Dear Htun Htun, 2024.8.14

The equation of the last your letter (2024.8.10) is for a just 3N system. The equation and code are nearly the same as the $\Sigma - \Sigma - \alpha$ model[1].

$$\psi_{12}^{\alpha_3}(p_3 q_3) \equiv \langle p_3 q_3 \alpha_3 | \psi_{12} \rangle = \frac{2}{E - \frac{p_3^2}{2\mu_3} - \frac{q_3^2}{2M_3}} \sum_{\alpha_1} \int_0^\infty dq_1 q_1^2 \times \int_{-1}^{1} dx t_{12}\left(p_3, \pi_2, E - \frac{q_3^2}{2M_3}\right) \times G_{\alpha_3\alpha_1}(q_3 q_1 x)\, \psi_{23}^{\alpha_2}(\pi_2', q_1), \qquad (2\cdot19)$$

$$\psi_{23}^{\alpha_1}(p_1 q_1) = \frac{1}{E - \frac{p_1^2}{2\mu_1} - \frac{q_1^2}{2M_1}} \sum_{\alpha_3} \int_0^\infty dq_3 q_3^2 \int_{-1}^{1} dx t_{23}\left(p_1, \pi_3, E - \frac{q_1^2}{2M_1}\right)$$

$$\times G_{\alpha_3\alpha_1}(q_3 q_1 x)\, \psi_{12}^{\alpha_3}(\pi_3' q_3) - \frac{1}{E - \frac{p_1^2}{2\mu_1} - \frac{q_1^2}{2M_1}} \sum_{\alpha_1'} \int_0^\infty dq_1' q_1'^2 \int_{-1}^{1} dx t_{23}\left(p_1, \pi_4, E - \frac{q_1^2}{2M_1}\right) \times G_{\alpha_1\alpha_1'}(q_1 q_1' x)\, \psi_{23}^{\alpha_1'}(\pi_4' q_1'). \qquad (2\cdot20)$$

$$\pi_2 = \sqrt{q_1^2 + \rho_1^2 q_3^2 + 2\rho_1 q_1 q_3 x}, \qquad \pi_3 = \sqrt{\rho_2^2 q_1^2 + q_3^2 + 2\rho_2 q_1 q_3 x},$$

$$\pi_2' = \sqrt{\rho_2^2 q_1^2 + q_3^2 + 2\rho_2 q_1 q_3 x}, \qquad \pi_3' = \sqrt{q_1^2 + \rho_1^2 q_3^2 + 2\rho_1 q_1 q_3 x},$$

$$\pi_4 = \sqrt{\rho^2 q_1^2 + q_1'^2 + 2\rho q_1 q_1' x}, \qquad \pi_4' = \sqrt{q_1^2 + \rho^2 q_1'^2 + 2\rho q_1 q_1' x},$$

where $\rho_1 = \frac{1}{2}, \rho_2 = \frac{M_\alpha}{M_\alpha + M_\Sigma}$ and $\rho = \frac{M_\Sigma}{M_\Sigma + M_\alpha}$.

The reduced masses are

$\mu_3 = \frac{1}{2} M_\Sigma$,

$M_3 = \frac{2M_\alpha M_\Sigma}{2M_\Sigma + M_\alpha}$,

$\mu_1 = \frac{M_\alpha M_\Sigma}{M_\alpha + M_\Sigma}$,

$M_1 = \frac{M_\Sigma (M_\Sigma + M_\alpha)}{2M_\Sigma + M_\alpha}$.

The spin of ^{8}Be has the cases ($\sigma = 0, 2$), therefore, the equation for $g_{\alpha\beta}$ must be changed as following.

$$G_{\alpha\beta} = \sum_k P_k(x) \sum_{l_1+l_2=l_\alpha} \sum_{l'_1+l'_2=l'_\beta} q_\alpha^{l_2+l'_2} q_\beta^{\prime l_1+l'_1} g_{\alpha\beta}^{kl_1l_2l'_1l'_2},$$

$$g_{\alpha\beta}^{kl_1l_2l'_1l'_2} = \frac{1}{2}\sqrt{\hat{l_\alpha}\hat{s_\alpha}\hat{j_\alpha}\hat{\lambda_\alpha}\hat{I_\alpha}\hat{l'_\beta}\hat{s'_\beta}\hat{j'_\beta}\hat{\lambda'_\beta}\hat{I'_\beta}} \sum_{LS} \hat{L}\hat{S} \left\{ \begin{array}{ccc} l_\alpha & s_\alpha & j_\alpha \\ \lambda_\alpha & \sigma_\alpha & I_\alpha \\ L & S & J \end{array} \right\} \left\{ \begin{array}{ccc} l'_\beta & s'_\beta & j'_\beta \\ \lambda'_\beta & \sigma'_\beta & I'_\beta \\ L & S & J \end{array} \right\} (-)^{l'_\beta+\sigma_\gamma+\sigma_\alpha-s'_\beta+2S}$$

$$\times\hat{k} \left\{ \begin{array}{c} \sigma_\alpha\sigma_\gamma s'_\beta \\ \sigma_\beta S s_\alpha \end{array} \right\} \sqrt{\frac{(2l_\alpha+1)!}{(2l_1)!(2l_2)!}} \sqrt{\frac{(2l'_\beta+1)!}{(2l'_1)!(2l'_2)!}} \sum_{ff'} \left\{ \begin{array}{c} l_1 l_2 l_\alpha \\ \lambda_\alpha L f \end{array} \right\} C(l_2\lambda_\alpha f,00) \left\{ \begin{array}{c} l'_2 l'_1 l'_\beta \\ \lambda'_\beta L f' \end{array} \right\}$$

$$\times C(l'_1\lambda'_\beta f',00) \left\{ \begin{array}{c} f l_1 L \\ f' l'_2 k \end{array} \right\} C(kl_1f',00)C(kl'_2f,00)\rho_\alpha^{l_2}\rho_\beta^{l'_1} \qquad (1)$$

with

$$\rho_\alpha = \frac{m_\beta}{m_\beta+m_\gamma}, \qquad \rho_\beta = \frac{m_\alpha}{m_\alpha+m_\gamma}. \qquad (2)$$

Table 1: Channels of ${}^{10}_{\Lambda\Lambda}$Be(0^+).

State nr.	α	J	I	λ	j	l	s	σ_α	σ_β	σ_γ
1	1	0	1/2	0	1/2	0	1/2	1/2	1/2	0
2	1	0	3/2	2	3/2	0	3/2	1/2	1/2	2
3	1	0	5/2	2	5/2	0	5/2	1/2	1/2	2
4	3	0	0	0	0	0	0	0	1/2	1/2
5	3	0	0	2	0	0	0	2	1/2	1/2

Table 2: Channels of ${}^{10}_{\Lambda\Lambda}$Be(2^+).

State nr.	α	J	I	λ	j	l	s	σ_α	σ_β	σ_γ
1	1	2	3/2	2	1/2	0	1/2	1/2	1/2	0
2	1	2	1/2	2	3/2	0	3/2	1/2	1/2	2
3	1	2	1/2	2	5/2	0	5/2	1/2	1/2	2
4	3	2	2	2	0	0	0	0	1/2	1/2
5	3	2	2	0	0	0	0	2	1/2	1/2

We prepare the partial wave quantum numbers set in Table 1 and Table 2 for 0^+ state and 2^+ state, respectively.

In addition, the differences between ${}^{6}_{\Sigma\Sigma}$He and ${}^{10}_{\Lambda\Lambda}$Be are corresponding to the masses.

$$\begin{aligned} M_\Sigma &\to M_\Lambda = 5.654 \text{ fm}^{-1}, \\ M_\alpha &\to M_{^8Be} = 20.47 \text{ fm}^{-1}. \end{aligned} \tag{3}$$

Do you have still your code?

Best regards, Hiroyuki

References

[1] Htun Htun Oo, Khin Swe Myint, Hiroyuki Kamada, Walter Glöckle, Progress of Theoretical Physics, Volume 113, Issue 4, April 2005, Pages 809. https://academic.oup.com/ptp/article/113/4/809/1810725?login=true