# Entanglement Suppression, Quantum Statistics and Symmetries in Spin-3/2 Baryon Scatterings

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# Contents

- Overview

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

  - Entanglement power
  - Entanglement suppression for $NN$ scattering

- SU(3) baryon decuplet scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025)

  - Spin 3/2 scattering
  - Application to $\Omega\Omega$ scattering

- Summary

# Basic concept of entanglement suppression

**Entanglement power (EP) of S-matrix** $E(\hat{S})$

**- ability of generating entanglement**

$|\psi_{\rm out}\rangle = \hat{S}\,|\psi_{\rm in}\rangle$

**entangle?**

## Entanglement suppression

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$E(\hat{S}) = 0$ —> **emergent symmetries of** $NN$ **scattering**

## Generalization to other hadrons and nuclei, …

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023);
T. Kirchner, W. Elkamhawy, H.-W. Hammer, Few-Body Syst. 65, 29 (2024);
T.R. Hu, S. Chen, F.K. Guo, PRD110, 014001 (2024); …
C.E.P. Robin, M.J. Savage, arXiv:2604.26376 [quat-ph]

# Symmetries of NN scattering

**Fundamental symmetries** $SU(2)_{\rm spin} \times SU(2)_{\rm isospin}$ **<— QCD**

- **spin** $SU(2)_{\rm spin}$ $\{|\uparrow\rangle, |\downarrow\rangle\}$
- **isospin** $SU(2)_{\rm isospin}$ $\{|p\rangle, |n\rangle\}$

$$\frac{1}{2} \otimes \frac{1}{2} = \underbrace{0}_{\rm A} \oplus \underbrace{1}_{\rm S}$$

**s-wave** $NN$ **scattering: spin** $J$ **× isospin** $I$ **should be** A

$|J=0, I=1\rangle$ $(^1S_0)$ **and** $|J=1, I=0\rangle$ $(^3S_1)$, **independent**

**Emergent symmetries <— entanglement suppression?**

- **large scattering length** $|a_0| \gg 1$ fm: **NR conformal symmetry**

T. Mehen, I.W. Stewart, M.B. Wise, PLB474, 145 (2000)

- **both** $^1S_0$ **and** $^3S_1$ **have large** $|a_0|$: **spin-flavor** $SU(4)$

$\{|p\uparrow\rangle, |p\downarrow\rangle, |n\uparrow\rangle, |n\downarrow\rangle\}$

E. Wigner, PR51, 106 (1937), …, D.B. Kaplan, M.J. Savage, PLB365, 244 (1996), …

# Entanglement measures

## Entanglement measure for bipartite state $|\psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_B$

- von Neumann entropy

$$\mathscr{E}_{\rm vN}(|\psi\rangle) = -\,\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$$

- linear entropy (expansion $\log \rho_A \sim (\rho_A - 1)$ )

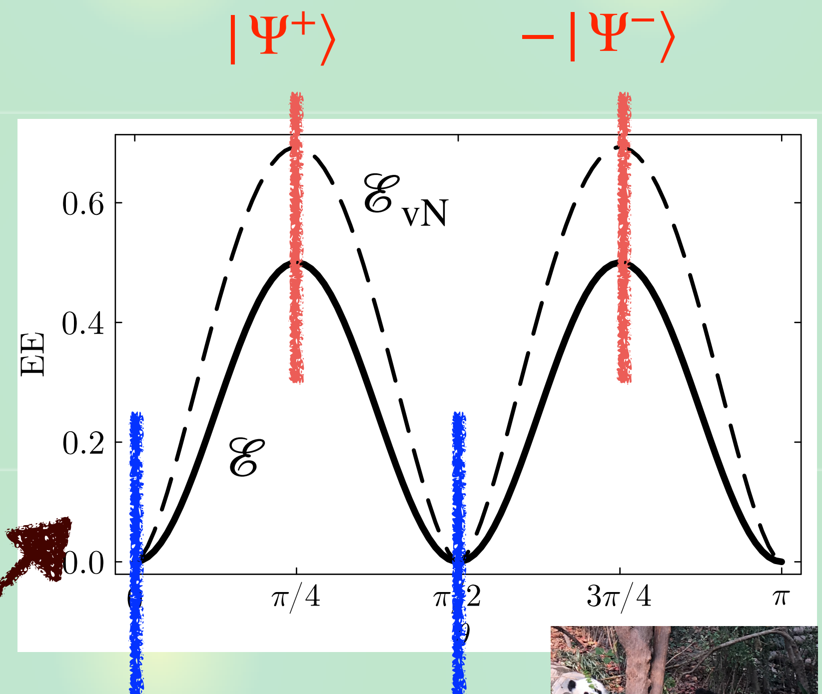
$$\mathscr{E}(|\psi\rangle) = 1 - \mathrm{Tr}_A[\rho_A^2]$$

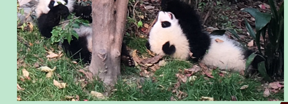
- product state: minimum (=0)

$$|\uparrow\downarrow\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle$$

- Bell state: maximum (=1/2)

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$



## Linear combination:

$$|\psi\rangle = \cos\theta |\uparrow\downarrow\rangle + \sin\theta |\downarrow\uparrow\rangle$$

# Entanglement power

## Entanglement power (EP) of operator $\hat{U}$ acting on $|\psi\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

- average over entanglement measure of $\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle$

$\mathscr{E}(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$

$$E(\hat{U}) = \overline{\mathscr{E}\left(\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle\right)}$$
$$= \int d\omega_A d\omega_B \; \mathscr{E}(\hat{U}|\psi_A(\omega_A)\rangle \otimes |\psi_B(\omega_B)\rangle)$$

**—> ability of $\hat{U}$ to generate entanglement (state independent)**

- **spin 1/2 (qubit):** rays in $\mathbb{C}^2 \simeq \mathbb{CP}^1$ **—> Fubini-Study measure**

$$|\psi(\omega_2)\rangle = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 e^{i\nu_1} \end{pmatrix}, \quad d\omega_2 = \frac{1}{\pi} d\theta_1 \cos\theta_1 \sin\theta_1 d\nu_1 \qquad \begin{matrix} \theta_1 \in [0,\pi/2) \\ \nu_1 \in [0,2\pi) \end{matrix}$$

**(equivalent to Bloch sphere with $\theta_1 = \theta/2,\ \nu_1 = \phi,\ d\omega_2 = d\Omega/4\pi$)**

# S-matrix for *NN* scattering

## S-matrix for *pn* scattering: 2 phase shifts $\delta_0, \delta_1$

$$\hat{S} = \underbrace{\exp\{2i\delta_0\}}_{J=0}\frac{1-\boldsymbol{\sigma}_A\cdot\boldsymbol{\sigma}_B}{4} + \underbrace{\exp\{2i\delta_1\}}_{J=1}\frac{3+\boldsymbol{\sigma}_A\cdot\boldsymbol{\sigma}_B}{4}$$

**(isospin is automatically determined by Pauli principle)**

## Entanglement power of *NN* scattering

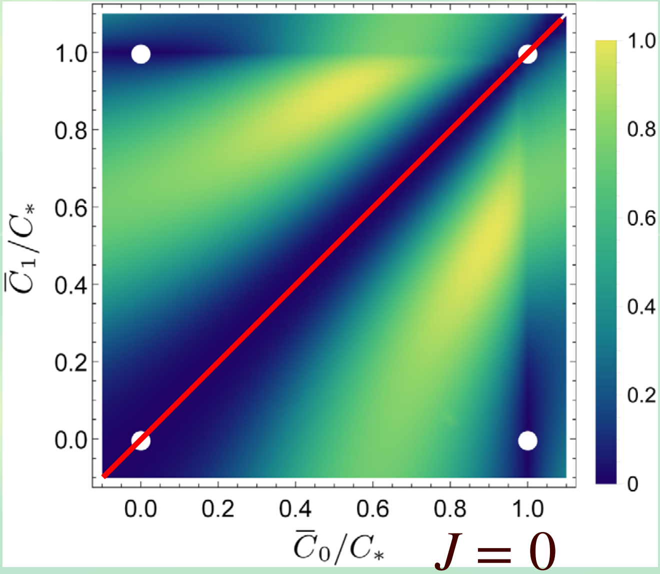
**S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)**

$$E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0-\delta_1)] \geq 0$$

$E(\hat{S}) = 0$ **is achieved if**

$$\begin{cases} \delta_0 = \delta_1 & \textbf{spin-flavor SU(4)} \\ (\delta_0, \delta_1) = (0,0), (0,\pi/2), (\pi/2,0), (\pi/2,\pi/2) \end{cases}$$

$|a_0| \to \infty$ **: NR conformal symmetry**



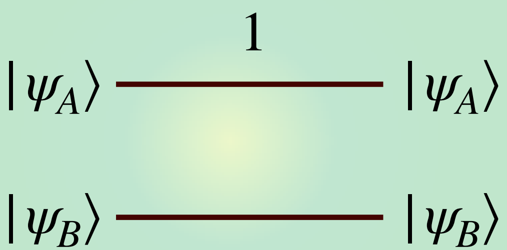
# Interpretation by quantum information

## S-matrix with $E(\hat{S}) = 0$: product state —> product state

I. Low, T. Mehen, PRD104, 074014 (2021)

- **Identity gate:** $|\delta_0 - \delta_1| = 0$

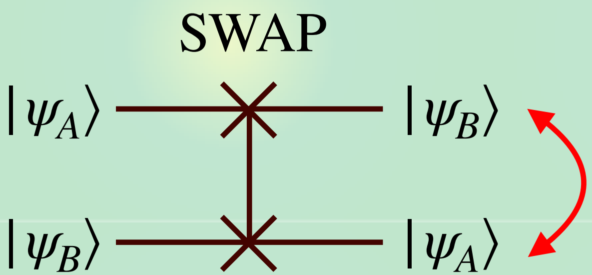
$$\hat{S} \propto \frac{1 - \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} + \frac{3 + \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} = 1$$




- **SWAP gate:** $|\delta_0 - \delta_1| = \pi/2$ **(spin exchange operator $P_s$)**

$$\hat{S} \propto -\frac{1 - \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} + \frac{3 + \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4}$$

$$= \frac{1 + \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{2} = \mathrm{SWAP}$$




**Identity and SWAP are the only minimal entanglers**

**( <— Cartan decomposition of $\mathrm{SU}(4)/(\mathrm{SU}(2) \times \mathrm{SU}(2))$ )**

# Further evidences of entanglement supression

## Spin-flavor symmetries with three flavors

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

- $\mathrm{SU}(6) \sim (u, d, s) \otimes (\uparrow, \downarrow)$ <— large Nc
- $\mathrm{SU}(16) \sim (n, p, \Lambda, \ldots) \otimes (\uparrow, \downarrow)$ <— Entanglement suppression
- Lattice QCD evaluation of LEC favors $\mathrm{SU}(16)$

M.L. Wagman, *et al.*, (NPLQCD), PRD96, 114510 (2017)

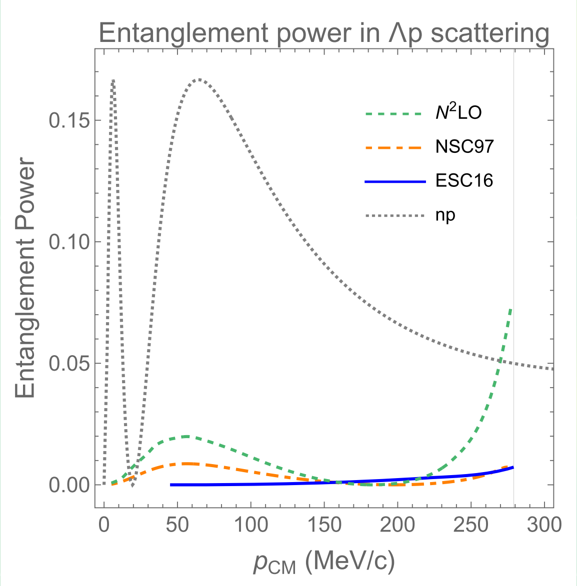
## Momentum dependence of Entangelement power

Q. Liu, I. Low, PLB, 856, 138899 (2024)

- $np, \Lambda p$ scatterings

$$E(p_{\mathrm{CM}}) = \frac{1}{6}\sin^2[2(\delta_0(p_{\mathrm{CM}}) - \delta_1(p_{\mathrm{CM}}))]$$

- EP suppressed at low energies



# Summary of overview part

- **Entanglement power** of S-matrix ~ ability of generating entanglement

$$E(\hat{S}) = \overline{\mathscr{E}(\hat{S}\,|\psi_A\rangle \otimes |\psi_B\rangle)}$$

- Entanglement suppression: **requireing** $E(\hat{S}) = 0$ for $NN$ scattering, the following symmetries emerge

  - spin-flavor $\mathrm{SU}(4)$ symmetry ($\hat{S} \propto 1$)

  - NR conformal symmetry ($\hat{S} \propto \mathrm{SWAP}$)

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

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- **Overview**

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I. Low, T. Mehen, PRD104, 074014 (2021)

  - Entanglement power
  - Entanglement suppression for $NN$ scattering

- $\mathrm{SU}(3)$ **baryon decuplet scattering**

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025)

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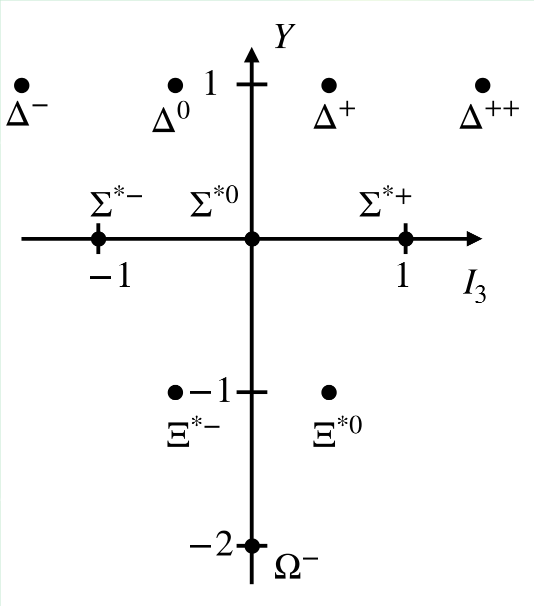
  - Application to $\Omega\Omega$ scattering

- **Summary**

# Motivation

## Decuplet baryons

- spin 3/2
- ground states with Octet in quark model




## Large scattering length $a_0$ ($\delta \sim \pi/2$)?

- ΔΔ dibaryon, $d^*(2380)$?

F. Dyson, N.H. Xuong, PRL13, 815 (1964);
P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for ΩΩ scattering: $a_0(J=0) = 4.6 \text{ fm} \gg 1 \text{ fm}$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

## Quantum information viewpoint

- spin 3/2 scattering = 4d-qudit quantum gates

# Spin 3/2 state

**Spin 3/2 (4d-qudit):** rays in $\mathbb{C}^4 \simeq \mathbb{CP}^3$

$$|\psi_{\mathbf{4}}\rangle = \begin{pmatrix} \cos\theta_1 \sin\theta_2 \sin\theta_3 \\ \sin\theta_1 \sin\theta_2 \sin\theta_3 e^{i\nu_1} \\ \cos\theta_2 \sin\theta_3 e^{i\nu_2} \\ \cos\theta_3 e^{i\nu_3} \end{pmatrix}, \quad d\omega_{\mathbf{4}} = \frac{3!}{\pi^3}\prod_{i=1}^{3} d\theta_i d\nu_i \cos\theta_i \sin^{2i-1}\theta_i,$$

$$\theta_i \in [0,\pi/2), \quad \nu_i \in [0,2\pi)$$

**Spin decomposition (2 symmetric, 2 anti-symmetric)**

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\mathrm{A}} \oplus \underbrace{1 \oplus 3}_{\mathrm{S}}$$

**S-matrix: 4 components, 4 phase shifts** $\delta_0, \delta_1, \delta_2, \delta_3$

$$\hat{S} = \mathcal{J}_0 \exp\{2i\delta_0\} + \mathcal{J}_1 \exp\{2i\delta_1\} + \mathcal{J}_2 \exp\{2i\delta_2\} + \mathcal{J}_3 \exp\{2i\delta_3\}$$

$$\mathcal{J}_0 = \frac{33}{128} + \frac{31}{96}\left(\boldsymbol{t}_{3/2}\cdot \boldsymbol{t}_{3/2}\right) - \frac{5}{72}\left(\boldsymbol{t}_{3/2}\cdot \boldsymbol{t}_{3/2}\right)^2 - \frac{1}{18}\left(\boldsymbol{t}_{3/2}\cdot \boldsymbol{t}_{3/2}\right)^3, \cdots$$

# Entanglement power

## Entanglement power (~3 days on mathematica)

$$E(\hat{S}) = \frac{1}{200000}\Big\{77482 - 2100\cos\left[2\left(\delta_0+\delta_1-\delta_2-\delta_3\right)\right]$$
$$-2100\cos\left[2\left(\delta_0-\delta_1+\delta_2-\delta_3\right)\right] - 2100\cos\left[2\left(\delta_0-\delta_1-\delta_2+\delta_3\right)\right]$$
$$-1200\cos\left[2\left(\delta_0-2\delta_1+\delta_2\right)\right] - 4200\cos\left[2\left(\delta_0+\delta_2-2\delta_3\right)\right]$$
$$-8400\cos\left[2\left(\delta_1-2\delta_2+\delta_3\right)\right]$$
$$-375\cos\left[4\left(\delta_0-\delta_1\right)\right] - 10800\cos\left[2\left(\delta_0-\delta_2\right)\right] - 625\cos\left[4\left(\delta_0-\delta_2\right)\right]$$
$$-875\cos\left[4\left(\delta_0-\delta_3\right)\right] - 2175\cos\left[4\left(\delta_1-\delta_2\right)\right] - 26376\cos\left[2\left(\delta_1-\delta_3\right)\right]$$
$$-5481\cos\left[4\left(\delta_1-\delta_3\right)\right] - 10675\cos\left[2\left(\delta_2-\delta_3\right)\right]\Big\}$$

**$E(\hat{S}) = 0$ is achieved if (all cos = +1)**

$$\delta_0 = \delta_2 = \delta_{\rm even}, \quad \delta_1 = \delta_3 = \delta_{\rm odd}, \quad \begin{cases} |\delta_{\rm even} - \delta_{\rm odd}| = 0 & \text{: Identity gate} \\ |\delta_{\rm even} - \delta_{\rm odd}| = \pi/2 & \text{: SWAP gate} \end{cases}$$

# Minimal entanglers

**S-matrix with** $E(\hat{S}) = 0$

- **Identity gate:** $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0$

$$\hat{S} \propto \mathcal{J}_0 + \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 = 1$$

- **SWAP gate:** $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$

$$\hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_1 - \mathcal{J}_2 + \mathcal{J}_3 = \sum_i \mathcal{S}_i - \sum_i \mathcal{A}_i$$

**—> Minimal entanglement is achived by Identity and SWAP, also for spin 3/2 scattering**

**Identity and SWAP are the only minimal entanglers for any bipartite system with same dimension**

E. Alfsen, F. Shultz, JMP51, 052201 (2010);…

# Decuplet-decuplet scattering

## Spin-flavor decomposition: spin⊗flavor should be A

- **spin** $\mathrm{SU}(2)_{\mathrm{spin}}$ $$\frac{3}{2}\otimes\frac{3}{2}=\underbrace{0\oplus 2}_{\mathrm{A}}\oplus\underbrace{1\oplus 3}_{\mathrm{S}}$$

- **flavor** $\mathrm{SU}(3)_{\mathrm{flavor}}$ $$\mathbf{10}\otimes\mathbf{10}=\underbrace{\mathbf{27}\oplus\mathbf{28}}_{\mathrm{S}}\oplus\underbrace{\overline{\mathbf{10}}\oplus\mathbf{35}}_{\mathrm{A}}$$

**8 phase shifts** $\delta_{0,\mathbf{27}},\ \delta_{0,\mathbf{28}},\ \delta_{1,\overline{\mathbf{10}}},\ \delta_{1,\mathbf{35}},\ \delta_{2,\mathbf{27}},\ \delta_{2,\mathbf{28}},\ \delta_{3,\overline{\mathbf{10}}},\ \delta_{3,\mathbf{35}}$

$$\hat{S}=\mathcal{J}_0\otimes\left(\mathcal{F}_{\mathbf{27}}e^{2i\delta_{0,\mathbf{27}}}+\mathcal{F}_{\mathbf{28}}e^{2i\delta_{0,\mathbf{28}}}\right)+\mathcal{J}_1\otimes\left(\mathcal{F}_{\overline{\mathbf{10}}}e^{2i\delta_{1,\overline{\mathbf{10}}}}+\mathcal{F}_{\mathbf{35}}e^{2i\delta_{1,\mathbf{35}}}\right)$$
$$+\mathcal{J}_2\otimes\left(\mathcal{F}_{\mathbf{27}}e^{2i\delta_{2,\mathbf{27}}}+\mathcal{F}_{\mathbf{28}}e^{2i\delta_{2,\mathbf{28}}}\right)+\mathcal{J}_3\otimes\left(\mathcal{F}_{\overline{\mathbf{10}}}e^{2i\delta_{3,\overline{\mathbf{10}}}}+\mathcal{F}_{\mathbf{35}}e^{2i\delta_{3,\mathbf{35}}}\right)$$

$E(\hat{S})=0$ **is achieved if**

$$\delta_{0,F}=\delta_{2,F'}=\delta_{\mathrm{even}},\quad \delta_{1,F}=\delta_{3,F'}=\delta_{\mathrm{odd}},\quad \begin{cases}|\delta_{\mathrm{even}}-\delta_{\mathrm{odd}}|=0\\ |\delta_{\mathrm{even}}-\delta_{\mathrm{odd}}|=\pi/2\end{cases}$$

# Emergent symmetries

## Emergent symmetry <— effective Lagrangian

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023)

- spin $i = 1,2,3,4$, flavor $T = (\Delta^{++}, \ldots, \Omega^{-})$

## Effective Lagrangian for $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0$

$$\mathscr{L} = c_1 (\boldsymbol{T}^{\dagger i} \cdot \boldsymbol{T}^{i})^2$$

- rotation of 40 component spin-flavor vector
  —> $\mathrm{SU}(40)$ spin-flavor symmetry

## Effective Lagrangian for $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$

$$\mathscr{L} = -\frac{2\pi}{\mu\Lambda}\left[-\frac{1}{4}\left(\boldsymbol{T}^{\dagger i} \cdot \boldsymbol{T}^{i}\right)^2 \pm \frac{1}{4}\left(\boldsymbol{T}^{\dagger i} \cdot \boldsymbol{T}^{j}\right)\left(\boldsymbol{T}^{\dagger j} \cdot \boldsymbol{T}^{i}\right)\right]$$

- independent rotation of 4 spin and 10 flavor
  —> $\mathrm{SU}(4)_{\text{spin}} \times \mathrm{SU}(10)_{\text{flavor}}$ + NR conformal symmetry

# Emergent symmetries

## Emergent symmetry <— degeneracy counting

| | $\delta_{\text{even}}(J=0,2,\ F=\mathbf{27},\mathbf{28})$ | | $\delta_{\text{odd}}(J=1,3,\ F=\overline{\mathbf{10}},\mathbf{35})$ | |
|---|---|---|---|---|
| **spin** | $1+5=6$ | | $3+7=10$ | |
| **flavor** | $27+28=55$ | | $10+35=45$ | |
| **total** | 330 | + | 450 | $=780$ |

**- Identity gate:** $|\delta_{\text{even}}-\delta_{\text{odd}}|=0$ **—> spin-flavor** $\mathrm{SU}(40)$

$$\mathbf{40}\otimes\mathbf{40}=\mathbf{820}\oplus\mathbf{780}$$

**- SWAP gate:** $|\delta_{\text{even}}-\delta_{\text{odd}}|=\pi/2$

**—>** $\mathrm{SU}(4)_{\text{spin}}\times\mathrm{SU}(10)_{\text{flavor}}$ **+ NR conformal symmetry**

$$\mathbf{4}\otimes\mathbf{4}=\mathbf{10}\oplus\mathbf{6},\quad \mathbf{10}\otimes\mathbf{10}=\mathbf{55}\oplus\mathbf{45}$$

**Justified by effective Lagrangian approach**

# Idntical particles and sub-unitary S-matrix

**Identical particles: symmetric spin states are forbidden**

**- $nn$ scattering/ΩΩ scattering: only antisymmetric component**

$$\hat{S}_{nn} = \mathscr{I}_0 \exp\{2i\delta_0\}$$ **spin 1 absent**

$$\hat{S}_{\Omega\Omega} = \mathscr{I}_0 \exp\{2i\delta_{0,\mathbf{28}}\} + \mathscr{I}_2 \exp\{2i\delta_{2,\mathbf{28}}\}$$ **spin 1 and 3 absent**

**S-matrix is not unitary in full space of** $|\psi_{\rm in}\rangle$ **:** $\hat{S}_{nn}\hat{S}^\dagger_{nn} = \mathscr{I}_0 \neq 1$

$$\hat{S} = \begin{pmatrix} (\text{Antisymmetric}) & 0 \\ 0 & (\text{Symmetric}) \end{pmatrix}, \quad \mathscr{I}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

**Density matrix of out state is not properly normalized**

$$\rho = |\psi_{\rm out}\rangle\langle\psi_{\rm out}| = \hat{S}_{nn}|\psi_{\rm in}\rangle\langle\psi_{\rm in}|\hat{S}^\dagger_{nn}, \quad {\rm Tr}[\rho] = \langle\psi_{\rm in}|\mathscr{I}_0|\psi_{\rm in}\rangle < 1$$

**—> Properly normalized Entanglement power**

# Normalized entanglement power

## Normalized density matrix

M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge

$$\tilde{\rho} = \frac{|\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|}{\langle\psi_{\text{out}}|\psi_{\text{out}}\rangle}$$

## k-weighted normalized entanglement power

$$E_k(\hat{S}) = 1 - \frac{\int \mathrm{d}\omega_A \mathrm{d}\omega_B \langle\psi_{\text{in}}|\mathscr{P}_A|\psi_{\text{in}}\rangle^k \ \mathrm{Tr}_A\left[\tilde{\rho}_A{}^2\right]}{\int \mathrm{d}\omega_A \mathrm{d}\omega_B \langle\psi_{\text{in}}|\mathscr{P}_A|\psi_{\text{in}}\rangle^k}$$

## For $nn$ scattering, result does not depend on $k$

$$E_k(\hat{S}_{nn}) = \frac{1}{2}$$

## $k = 2$ is practically convenient

$$E_2(\hat{S}) = 1 - \frac{\int \mathrm{d}\omega_A \mathrm{d}\omega_B \, \mathrm{Tr}_A\left[\rho_A{}^2\right]}{\int \mathrm{d}\omega_A \mathrm{d}\omega_B \langle\psi_{\text{in}}|\mathscr{P}_A|\psi_{\text{in}}\rangle^2}$$

# ΩΩ scattering

## ΩΩ S-matrix

$$\hat{S}_{\Omega\Omega} = \mathcal{J}_0 \exp\{2i\delta_{0,\mathbf{28}}\} + \mathcal{J}_2 \exp\{2i\delta_{2,\mathbf{28}}\}$$

## Properly normalized EP of ΩΩ S-matrix

$$E_2(\hat{S}_{\Omega\Omega}) = \frac{1}{48}\left\{25 - \cos\left[4\left(\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}\right)\right]\right\}$$

$E_2(\hat{S}_{\Omega\Omega})$ **is minimized (1/2, not 0) if**

$$\begin{cases} |\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0 & \hat{S} \propto \mathcal{J}_0 + \mathcal{J}_2 \equiv \mathcal{P}_{\mathrm{A}} \\ |\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2 & \hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_2 \equiv \mathrm{SWAP}_{\mathrm{A}} \end{cases}$$

$a_0(J=0) \gg 1\ \mathrm{fm}$ **by lattice QCD suggests either**

- $|a_0(J=2)| \gg 1\ \mathrm{fm}$ **(unitary limit,** $|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0$**) or**
- $a_0(J=2) \sim 0$ **(noninteracting,** $|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2$**)**

# Summary

- Entanglement suppression for baryon decuplet
- $E(\hat{S}) = 0$ is achieved only by $\hat{S} \propto 1$ or $\hat{S} \propto \mathrm{SWAP}$ for spin 3/2 scattering
- Largest emergent symmetries
  - spin-flavor $\mathrm{SU}(40)$ symmetry ($\hat{S} \propto 1$)
  - $\mathrm{SU}(4)_{\mathrm{spin}} \times \mathrm{SU}(10)_{\mathrm{flavor}}$ + NR conformal ($\hat{S} \propto \mathrm{SWAP}$)
- Sub-unitary S-matrix
  - Final state should be properly normalized
  - $\Omega\Omega$ scattering: implication to spin 2 channel

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025)