# Resonance contributions in momentum correlation functions in high-energy collisions


**Tetsuo Hyodo**

*RCNP, The University of Osaka*


2026, Aug. 23rd

# Contents

- Introduction — resonances
  - Complex eigenenergy and pole
- Resonance peak in correlation function
  - Wavefunction behavior (KP formula)
  - Difference from cross section (LL formula)

S. Watanabe, T. Hyodo, in preparation

  - Resonances with $l > 0$

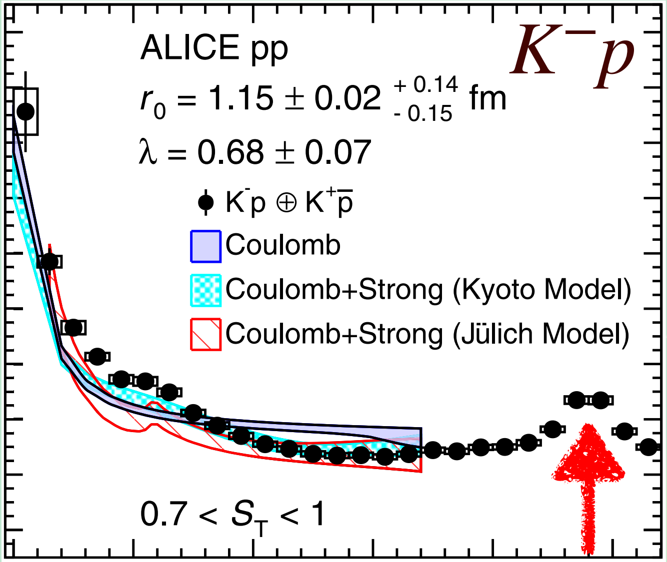
K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]

- Summary

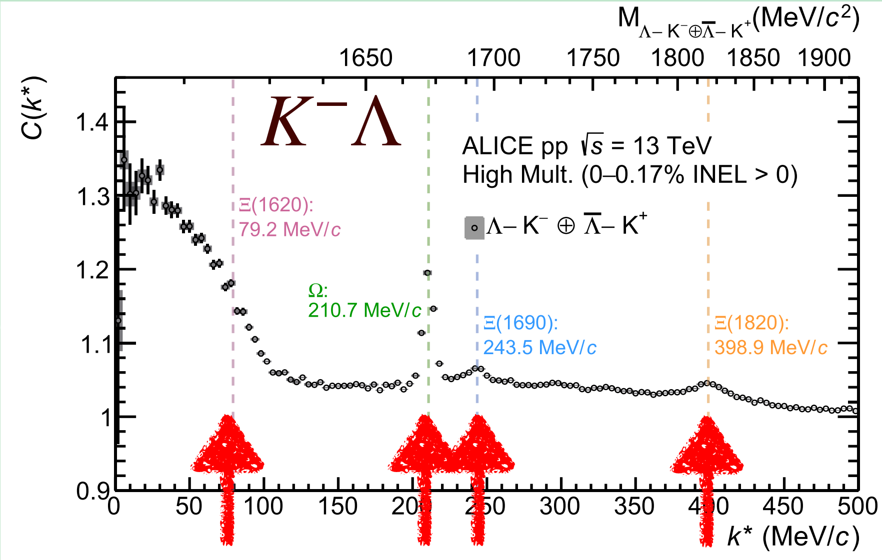
# Resonances and higher partial waves

## Resonance peaks in experimental data ( $l = 0$ and $l > 0$ )

ALICE collaboration, PRL 124, 092301 (2020); PLB845, 138145 (2023)




$\Lambda(1520)$: d wave




$\Xi(1620)$, $\Xi(1690)$: s wave,

$\Omega$: p wave (weak decay),

$\Xi(1820)$: d wave

**Questions:**

- **Origin** of resonance peak?
- **Condition** for resonance peak?

# Resonance in quantum mechanics

## Resonance as an "eigenstate" of Hamiltonian

- **Complex eigenenergy**

G. Gamow, Z. Phys. 51, 204 (1928)

**Zur Quantentheorie des Atomkernes.**

Von **G. Gamow**, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

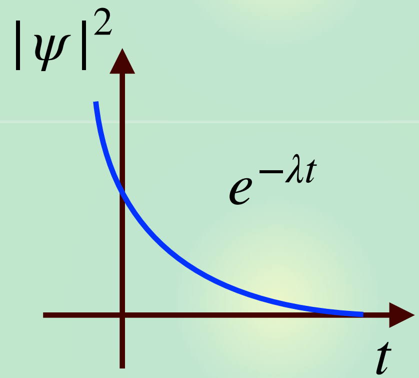
Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und $E$ komplex setzen:

$$E = E_0 + \boxed{i\,\frac{h\lambda}{4\pi}},$$

wo $E_0$ die gewöhnliche Energie ist und $\lambda$ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- **Time evolution: decreasing probability**

$$\psi = \Psi(q)\cdot e^{+\frac{2\pi i E}{h}t}, \quad \propto e^{+2\pi i E_0 t/h} e^{-(\lambda/2)t}, \quad |\psi|^2 \propto e^{-\lambda t}$$




## Equivalent to pole of the scattering amplitude

T. Hyodo, M. Niiyama, PPNP 120, 103868 (2021)

**Schrödinger eq. + outgoing b.c. at energy** $E$ $(p = \sqrt{2\mu E})$

- **bound states** $(E < 0)$

  $p = i\kappa \quad (\kappa > 0)$

- **resonances** $(E \in \mathbb{C})$

  $p \in \mathbb{C} \quad (\mathrm{Im}\, p < 0)$

$\Leftrightarrow$

**zero of Jost function**

$\mathscr{f}_\ell(p) = 0$

$\Leftrightarrow$

**pole of s-matrix/ scattering amplitude**

$|f_\ell(p)| \to \infty$

$|s_\ell(p)| \to \infty$

# Resonance in effective range expansion

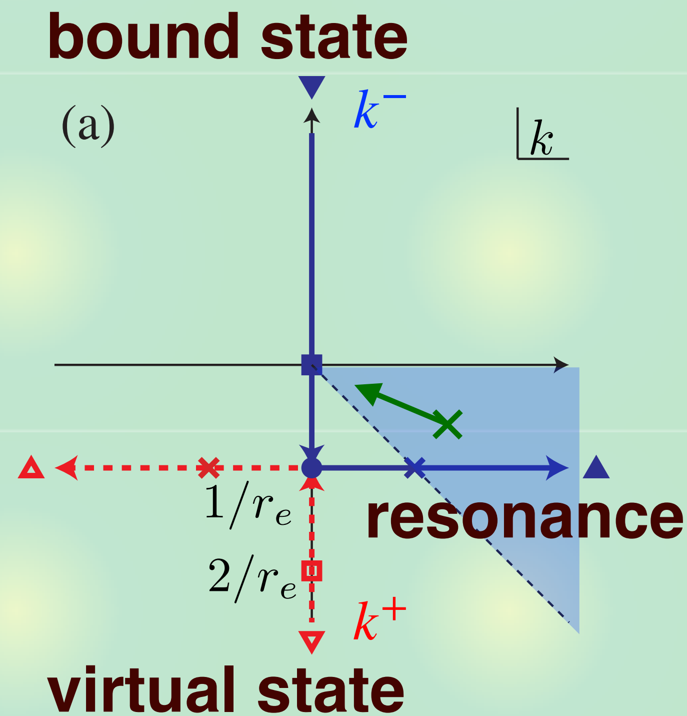
## Effective range expansion (ERE): valid for small $k$

$$f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik\right]^{-1}$$

- **Pole positions** $k^{\pm}$ **<—>** $(a_0, r_e)$

T. Hyodo, PRL111, 132002 (2013);
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

$$k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1 + i0^+}$$




## Resonance solution ($r_e < 0$)

$$\frac{1}{|r_e|}\sqrt{\frac{2r_e}{a_0} - 1} \geq \frac{1}{|r_e|}, \quad \Rightarrow \quad \frac{r_e}{a_0} \geq 1, \quad \Rightarrow \quad |a_0| \leq |r_e|$$

- **Resonance with** $|k^-| \to 0$ **: not only** $|a_0| \to \infty$ **but also** $|r_e| \to \infty$
- **Eigenenergy** $E_R = \frac{(k^-)^2}{2\mu} = M_R - i\frac{\Gamma_R}{2}$ **<—>** $(a_0, r_e)$

# Contents

- Introduction — resonances
  - Complex eigenenergy and pole
- Resonance in correlation function
  - Wavefunction behavior (KP formula)
  - Difference from cross section (LL formula)

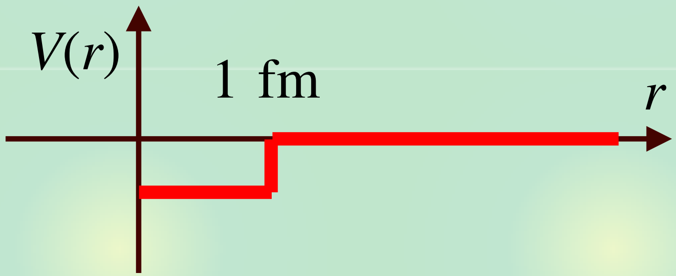
  S. Watanabe, T. Hyodo, in preparation

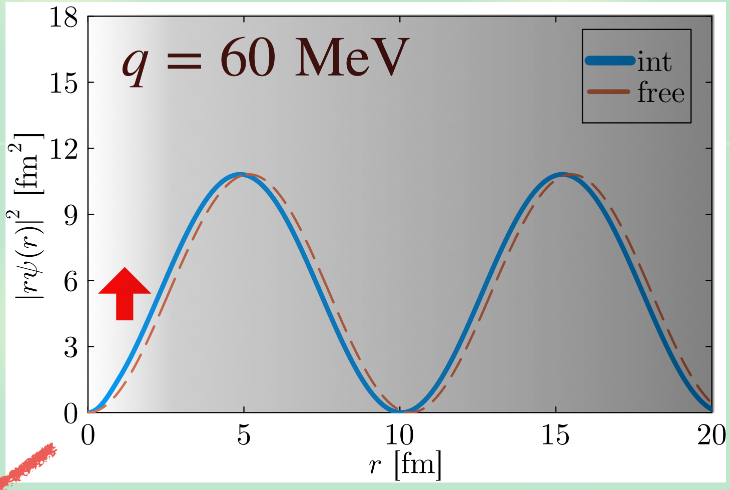
  - Resonances with $l > 0$

  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]
- Summary

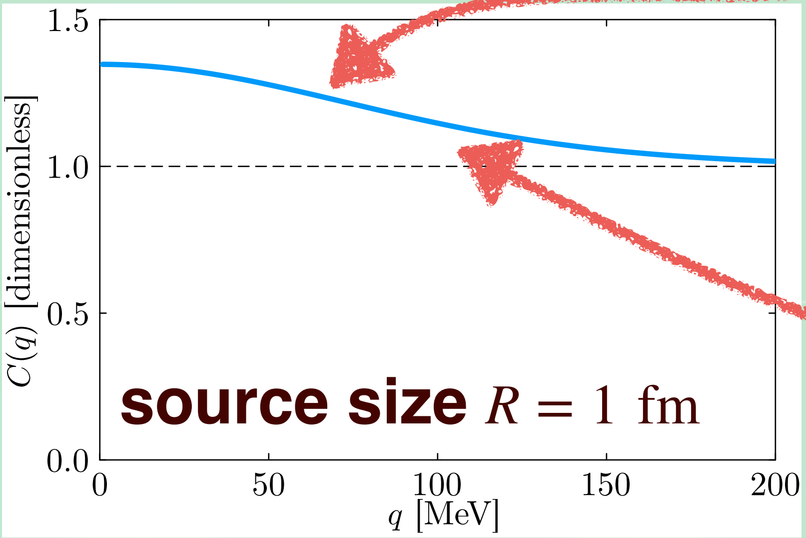
# Wave function and correlation (attraction)

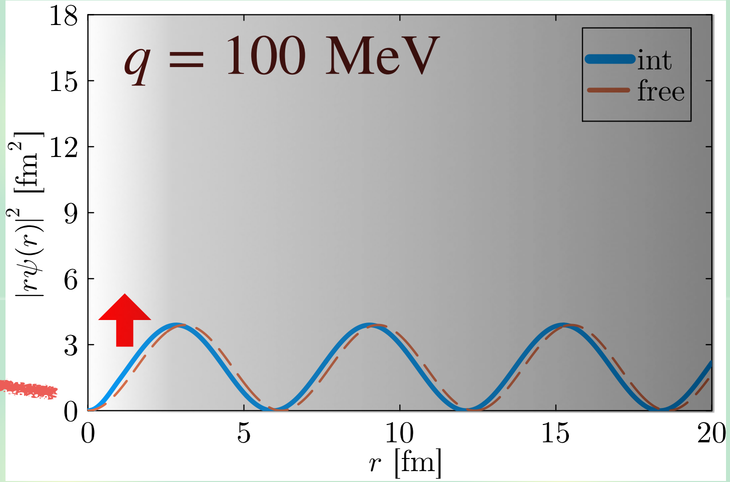
## Attractive well (no bound state)




$$C(q) \simeq 1 + \int_0^\infty dr\ \frac{4\pi}{q^2} S(r)\{\,||rq\psi_q(r)|^2 - \sin^2(qr)\}$$

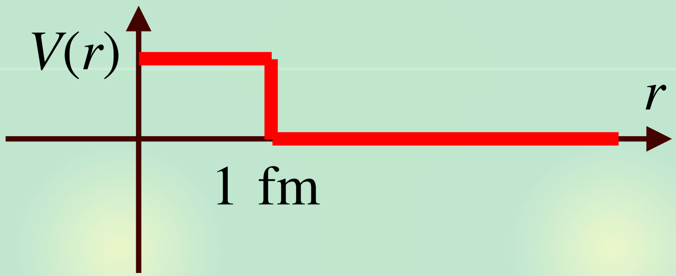


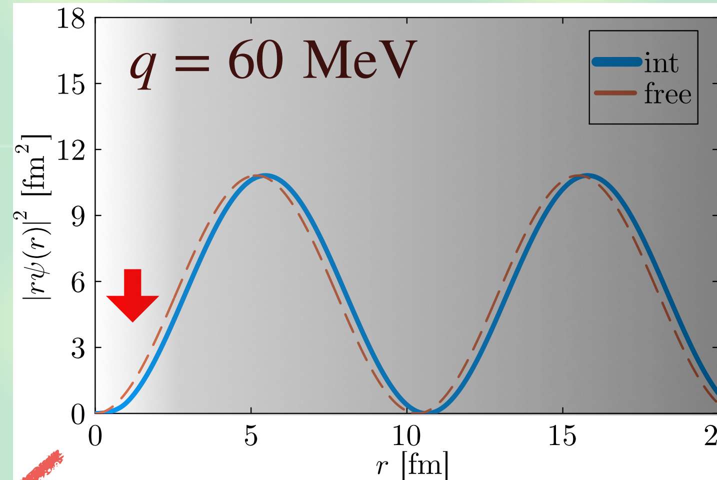

$\psi_q(r)$ **is pulled in, increased at** $r \lesssim R$ **—>** $C(q)$ **enhancement**

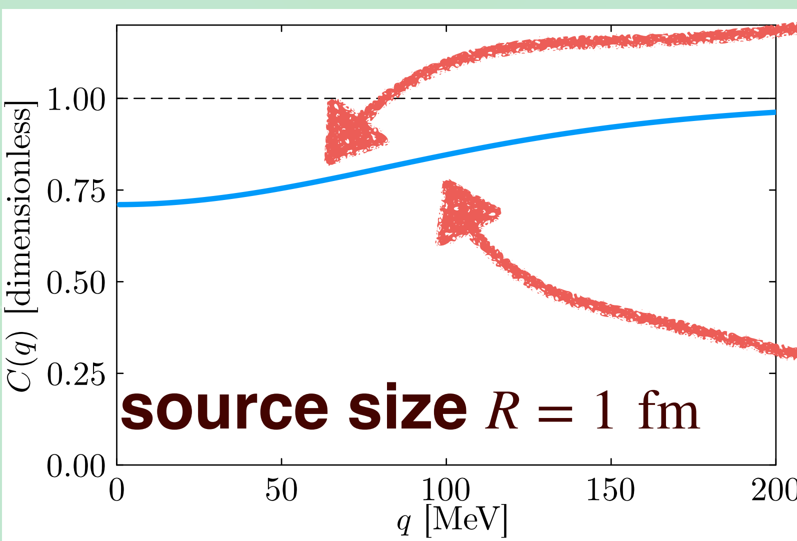
# Wave function and correlation (repulsion)

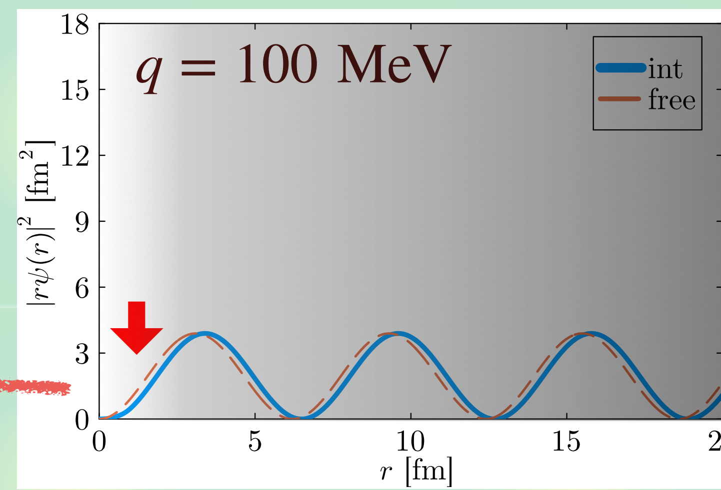
## Repulsive rectangular potential




$$C(q) \simeq 1 + \int_0^\infty dr\ \frac{4\pi}{q^2} S(r)\{\,|\,|r q \psi_q(r)|^2 - \sin^2(qr)\}$$



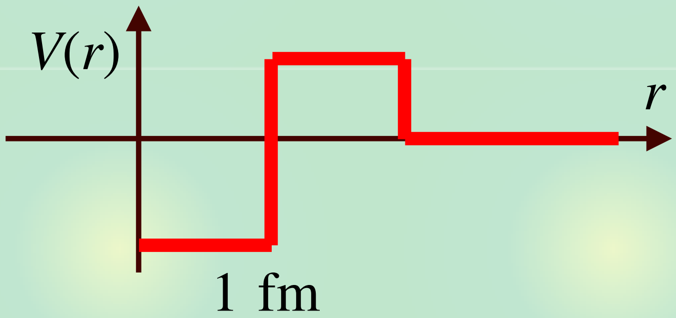



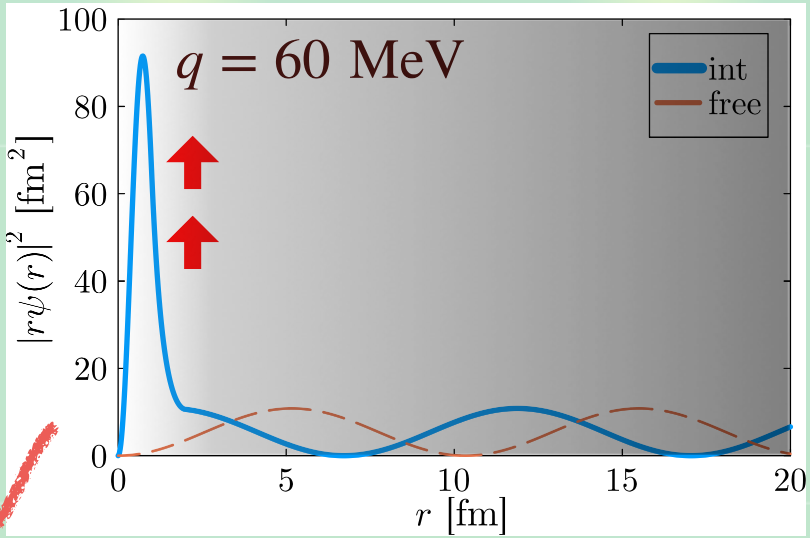

$\psi_q(r)$ **is pushed out, decreased at** $r \lesssim R$ **—>** $C(q)$ **suppression**

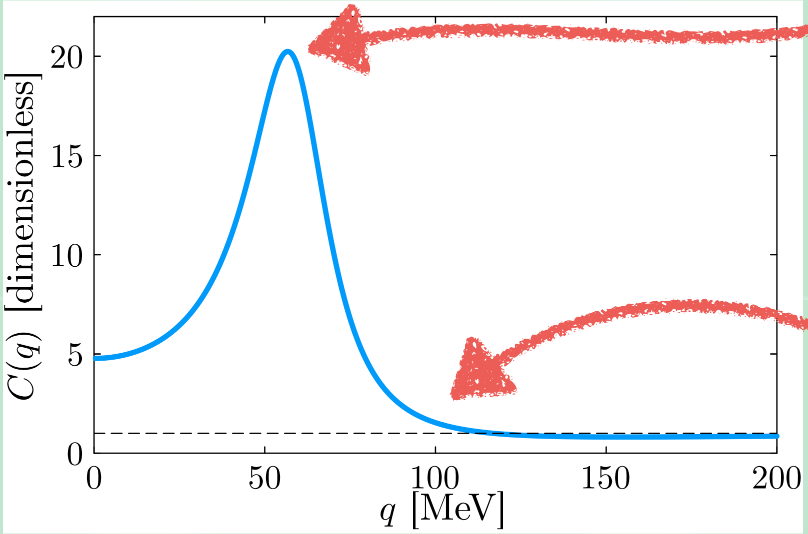
# Wave function and correlation (resonance)

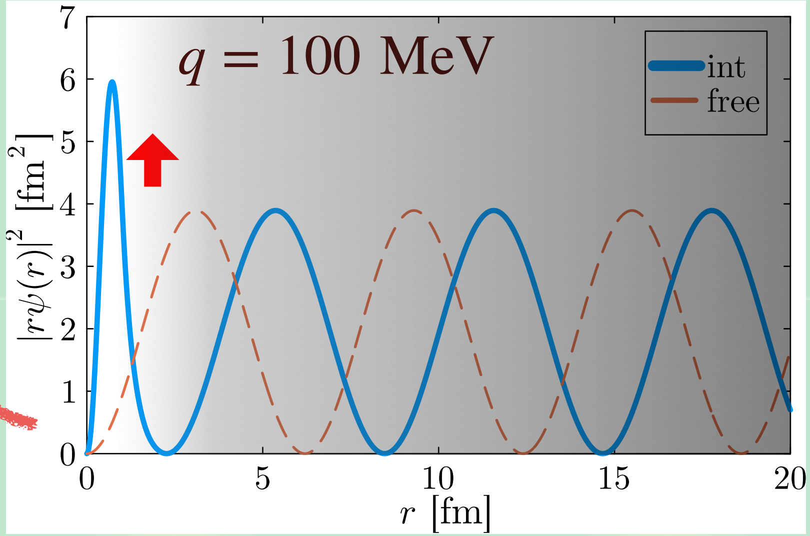
## Well + barrier potential




- **resonance @** $q^{-} = 59 - 14i$ MeV




**W.f. is localized in $r \lesssim R$ at pole momentum —> peak in $C(q)$**

# Resonance in cross section

## Pole of scattering amplitude with effective range expansion

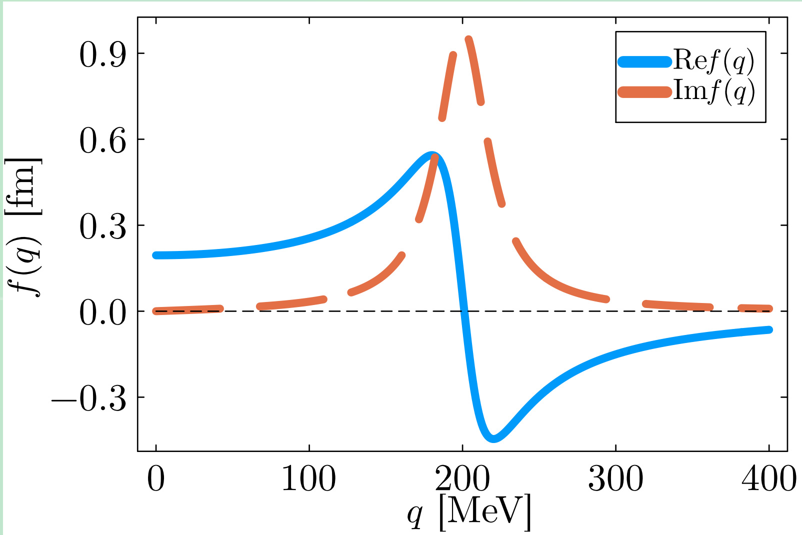
T. Hyodo, PRL 111, 132002 (2013),
T. Kinugawa, T. Hyodo, arXiv:2403.12635 [hep-ph]

$$f(q) = \left[-\frac{1}{a_0} + \frac{r_e}{2}q^2 - iq\right]^{-1}, \quad q^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1 + i0^+}$$

**With** $a_0 = -0.195 \text{ fm}, r_e = -9.87 \text{ fm}$

$$q^- = 200 - 20i \text{ MeV}$$

- $\mathrm{Re}\, f$ **~ zero,** $\mathrm{Im}\, f$ **~ peak at** $\mathrm{Re}\, q^-$




## Optical theorem

$$\sigma(q) = \frac{4\pi}{q}\mathrm{Im}\, f(q, \theta = 0)$$

**—> Peak in cross section** $\sigma$

# Resonance in correlation (LL formula)

## LL formula: correlation function <— scattering amplitude

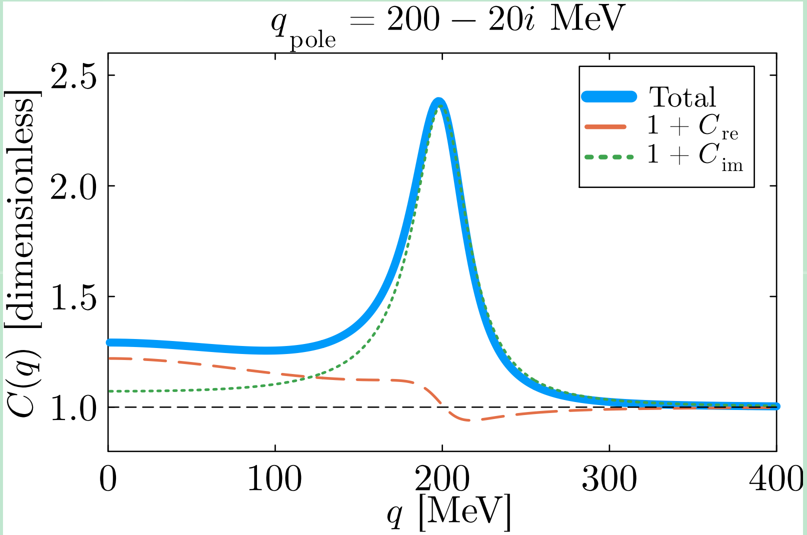
R. Lednicky, V.L. Lyuboshits, Yad. Fiz. 35, 1316 (1981);
K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025)

$$C(q) = 1 + \frac{|f(q)|^2}{2R^2}F_3(r_e/R) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R}F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R}F_2(2qR)$$

$$= 1 + \underbrace{\frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R}F_1(2qR)}_{C_{\mathrm{re}}(q)} + \underbrace{\frac{\mathrm{Im}\, f(q)}{2qR^2}\left(e^{-(2qR)^2} - \frac{r_e}{2\sqrt{\pi}R}\right)}_{C_{\mathrm{im}}(q)}$$

## With resonance amplitude

$$q^- = 200 - 20i \ \ \mathrm{MeV}$$




## Correlation = Peak (im) + background (re)

# Higher partial waves

## KP formula with $l > 0$ (spherical source)

K. Murase, T. Hyodo, JSPC 3, 100017 (2025);
K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]

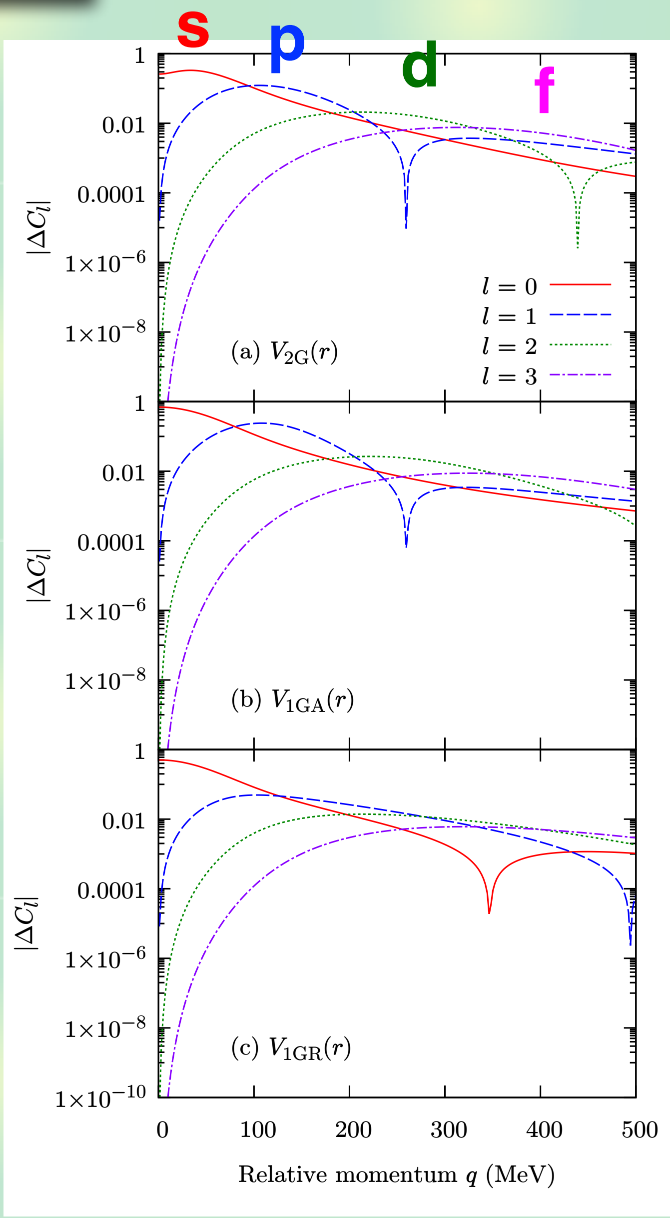
$$C(q) = 1 + \sum_{l=0}^{\infty} \Delta C_l(q)$$

$$\Delta C_l(q) = (2l+1) \times \int_0^{\infty} dr\, 4\pi r^2 S(r)[\,|R_l(r)|^2 - |j_l(qr)|^2\,]$$

- sum of partial wave contributions
- interacting w.f. $R_l(r)$ — free w.f. $j_l(qr)$

## Gaussian potentials (range ~ 1.25 fm)

- $l > 0$ components at **larger** $q$
- $l$-th wave dominant at $q \sim l/r \sim 160l$ MeV



# Resonance contribution in higher partial waves

**With $q \to \infty$, $C(q)$ approaches unity**

**- How can resonances be seen?**

**Resonances in higher partial waves**

**- p-wave resonance at**

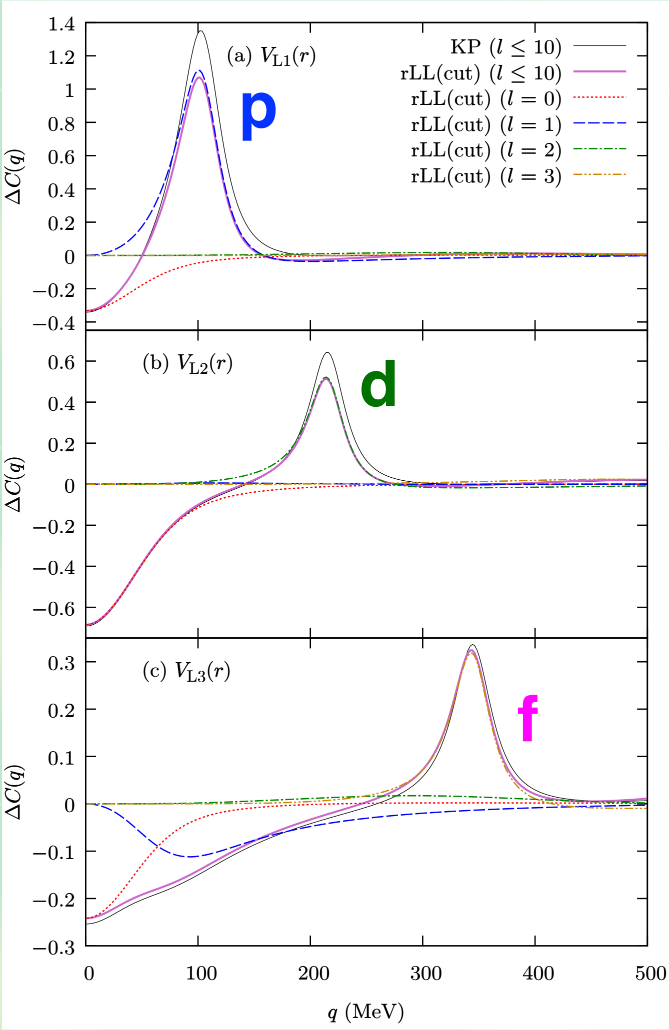
$$q^- \sim 105 - 23i \text{ MeV}$$

**- d-wave resonance at**

$$q^- \sim 216 - 20i \text{ MeV}$$

**- f-wave resonance at**

$$q^- \sim 345 - 21i \text{ MeV}$$




**With resonances, $l > 0$ components is enahnced**

# Summary

- Resonace peaks in correlation functions

- s-wave resonance peak <— localization of wave function at interacting region

- LL formula: not only peak in $\mathrm{Im}\, f$ but also background from $\mathrm{Re}\, f$

  S. Watanabe, T. Hyodo, in preparation

- Resonance enhances the higher partial waves ($l > 0$) contributions

  K. Murase, T. Hyodo, J. Subatomic Part. Cosmol. 3, 100017 (2025);
  K. Murase, T. Hyodo, arXiv:2509.22844 [nucl-th]

# NPA special issue

- **Special issue** in Nuclear Physics A
- **Invitation-based**: primarily targeting participants of this workshop $+\alpha$
- Contributions: either original research or a details of results already published elsewhere
- Not a proceedings issue; contributions will undergo the standard peer-review process