Quantum entanglement in hadron and nuclear physics





Tetsuo Hyodo

RCNP, The University of Osaka

2026, Oct. 27th

Contents

Overview

- Entanglement in hadron and nuclear physics

Entanglement suppression

- Emergent symmetries in nuclear force

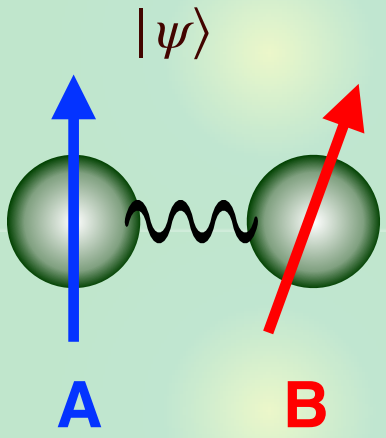
S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

- SU(3) baryon decuplet ($\Omega\Omega$) scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025);
K. Sone, T.R. Hu, F.K. Guo, T. Hyodo, I. Low, MPLA 41, 2642007 (2026)

Summary

Quantum entanglement



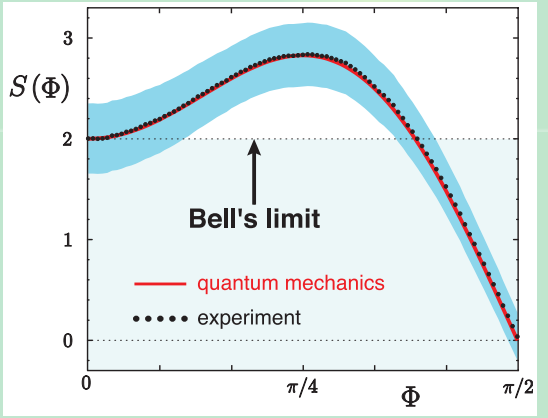
Entanglement

- Correlation between subsystem A and B

$$\mathcal{E}(|\psi\rangle) = -\,\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B\,|\psi\rangle\langle\psi|$$

- Quantum (not classical) correlation

H. Sakai, *et al.*, PRL97, 150405 (2006)



- Number of effective degrees of freedom

Applications

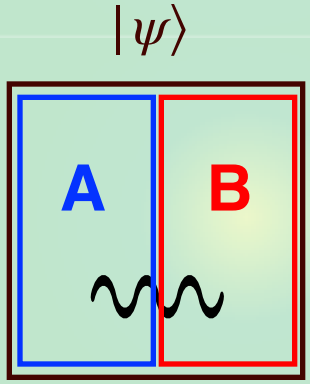
- QFT (vacuum) and phase transition

Talk by Takeda-san, …

- Few-body and many-body systems

Talk by Fukui-san, Wakasa-san, …

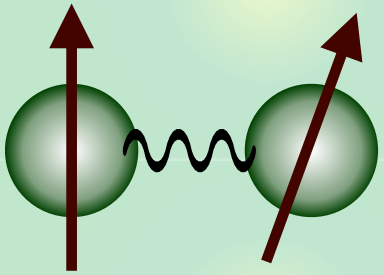
see, e.g. C.E.P. Robin, M.J. Savage, arXiv:2604.26376 [quat-ph]



Example: two-qubit system

Entanglement entropy for two-qubit system

$$\mathscr{E}_{\mathrm{vN}}(|\psi\rangle) = -\,\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B\,|\psi\rangle\langle\psi|$$



- **product state: minimum** $\mathscr{E} = 0$

$$|\uparrow\downarrow\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle$$

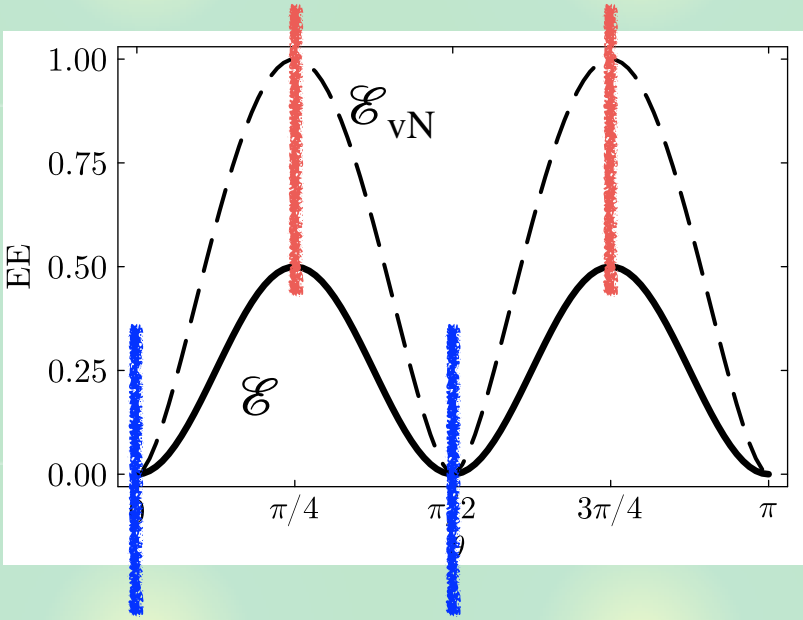
- **Linear combination** $1 \le \mathscr{E} \le 0$

$$|\psi\rangle = \cos\theta\,|\uparrow\downarrow\rangle + \sin\theta\,|\downarrow\uparrow\rangle$$

- **Bell state: maximum** $\mathscr{E} = 1$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

$|\Psi^+\rangle$ $\quad -|\Psi^-\rangle$



$|\uparrow\downarrow\rangle$ $\quad |\downarrow\uparrow\rangle$

Note: defined for pure states.

—> For mixed states, other entanglement measure is needed

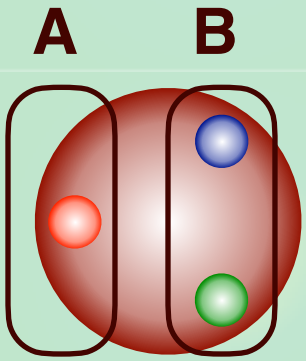
Quark-quark correlations: various d.o.f.

Color entanglement of quarks in a proton

A. Dumitru, E. Kolbusz, PRD105, 074030 (2022)

- q and (qq) are maximally entangled in color

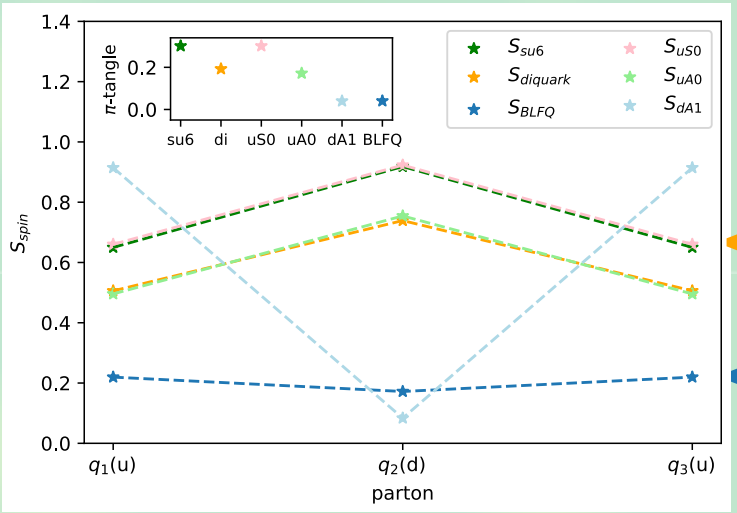
$$\rho_A = \mathrm{Tr}_B^{\mathrm{color}} |qqq\rangle\langle qqq| = \frac{1}{3} I_{3\times 3} \quad \Rightarrow \quad \mathscr{E}_{\mathrm{vN}} = \log 3$$



Spin entanglement of quarks in a proton

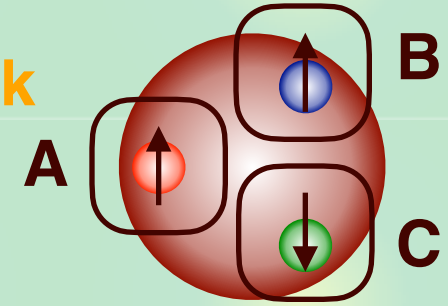
C. Qian, S. Xu, Y.-G. Yang, X. Zhao, arXiv:2603.16681 [hep-ph]

Entanglement



<— quark-diquark

<— BLFQ



Entanglement ~ diagnostic of emergent diquark correlations

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Summary

Symmetries of NN scattering

Fundamental symmetries $\mathrm{SU(2)_{spin}} \times \mathrm{SU(2)_{isospin}}$ **<— QCD**

- **spin** $\mathrm{SU(2)_{spin}}$ $\{|\uparrow\rangle, |\downarrow\rangle\}$
- **isospin** $\mathrm{SU(2)_{isospin}}$ $\{|p\rangle, |n\rangle\}$

$$\frac{1}{2} \otimes \frac{1}{2} = \underbrace{0}_{\mathrm{A}} \oplus \underbrace{1}_{\mathrm{S}}$$

s-wave NN **scattering: spin** J **× isospin** I **should be** A

$|J=0, I=1\rangle$ $(^1\mathrm{S}_0)$ **and** $|J=1, I=0\rangle$ $(^3\mathrm{S}_1)$**, independent**

Emergent symmetries **<— entanglement suppression?**

- **large scattering length** $|a_0| \gg 1$ fm**: NR conformal symmetry**

  T. Mehen, I.W. Stewart, M.B. Wise, PLB474, 145 (2000)

- **both** $^1\mathrm{S}_0$ **and** $^3\mathrm{S}_1$ **have large** $|a_0|$**: spin-flavor** $\mathrm{SU(4)}$

  $\{|p\uparrow\rangle, |p\downarrow\rangle, |n\uparrow\rangle, |n\downarrow\rangle\}$

  E. Wigner, PR51, 106 (1937), …, D.B. Kaplan, M.J. Savage, PLB365, 244 (1996), …

Entanglement suppression

Entanglement power (EP) of S-matrix $E(\hat{S})$

- ability of generating entanglement



$$|\psi_{\text{out}}\rangle = \hat{S}\,|\psi_{\text{in}}\rangle$$

$E(\hat{S})$

entangle?



Entanglement power of nuclear force (δ_J : phase shift)

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$$E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0 - \delta_1)] \geq 0$$

$E(\hat{S}) = 0$ **is achieved if (in the low-energy limit)**

$$\begin{cases} \delta_0 = \delta_1 \\ (\delta_0, \delta_1) = (0,\pi/2), (\pi/2,0) \end{cases}$$

spin-flavor SU(4) symmetry $\hat{S} \propto I$

NR conformal symmetry $\hat{S} \propto \text{SWAP}$

I. Low, T. Mehen, PRD104, 074014 (2021)

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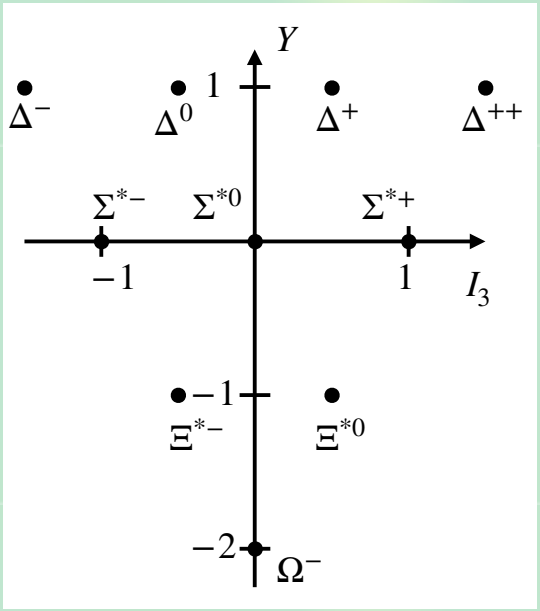
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Summary

Motivation

Decuplet baryons

- spin 3/2
- ground states with Octet in quark model



Large scattering length a_0 ($\delta \sim \pi/2$)?

- ΔΔ dibaryon, $d^*(2380)$?

F. Dyson, N.H. Xuong, PRL13, 815 (1964);
P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for ΩΩ scattering: $a_0(J=0) = 4.6 \text{ fm} \gg 1 \text{ fm}$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Quantum information viewpoint

- spin 3/2 scattering = 4d-qudit quantum gates

Results

Spin 3/2 scattering (δ_J with $J = 0,1,2,3$)

$$\delta_0 = \delta_2 = \delta_{\text{even}}, \quad \delta_1 = \delta_3 = \delta_{\text{odd}}, \quad \begin{cases} |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 & \text{: Identity gate} \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 & \text{: SWAP gate} \end{cases}$$

With flavor degrees of freedom

- **spin-flavor** $\mathrm{SU}(40)$ **symmetry** ($\hat{S} \propto 1$)
- $\mathrm{SU}(4)_{\text{spin}} \times \mathrm{SU}(10)_{\text{flavor}}$ **+ NR conformal** ($\hat{S} \propto \mathrm{SWAP}$)

ΩΩ scattering

- **Identical particle —> normalization of final state**

$a_0(J = 0) \gg 1$ fm **by lattice QCD suggests either**

- $|a_0(J = 2)| \gg 1$ fm **(unitary limit,** $|\delta_{0,28} - \delta_{2,28}| = 0$**) or**
- $a_0(J = 2) \sim 0$ **(noninteracting,** $|\delta_{0,28} - \delta_{2,28}| = \pi/2$**)**

Summary

- Entanglement sheds new light on hadron and nuclear physics
- Nature of vacuum ~ number of effective degrees of freedom
- Entanglement suppression allows prediction for unobserved $J = 2$ $\Omega\Omega$ scattering

  T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025); K. Sone, T.R. Hu, F.K. Guo, T. Hyodo, I. Low, MPLA 41, 2642007 (2026)

- Properties characterized solely by quantum entanglement?