Dear Htun Htun, 2024.7.28

The ^{8}Be-Λ potential is introduced as the Yamaguchi separable type;

$$\begin{aligned}
V^{\frac{1}{2}^+}_{^8\mathrm{Be}(0^+)-\Lambda}(p,p') &= -\lambda_{\frac{1}{2}^+}\ g_{\frac{1}{2}^+}(p)g_{\frac{1}{2}^+}(p'), \\
V^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda}(p,p') &= -\lambda_{\frac{3}{2}^+}\ g_{\frac{3}{2}^+}(p)g_{\frac{3}{2}^+}(p'), \\
V^{\frac{5}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda}(p,p') &= -\lambda_{\frac{5}{2}^+}\ g_{\frac{5}{2}^+}(p)g_{\frac{5}{2}^+}(p').
\end{aligned} \tag{1}$$

with

$$g_s(p) = \frac{1}{p^2+\beta^2} \tag{2}$$

We assume the potential has no dependence of spin operator, the states $\frac{3}{2}^+$ and $\frac{5}{2}^+$ are degenerated.

$$V^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda}(p,p') = V^{\frac{5}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda}(p,p'). \tag{3}$$

The Schrödinger eqs are given as

$$\begin{aligned}
\left(\frac{p^2}{2m_{(0^+)}} + V_{^8\mathrm{Be}(0^+)-\Lambda}\right)\psi^{\frac{1}{2}^+}_{^8\mathrm{Be}(0^+)-\Lambda} &= (E^{\frac{1}{2}^+}_{^8\mathrm{Be}(0^+)-\Lambda} - \Delta_{(0^+)})\psi^{\frac{1}{2}^+}_{^8\mathrm{Be}(0^+)-\Lambda}, \\
\left(\frac{p^2}{2m_{(2^+)}} + V_{^8\mathrm{Be}(2^+)-\Lambda}\right)\psi^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda} &= (E^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda} - \Delta_{(2^+)})\psi^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)-\Lambda}.
\end{aligned} \tag{4}$$

where $\Delta_{(0^+)}$ and $\Delta_{(2^+)}$ are the resonance energies for ^{8}Be(0^+) and ^{8}Be(2^+), respectively,

$$\begin{aligned}
\Delta_{(0^+)} &= +0.09\ \mathrm{MeV}, \\
\Delta_{(2^+)} &= +3.00\ \mathrm{MeV},
\end{aligned} \tag{5}$$

and $E^{\frac{1}{2}^+}_{^8\mathrm{Be}(0^+)}$ and $E^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)}$ are the binding energies bwtween ^{8}Be(resonance state) and Λ.

$$E^{\frac{1}{2}^+}_{^8\mathrm{Be}(0^+)} = -6.71\ \mathrm{MeV},$$

$$E^{\frac{3}{2}^+}_{^8\mathrm{Be}(2^+)} = -3.66 \text{ MeV}, \tag{6}$$

Reduced masses $m_{(0^+)}$ and $m_{(2^+)}$ are calculated as

$$m_{(0^+)} \approx m_{(2^+)} = 4.443 \text{ fm}^{-1} \tag{7}$$

Representing Eq.(4) we have

$$\left(\frac{p^2}{2m} + V\right)\psi = E_2\psi \tag{8}$$

$$E_2 = (E_{^8\mathrm{Be}(*)-\Lambda} - \Delta_{(*)}), \tag{9}$$

then LS eq for bound state is given

$$\psi = \frac{1}{E_2 - p^2/(2m)} V\psi \tag{10}$$

namely,

$$\begin{aligned}\psi(p) &= \frac{1}{E_2 - p^2/(2m)} \int_0^\infty V(p,p')\psi(p')p'^2 dp' \\ &= \frac{1}{E_2 - p^2/(2m)} \int_0^\infty \{-\lambda g(p)g(p')\}\psi(p')p'^2 dp' \\ &= \frac{-\lambda}{E_2 - p^2/(2m)} g(p) I \end{aligned}\tag{11}$$

with

$$\begin{aligned} I &\equiv \int_0^\infty g(p')\psi(p')p'^2 dp' \\ &= \int_0^\infty g(p')\{\frac{-\lambda}{E_2 - p'^2/(2m)} g(p') I\} p'^2 dp'. \end{aligned}\tag{12}$$

When I is dropped we have the condition of λ

$$\begin{aligned} &-\lambda \int_0^\infty \frac{g^2(p)}{E_2 - p^2/(2m)} p^2 dp \\ &= -\lambda \int_0^\infty \frac{1}{(p^2+\beta^2)^2} \frac{1}{E_2 - p^2/(2m)} p^2 dp \end{aligned}$$

$$
\begin{aligned}
&= \lambda \frac{m\pi}{2\beta(\beta+k)^2} \\
&= 1.
\end{aligned} \tag{13}
$$

Therefore, we have

$$
\lambda = \frac{2\beta(\beta+k)^2}{m\pi} \tag{14}
$$

where $k = \sqrt{-2mE_2} = \sqrt{2m|E_b|}$. $E_b < 0$ is bound state energy.

Our LS eq. for scattering is

$$
t(p,p';E_2) = V(p,p') + \int_0^\infty V(p,p'') \frac{1}{E_2 - p''^2/2m + i\epsilon} t(p'',p';E_2) p''^2 dp'', \tag{15}
$$

with

$$
t(p,p';E_2) \equiv g(p)\ \tau(E_2)\ g(p'). \tag{16}
$$

and we have t-matrix

$$
\begin{aligned}
\tau(E_2) &= -\lambda + \lambda\ \tau(E_2) \int_0^\infty \frac{1}{(p^2+\beta^2)^2(E_2 - p^2/2m + i\epsilon)} p^2 dp \\
&= -\lambda + \lambda\ \tau(E_2)\ \frac{m\pi}{2\beta(\beta+\sqrt{-2mE_2})^2},
\end{aligned} \tag{17}
$$

then,

$$
\tau(E_2) = \frac{1}{-\frac{1}{\lambda} + \frac{m\pi}{2\beta(\beta+\sqrt{-2mE_2})^2}} \tag{18}
$$

and

$$
t(p,p';E_2) = \frac{1}{p^2+\beta^2} \frac{1}{-\frac{1}{\lambda} + \frac{m\pi}{2\beta(\beta+\sqrt{-2mE_2})^2}} \frac{1}{p'^2+\beta^2}. \tag{19}
$$

When $E_2 > 0$ we need $\sqrt{-2mE_2} = -i\sqrt{2mE_2}$.

1 Table

We take the parameter β to fitting to the reduced wave function of α-α-Λ Faddeev calculation [1].

$$
\beta = 0.473\ \text{fm}^{-1} \tag{20}
$$

State	β [fm^{-1}]	E_2 [MeV]	λ [fm^{-2}]
$\frac{1}{2}^+$	0.473	- 6.80	3.66658196E-02
$\frac{3}{2}^+$ or $\frac{5}{2}^+$	0.473	- 6.66	3.63954790E-02

2 $^8\mathrm{Be} - \Lambda$ scattering

As the exercise, please plot the phase shift as Fig. 1. The energies E_2 of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ are so close that these phrases are similar.

You can peep the code [2]

Best regards, Hiroyuki

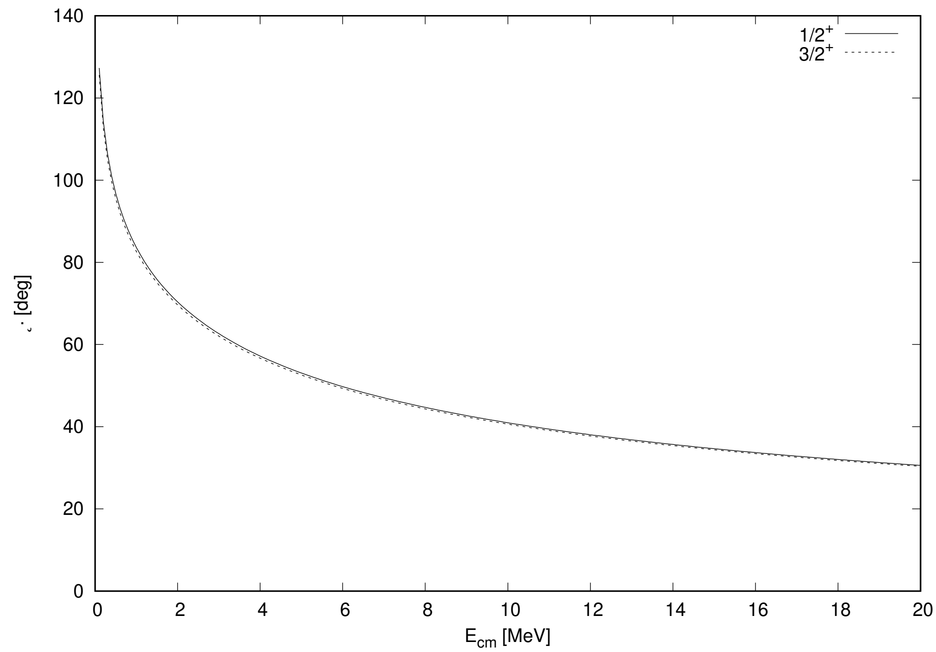

Figure 1: Phase shifts for $^8\mathrm{Be} - \Lambda$ scattering; The solid (dashed) line is the phase shift of $J^\pi = \frac{1}{2}^+$ $(\frac{3}{2}^+)$.

References

[1] S. Oryu *et al.*, Few-Body Syst. **28**, 103 (2000). `https://link.springer.com/article/10.1007/s006010070026`

[2] ^{8}Be-Λ scattering code. `https://www.rcnp.osaka-u.ac.jp/~kamada/Myanmar3/date/scatter.f90.pdf`