

Isospin symmetry breaking effects for near-threshold states



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Introduction — $X(3872)$ and η_{c2}



Formulation

- Coupled-channel model and compositeness
- Spin and partial-wave structure



Results

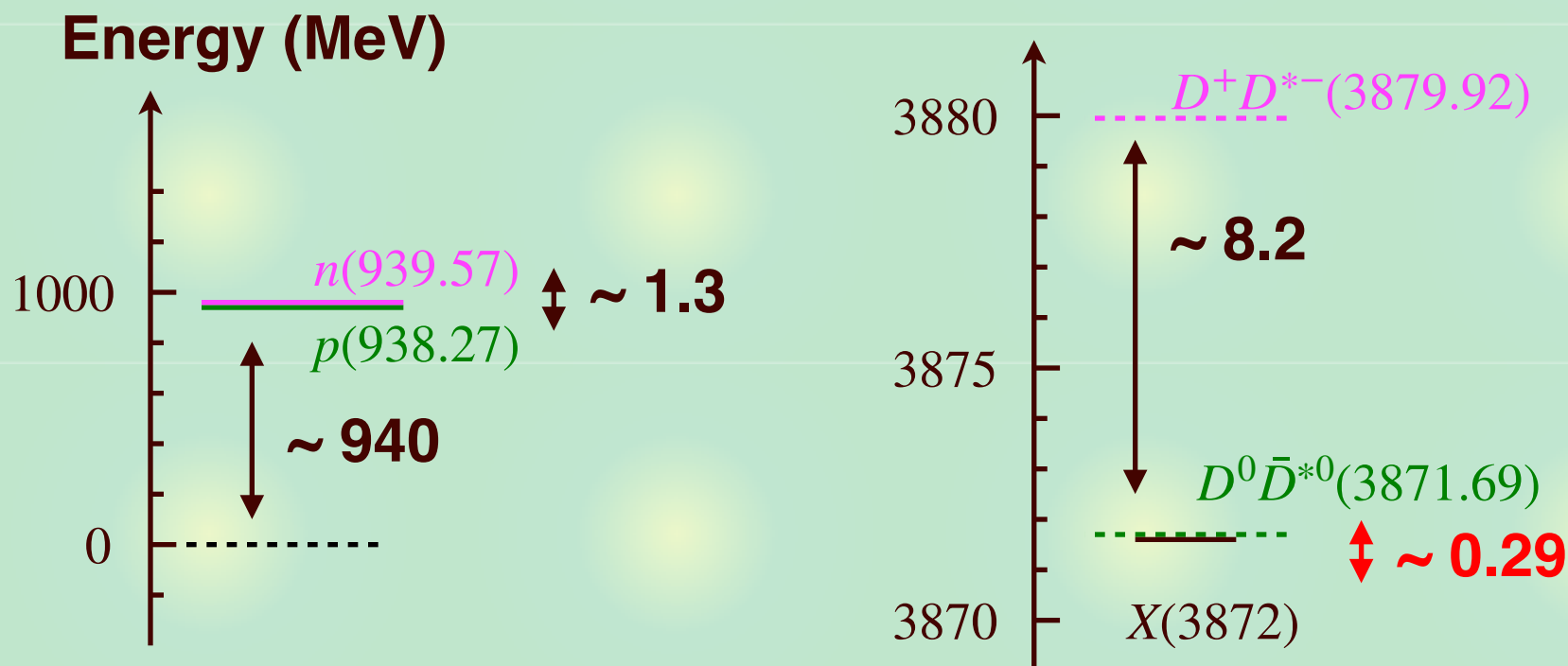
- Compositeness of near-threshold states
- Isospin symmetry breaking
- Realistic setup



Summary

Isospin symmetry breaking

Isospin symmetry holds approximately well (up to a few MeV)



- Nucleon: 1.3 MeV compared with mass (940 MeV)
- X(3872): 8.2 MeV compared with binding energy (**0.29 MeV**)

Important **symmetry breaking** for near-threshold eigenstates

$X(3872)$ and η_{c2}

Isospin symmetry breaking for $X(3872)$

$$\text{Br}(X(3872) \rightarrow \rho J/\psi) = 3.4 \pm 1.1 \% , \quad \text{Br}(X(3872) \rightarrow \omega J/\psi) = 5.0 \pm 1.9 \%$$

← **Dominance of neutral channel:** $X_0 \gg X_c$

D. Gamermann, J. Nieves, E. Oset, E. Ruiz Arriola, PRD 81, 014029 (2010);

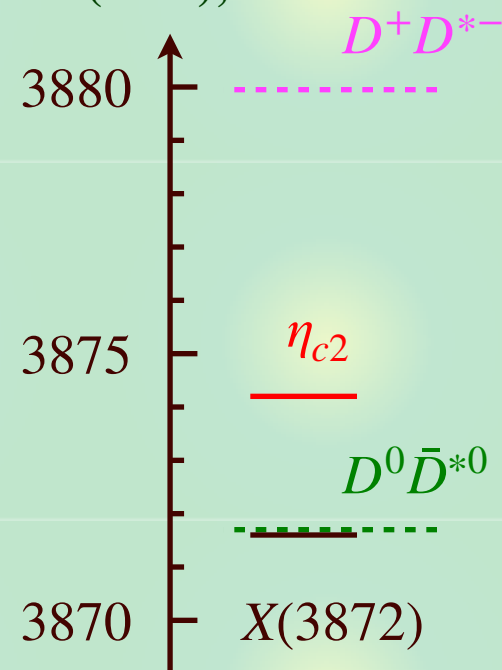
T. Kinugawa, T. Hyodo, PRC 109, 045205 (2024);

J. Song, L.R. Dai, E. Oset, PRD 108, 114017 (2023)

$\eta_{c2} : J^{PC} = 2^{-+}$ (not yet identified)

- η_{c2} **near $D\bar{D}^*$ threshold** in addition to $X(3872)$

S.X. Nakamura, PRD 113, 114031 (2026)



Questions

- How is the isospin symmetry breaking for η_{c2} ?
- $X(3872)$ couples to s-wave $D\bar{D}^*$, while η_{c2} to p-wave $D\bar{D}^*$

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Summary

Hamiltonian and self-energy

$c\bar{c}$ state $|\psi_0\rangle$ coupled with $D^0\bar{D}^{*0}$ state $|p,0\rangle$ and D^+D^{*-} state $|p,c\rangle$

V. Baru, *et al.*, PLB 586, 53 (2004); T. Hyodo, PRC 90, 055208 (2014)

$$\begin{pmatrix} M_0 & V & V \\ V & p^2/(2\mu_0) & 0 \\ V & 0 & p^2/(2\mu_c) + \Delta \end{pmatrix} |\Psi\rangle = E |\Psi\rangle, \quad |\Psi\rangle = \begin{pmatrix} c |\psi_0\rangle \\ \int d\mathbf{q} \chi_0(\mathbf{q}) |q,0\rangle \\ \int d\mathbf{q} \chi_c(\mathbf{q}) |q,c\rangle \end{pmatrix}$$

T-matrix

$$T_{ij}(p, p', E) = \frac{\langle p, i | V | \psi_0 \rangle \langle \psi_0 | V | p', j \rangle}{E - M_0 - \Sigma(E)} \sim \text{diagram 1} + \text{diagram 2} + \dots$$

Self-energy

$$\Sigma(E) = \Sigma_0(E) + \Sigma_c(E), \quad \Sigma_i(E) = \int d\mathbf{q} \frac{\langle \psi_0 | V | q, i \rangle \langle q, i | V | \psi_0 \rangle}{E_i - \Delta\delta_{ic} - q^2/(2\mu_i) + i0^+} \sim \text{diagram 3}$$

Pole condition: zero of the denominator

$$E_h - M_0 - \Sigma(E_h) = 0$$

Compositeness

Pole corresponds to discrete eigenstate (outgoing b.c.)

$$H |\Psi_h\rangle = E_h |\Psi_h\rangle$$

- Left eigenstate for resonances and normalization

$$\langle \widetilde{\Psi}_h | H = E_h \langle \widetilde{\Psi}_h |, \quad \langle \widetilde{\Psi}_h | \Psi_h \rangle = 1.$$

Compositeness X_i (weight of scattering channel i)

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, IJMPA 28, 1330045 (2013); F.K. Guo *et al.*, RMP 90, 015004 (2018);

T. Kinugawa, T. Hyodo, EPJA 61, 154 (2025)

$$X_i = \int d\mathbf{q} \langle \widetilde{\Psi}_h | \mathbf{q}, i \rangle \langle \mathbf{q}, i | \Psi_h \rangle, \quad Z = \langle \widetilde{\Psi}_h | \psi_0 \rangle \langle \psi_0 | \Psi_h \rangle \in \mathbb{C}, \quad \sum_i X_i + Z = 1$$

- Expression in terms of the self-energy

$$X_i = \left. \frac{-\Sigma'_i(E)}{1 - \Sigma'_0(E) - \Sigma'_c(E)} \right|_{E=E_h}, \quad Z = \left. \frac{1}{1 - \Sigma'_0(E) - \Sigma'_c(E)} \right|_{E=E_h}$$

Transition matrix element

Angular momentum structure of transition

- Bare state $|\psi_0\rangle$: spin J , polarization M
- $|p\rangle$: orbital angular momentum ℓ , D^* polarization λ

$$\langle p, \ell, \lambda, i | V | \psi_0; JM \rangle = g_i^{(\ell)} \sqrt{4\pi} p^\ell F^{(\ell)}(p) \sum_m \langle \ell m, 1\lambda | JM \rangle Y_{\ell m}(\hat{p})$$

- Coupling constant: $g_i^{(\ell)}$
- Form factor normalized as $F^{(\ell)}(0) = 1$

For s-wave $X(3872)$ and p-wave η_{c2}

- $X(3872)$: $\langle p, 0, \lambda, i | V | \chi_{c1}; 1M \rangle = g_i^{(0)} F^{(0)}(p) \delta_{\lambda M}$
- η_{c2} : $\langle p, 1, \lambda, i | V | \eta_{c2}; 2M \rangle = g_i^{(1)} \sqrt{4\pi} p F^{(1)}(p) \sum_m \langle 1m, 1\lambda | 2M \rangle Y_{1m}(\hat{p})$

Explicit model

Isospin symmetric coupling and sharp cutoff form factor

$$g_0^{(\ell)} = g_c^{(\ell)} \equiv g^{(\ell)}, \quad F^{(\ell)}(p) = \Theta(\Lambda - p)$$

- For a given M , reduced matrix element reads

$$\langle \mathbf{p}, i | V | \psi_0 \rangle = \begin{cases} g^{(0)} \Theta(\Lambda - p) & X(3872) \\ g^{(1)} \mathbf{p} \Theta(\Lambda - p) & \eta_{c2} \end{cases}$$

Explicit expression of self-energy

$$\Sigma_i(E) = \begin{cases} -8\pi\mu_i(g^{(0)})^2 \left[\Lambda + ik_i \arctan \left(-\frac{\Lambda}{ik_i} \right) \right] & X(3872) \\ -8\pi\mu_i(g^{(1)})^2 \left[\frac{\Lambda^3}{3} + k_i^2 \Lambda + ik_i^3 \arctan \left(-\frac{\Lambda}{ik_i} \right) \right] & \eta_{c2} \end{cases}$$

$$k_i = \sqrt{2\mu_i(E - \Delta\delta_{ic})}$$

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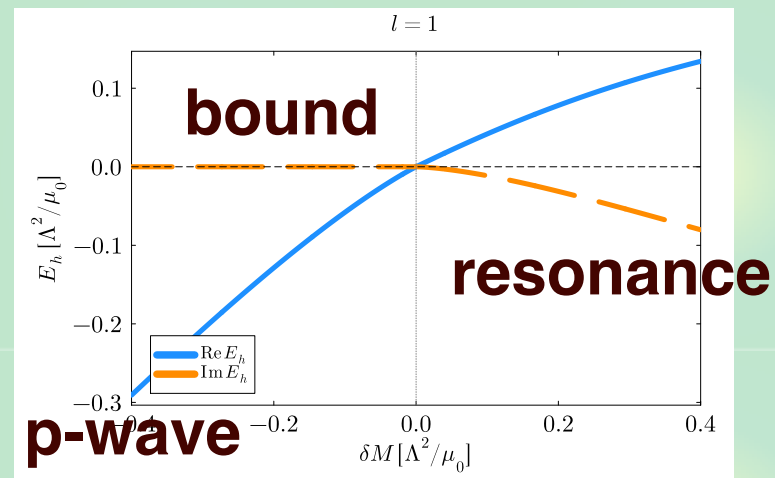
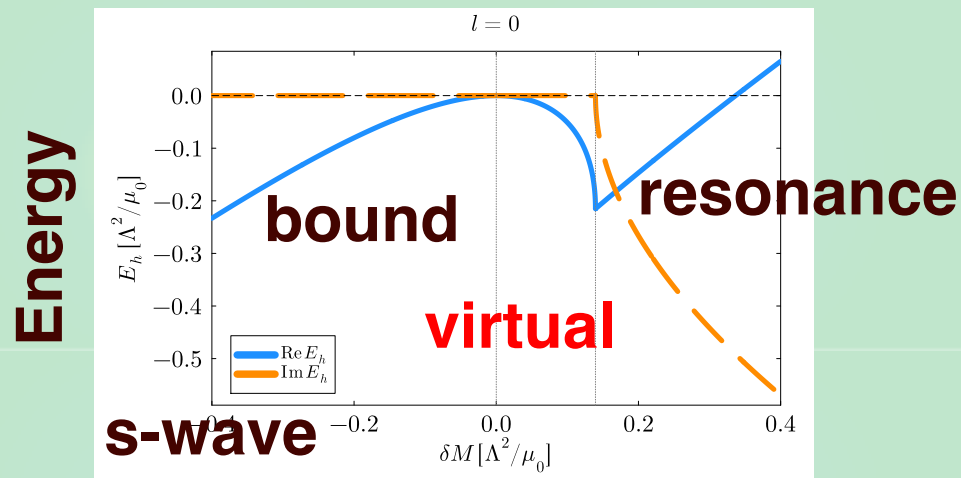


Summary

Near-threshold eigenenergy (single-channel)

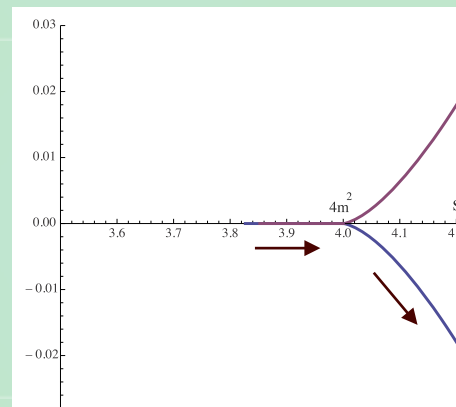
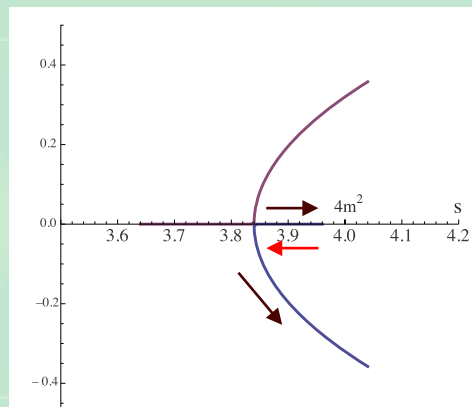
Eigenenergy as function of bare mass for fixed $g^{(\ell)}$

T. Hyodo, PRC 90, 055208 (2014)



c.f.) pole trajectory

C. Hanhart, J.R. Pelaez, G. Rios, PLB 739, 375 (2014)

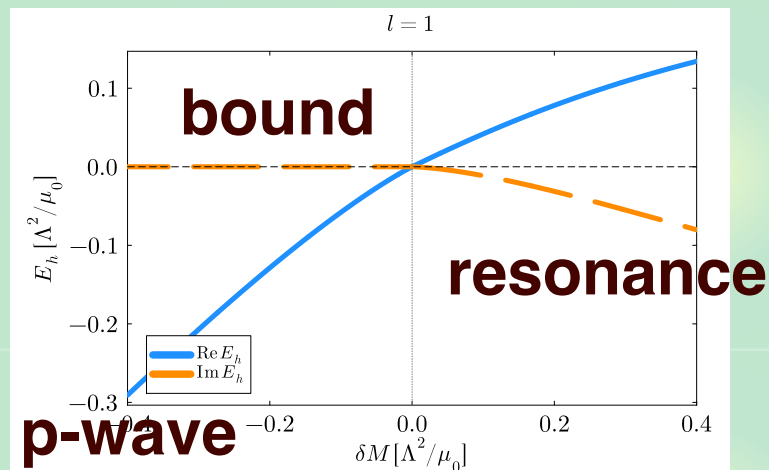
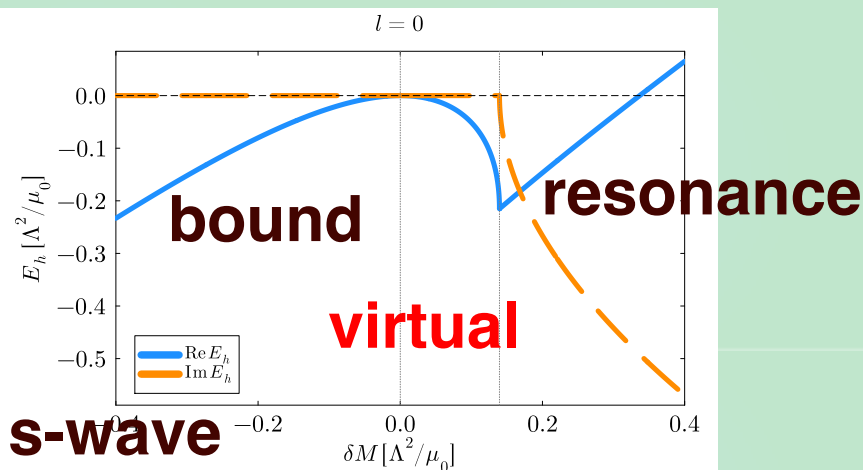


Near-threshold eigenenergy (single-channel)

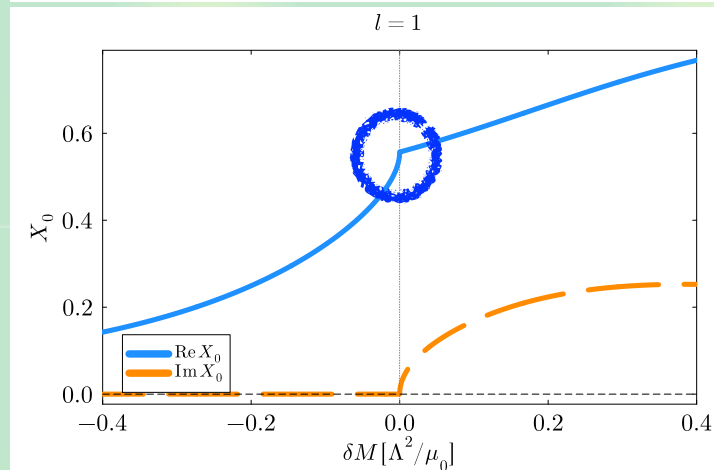
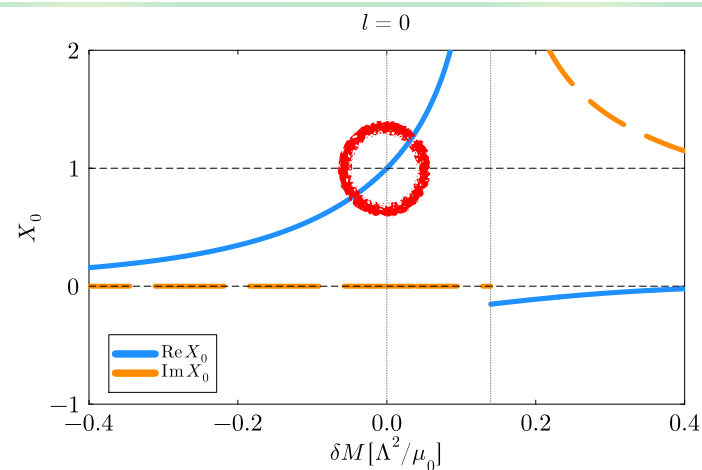
Eigenenergy as function of bare mass for fixed $g^{(\ell)}$

T. Hyodo, PRC 90, 055208 (2014)

Energy



Compositeness

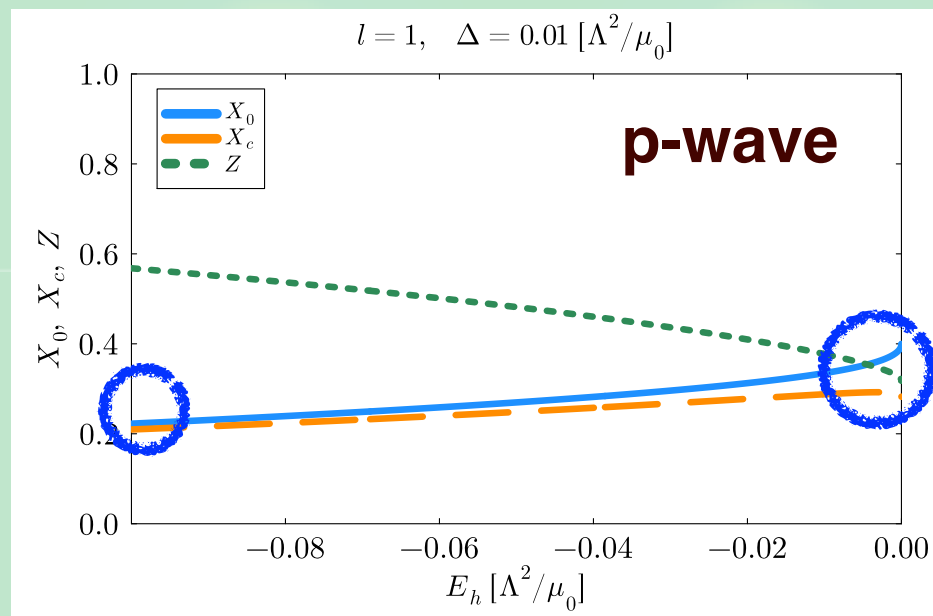
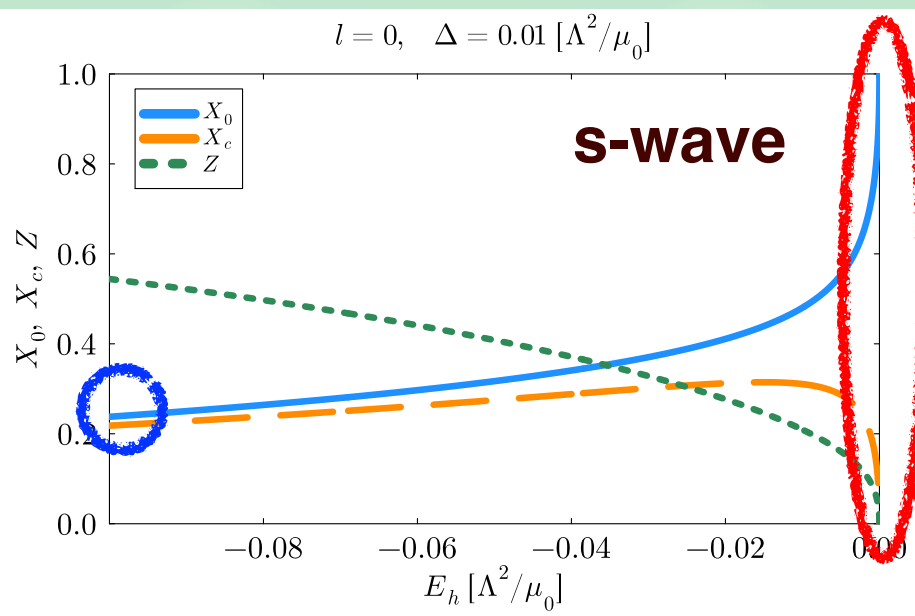


s-wave: $X \rightarrow 1$ at threshold, p-wave: arbitrary X at threshold

Isp spin symmetry breaking (weak)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.01 [\Lambda^2/\mu_0^2]$

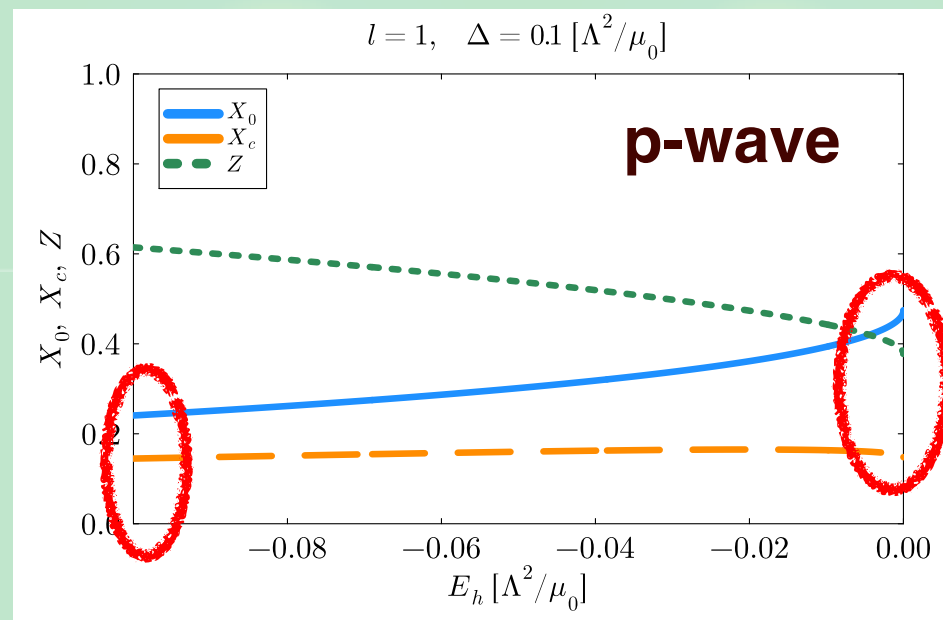
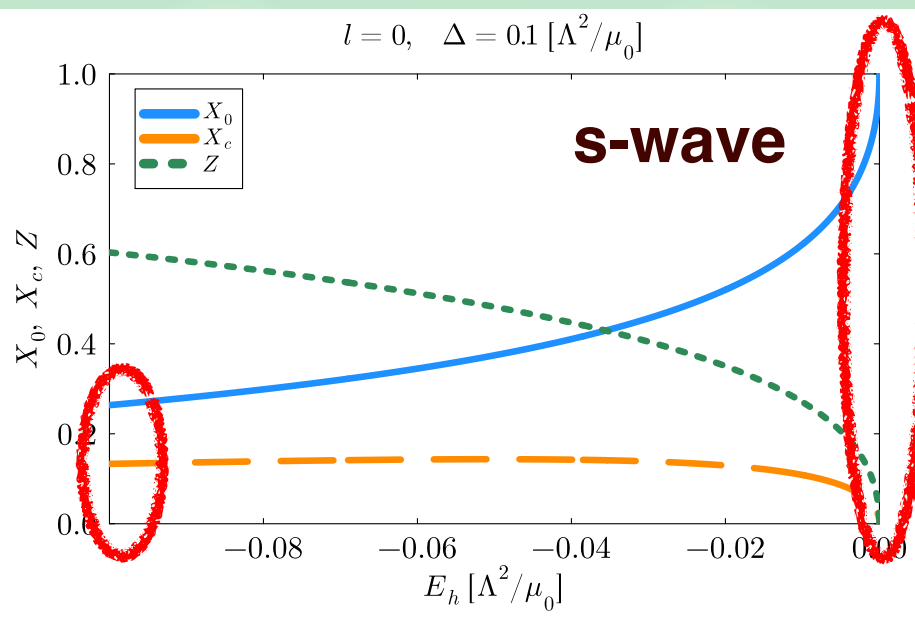


- Deep bound state: almost symmetric ($X_0 \approx X_c$)
- s-wave shallow bound state: large breaking ($X_0 \gg X_c$)
- p-wave shallow bound state: almost symmetric ($X_0 \approx X_c$)

Isospin symmetry breaking (large)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.1 [\Lambda^2/\mu_0^2]$



- s-wave shallow bound state: large breaking ($X_0 \gg X_c$)
- p-wave shallow bound state: substantial breaking ($X_0 > X_c$)
- Deep bound states: substantial breaking ($X_0 > X_c$)

Realistic setup

Input

- **Eigenenergy** E_h : $X(3872)$ from PDG, η_{c2} from Nakamura
- **Bare mass** M_0 : **constituent quark model**
S. Godfrey and N. Isgur, PRD 32, 189 (1985)
- **Cutoff** $600 \text{ MeV} \leq \Lambda \leq 1000 \text{ MeV}$

Input Resonance	Bare Mass (GeV)	Z	X_0	X_c
$X(3872)$	3.95	0.19 ± 0.04	0.69 ± 0.04	0.117 ± 0.004
η_{c2}	3.84	(1.49 ± 0.29) $-(0.14 \pm 0.12)i$	(-0.29 ± 0.17) $+(0.16 \pm 0.09)i$	(-0.20 ± 0.11) $+(0.03 \pm 0.02)i$
η_{c2}	3.95	(0.61 ± 0.11) $(0.03 \pm 0.01)i$	$+(0.23 \pm 0.07)$ $-(0.04 \pm 0.02)i$	(0.16 ± 0.04) $+(0.01 \pm 0.01)i$

$X(3872)$: **large isospin symmetry breaking** $X_0 \gg X_c$

η_{c2} : **almost isospin symmetric** $X_0 \approx X_c$

Summary

-  We study **isospin symmetry breaking** for near-threshold states.
T. Hyodo, PRC 90, 055208 (2014)
-  Near-threshold s-wave bound state is complete composite at threshold, while p-wave state has arbitrary structure even at threshold
T. Hyodo, PRC 90, 055208 (2014)
-  **Isospin symmetry** is largely broken for near-threshold s-wave states, while it is **not always broken for p-wave states**
-  Near-threshold η_{c2} follows isospin symmetry
M. Bradshaw, T. Hyodo, in preparation