

Higher partial waves and resonance contributions in femtoscopy



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2026, Jun. 3rd

Contents




Introduction — motivation



Contribution from s-wave resonance

[S. Watanabe, T. Hyodo, in preparation](#)

- Origin of resonance peak



Contribution from higher partial waves

[K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 \(2025\);](#)

[K. Murase, T. Hyodo, arXiv:2509.22844 \[nucl-th\]](#)

- Correlation function with $l > 0$
- Regularized LL formula for $l > 0$

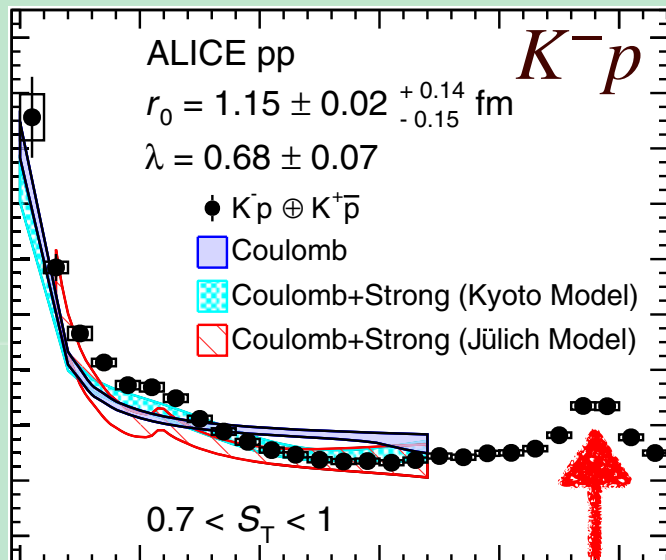


Summary

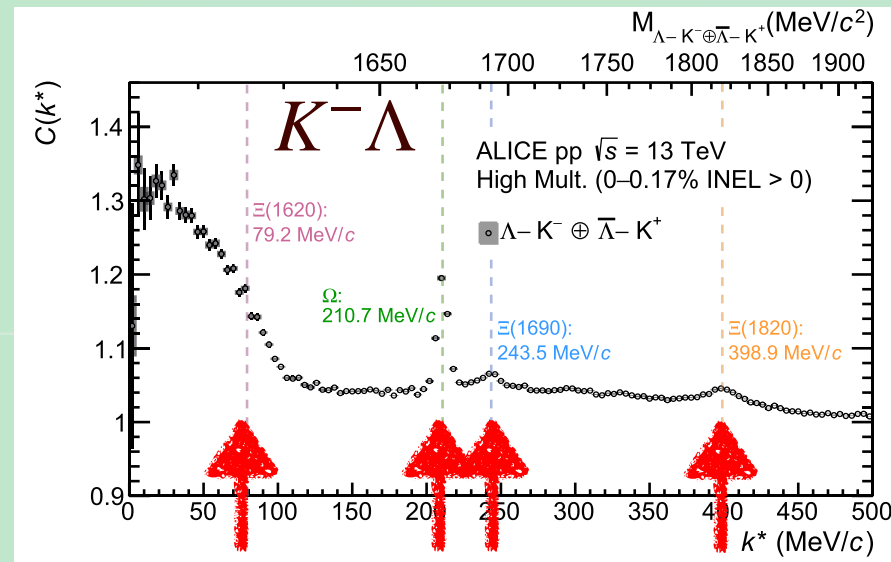
Resonances and higher partial waves

Resonance peaks in experimental data ($l = 0$ and $l \neq 0$)

ALICE collaboration, PRL 124, 092301 (2020); PLB845, 138145 (2023)



$\Lambda(1520)$: d wave




$E(1620), E(1690)$: s wave,
 Ω : p wave (weak decay),
 $E(1820)$: d wave

Questions:

- **Origin** of resonance peak?
- Usually s-wave only. How about **higher partial waves**?


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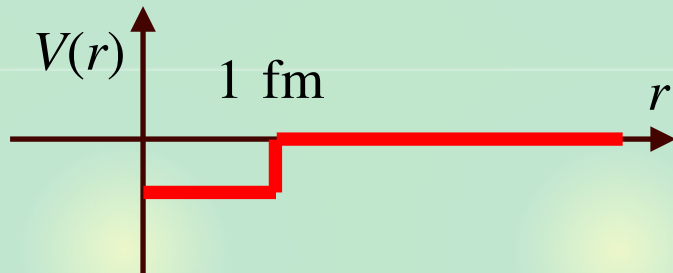
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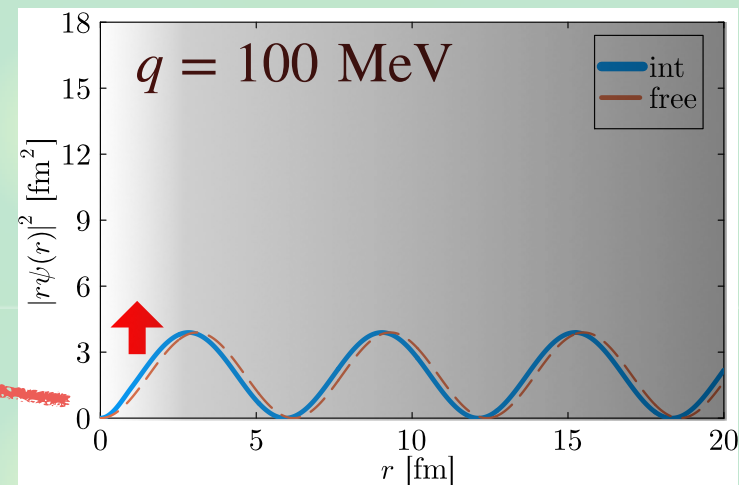
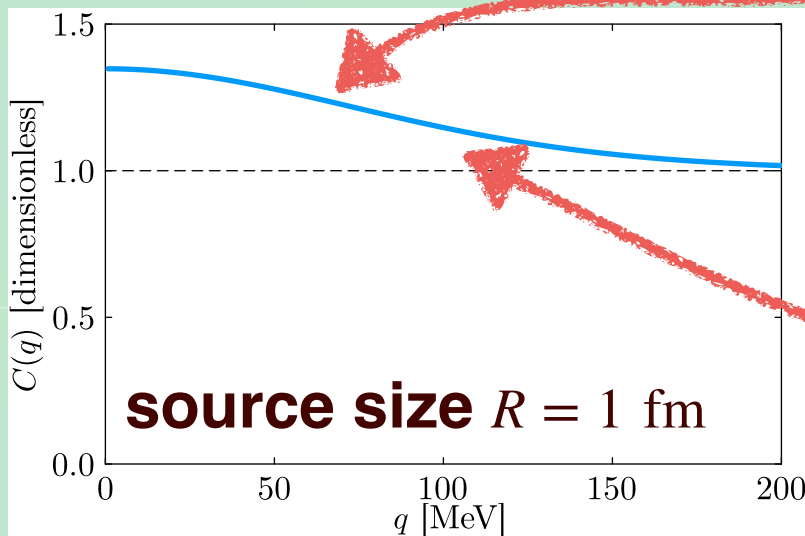
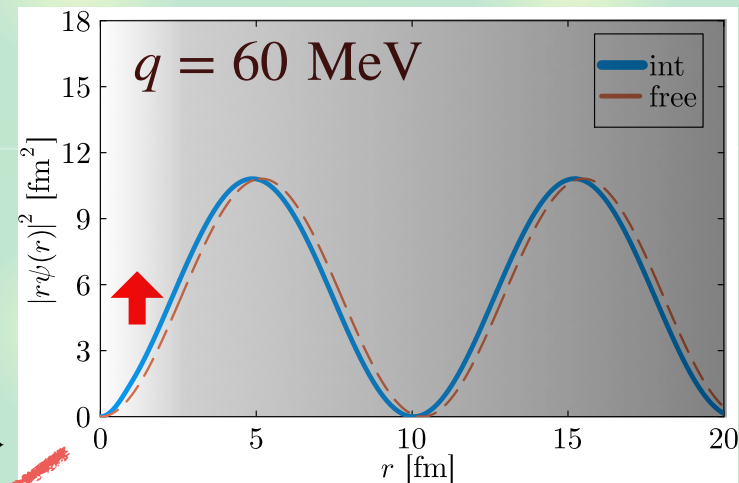
 Summary

Wave function and correlation (attraction)

Attractive well (no bound state)



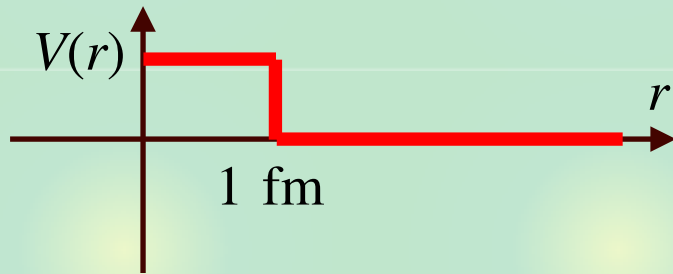
$$C(q) \simeq 1 + \int_0^\infty dr \frac{4\pi}{q^2} S(r) \{ ||rq\psi_q(r)||^2 - \sin^2(qr) \}$$



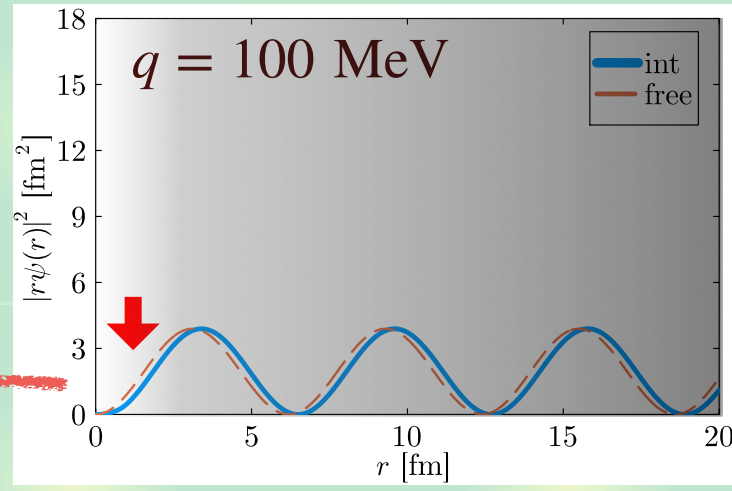
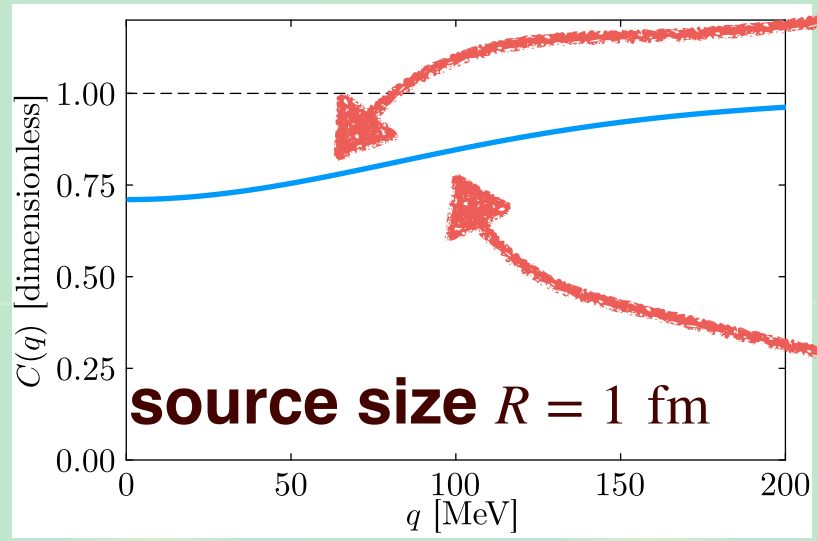
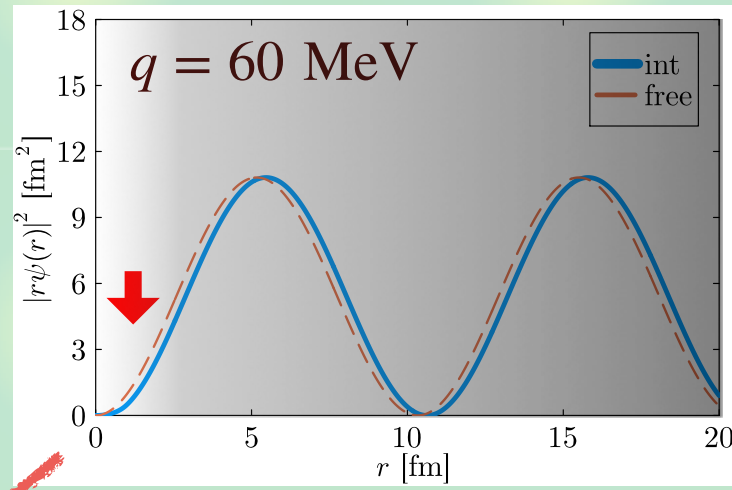
W.f. is pulled in, increased at $r \lesssim R \rightarrow C(q)$ enhancement

Wave function and correlation (repulsion)

Repulsive rectangular potential



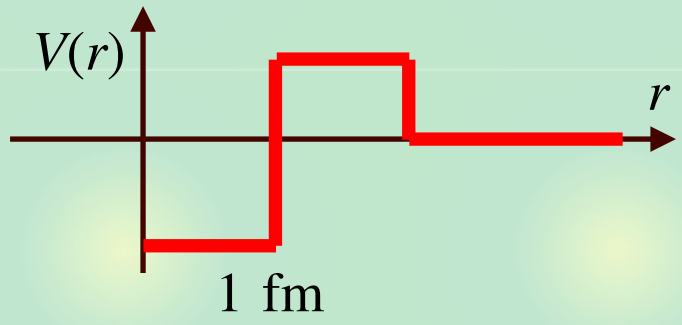
$$C(q) \simeq 1 + \int_0^\infty dr \frac{4\pi}{q^2} S(r) \{ ||rq\psi_q(r)||^2 - \sin^2(qr) \}$$



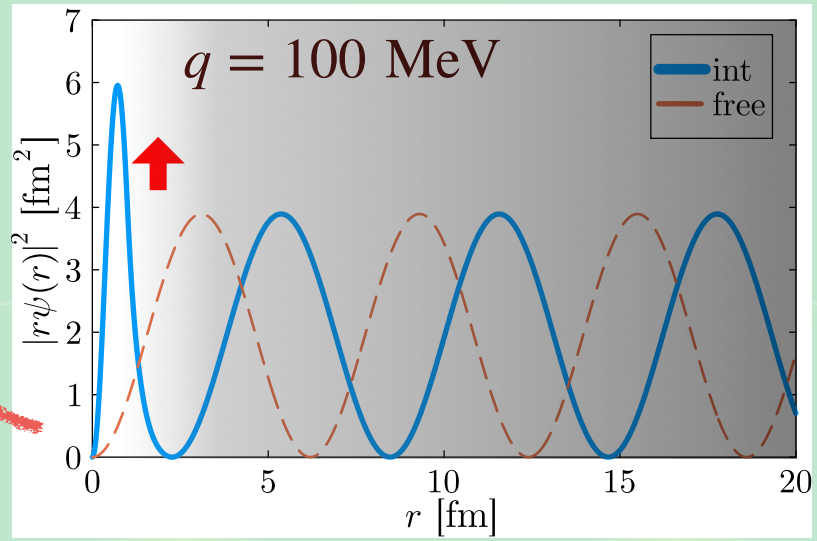
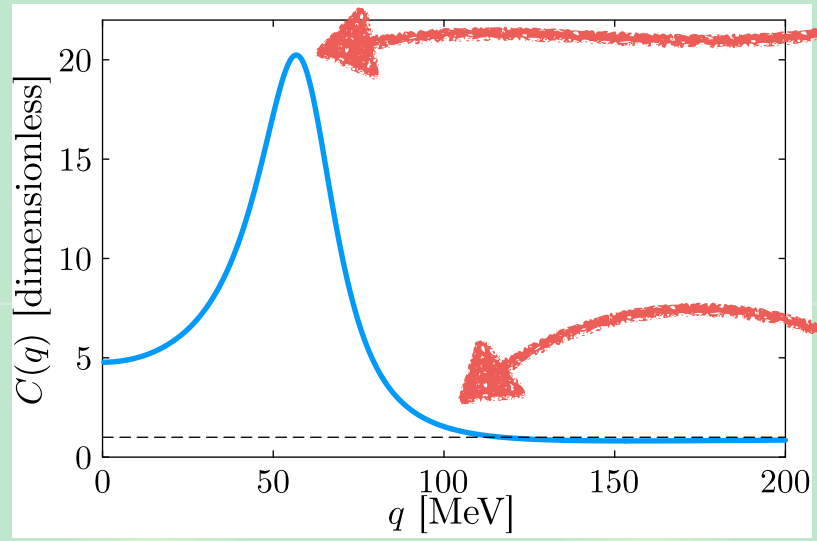
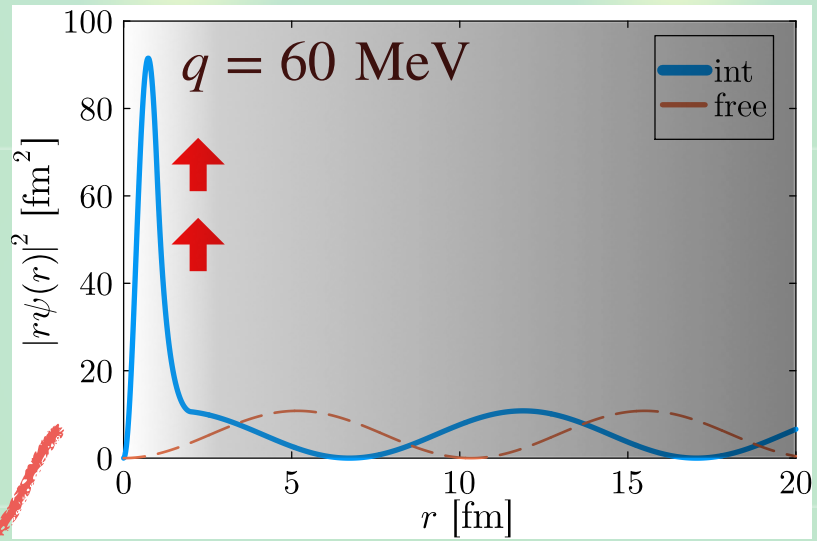
W.f. is pushed out, decreased at $r \lesssim R \rightarrow C(q)$ suppression

Wave function and correlation (resonance)

Well + barrier potential





- resonance @ $q^- = 59 - 14i$ MeV



W.f. is **localized** in $r \lesssim R$ at pole momentum \rightarrow peak in $C(q)$

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-  Summary

Higher partial waves

KP formula with $l > 0$ (spherical source)

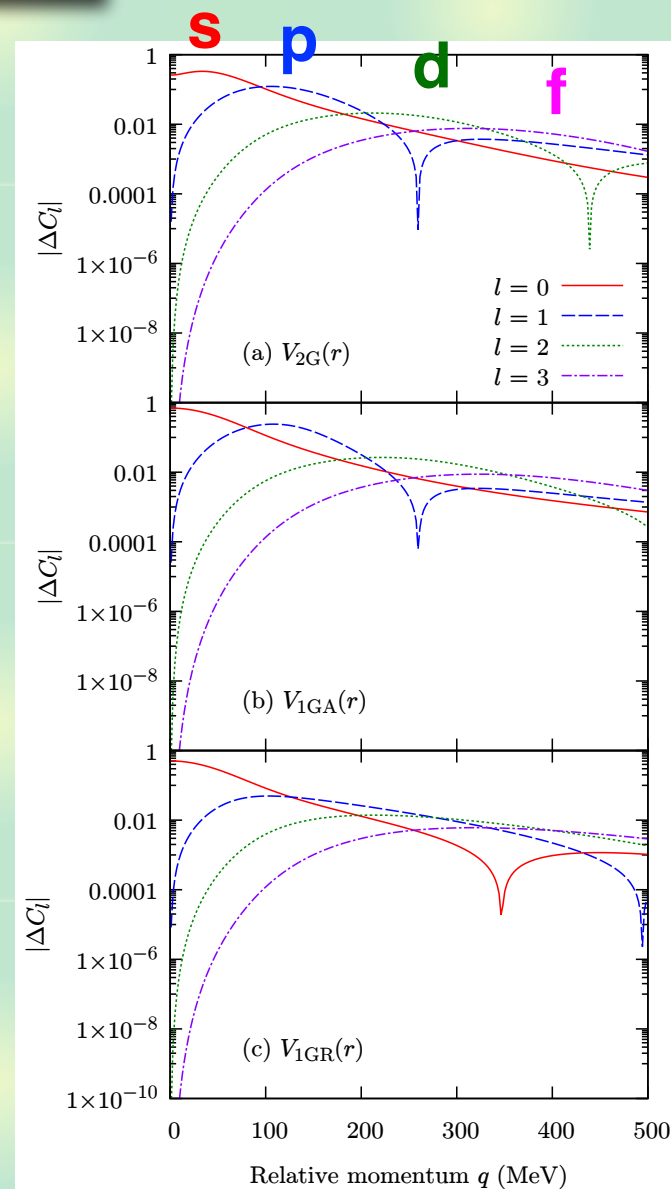
$$C(q) = 1 + \sum_{l=0}^{\infty} \Delta C_l(q)$$

$$\Delta C_l(q) = (2l + 1) \times \int_0^{\infty} dr 4\pi r^2 S(r) [|R_l(r)|^2 - |j_l(qr)|^2]$$

- sum of partial wave contributions
- interacting w.f. $R_l(r)$ — free w.f. $j_l(qr)$

Gaussian potentials (range ~ 1.25 fm)

- $l > 0$ components at larger q
- l -th wave dominant at $q \sim l/r \sim 160l$ MeV



Resonance contribution

With $q \rightarrow \infty$, $C(q)$ approaches unity

- How can resonances be seen?

Resonances in higher partial waves

- p-wave resonance at

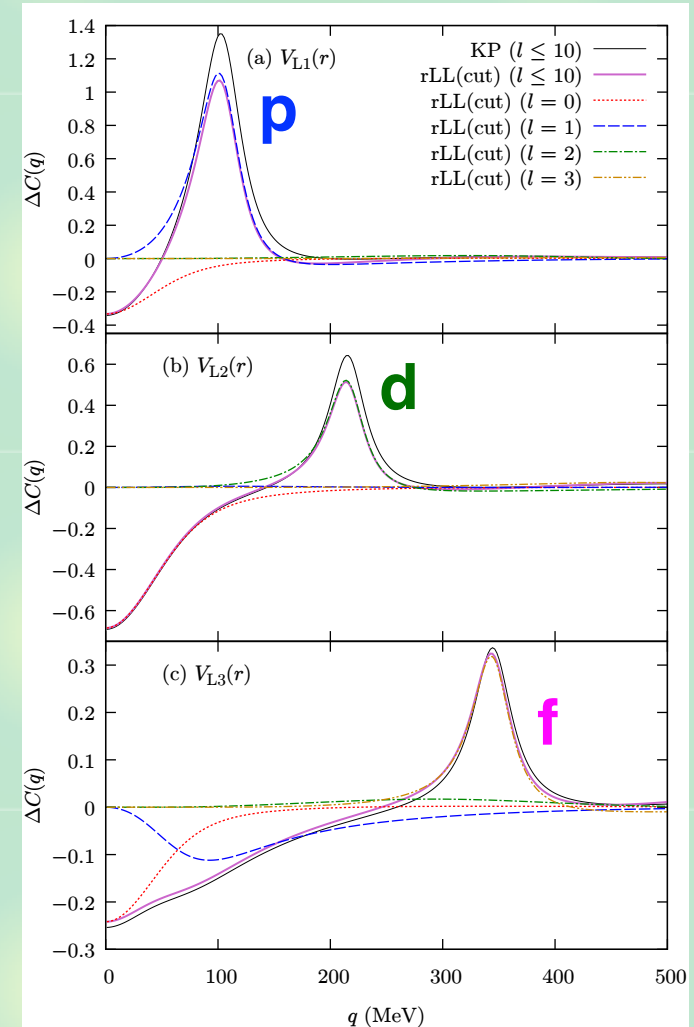
$$q^- \sim 105 - 23i \text{ MeV}$$

- d-wave resonance at

$$q^- \sim 216 - 20i \text{ MeV}$$

- f-wave resonance at

$$q^- \sim 345 - 21i \text{ MeV}$$



With **resonances**, $l > 0$ components is enhanced

LL formula

Lednicky-Lyuboshits (LL) formula for s-wave

R. Lednicky, V.L. Lyuboshits, *Yad. Fiz.* **35**, 1316 (1981);

K. Murase, T. Hyodo, *J. Subatomic Part. Cosmol.* **3**, 100017 (2025)

- **Gaussian (relative) source** $S(r) = \exp(-r^2/4R^2)/(4\pi R^2)^{3/2}$
- **Zero-range limit of interaction (asymptotic w.f. for entire r)**

$$C(q) = 1 + \int_0^\infty dr \frac{4\pi}{q^2} S(r) [\sin^2(qr - \delta(q)) - \sin^2(qr)]$$

(Semi) analytic formula with only on-shell observable

$$\begin{aligned} C(q) &= 1 + \frac{|f(q)|^2}{2R^2} F_3(r_e/R) + \frac{2\text{Re } f(q)}{\sqrt{\pi R}} F_1(2qR) - \frac{\text{Im } f(q)}{R} F_2(2qR) \\ &= 1 + \frac{2\text{Re } f(q)}{\sqrt{\pi R}} F_1(2qR) + \frac{\text{Im } f(q)}{2qR^2} \left(e^{-(2qR)^2} - \frac{r_e}{2\sqrt{\pi R}} \right) \end{aligned}$$

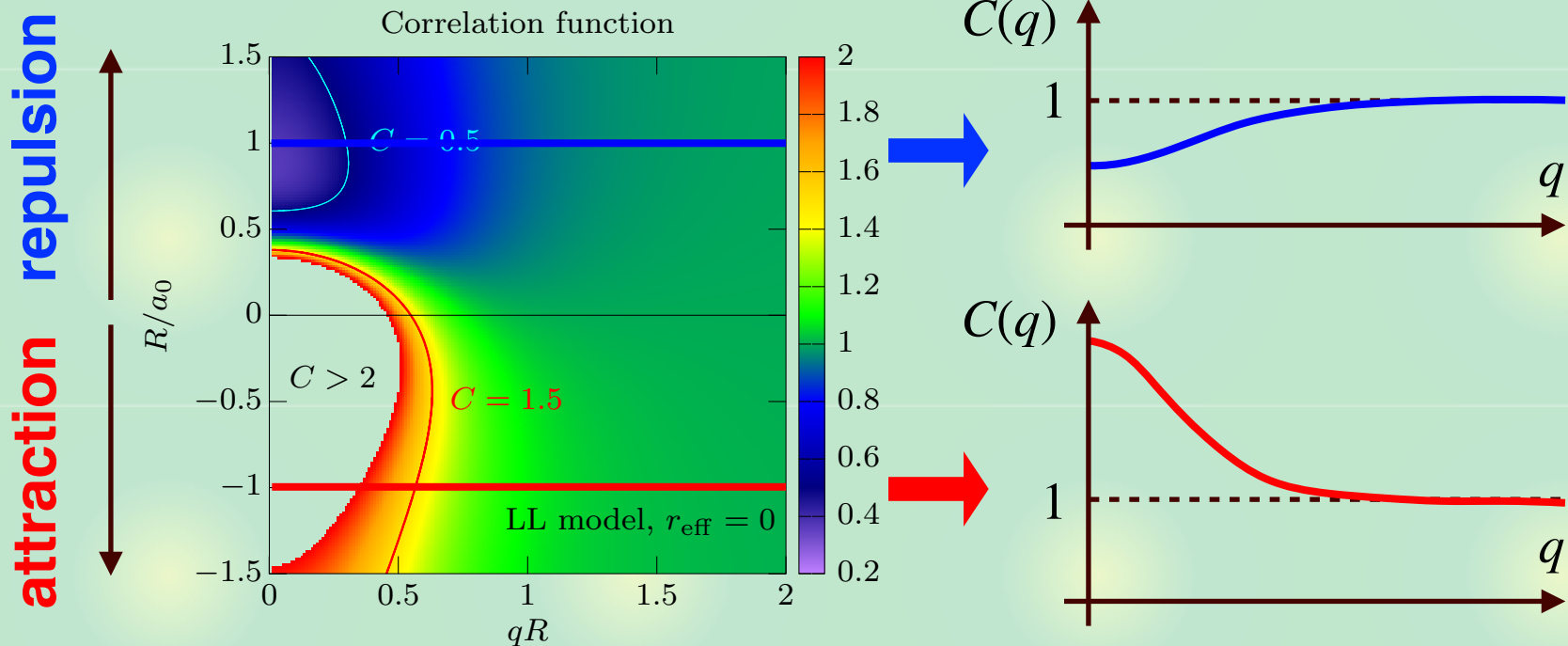
- **Direct extraction of a_0 and r_e from experimental data**

LL formula and correlations

LL formula with $r_e = 0$

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, T. Hatsuda, PRC 105, 014915 (2022)

- fixed $R > 0$
- **repulsion**: $R/a_0 > 0$, **attraction**: $R/a_0 < 0$



Consistent with KP formula

LL formula for higher partial wave?

Naive generalization for $l > 0$ is **not possible**

K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025)

$$\Delta C_l(q) = (2l + 1) \int_0^\infty dr 4\pi r^2 S(r) [|R_l^{\text{aym}}(r)|^2 - |j_l(qr)|^2]$$

- $R_l^{\text{aym}}(r)$ is too singular for $l > 0$ ($n_l(qr) \sim r^{-l-1}$) at $r \rightarrow 0$

- No sensible zero-range limit for $l > 0$

Y. Nishida, Phys. Rev. A 86, 012710 (2012)

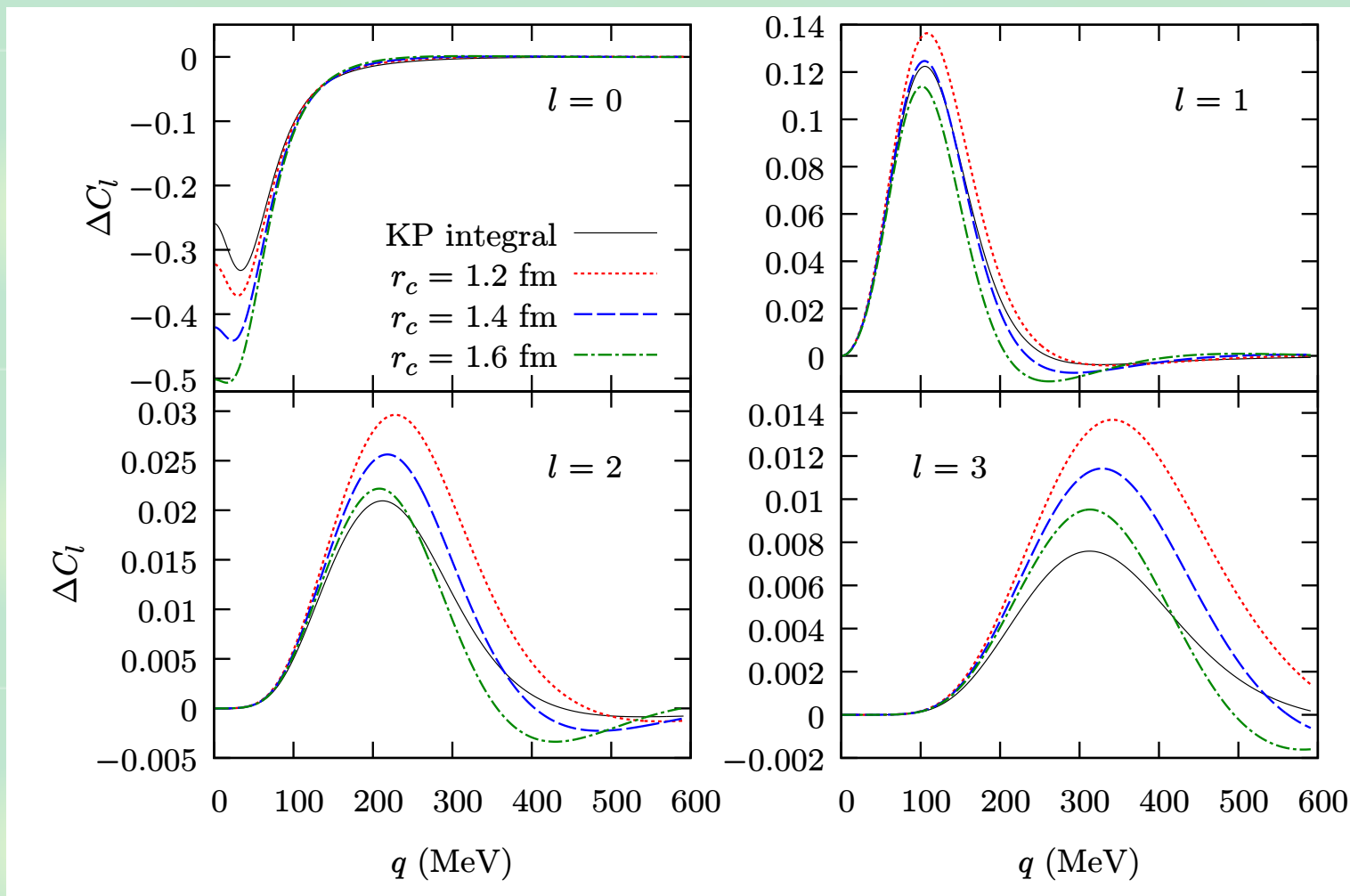
Some **regularization** for $r \rightarrow 0$ is needed

- a simple prescription: introducing **cutoff** r_c

$$\Delta C_l(q) = (2l + 1) \int_{r_c}^\infty dr 4\pi r^2 S(r) [|R_l^{\text{aym}}(r)|^2 - |j_l(qr)|^2]$$

Regularized LL formula

Dependence on **cutoff** r_c



Works for $l > 0$ with $r_c \sim$ interaction range (1.25 fm)

Preliminary result for Xi p

K. Murase, talk @ WPCF 2026

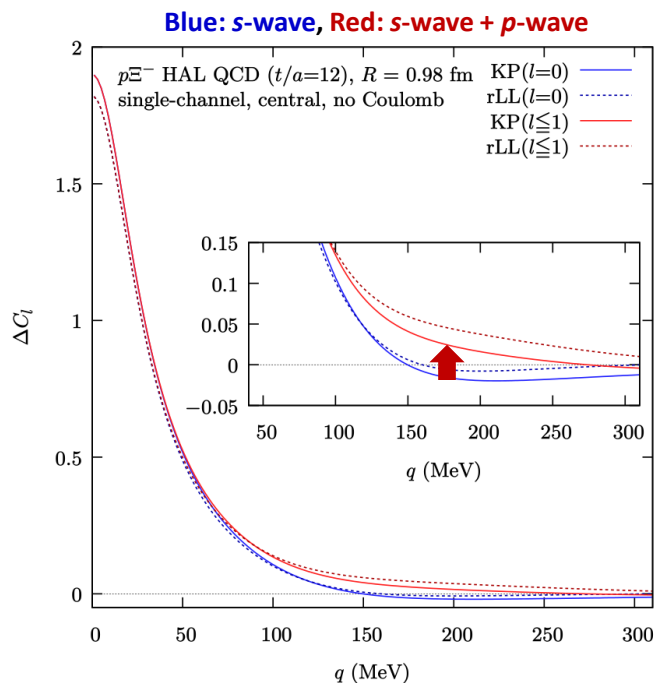
Effect of p wave in $p\text{-}\Xi^-$ correlation

KP and rLL formula with partial waves

Simplified analysis (single-channel, no Coulomb)

based on the s -wave HAL QCD potentials:

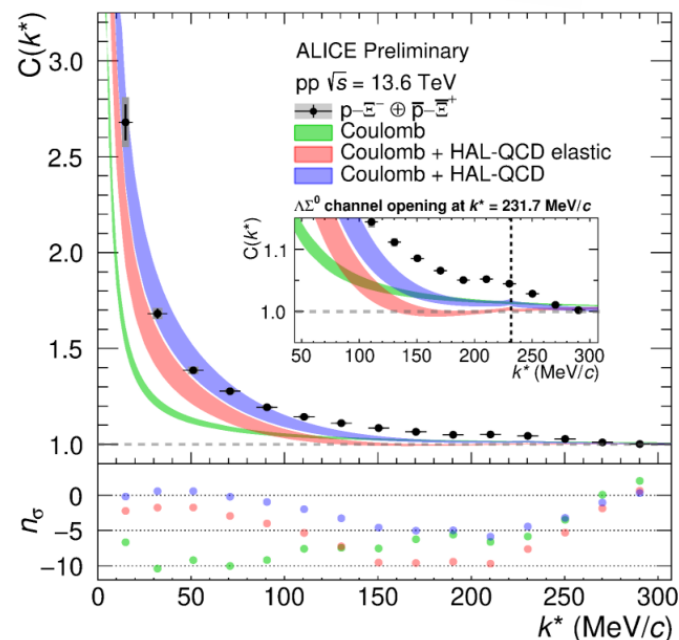
K. Sasaki et al. [HAL QCD] NPA 998 (2020) 121737



ALICE Preliminary


[talk by Raffaele D. Grande on Monday 9:30]


HAL QCD: Only s -wave




The overall discrepancy in the data may *qualitatively* be explained by the p -wave contribution


Summary

 **Resonance peaks** in correlation functions

 **s-wave resonance peak** ← **localization** of wave function at interacting region

[S. Watanabe, T. Hyodo, in preparation](#)

 **Higher partial wave ($l > 0$) contributions** becomes important for larger q

 **Regularized LL formula with suitable cutoff r_c** works for $l > 0$

[K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 \(2025\);](#)
[K. Murase, T. Hyodo, arXiv:2509.22844 \[nucl-th\]](#)