Nucleon Pair Approximation to the nuclear Shell Model

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The shell model

SM gives a good description to the level spectra, electromagnetic properties, and transition rates for nuclei close to a double-magic nucleus.

a) As the number of valence nucleons and the number of single-$j$ orbits increase, the dimension of the SM space increase dramatically.

b) If diagonalization can be performed, it is usually difficult to obtain a simple picture from the complicated wave function.

NPA is a pair truncation scheme of the SM.
Outline

- A brief introduction of the NPA

- Like-nucleon pair approximation
  
  Simple structures of semi-magic nuclei

- NPA with isospin symmetry
  
  Isoscalar spin-aligned proton-neutron pairs in heavy $N=Z$ nuclei
  
  Dual description by isovector and isoscalar pair approximation

- Summary

- Future and perspective
Why we use Pair Approximation?

- *S pair condensation* is found in ground states of semi-magic even-even nuclei!

- In a single-\(j\) shell calculation, binding energy of a semi-magic nucleus can be written by

\[
BE(j^n\text{ g.s.}) = BE(n=0) + nA + n(n-1)B/2 + \lceil n/2 \rceil C
\]

\(A\) is S.P.E.; \(B\) and \(C\) are combinations of the shell model two-body interaction parameters
Why we use Pair Approximation?

Nuclear force: attractive and short range

\[ \delta \text{ Interaction} \]

\[ V_{12} (\delta) = -V_0 \delta (r_1 - r_2) \]

\[ = \frac{-V_0}{r_1 - r_2} \delta (r_1 - r_2) \delta (\cos \theta_1 - \cos \theta_2) \delta (\Phi_1 - \Phi_2) \]

Interacting boson model (\textit{sd} boson model), \textit{successful}!
The NPA of the shell model

shell model basis:

\[ |\varphi\rangle = \left[ \left( (C_{j_1}^+ C_{j_2}^+ C_{j_3}^+) \right)_{j_3} ... C_{j_n}^+ \right]_{J_n M_n} |0\rangle. \]

nucleon pair:

\[ A^{r\dagger} = \sum_{ab} y(ab) A^{r\dagger} (ab), \quad A^{r\dagger} (ab) = (C_a^+ \times C_b^+)^r \]

pair basis:

\[ |\varphi\rangle = \left[ \left( (A^{(r_1)} \times A^{(r_2)} \times A^{(r_3)}) \right)^{(J_3)} ... A^{(r_N)} \right]^{(J_N)}_{M_N} |0\rangle. \]

If all possible nucleon pairs are considered, the NPA space = the SM space

In the NPA, the same Hamiltonian and transition operators are used as in the SM
Truncation

pair basis: \(|\varphi\rangle = \left[\left(\left( A^{(r_1)} + A^{(r_2)} \right)^{(J_2)} A^{(r_3)} \right)^{(J_3)} \cdots A^{(r_N)} \right]^{(J_N)}_{M_N} |0\rangle\).

1. Only a few pairs with certain spin and parity are considered, e.g., \(S\) pair with spin zero and positive parity, \(D\) pair with spin two and positive parity.

2. Using collective pairs

\[ A^{r\dagger} |0\rangle = \sum_{ab} y(ab)r A^{r\dagger}(ab) |0\rangle \]

As more orbits are considered,

a) dimension of NPA space constructed by collective pairs does not change;
b) time for calculating matrix elements increases not so much.
Although the NPA and the IBM have the similar idea (pair truncation), the NPA is a fermion model (like shell model), not a boson model.

Based on fermion’s commutation relation and angular momentum algebra:

\[ \left[ \tilde{A}_k^r, A_{s}^{t\dagger} \right]^{(t)} = 2 \hat{r}_k \delta_{t,0} \delta_{s, r_k} \sum_{ab} y(abr_k)y(abs) - \sum_{ad} \left( 4 \hat{r}_k \delta \sum_{b} y(abr_k)y(bds) \left\{ \begin{array}{c} r_k \\ d \\ a \\ b \end{array} \right\} \right) \left( c_d^t \times \tilde{c}_a \right)^{(t)} , \]

\[ A_{k}^{r,s} = [Q^{t}, A_{k}^{r,s}]^{(r')} = (-)^{r_k+r'_k} \left\{ \sum_{da} y'(dar'_k)A_{k}^{r,s}(da), \quad 1 \leq k \leq N, \right. \]

\[ \left( (-)^{t-j'-j} q(j'j)t/\tilde{j'} c_{j'}^t, \quad k = 0 (r_k = j, r'_k = j'), \right. \]

\[ y'(dar'_k) = z(dar'_k) - \theta (dar'_k) z(adr'_k), \quad z(dar'_k) = \hat{r}_k \tilde{t} \sum_{b} y(abr_k)q(b_k dt) \left\{ \begin{array}{c} r_k \\ d \\ a \\ b_k \end{array} \right\}. \]
The key technique of the NPA is the Wick theorem of coupled fermion cluster developed by Chen. Recursive formulas for calculating the matrix elements.

This method was refined by Zhao et. al., and calculations for odd-number nucleon system become practical.

The generalized version of the NPA with isospin symmetry has been recently presented by Fu et. al.

The coupling for angular momentum and that for isospin are both needed.

For a comprehensive review, see Y. M. Zhao and A. Arima, Phys. Rep. 545, 1 (2014)
<table>
<thead>
<tr>
<th>References</th>
<th>Collective pairs</th>
<th>Nuclei</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caprio, Luo et al. [54]</td>
<td>$v_s \leq 3, v_v \leq 2$</td>
<td>even and odd $^{21-39}$Ca</td>
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<tr>
<td>Caprio, Luo et al. [55]</td>
<td></td>
<td>even-even $^{42-58}$Ca, even-even $^{44-60}$Ti,</td>
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<td>Rowe, Rosenstreet [56]</td>
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<td>even-even $^{46-62}$Cr</td>
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<td>Monnoye, Pitrel et al. [65]</td>
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<td>$^{92}$Mo, $^{94}$Ru, $^{95}$Pd,</td>
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<tr>
<td>Sandulescu, Blomqvist et al. [69]</td>
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<td>odd-mass $^{57-60}$Ni</td>
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<tr>
<td>Morales, Isacker et al. [70]</td>
<td>$s^N + s^N(-1)D$</td>
<td>even-even $^{104-112}$Sn</td>
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<tr>
<td>Qi, Blomqvist et al. [116]</td>
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<td>even-even $^{102-130}$Sn</td>
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<tr>
<td>Xu, Qi et al. [117]</td>
<td>$(g_9/2)_{9/2}, T = 0$</td>
<td>$^{92}$Pd, $^{94}$Ag, $^{96}$Cd,</td>
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<tr>
<td>Fu, Shen et al. [115]</td>
<td>$(g_9/2)_{9/2}, T = 0$</td>
<td>$^{92}$Pd, $^{94}$Ag, $^{96}$Cd,</td>
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<td>Kwasniewicz, Brzostowski et al. [121]</td>
<td>$(g_9/2)_{9/2}, T = 0$</td>
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<td>Kartamyshov, Engelard et al. [149]</td>
<td>$SS' + DD' + D''$</td>
<td>$^{112}$Cd, $^{112}$Cd, $^{114}$Cd,</td>
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<td>Xu, Lei et al. [150]</td>
<td>$v \leq 2$</td>
<td>$^{112}$Cd, $^{114}$Cd, $^{116}$Cd,</td>
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<td>Jia, Zhang et al. [143, 151]</td>
<td>$SD + G_n L_n K_n G_v$</td>
<td>$^{116}$Xe, $^{118}$Ba, $^{120}$Te, $^{122-138}$Ce,</td>
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<tr>
<td>Jia, Zhang et al. [151]</td>
<td>$SD + G_n L_n K_n G_v$</td>
<td>$^{134}$Ba, $^{116}$Xe, $^{118}$Ba</td>
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<td>Pan, Ping et al. [152]</td>
<td>$SO(6) + SD$</td>
<td>$^{114}$Xe, $^{116}$Ba, $^{118}$Te, $^{120}$Te, $^{122-138}$Ce, $^{134}$Ba, $^{116}$Xe, $^{118}$Ba</td>
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<tr>
<td>Chen, Luo et al. [153, 154]</td>
<td>$SD + (h \frac{1}{2} \frac{1}{2} \frac{1}{2}) (2i)$</td>
<td>$^{112}$Ce, $^{114}$Ce, $^{116}$Ce, $^{118}$Ce, $^{120}$Nd,</td>
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<td>Zhao, Yamaji et al. [157]</td>
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<td>Yoshinaga, Higashiyama [158]</td>
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<td>Takahashi, Yoshinaga et al. [160]</td>
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<td>Higashiyama, Yoshinaga [161]</td>
<td>$SD + A_1^{(3)} J = 5, 6$</td>
<td>$^{110, 112}$Cs, $^{112, 114}$La,</td>
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<tr>
<td>Lei, Xu et al. [162]</td>
<td>$SD + A_1^{(3)} J = 5, 6$</td>
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<td>Lei, Fu et al. [163]</td>
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<td>Jiang, Fu et al. [166]</td>
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<td>Zerguine, Isacker [170]</td>
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<td>Higashiyama, Yoshinaga [171]</td>
<td>$SD + (g_9/2)_{9/2} (0)$</td>
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<td>Jiang, Qi et al. [172]</td>
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<td>$^{110, 112}$Cs, $^{112, 114}$La,</td>
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<td>Jiang, Lei et al. [173]</td>
<td>$SD + (g_9/2)_{9/2} (0)$</td>
<td>$^{110, 112}$Cs, $^{112, 114}$La,</td>
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</table>

A = 59 – 62, nuclei, with T = 0 and 1.

even and odd mass $^{136-142}$Sn.

even-even $^{202-206}$Pb, $^{205-210}$Pb, $^{212-215}$Bi, $^{219-224}$Ra.

odd-A $^{203-207}$Bi, $^{209-211}$At, $^{217-219}$Fr.

odd-A $^{203-207}$Pb, $^{205-209}$Pb, $^{207-211}$Rn, $^{211, 213}$Ra.

Both even and odd-A $^{124-135}$Sn, $^{126-142}$Te, $^{128-144}$Xe.

Both even and odd-A $^{130-140}$Ba, $^{132-148}$Ce.

Odd-mass $^{125-141}$Sb, $^{127-143}$I, $^{129-149}$Cs, $^{129-149}$La.

$^{126-135}$Xe, $^{131-139}$Ba.

even-even $^{128-134}$Xe, $^{130-136}$Ba, $^{126-132}$Te, $^{132-138}$Ce, $^{134}$Ba.

$^{116}$Xe, $^{128}$Ba.

even-even $^{124-140}$Sn, $^{126-132}$Te, $^{128-144}$Xe, $^{130-136}$Ba, $^{132-138}$Ce.

even-even $^{126-134}$Xe, $^{128-136}$Ba, $^{130-138}$Ce, $^{132-140}$Nd, $^{132}$Ba.

Odd-mass $^{129-135}$Xe, $^{131-135}$Ba, $^{133-137}$Ce.

$^{112}$Ba, $^{116}$Ba, $^{118}$Ba, $^{120}$Ba, $^{128}$Ba.
Applications of like-nucleon pair approximation

- Description of low-lying states of nuclei
- Backbending in yrast states of even–even nuclei (Yoshinaga, Lei et al.)
- Negative parity states of even–even nuclei (Lei et al.)
- Chiral bands in odd–odd nuclei (Yoshinaga et al.)
- B(E2) and g factors of even–even Sn isotopes (Jiang et al.)
- Shape phase transitions in the NPA (Luo, Lei et al.)
- Nucleon-pair approximation with random interactions (Zhao et al.)
A brief introduction of the NPA

Like-nucleon pair approximation

Simple structures of semi-magic nuclei

NPA with isospin symmetry

Isoscalar spin-aligned proton-neutron pairs in heavy $N=Z$ nuclei

Dual description by isovector and isoscalar pair approximation

Summary

Future and perspective
Semi-magic nuclei: Z=50 isotopes $^{128,126,124}$Sn

Effective interaction based on realistic CD-Bonn nucleon-nucleon potential, the monopole part of which is refined [Chong Qi and Z. X. Xu, Phys. Rev. C 86, 044323 (2012)]

$$H = \sum_\alpha \varepsilon_\alpha \hat{N}_\alpha + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle j_\alpha j_\beta | V | j_\gamma j_\delta \rangle_{JT} A^\dagger_{JT:j_\alpha j_\beta} A_{JT:j_\delta j_\gamma}$$

NPA configuration space

Positive-parity states: positive-parity pairs with spin zero, two, four, six, eight and ten, denoted by $S, D, G, I, K$ and $M$.

Negative-parity states: the $SDGIKM$ pairs coupled to one negative-parity pair with spin four, five, six and seven, denoted as $\mathcal{G}, \mathcal{H}, \mathcal{I}$ and $\mathcal{J}$.
The experimental level energies are well reproduced by SM calculation.

The level spectra given by NPA is almost the same as that given by SM.
Validity of NPA: comparison with shell-model (SM) calculation

(a) positive-parity, even-spin
(b) positive-parity, odd-spin
(c) negative-parity, even-spin
(d) negative-parity, odd-spin

$B(E2, I \rightarrow I-2)$ (W.u.)

spin

$^{128}$Sn
For most states the overlap is close to 1. SM w. f. are well approximated by NPA w. f.
Dominant configuration for yrast states of $^{128}$Sn

overlap-1: with NPA w.f.

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>dominant configuration</th>
<th>overlap-1</th>
<th>overlap-2</th>
</tr>
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<tbody>
<tr>
<td>$0^+_1$</td>
<td>$</td>
<td>S^2\rangle$</td>
<td>0.99</td>
</tr>
<tr>
<td>$1^+_1$</td>
<td>$</td>
<td>SP\rangle$</td>
<td>1</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>$</td>
<td>SD\rangle$</td>
<td>0.99</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>$</td>
<td>SG\rangle$</td>
<td>0.96</td>
</tr>
<tr>
<td>$6^+_1$</td>
<td>$</td>
<td>SI\rangle$</td>
<td>0.99</td>
</tr>
<tr>
<td>$8^+_1$</td>
<td>$</td>
<td>SK\rangle$</td>
<td>0.99</td>
</tr>
<tr>
<td>$10^+_1$</td>
<td>$</td>
<td>SM\rangle$</td>
<td>0.99</td>
</tr>
<tr>
<td>$4^-_1$</td>
<td>$</td>
<td>SG\rangle$</td>
<td>0.96</td>
</tr>
<tr>
<td>$5^-_1$</td>
<td>$</td>
<td>SH\rangle$</td>
<td>0.98</td>
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<tr>
<td>$6^-_1$</td>
<td>$</td>
<td>SI\rangle$</td>
<td>0.78</td>
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<tr>
<td>$7^-_1$</td>
<td>$</td>
<td>SJ\rangle$</td>
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</table>

overlap-2: with SM w.f.

<table>
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<th>$J^P$</th>
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<th>overlap-1</th>
<th>overlap-2</th>
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<td>$5^+_1$</td>
<td>$</td>
<td>DG\rangle$</td>
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<td>$7^+_1$</td>
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<td>DI\rangle$</td>
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<td>$9^+_1$</td>
<td>$</td>
<td>DM\rangle$</td>
<td>0.92</td>
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<td>$12^+_1$</td>
<td>$</td>
<td>DM\rangle$</td>
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</tr>
<tr>
<td>$13^+_1$</td>
<td>$</td>
<td>IM\rangle$</td>
<td>0.99</td>
</tr>
<tr>
<td>$14^+_1$</td>
<td>$</td>
<td>GM\rangle$</td>
<td>$\approx$1</td>
</tr>
<tr>
<td>$16^+_1$</td>
<td>$</td>
<td>IM\rangle$</td>
<td>$\approx$1</td>
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<tr>
<td>$8^-_1$</td>
<td>$</td>
<td>DJ\rangle$</td>
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<td>$16^-_1$</td>
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<td>MJ\rangle$</td>
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overlap-1: NPA w.f. can be well represented by a one-dimension pair configuration.

overlap-2: as SM w.f. are well approximated by NPA w.f., SM w.f. can also be represented by the one-dimension pair configuration!
Features of yrast states based on dominant configuration

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<td>SJ\rangle$</td>
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Having only one non-$S$ pair
Seniority-two states
Positive-parity yrast states with spin larger than ten:
  having one $M$ pair (positive-parity and spin-ten pair);
  can be described by breaking one $S$ pair of $10^+_1$ (positive-parity, spin-maximum and seniority-two state)

Negative-parity yrast states with spin larger than seven:
  having one $J$ pair (negative-parity and spin-seven pair);
  can be described by breaking one $S$ pair of $7^-_1$ (negative-parity, spin-maximum and seniority-two state)
NPA calculation for yrast states of $^{126}\text{Sn}$ and $^{124}\text{Sn}$
For $^{126}\text{Sn}$ and $^{124}\text{Sn}$ as well as $N=82$ isotones $^{136}\text{Xe}$ and $^{138}\text{Ba}$:

1. NPA wave function for their yrast states can also be well represented by one-dimension configuration in terms of collective pairs.

2. Features for their yrast states based on the one-dimension configuration are the same as those for $^{128}\text{Sn}$
Outline

- A brief introduction of the NPA
- Like-nucleon pair approximation
  
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- NPA with isospin symmetry
  
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- Future and perspective
Nuclear force is charge independent

The Coulomb interaction doesn’t play an important role in low-lying states

A proton and a neutron are the same kind of nucleons, with different $z$-components of isospin

$T = 1/2, \quad T_z = +1/2 \quad \text{for} \quad = -1/2 \quad \text{for} \quad $}

Analogizing singlet and triplet states in spin space, one have

$T = 1, \quad T_z = 1$

isospin triplet states

$T = 1, \quad T_z = -1$

isovector pairs

$T = 1, \quad T_z = 0$

isospin singlet state

$T = 0, \quad T_z = 0$ → isoscalar pair
How about isoscalar proton-neutron pairs?

Nuclear force:
- attractive and short range

\[ V_{12}(\delta) = -V_0 \delta(r_1 - r_2) \]
\[ = -\frac{V_0}{r_1 - r_2} \delta(\cos \theta_1 - \cos \theta_2) \delta(\Phi_1 - \Phi_2) \]

\[ T = 1 \]
\[ j = 9/2 \]

\[ 4^{+}Sc \]

\[ 4^{+} \]
\[ 2^{+} \]
\[ 3^{+} \]
\[ 5^{+} \]
\[ 7^{+} \]
\[ 0^{+} \]

spin-aligned pair

Even J

Odd J

P pair (J=1 pair)
Evidence for a spin-aligned neutron-proton paired phase from the level structure of $^{92}$Pd

B. Cederwall et al.

$^{92}$Pd: a system of four-proton holes and four-neutron holes below $^{100}$Sn
“Number of pairs” in the Shell Model

Spin-aligned neutron-proton pair mode in atomic nuclei

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(Received 20 January 2011; revised manuscript received 28 June 2011; published 10 August 2011)

FIG. 3. (Color online) Average number of isoscalar \( (0^+_2)_{\frac{1}{2}+} \) pairs, \( C^I_J \), as a function of total angular momentum \( I \) for the wave functions of the yrast states of \(^{92}\text{Pd}\), compared with those of the normal isovector \( J = 0 \) pair.
Using the NPA with isospin symmetry, we calculate low-lying states of $^{96}\text{Cd}$, $^{94}\text{Ag}$ and $^{92}\text{Pd}$ in the $pfg$ shell below doubly-magic nucleus $^{100}\text{Sn}$, with the JUN45 interaction.

- **We study nucleon pairs including**
  - $S$ pair: $J=0$
  - $D$ pair: $J=2$
  - $K$ pair: $J=8$
  - $A^{(9)}$ pair: $J=9$
$^{96}$Cd: the lowest $T=0$ states

- **S pair**: $J=0, T=1$
- **D pair**: $J=2, T=1$
- **K pair**: $J=8, T=1$
- **$A^{(9)}$ pair**: $J=9, T=0$

**Dual description:** Non-orthogonality feature of nucleon-pair basis!
$^{96}$Cd: the lowest $T=0$ states

The lowest $8^+$ state cannot be well described by the $A^{(9)}$ pair. It can be described by the seniority two configuration.
Deformed $sd$ shell nuclei

$PFH \ (T=0, \ J=1,3,5)$ pairs /$SDG \ (T=1, \ J=0,2,4)$ pairs give us similar energy spectra.

For the ground band:

a) the overlap between SM w.f. and w.f. given by isovector $SDG$ pair approximation, and the overlap between SM w.f. and w.f. given by isoscalar $PFH$ pair approximation, are both large ---- dual description

b) the overlap between w.f. given respectively by $SDG$ and $PFH$ pair approximations is 0.800, 0.800, 0.845, 0.889, and 0.991 for 0+, 2+, 4+, 6+ and 8+ ---- non-orthogonality feature of nucleon-pair basis

FIG. 6. (Color online) Overlaps between wave functions obtained in nucleon-pair truncated subspaces with the USDB interaction (or the $P + Q$ interaction) and those obtained by the shell model with the USDB interaction, for (a) the lowest and (b) the second lowest states of $T = 0$ in $^{20}$Ne. “USDB” and “$P + Q$” are the same as in Fig. 5.
$^{24}\text{Mg}$: dual description
Summary

- For most **yrast states of semi-magic nuclei**, shell-model wave function can be well approximated by our NPA wave function; NPA wave function can be further **represented by a one-dimension configuration in terms of collective pairs**.

- For semi-magic nuclei, if the positive-parity yrast state with spin $J$ is the spin-maximum seniority-two state, the positive-parity yrast states with spin larger than $J$ can be described by breaking $S$ pairs of this spin-$J$ state; same for the negative-parity states.

- Many-$j$ shell calculation confirms the importance of the spin-aligned isoscalar pair on the $1g_{9/2}$ orbit in $^{96}$Cd. The traditional isovector $SD$ pairs also provide a good description for $0^+$ and $2^+$. The **dual description** is due to the **non-orthogonality feature** of the pair basis. The same feature is also found for $sd$-shell nuclei.
Future and perspective

- Beta decay and double-beta decay
- Description of heavy nuclei
- Quartet (or alternatively like- $\alpha$ ) correlations in self-conjugate nuclei
- Multiple particle–hole excitations across closed shells
- Nuclear continuum and resonance states
Thanks for your attentions!
\[ [F^a, F^b]_e^{(e)} = \sum_{\alpha \beta} C_{\alpha \beta}^{e e} [F^\alpha, F^\beta] \]

\[ [(F^a \times F^b)^e, F^c]^{(d)} = \sum_f U(abdc; ef)(F^a \times [F^b, F^c]^{(f)})^{(d)} \]

\[ + \sum_f U(abcd; ef)(-1)^{a+d-f-e} \theta_{bc}([F^a, F^c]^{(f)} \times F^b)^{(d)}, \]

[1] Factorization of commutators: The Wick theorem for coupled operators

\[
\begin{align*}
\left[ A_{rk}^r, A_{s}^{s\dagger} \right]^{(t)} &= 2\hat{r}_k \delta_{r0} \delta_{s,r_k} \sum_{ab} y(abr_k)y(abs) - \sum_{ad} \left( 4\hat{r}_k \hat{s} \sum_{b} y(abr_k)y(bds) \right) \left\{ \begin{array}{ccc} r_k & s & t \\ d & a & b \end{array} \right\} \left( C_d^{\dagger} \times \tilde{C}_a \right)^{(t)}, \\
A_{rk}^{r'} &= [Q^r, A_{rk}^{r\dagger}]^{(r_k')} = (-)^{r_k + r_k'} \left\{ \begin{array}{l}
\sum_{da} y'(dar_k')A_{rk}^{r\dagger}(da), \\
(-)^{t-j-j'} q(jjt') \hat{t}/j' C_j^{\dagger},
\end{array} \right. \\
&\quad 1 \leq k \leq N, \\
&\quad k = 0 \ (r_k = j, r'_k = j'), \\
y'(dar_k') = z(dar_k') - \theta(dar_k')z(dar_k'), \\
z(dar_k') = \hat{r}_k \hat{t} \sum_{b_k} y(ab_k r_k) q(b_k dt) \left\{ \begin{array}{ccc} r_k & t & r_k' \\ d & a & b \end{array} \right\}.
\end{align*}
\]
\[
\langle j_0 s_1 \cdots s_N J_1' \cdots J_N' | j_1 \cdots r_N, J_1 \cdots J_N \rangle = \frac{\hat{J}^{N-1}}{\hat{J}_N} (-1)^{J^{N-J_{N-1}+SN}} \sum_{k=N}^1 \sum_{L_{N-2}} H_N(s_N) \cdots H_{k+1}(s_N) \\
\times \left[ \psi_k \delta_{s_N} \delta_{L_{k-1}j_{k-1}} \langle j_0 s_1 \cdots s_{N-1} J_1' \cdots J_{N-1}' | j_1 \cdots r_{k-1} r_{k+1} \cdots r_N, J_1 \cdots J_{k-1} L_k \cdots J_{N-1} \rangle \\
\times \langle j_0 s_1 \cdots s_{N-1} J_1' \cdots J_{N-1}' | j_1 \cdots r_i' \bar{b} \cdots r_{k-1} r_{k+1} \cdots r_N, J_1 \cdots J_{k-1} L_i \cdots J_{N-1} \rangle \right]
\]

where

\[
\psi_k = 2(-)^{j_{k-1}-r_k^j_{k-1}} \frac{\hat{J}_k}{\hat{J}_{k-1}} \sum_{ab} y(abr_k) y(abs_N),
\]

\[
H_k(s_N) = (-)^{l_{k-1}+l_{k-2}} U(r_k L_{k-1} J_{k-1} S_N; L_{k-2} J_k),
\]

\[
\tilde{G}_k(s_N t) = -U(r_k s_N J_{k-1} L_k, t_j),
\]

with the convention

\[
H_N(s_N) \cdots H_{k+1}(s_N) = \begin{cases} 1, & \text{for } k = N, \\ H_N(s_N), & \text{for } k = N - 1. \end{cases}
\]

\[
r_0 = J_0 = j, \quad r_0' = J_0 = j', \quad J_{-1} \equiv 0,
\]

\[
\tilde{G}_0(s t) = -U(r_0 s_{-1} L_{-1}; t_j_0) = -1, \quad \tilde{M}_0(t r_0') = U(r_0 t_{-1} L_0; r_0' j_0) = 1.
\]

\[
\tilde{B}_i = -[\tilde{A}_i, [\tilde{A}_k, A^{SN \dagger}]]_{i} = \begin{cases} \sum_{aa'} \left[ z(aa' r_i') - (-)^{a+a'+r_i' z(aa' r_i')} \right] \tilde{A}_i (aa'), & 1 \leq i \leq N, \\
(-1)^{t-j-j'} 4 \hat{f}_k \hat{s}_N \hat{t} \sum_b y(j' b r_k) y(b s_N) \begin{bmatrix} r_k & s_N \\ j & j' \\ b & t \end{bmatrix} \tilde{C}_{j'} \hat{j'}, & i = 0 (r_i' = j'), \end{cases}
\]