Dear Htun Htun, 2024.8.5 ver2

We take a Yamaguchi type separable potential for Λ-Λ interaction.

$$V^{^1S_0}_{\Lambda\Lambda}(p,p') = -\lambda_{\Lambda\Lambda}\ g_{\Lambda\Lambda}(p)g_{\Lambda\Lambda}(p'), \tag{1}$$

with

$$g(p)_{\Lambda\Lambda} = \frac{1}{p^2+\beta^2} \tag{2}$$

From scattering length a and effective range r

$$\beta = \frac{3+\sqrt{9-16\frac{r}{a}}}{2r},$$
$$\lambda_{\Lambda\Lambda} = \frac{4\beta_{\rm Y}^3}{\pi\mu(r\beta_{\rm Y}-1)}, \tag{3}$$

with the reduced mass $\mu = 2.827{\rm fm}^{-1}$.

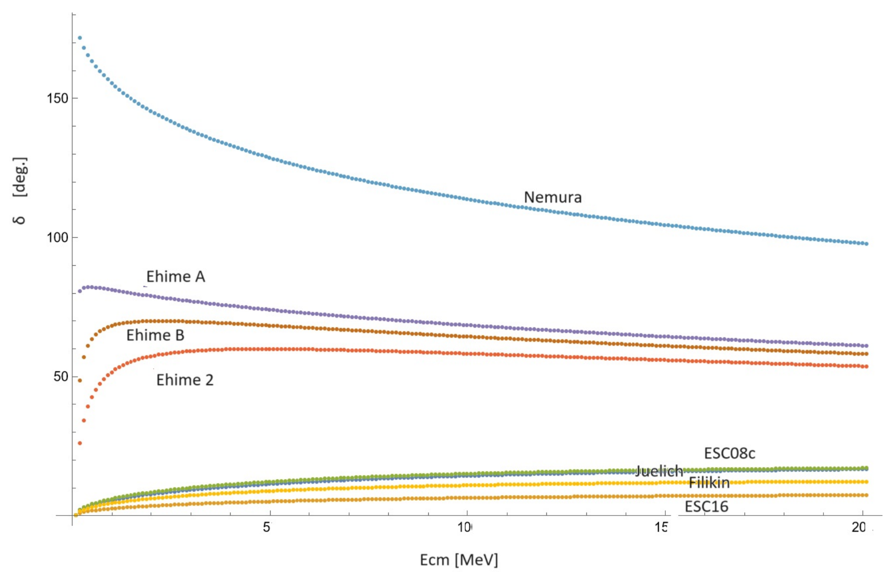

Figure 1: Phase shifts for Λ-Λ scattering ($^1S_0$).

Best regards, Hiroyuki

Table 1: β and $\lambda_{\Lambda\Lambda}$ from a and r. *In the original paper it has the opposite sign.

Model	a[fm]	r [fm]	β [fm^{-1}]	λ [fm^{-2}]
Jülich [2]	-0.66	5.05	1.43208	0.212256
Jülich [3]	-3.00*	4.95*	0.904019	0.0957581
ESC16 [4]	-0.439	9.533	1.14758	0.068478
ESC08c [5]	-0.85	5.13	1.29381	0.173032
Ehime 2 [6]	-3.09	2.89	1.3659	0.389435
Ehime A [6]	-3.64	2.73	1.3887	0.432167
Ehime B [6]	-3.40	2,79	1.3806	0.415622
Nemura [7]	-3.011	2.159	1.74263	0.862824
Filikin02 [8, 9]	-0.778	6.564	1.14257	0.103355
ESC00 [9, 11]	-10.6	2.23	1.46111	0.622092
ND(G) [9, 10]	-5.37	2.40	1.46225	0.561152
ND [9]	-2.81	2.95	1.36934	0.380458
ND [9]	-0.77	2.92	1.94301	0.706901
ND [9]	-0.31	3.12	2.57046	1.08965
NSC97e [9]	-0.50	10.6	1.0217	0.0488656
NSC97b [9]	-0.38	15.2	0.936693	0.0279617
ND [9]	-2.80	2.81	1.4245	0.433551

References

[1] I. R. Afnan, B. F. Gibson, Phys. Rev. C **92**, 054608 (2015). `https://journals.aps.org/prc/abstract/10.1103/PhysRevC.92.054608`

[2] J. Haidenbauer, Nucl. Phys. A**981**, 1 (2019).`https://www.sciencedirect.com/science/article/pii/S0375947418303774?via%3Dihub`

[3] J. Haidenbauer, U.-G. Meißner, Nucl. Phys. A **881**, 44 (2012). `https://www.sciencedirect.com/science/article/pii/S0375947412000528`

[4] M. M. Nagel *et al.*, Phys. Rev. C**99**, 044003 (2019).`https://nn-online.org/eprints/pdf/20.07.pdfhttps://journals.aps.org/prc/abstract/10.1103/PhysRevC.99.044003`

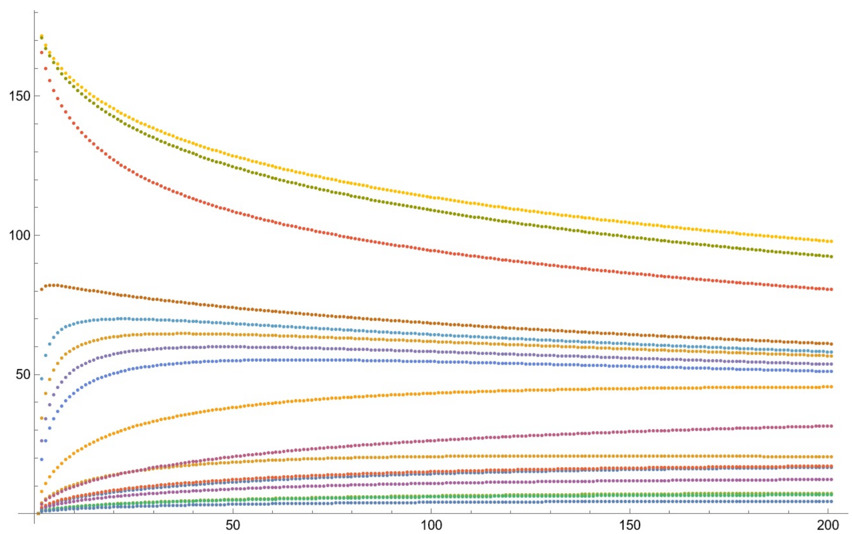

Figure 2: Phase shifts for Λ-Λ scattering ($^1S_0$) for 17 potentials.

[5] M. M. Nagels, Th. A. Rijken, and Y. Yamamoto, `https://arxiv.org/abs/1504.02634`

[6] K. Tominaga *et al.*, Nucl. Phys. A**642**, 483 (1998). `https://www.sciencedirect.com/science/article/abs/pii/S0375947498004850`

[7] H. Nemura, Y. Suzuki, Y. Fujiwara, C. Nakamoto, Prog. Theor. Phys. 103 (2000) 929.

[8] V.B. Belyaev, S.A. Rakityansky, W. Sandhas, Nucl. Phys. A **803** 210 (2008). `https://www.sciencedirect.com/science/article/pii/S0375947408003126?via%3Dihub`

[9] I.N. Filikhin, A. Gal, Nucl. Phys. A 707 (2002) 491. `https://www.sciencedirect.com/science/article/abs/pii/S0375947402010084`

[10] D.H. Davis, J. Pniewski, Contemp. Phys. 27 (1986) 91; See also: D.H. Davis, in: B.F. Gibson, W.R. Gibbs, M.B. Johnson (Eds.), LAMPF Workshop on (π, K) Physics, AIP Conf. Proc., Vol. 224, Amer. Inst. Physics, New York, 1991, pp. 38–48.

[11] Th.A. Rijken, Nucl. Phys. A **691**,322c (2001).

Table 5

Calculated ground-state energies of $\Lambda\Lambda$ hypernuclei $E_{\Lambda\Lambda}$ (in MeV) with respect to the breakup threshold of the free clusters, using $\alpha\alpha$ and $\Lambda\alpha$ potentials of the form (25) with parameters listed in Table 3, for $\Lambda\Lambda$ potentials (24) simulated by fitting to the scattering length a and effective range r (in fm) of several Nijmegen interaction models

Model	${}^{5}_{\Lambda}$He	${}^{9}_{\Lambda}$Be	$a_{\Lambda\Lambda}$	$r_{\Lambda\Lambda}$	${}^{6}_{\Lambda\Lambda}$He	${}^{10}_{\Lambda\Lambda}$Be
ESC00	−3.09	−6.55	−10.6	2.23	−10.7	−19.4
ND(G)[a]			−5.37	2.40	−10.1	−18.7
ND			−2.81	2.95	−9.10	−17.7
–			−0.77	2.92	−7.70	−16.4
–			−0.31	3.12	−6.98	−15.6
NSC97e			−0.50	10.6	−6.82	−15.4
NSC97b			−0.38	15.2	−6.60	−15.2
$V_{\Lambda\Lambda} = 0$			0.0	–	−6.27	−14.8
[14] ND	−3.12	−6.67	−2.80	2.81	−9.34	−17.15
Exp.	-3.12 ± 0.02[b]	-6.62 ± 0.04[b]	–	–	-10.9 ± 0.6[c]	-17.6 ± 0.4[d]
					$-7.25 \pm 0.19^{+0.18}_{-0.11}$[e]	-14.5 ± 0.4[f]

[a] Ref. [37].
[b] Ref. [32].
[c] Ref. [3].
[d] Ref. [17].
[e] Ref. [9].
[f] Assuming ${}^{10}_{\Lambda\Lambda}\mathrm{Be} \to \pi^- + p + {}^{9}_{\Lambda}\mathrm{Be}^*$.

Figure 3: From [9]

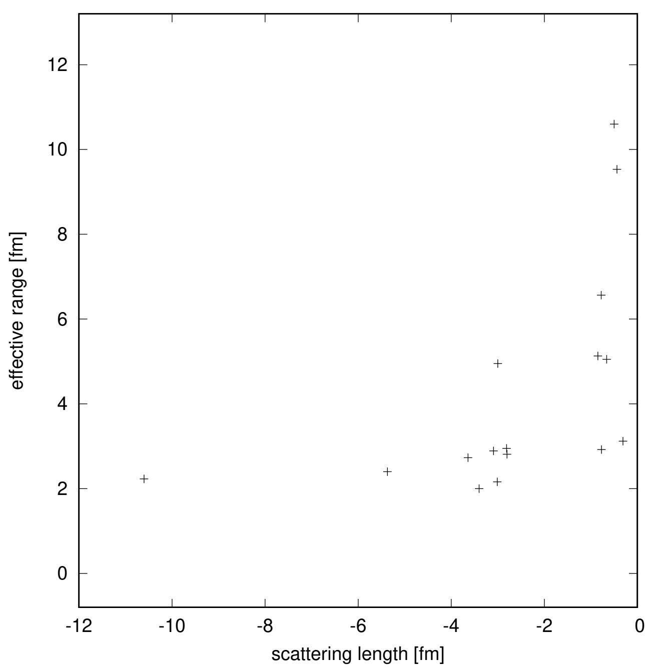

Figure 4: Many cases