10 years of the shell evolution by nuclear forces and beyond

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HPCI project field 5
“The origin of matter and the universe”
Outline

1. Shell evolution and tensor force

2. Shell evolution and QCD

3. Dual quantum liquid picture

4. Summary and perspectives
Spin-orbit splitting

Eigenvalues of HO potential

Magic numbers by Mayer and Jensen (1949)

- $5\hbar\omega$
- $4\hbar\omega$
- $3\hbar\omega$
- $2\hbar\omega$
- $1\hbar\omega$

$1h\omega$: 8
$2h\omega$: 20
$3h\omega$: 28
$4h\omega$: 82
$5h\omega$: 126
As $N$ or $Z$ is changed in an open shell, the shell structure is changed (evolved), and the change can be described by

- **Monopole component of the $NN$ interaction**

$$
\nu_{m;j,j'} = \sum_{k,k'} \langle jk'k'|V|jk'k'\rangle / \sum_{k,k'} 1,
$$

→ Averaged over possible orientations

This can be substantial change in exotic nuclei. For $j' = 9/2$, the multiplication by a factor of 10!

**Linearity: Shift**

$$
\Delta \epsilon_j = \nu_{m;j,j'} n_{j'}
$$

$n_{j'}$: # of particles in $j'$

Poves and Zuker made a major contribution in initiating systematic use of the monopole interaction. (Poves and Zuker, Phys. Rep. 70, 235 (1981))
Evolution of shell structure due to the tensor force

Tensor Interaction by pion exchange

\[ V_T = (\tau_1 \tau_2) \left[ \sigma_1 \sigma_2 \right]^{2} Y^{(2)}(\Omega) \right) Z(r) \]

\[ \text{contributes only to } S=1 \text{ states} \]

\[ \pi \text{ meson: primary source} \]

\[ \rho \text{ meson (} \sim \pi+\pi) \text{: minor (} \sim1/4 \text{) cancellation} \]

Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)
How does the tensor force work?

Spin of each nucleon \( \uparrow \) is parallel, because the total spin must be \( S=1 \)

The potential has the following dependence on the angle \( \theta \) with respect to the total spin \( \vec{S} \).

\[
V \sim Y_{2,0} \sim 1 - 3 \cos^2 \theta
\]

\( \theta = 0 \) attraction

\( \theta = \pi/2 \) repulsion

\( \theta \)

relative coordinate

\( \vec{S} \)

nucleons
Monopole effect of tensor force

One-dimensional collision model

At collision point:

\[ k = k_1 - k_2, \quad K = k_1 + k_2 \]

TO, Suzuki et al. PRL 95, 232502 (2005)
TO, Phys. Scr. T152, 014007 (2013)

- At collision point:
  \[ \Psi \propto e^{ik_1x_1}e^{ik_2x_2} + e^{ik_2x_1}e^{ik_1x_2} = 2e^{iKx}\cos(kx) \]

- Large relative momentum \( k \) leads to strong damping.

- Small relative momentum \( k \) leads to loose damping.

Wave function of relative coordinate.

Proton and neutron.
Appearance of N=32 and 34 magic structures

Shell structure for neutrons in Ni isotopes (f\(_{7/2}\) fully occupied)

N=34 magic number may appear if proton f\(_{7/2}\) becomes vacant (Ca) (f\(_{5/2}\) becomes less bound)

Predicted by TO et al., PRL 87, 082502 (2001)

Mayer-Jensen

ISOLDE experiment Huck et al., PRC 31, 2226 (1985).
FIG. 4: Systematics of excited-state energies in even-even Ca isotopes and neighbouring nuclei. a, Energies of first $2^+$ (closed symbols) and $3^-$ (open symbols) levels for even-even $^{42-54}\text{Ca}$ isotopes [28]. The results of the present study are indicated by triangular markers, and solid and dashed lines are shell-model predictions of $E(3^1_1)$ and $E(2^+_2)$, respectively (see text for details). Tentative wave-function assignments are enclosed by parentheses. b, $E(2^+_2)$ along the $N = 30, 32$ and $34$ isotonic chains. The solid and dashed lines are intended to guide the eye. Vertical dotted lines represent the traditional magic numbers in both plots.

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Input from chiral Effective Field Theory (EFT) of QCD

N3LO \( NN \) interaction

\( V_{\text{low } k} \) treatment of high momentum part

Fujita-Miyazawa type \( 3N \) interaction

Effective \( NN \) int.
Novel method for \textit{in-medium correction}

Kuo-Krenciglowa method *

\textbf{KK method}

Divergence problem in multi-shell

\[ H = H_0 + V = \left( \begin{array}{cc} PH_0 P & 0 \\ 0 & QH_0 Q \end{array} \right) + \left( \begin{array}{cc} PVP & PVQ \\ QVP & QVQ \end{array} \right) \]

\[ \hat{Q}(E) = PVP + PVQ \frac{1}{E - QHQ} QVP \]

\[ V_{\text{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0) \{ V_{\text{eff}}^{(n-1)} \}^k. \]

Extended KK method **

\textbf{EKK method}

New parameter \( E \) (arbitrary parameter)

\[ H = H'_0 + V' = \left( \begin{array}{cc} E & 0 \\ 0 & QH_0 Q \end{array} \right) + \left( \begin{array}{cc} P\hat{H}P & PVQ \\ QVP & QVQ \end{array} \right), \]

\[ H_{BH}(E) = PHP + PVQ \frac{1}{E - QHQ} QVP. \]

\[ \tilde{H}_{\text{eff}}^{(n)} = \tilde{H}_{BH}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E) \{ \tilde{H}_{\text{eff}}^{(n-1)} \}^k. \]


EFT $NN$ int. + Fujita-Miyazawa $3N$ int. with averaging
(to be replaced by EFT N2LO $3N$ int.)

$V_{low\, k}$ : treatment of high-momentum components

EKK : in-medium correction (core polarization)

Shell model Hamiltonian

- Effective single-particle energy
  ($N$ or $Z$ dependence of effects of monopole int.)
- Energy levels, electromagnetic matrix elements
  (diagonalization of Hamiltonian matrix)
Island of Inversion: neutron effective single-particle energies

Present work (EFT + EKK)

Meson exchange potential

Shell evolution arises also from QCD

... and other properties obtained by the shell-model diagonalization in the $sd + pf$ shell.
Ca isotopes in the pf + sdg shell

The prediction of N=34 magic number (2001) is consistent with EFT (QCD) + EKK theoretical calculation.

TO et al, PRL 87, 082502 (2001)
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shape coexistence

Island of Inversion
(Z=10~12, N=20)

$^{16}\text{O}$
H. Morinaga (1956)

$^{186}\text{Pb}$

REVIEW OF MODERN PHYSICS. VOLUME 83, Shape coexistence in atomic nuclei
Kris Heyde* John L. Wood†
Monte Carlo Shell Model (MCSM) calculation on Ni isotopes

This model space is wide enough to discuss how **magic numbers 28, 50 and semi-magic number 40** are visible or smeared out.

Interaction:
A3DA interaction is used with minor corrections

Y. Tsunoda et al.
Energy levels and B(E2) values of Ni isotopes

Description by the same Hamiltonian

Shape coexistence in $^{68}\text{Ni}$

Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)
Effective s.p.e. by actual occupation numbers

Occupation numbers

(c) proton
- $d_{5/2}$
- $g_{9/2}$
- $p_{3/2}$
- $f_{5/2}$
- $p_{1/2}$
- $f_{7/2}$

(d) neutron
- $f_{7/2}$
- $f_{5/2}$
- $p_{3/2}$
- $p_{1/2}$
- $g_{9/2}$

0$^+$ state
- 1st modest correlations
- 2nd 2p-2h
- 3rd 6p-6h
Underlying mechanism of the appearance of low-lying deformed states:

**Type II Shell Evolution**

- Different isotopes
- Within the same nucleus

Monopole effects on the shell structure from the tensor interaction

Type I Shell Evolution: different isotopes

- $j_\geq$ : particles
- $j_\leq$ : holes

Type II Shell Evolution: within the same nucleus

- $j_\geq$ : particles
- $j_\leq$ : holes
Type II Shell Evolution in $^{68}$Ni ($Z=28$, $N=40$)

Spin-orbit splitting works against quadrupole deformation (cf. Elliott’s SU(3)).

Weakening of spin-orbit splitting

Type II shell evolution

Stronger deformation of protons → more neutron p-h excitation

PES along axially symmetric shape

Reset Hamiltonian with Type II SE suppressed

The local minima become much less pronounced.

Shape coexistence is enhanced by type II shell evolution as the same quadrupole interaction works more efficiently.

Type II shell evolution is suppressed by resetting monopole interactions as

$$\pi f_{7/2} - \nu g_{9/2} = \pi f_{5/2} - \nu g_{9/2}$$

$$\pi f_{7/2} - \nu f_{5/2} = \pi f_{5/2} - \nu f_{5/2}$$
Effect of the tensor force

Bohr-model calc. by HFB with Gogny force, Girod, Dessagne, Bernes, Langevin, Pougheon and Roussel, PRC 37,2600 (1988)

no (explicit) tensor force
The atomic nucleus is a Quantum (Fermi) Liquid (of Landau) described by interplay between single-particle energies and “residual” interaction - in a way like free particles -

For most of states, there may have been Ansatz that

Spherical single particle energies remain basically unchanged. -> spherical part of Nilsson model

Correlations originating in nuclear forces (residual interaction) produce various features, including shape evolution and shape coexistence.
Dual quantum liquids in the same nucleus

Certain configurations produce different shell structures owing to (i) tensor force and (ii) proton-neutron compositions

Liquid 1 (~constant spherical SPE)

relevant to normal states in general

Liquid 2 (varying spherical SPE)

relevant to specific intruder states

Note: density -> single-particle energies in other many-body systems
Shape evolution and phase transitions

68Ni

~6p6h

prolate 0^+

~2p2h

0^+

2^+

N

E (MeV)

70Ni

critical phenomenon

prolate

oblate

spherical

74Ni

large fluctuation

prolate

oblate

spherical

78Ni

oblate

prolate

spherical

⇔ 2^+ states in the g_{9/2} seniority scheme
Other cases ….. just an example

\( ^{186}\text{Pb} \) Andreyev et al., Nature 405, 430 (2000)
Type II shell evolution reduces the barrier and make the local minimum more profound

-> passage to fission … (long-term open question)
Summary and perspectives

Quantum Liquid picture (of Landau) with stable shell structure (a la Mayer-Jensen) has been a good conceptual guidance for most of states of many nuclei near the stability line on the Segre chart.

The central and tensor parts of nuclear force produce Type I Shell Evolution in many domains on the nuclear chart particularly away from the stability line, presenting a paradigm shift. Type I Shell Evolution is shown to occur in ab initio-type approaches with EFT (QCD) + EKK.

There is another new aspect, Type II Shell Evolution, resulting in Dual Quantum Liquid picture. This produces dynamical (softer) shell structure in a non-linear way, where the two essential ingredients may be:
- force that can change $ls$-splitting, like the tensor force
- proton-neutron contents of quantum liquids

Could we solve the problem of (spontaneous) fission?
Could it be a way to Island of Stability?
Collaborators

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$^{68}\text{Ni} \ 0^+$ states

occupation numbers

(a) proton
(b) neutron

Gaps

ESPE

effective single-particle energies (ESPE) for correlated eigenstate

$$\epsilon_j = \langle \frac{\partial H_m}{\partial n_j} \rangle$$

monopole part of $H$

$\langle >$ : by actual occup. numbers