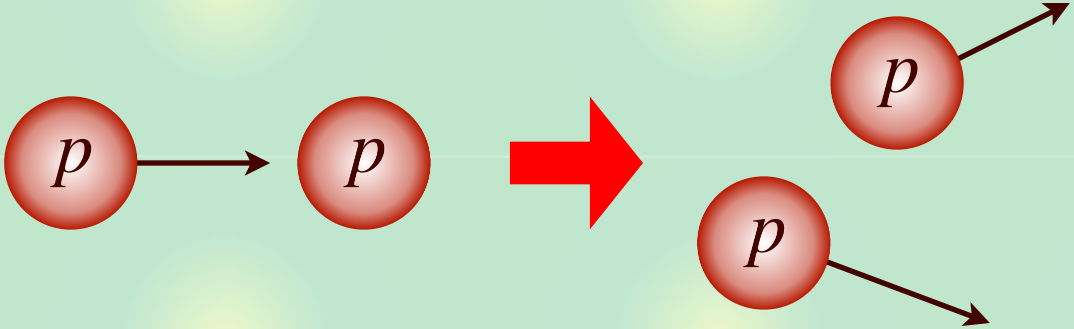
F. K. Guo, *et al.*, Rev. Mod. Phys. 90, 072501 (2018)

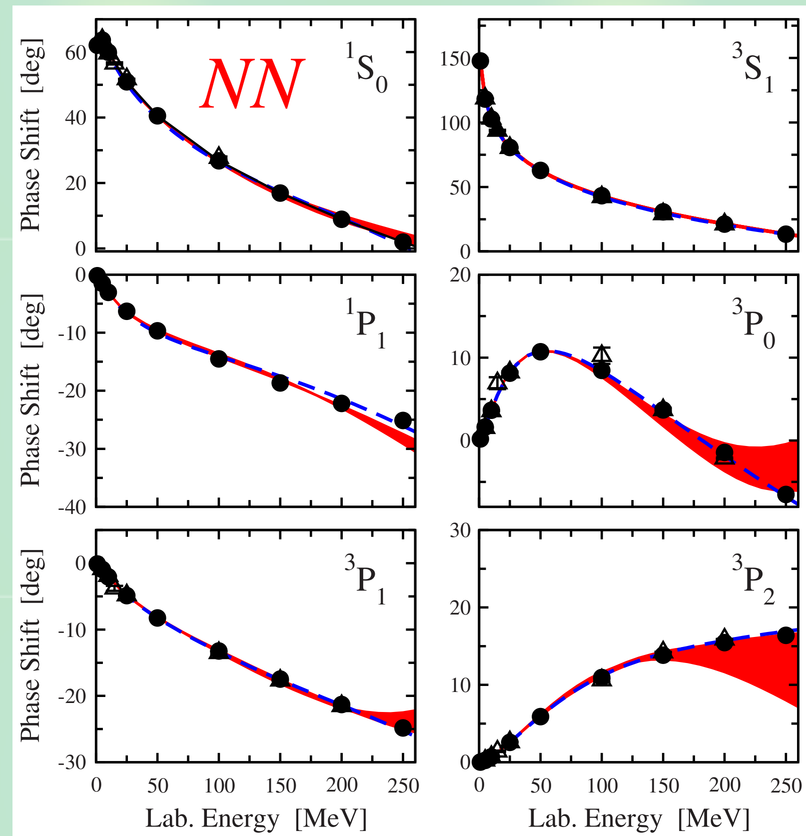
Study of hadron interactions

Traditional methods: scattering experiments

- differential cross sections -> phase shift

E. Epelbaum, H.W. Hammer, U.-G Meißner RMP 81, 1773 (2009)





Problems

- **Stable** beam/target particles
- Limited channels: $NN, \pi N, KN, \bar{K}N, \cdots$
- Heavy (c, b) hadrons: impossible

NN scattering is fine, but…

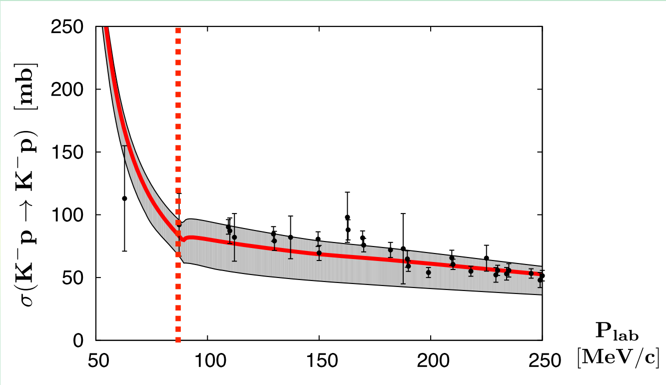
Scattering experiments and femtoscopy

$K^{-}p$ scattering

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

- low-energy beam is unstable

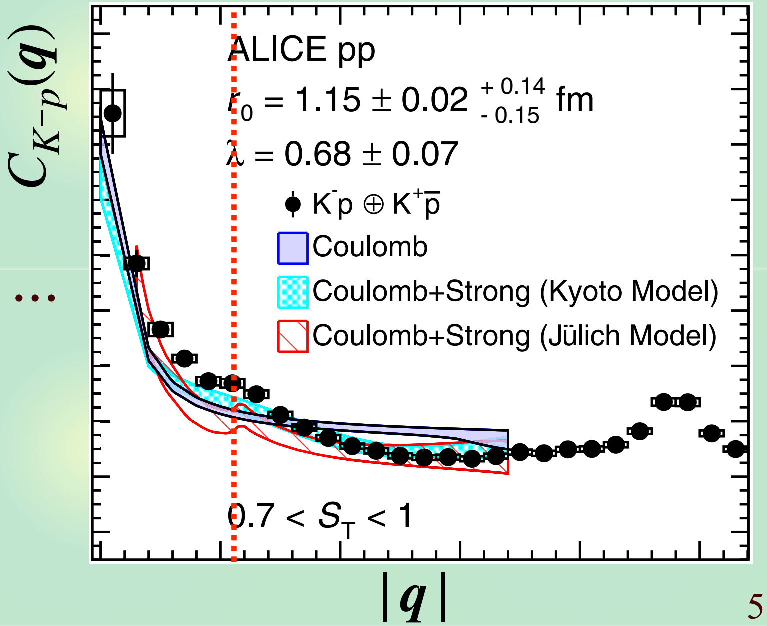
-> limited statistics



Femtoscopy: correlation function

ALICE collaboration, PRL 124, 092301 (2020)

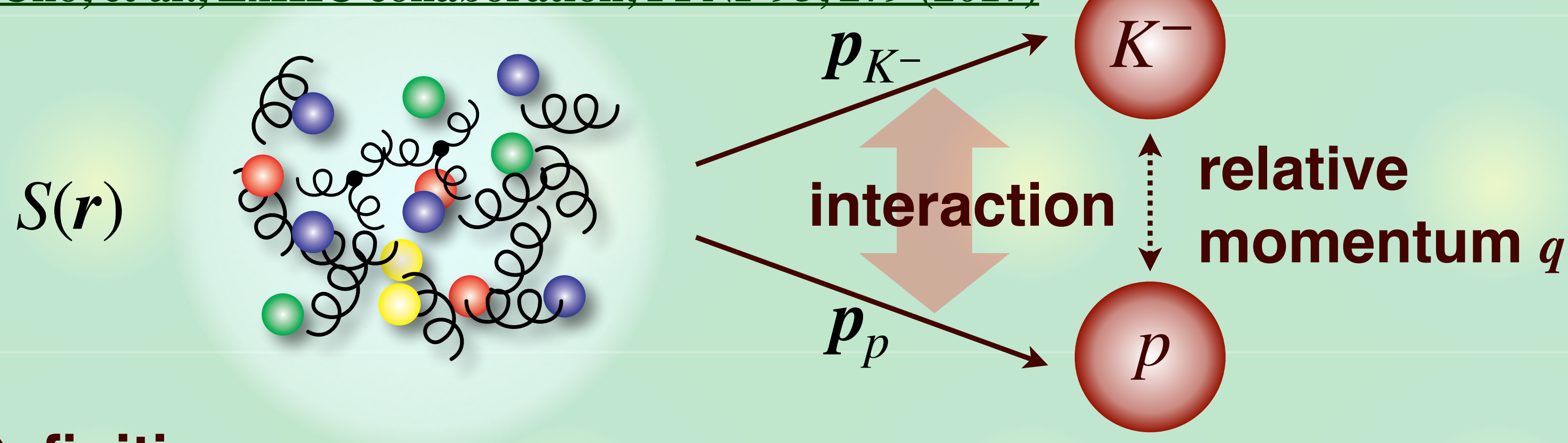
- **Various** systems: $\Lambda\Lambda, N\Omega, \phi N, \bar{K}\Lambda, DN, \cdots$
- Heavy hadrons: **possible!**
- Excellent **precision** ($\bar{K}^0 n$ cusp)



Correlation function and KP formula

High-energy collision: chaotic source $S(r)$ of hadron emission

S. Cho, et al., ExHIC collaboration, PPNP 95, 279 (2017)



- Definition

$$C(\boldsymbol{q}) = \frac{N_{K^-p}(\boldsymbol{p}_{K^-}, \boldsymbol{p}_p)}{N_{K^-}(\boldsymbol{p}_{K^-})N_p(\boldsymbol{p}_p)}$$

(= 1 in the absence of FSI/QS)

- Theory: KP (Koonin-Pratt) formula

S.E. Koonin, PLB 70, 43 (1977); S. Pratt, PRD 33, 1314 (1986)

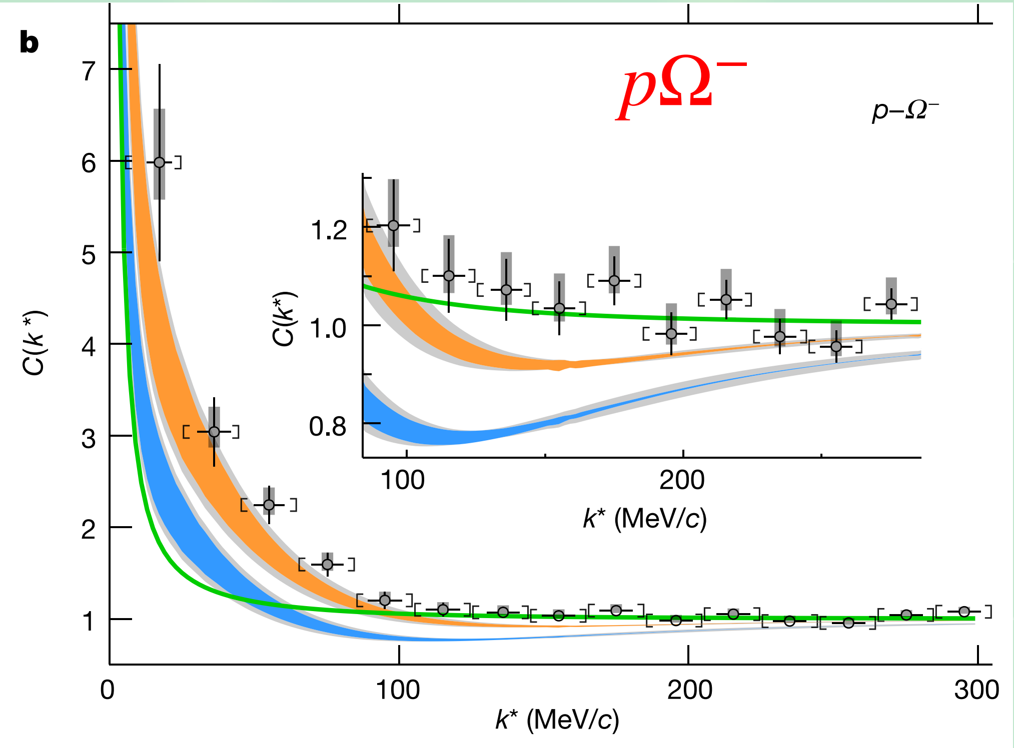
$$C(\boldsymbol{q}) \simeq \int d^3\boldsymbol{r}\, S(\boldsymbol{r})\, |\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})|^2$$

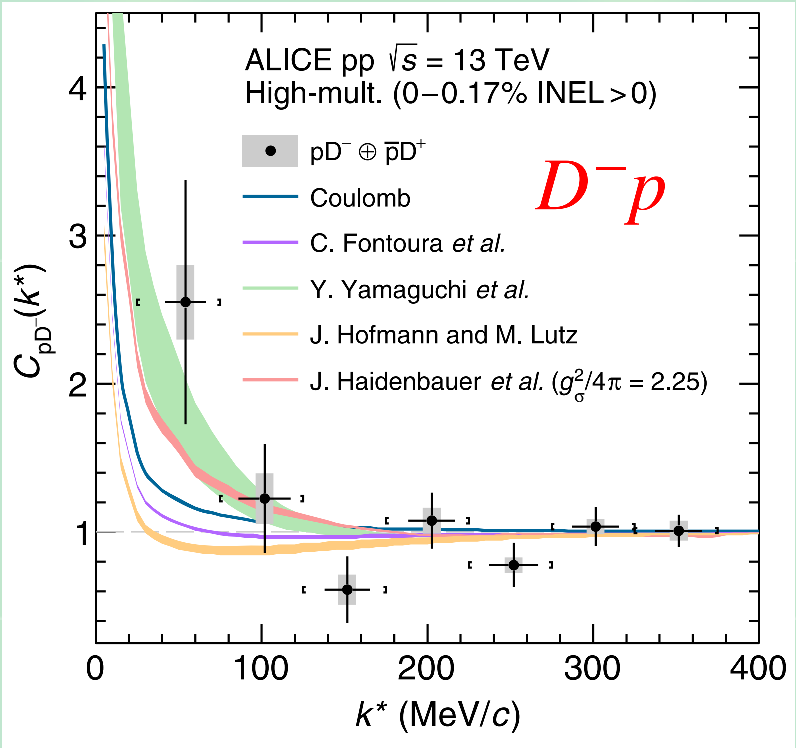
Source function $S(\boldsymbol{r})$ <—> wave function $\Psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r})$ (interaction)

Experimental data with strangeness and charm

Correlation functions observed by ALICE@LHC

ALICE collaboration, Nature 588, 232 (2020);
ALICE collaboration, PRD 106, 052010 (2022)





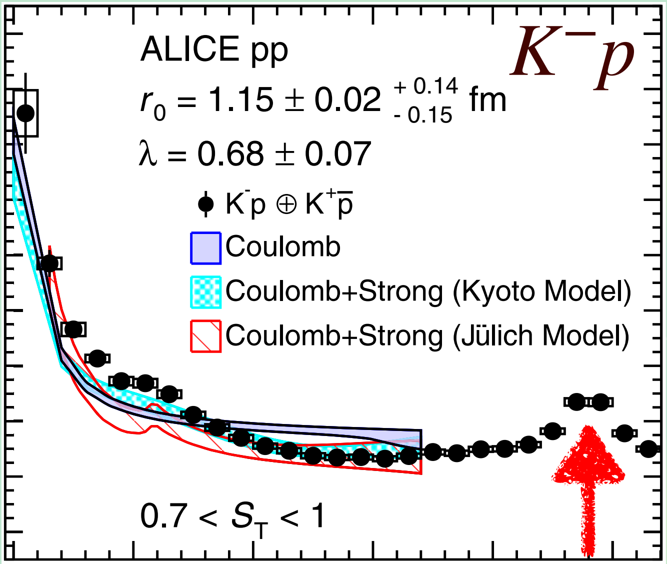
$\Omega^- \sim sss$: **strangeness** $S = -3$, $D^- \sim \bar{c}d$: **charm** $C = -1$

Complementary to scattering experiments

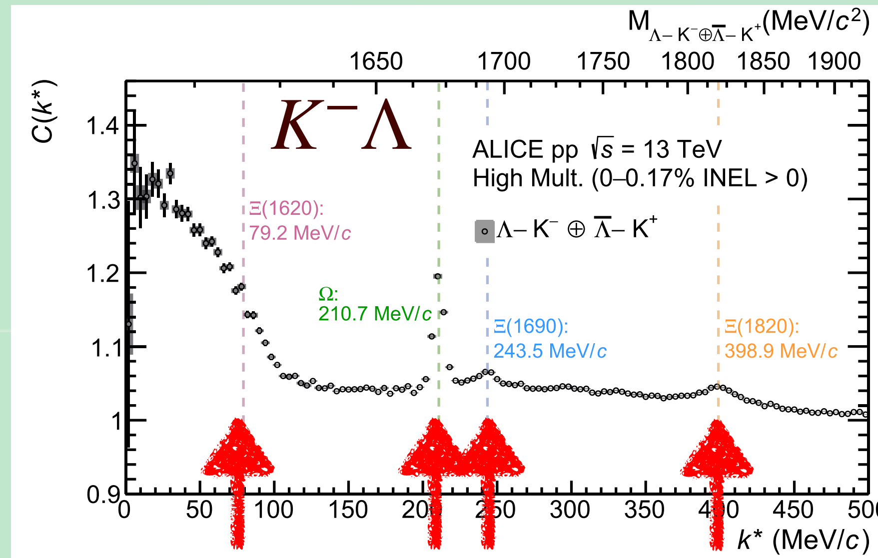
Resonances and higher partial waves

Resonance peaks in experimental data ($l = 0$ and $l \neq 0$)

ALICE collaboration, PRL 124, 092301 (2020); PLB845, 138145 (2023)



$\Lambda(1520)$: d wave



$\Xi(1620)$, $\Xi(1690)$: s wave,

Ω : p wave (weak decay),

$\Xi(1820)$: d wave

Questions:

- **Origin** of resonance peak?
- Usually s-wave only. How about **higher partial waves**?

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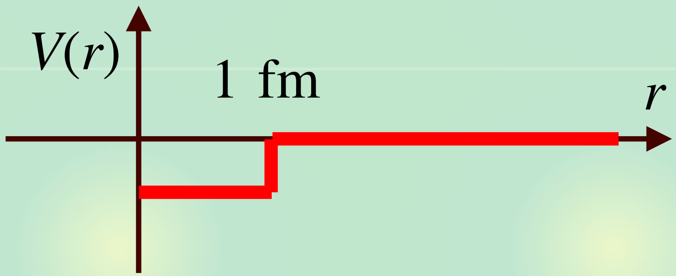
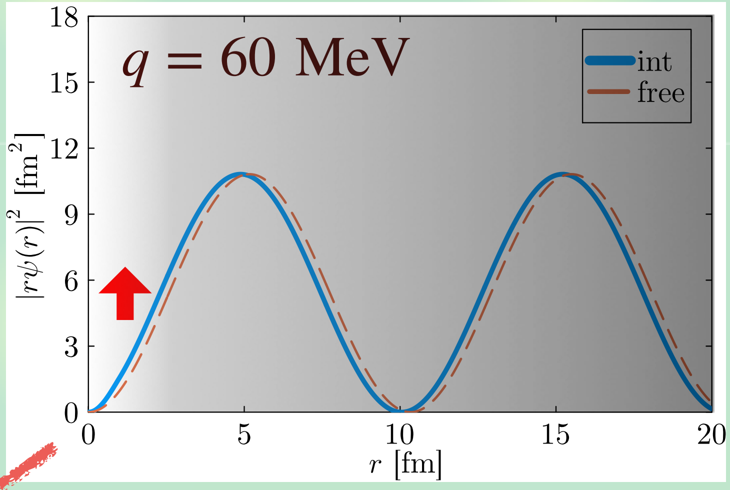
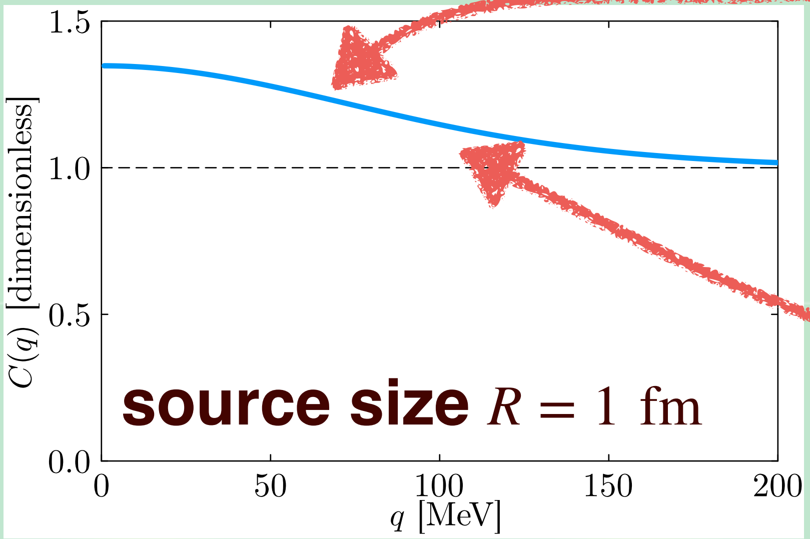
Wave function and correlation (attraction)

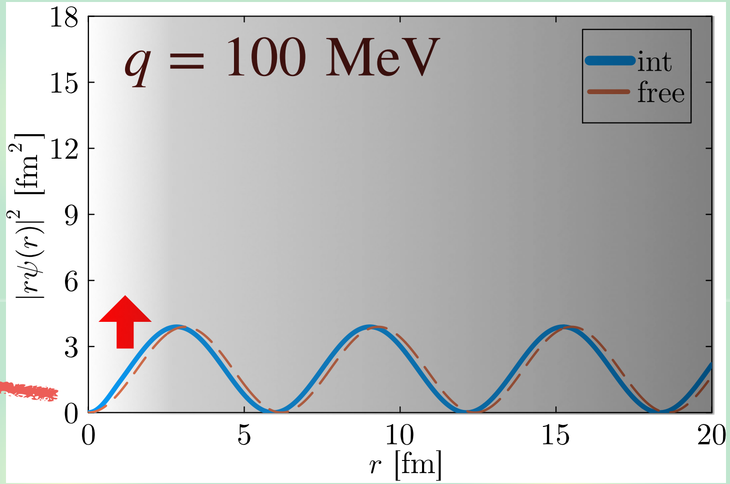
Attractive well (no bound state)



$$C(q) \simeq 1 + \int_0^\infty dr \, \frac{4\pi}{q^2} S(r) \{ \, | \, r q \psi_q(r) \, |^2 - \sin^2(qr) \}$$



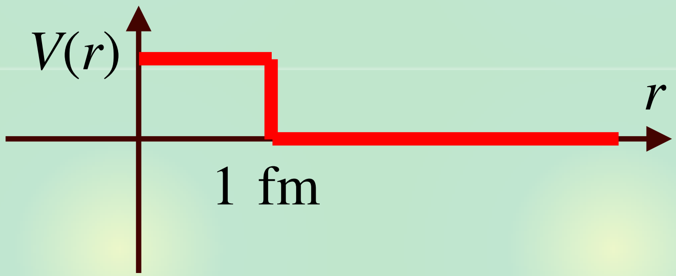


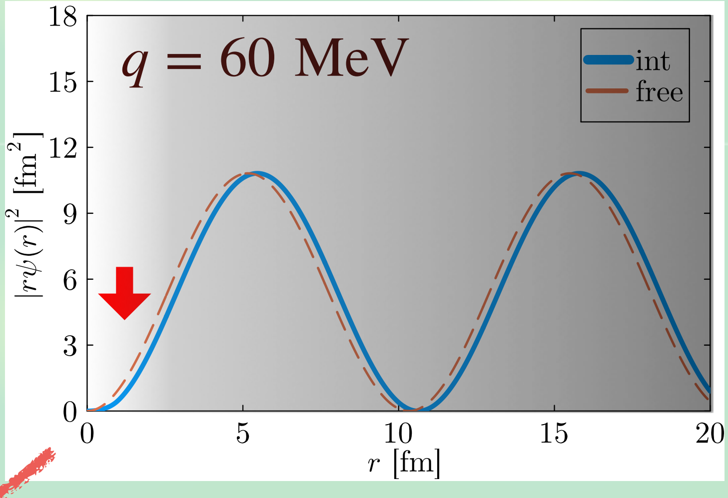


W.f. is pulled in, increased at $r \lesssim R$ —> $C(q)$ enhancement

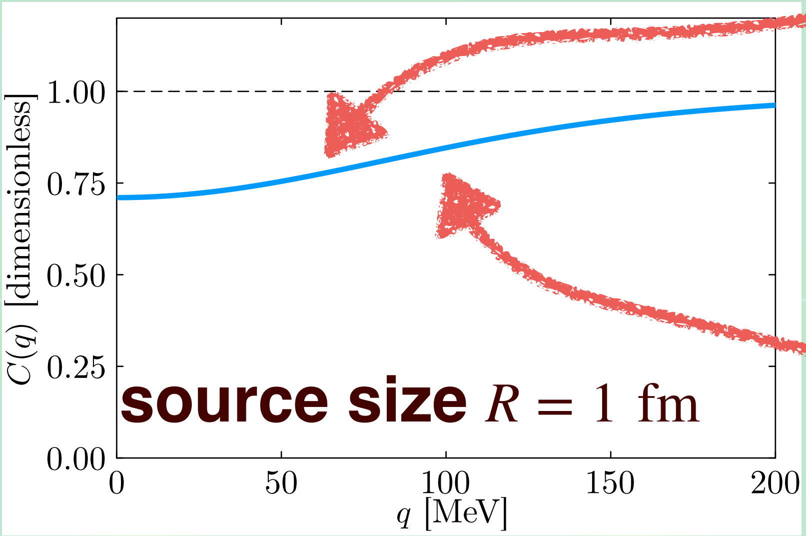
Wave function and correlation (repulsion)

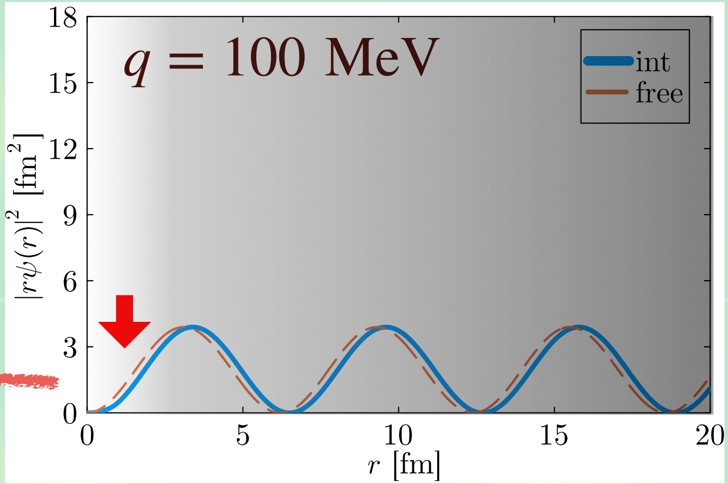
Repulsive rectangular potential





$$C(q) \simeq 1 + \int_0^\infty dr\ \frac{4\pi}{q^2} S(r)\{\,|\,|rq\psi_q(r)|^2 - \sin^2(qr)\}$$

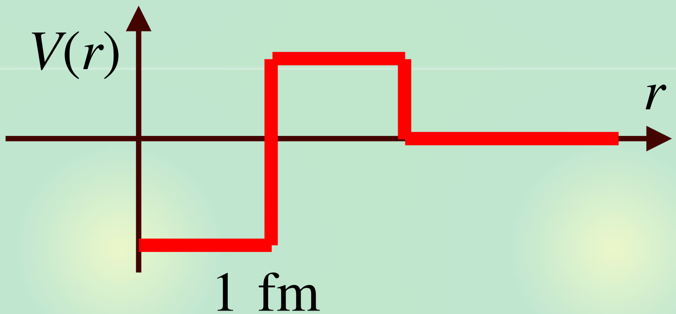




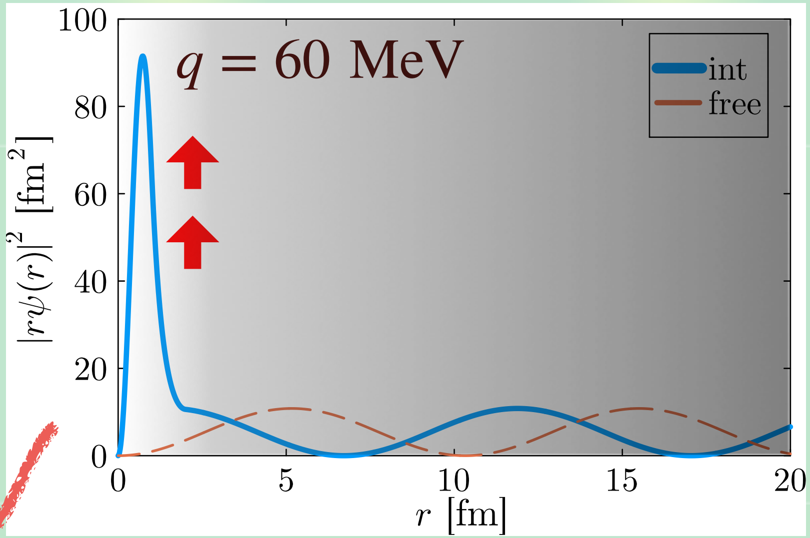
W.f. is pushed out, decreased at $r \lesssim R$ —> $C(q)$ suppression

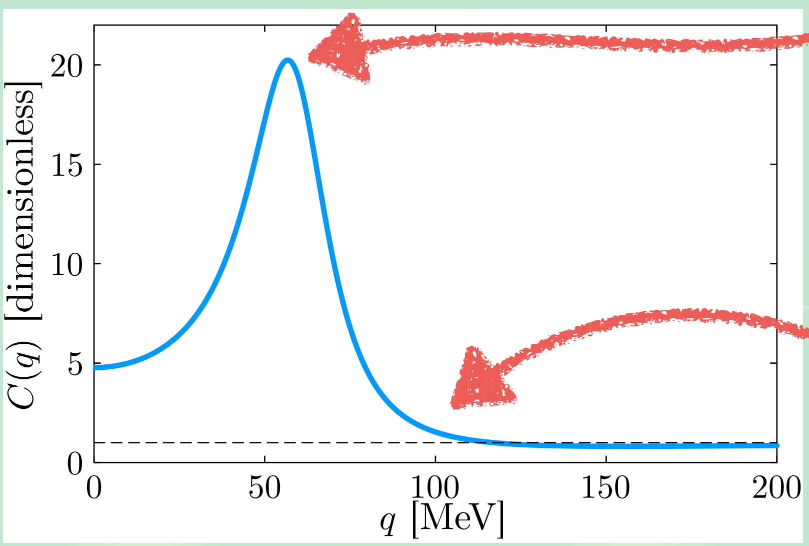
Wave function and correlation (resonance)

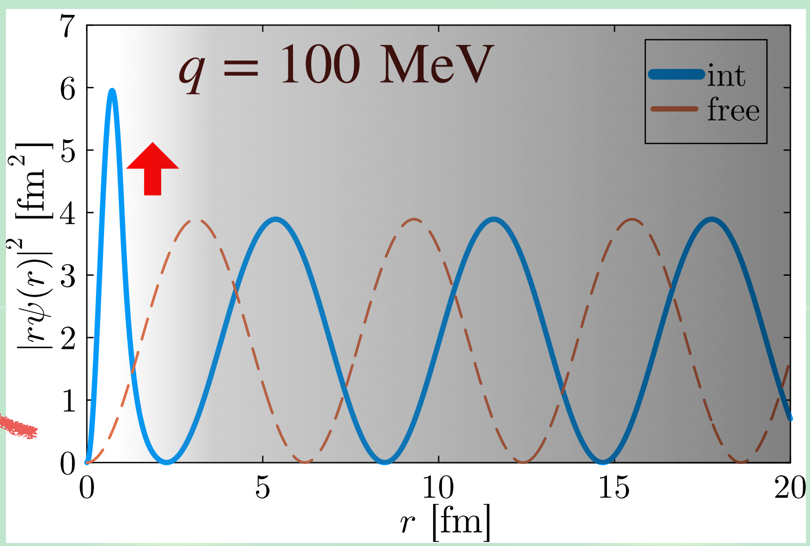
Well + barrier potential



- **resonance @** $q^{-} = 59 - 14i$ MeV







W.f. is localized in $r \lesssim R$ at pole momentum —> peak in $C(q)$

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Higher partial waves

KP formula with $l > 0$ (spherical source)

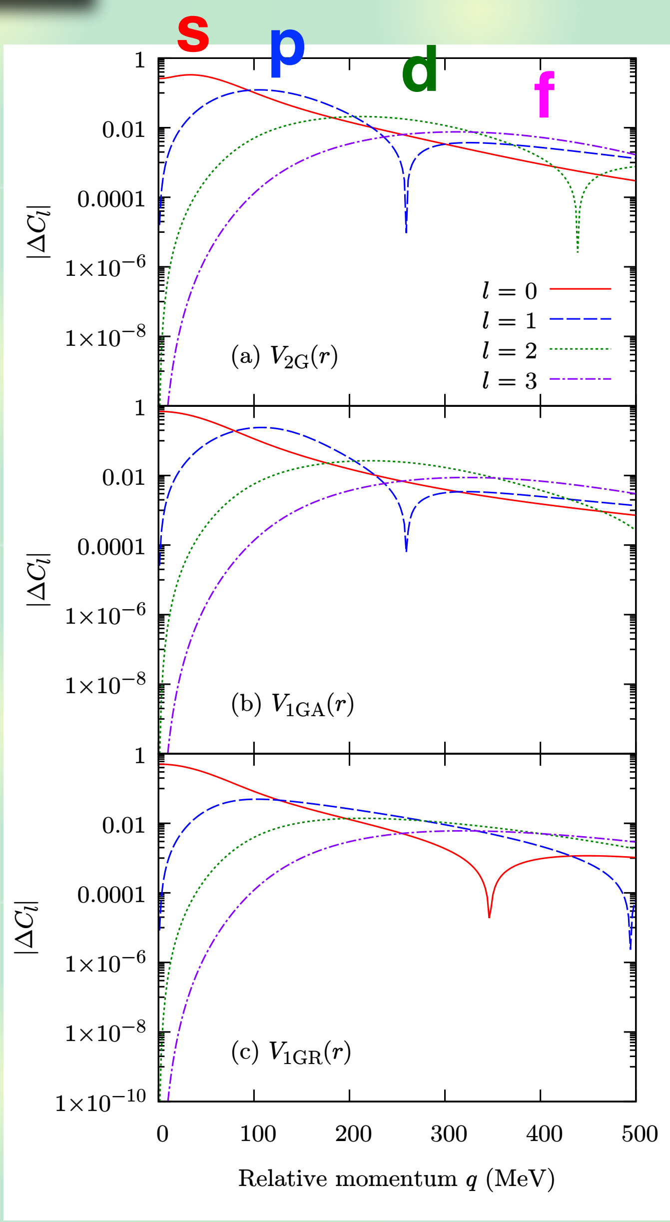
$$C(q) = 1 + \sum_{l=0}^{\infty} \Delta C_l(q)$$

$$\Delta C_l(q) = (2l+1) \times \int_0^{\infty} dr\, 4\pi r^2 S(r)[\,|R_l(r)|^2 - |j_l(qr)|^2]$$

- sum of partial wave contributions
- interacting w.f. $R_l(r)$ — free w.f. $j_l(qr)$

Gaussian potentials (range ~ 1.25 fm)

- $l > 0$ components at **larger** q
- l-th wave dominant at $q \sim l/r \sim 160l$ MeV



Resonance contribution

With $q \to \infty$, $C(q)$ approaches unity

- How can resonances be seen?

Resonances in higher partial waves

- p-wave resonance at

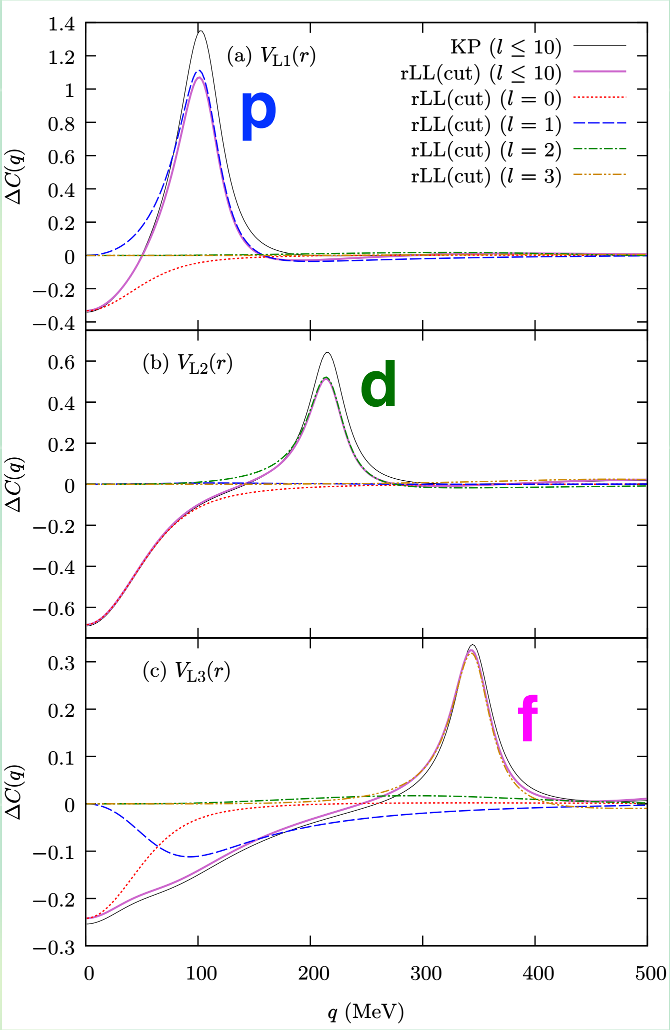
$$q^{-} \sim 105 - 23i \ \mathrm{MeV}$$

- d-wave resonance at

$$q^{-} \sim 216 - 20i \ \mathrm{MeV}$$

- f-wave resonance at

$$q^{-} \sim 345 - 21i \ \mathrm{MeV}$$



With resonances, $l > 0$ components is enahnced

LL formula

Lednicky-Lyuboshits (LL) formula for s-wave

R. Lednicky, V.L. Lyuboshits, Yad. Fiz. 35, 1316 (1981);

K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025)

- Gaussian (relative) source $S(r) = \exp(-r^2/4R^2)/(4\pi R^2)^{3/2}$
- Zero-range limit of interaction (asymptotic w.f. for entire r)

$$C(q) = 1 + \int_0^\infty dr\, \frac{4\pi}{q^2} S(r)[\sin^2(qr - \delta(q)) - \sin^2(qr)]$$

(Semi) analytic formula with only on-shell observable

$$C(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3(r_e/R) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R} F_2(2qR)$$

$$= 1 + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) + \frac{\mathrm{Im}\, f(q)}{2qR^2} \left(e^{-(2qR)^2} - \frac{r_e}{2\sqrt{\pi}R} \right)$$

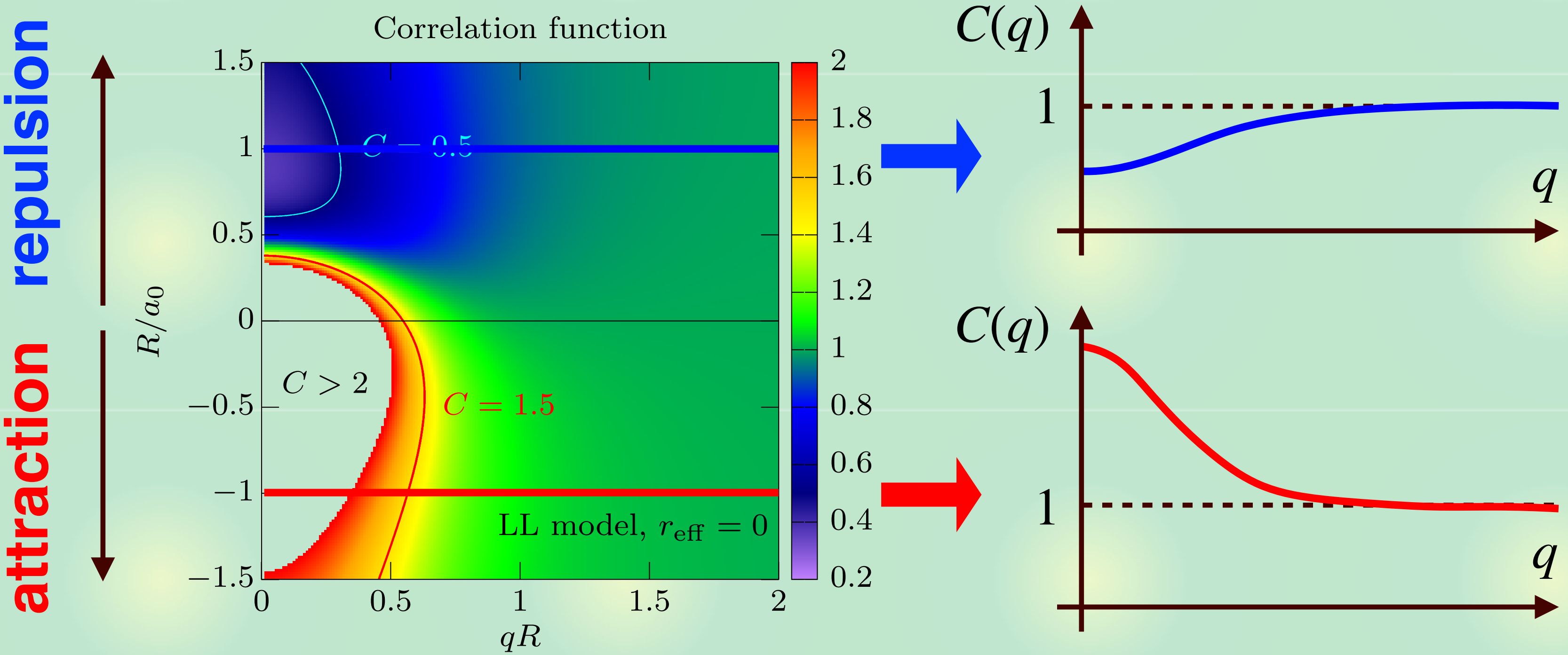
- Direct extraction of a_0 and r_e from experimental data

LL formula and correlations

LL formula with $r_e = 0$

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, T. Hatsuda, PRC 105, 014915 (2022)

- fixed $R > 0$
- **repulsion**: $R/a_0 > 0$, **attraction**: $R/a_0 < 0$



Consistent with KP formula

LL formula for higher partial wave?

Naive generalization for $l > 0$ is not possible

K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025)

$$\Delta C_l(q) = (2l+1)\int_0^\infty dr\, 4\pi r^2 S(r)[\,|R_l^{\text{aym}}(r)|^2 - |j_l(qr)|^2\,]$$

- $R_l^{\text{aym}}(r)$ **is too singular for $l > 0$ ($n_l(qr) \sim r^{-l-1}$) at $r \to 0$**
- **No sensible zero-range limit for $l > 0$**

Y. Nishida, Phys. Rev. A 86, 012710 (2012)

Some regularization for $r \to 0$ is needed

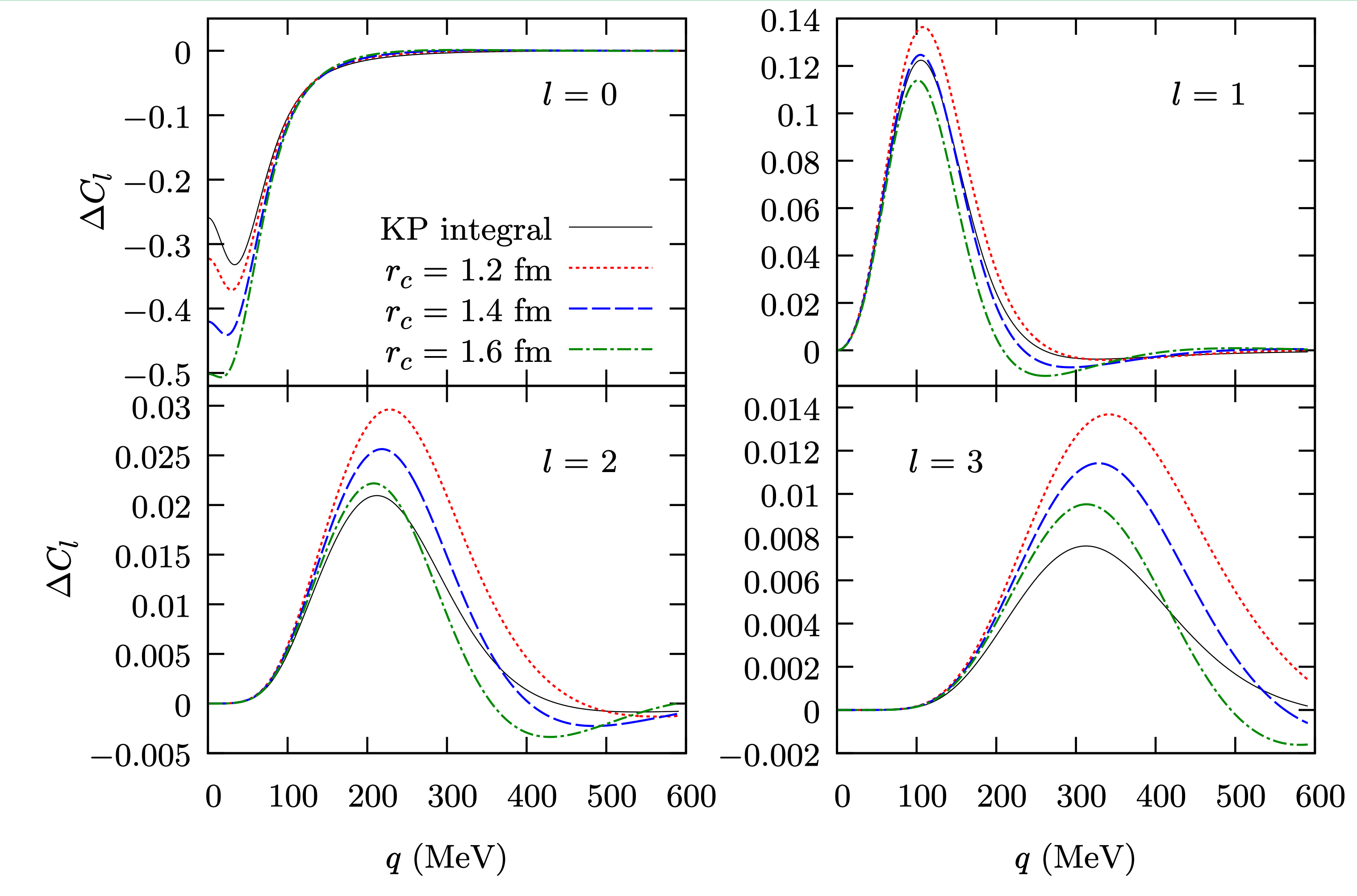
- **a simple prescription: introducing cutoff r_c**

$$\Delta C_l(q) = (2l+1)\int_{r_c}^\infty dr\, 4\pi r^2 S(r)[\,|R_l^{\text{aym}}(r)|^2 - |j_l(qr)|^2\,]$$

Contribution from higher partial waves

Regularized LL formula

Dependence on cutoff r_c



Works for $l > 0$ with r_c ~ interaction range (1.25 fm)

Summary

- **Resonace peaks in correlation functions**

- **s-wave resonance peak <— localization of wave function at interacting region**

  S. Watanabe, T. Hyodo, in preparation

- **Higher partial wave ($l > 0$) contributions becomes important for larger q**

- **Regularized LL formula with suitable cutoff r_c works for $l > 0$**

  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]