Λnn系における３体Faddeev理論による共鳴状態の研究（II）

鎌田裕之　（九州工業大学・工、阪大RCNP）
河野通郎　（大阪大学・RCNP）
宮川和也　（大阪大学・RCNP）

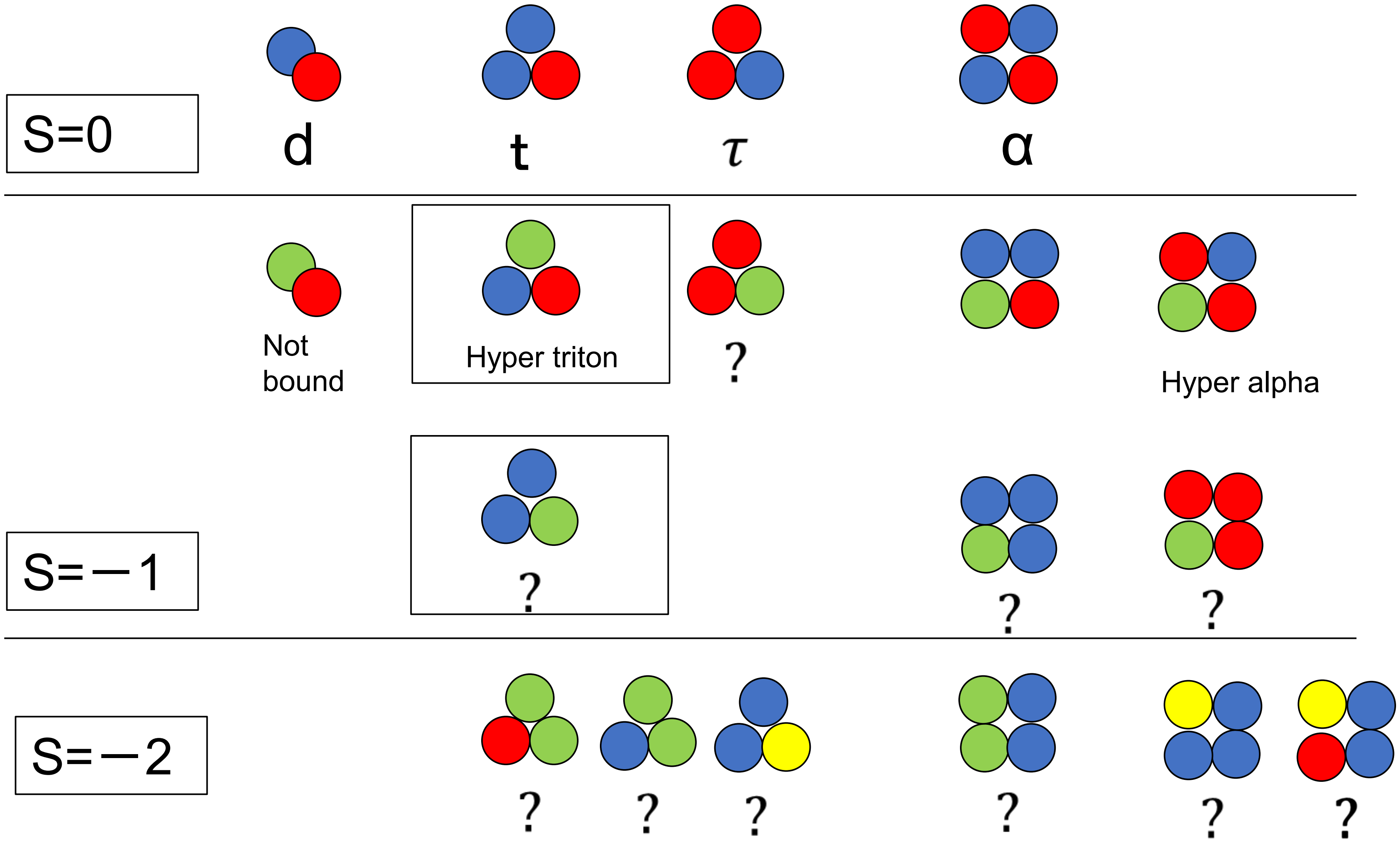
Kyutech
Kyushu Institute of Technology
RCNP
大阪大学 核物理研究センター
LENPIC

日本物理学会
第７４回年次大会
於　北海道大学
R６．９．１７

Outline

- Faddeev eq. for YNN system
- Hypertriton (J=1/2,T=0)
- Effective Field Theoretical Chiral NN Potential
- ΛNN Resonance (J=1/2,T=1)
- Complex Energy Method
- Summary

Hyper nuclei



Faddeev Equations for the Hypertriton bound state (no 3-body force)

[K. Miyagawa, H. Kamada, W. Glöckle, and V. Stoks, Phys. Rev. C 51, 2905 (1995)]

- ΛNN Schrödinger Eq. ; 2body force V_{ij}, Kinetic term H_0

$$(H_0+V_{12}+V_{23}+V_{31})\Psi = E\Psi \quad \xrightarrow{\text{L-S eq.}} \quad \Psi = \frac{1}{E-H_0}(V_{12}+V_{23}+V_{31})\Psi \equiv \psi_{12}+\psi_{23}+\psi_{31}$$

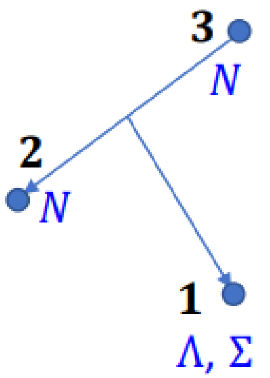
- $G_0 \equiv \frac{1}{E-H_0}$, t-matrix $t_{ij} = V_{ij} + V_{ij}G_0t_{ij}$ $\qquad P_{23}\psi_{12} = -\psi_{31}$

$$\psi_{12} = G_0t_{12}(\psi_{23} - P_{23}\psi_{12}) \quad \text{and} \quad \psi_{23} = G_0t_{23}(1-P_{23})\psi_{12}$$

- ψ_{ij} consists of Λ channel and Σ channel

$$\begin{pmatrix}\psi_\Lambda^{(12)}\\ \psi_\Sigma^{(12)}\end{pmatrix} = \begin{pmatrix}\{E-H_0(NN\Lambda)\}^{-1} & 0\\ 0 & \{E-H_0(NN\Sigma)\}^{-1}\end{pmatrix}\begin{pmatrix}t_{12}^\Lambda & 0\\ 0 & t_{12}^\Sigma\end{pmatrix}\left[\begin{pmatrix}\psi_\Lambda^{(23)}\\ \psi_\Sigma^{(23)}\end{pmatrix} - \begin{pmatrix}P_{23}\psi_\Lambda^{(12)}\\ P_{23}\psi_\Sigma^{(12)}\end{pmatrix}\right]$$

$$\begin{pmatrix}\psi_\Lambda^{(23)}\\ \psi_\Sigma^{(23)}\end{pmatrix} = \begin{pmatrix}\{E-H_0(NN\Lambda)\}^{-1} & 0\\ 0 & \{E-H_0(NN\Sigma)\}^{-1}\end{pmatrix}\begin{pmatrix}t_{23}^\Lambda & 0\\ 0 & t_{23}^\Sigma\end{pmatrix}\begin{pmatrix}(1-P_{23})\psi_\Lambda^{(12)}\\ (1-P_{23})\psi_\Sigma^{(12)}\end{pmatrix}$$



Faddeev Equations including with 3body force

- ΛNN Schrödinger Eq. ; 2body force V_{ij}, Kinetic term H_0 , 3-body force W

$$(H_0+V_{12}+V_{23}+V_{31}+W)\Psi = E\Psi$$

- Green func. $G_0 \equiv \frac{1}{E-H_0}$ Lippmann-Schwinger eq. $\Psi = G_0(V_{12}+V_{23}+V_{31}+W)\Psi$
- Faddeev Components : $\Psi = \psi_{12}+\psi_{23}+\psi_{31}$ $i=1$ = Hyperon

$$\psi_{12} \equiv G_0V_{12}\Psi, \quad \psi_{23} \equiv G_0(V_{23}+W)\Psi, \quad \psi_{31} \equiv G_0V_{31}\Psi$$

- t-matrix ; $t_{ij} = V_{ij}+V_{ij}G_0t_{ij}$ Möller Oper.; $\Omega_{ij} \equiv (1-G_0V_{ij})^{-1} = (1+G_0t_{ij})$

$$\begin{cases} \psi_{12} = G_0t_{12}(\psi_{23}+\psi_{31}) \\ \psi_{23} = G_0t_{23}(\psi_{12}+\psi_{31}) + (G_0+G_0t_{23}G_0)W(\psi_{12}+\psi_{23}+\psi_{31}) \\ \psi_{31} = G_0t_{31}(\psi_{12}+\psi_{23}) \end{cases}$$

$[\psi_{31} = -P_{12}\psi_{23}]$

: $\psi_{ij} = \begin{pmatrix} \psi_\Lambda^{(ij)} \\ \psi_\Sigma^{(ij)} \end{pmatrix}$

INPUT

- 2body force (NN) : Chiral forces N^4LO+ [Cut off scale; 400,450,500,550 MeV]
- 2body force (YN): Chiral forces (Juelich) NLO13,NLO19 [Cut off 550, 600, 650 MeV]
- 3body force (YNN): 2pi exchange type

J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, and W. Weise, Hyperon-nucleon interaction at next-to-leading order in chiral effective field theory, Nucl. Phys. A **915**, 24 (2013). — NLO13(550)

J. Haidenbauer, U.-G. Meißner, and A. Nogga, Hyperon-nucleon interaction within chiral effective field theory revisited, Eur. Phys. J. A **56**, 91 (2020). — **NLO19(550),NLO19(600), NLO19(650)**

P. M. M. Maessen, Th. A. Rijken, and J. J. de Swart, Soft-core baryon-baryon one-boson-exchange models. II. Hyperon-nucleon potential, Phys. Rev. C **40**, 2226 (1989). — NSC89

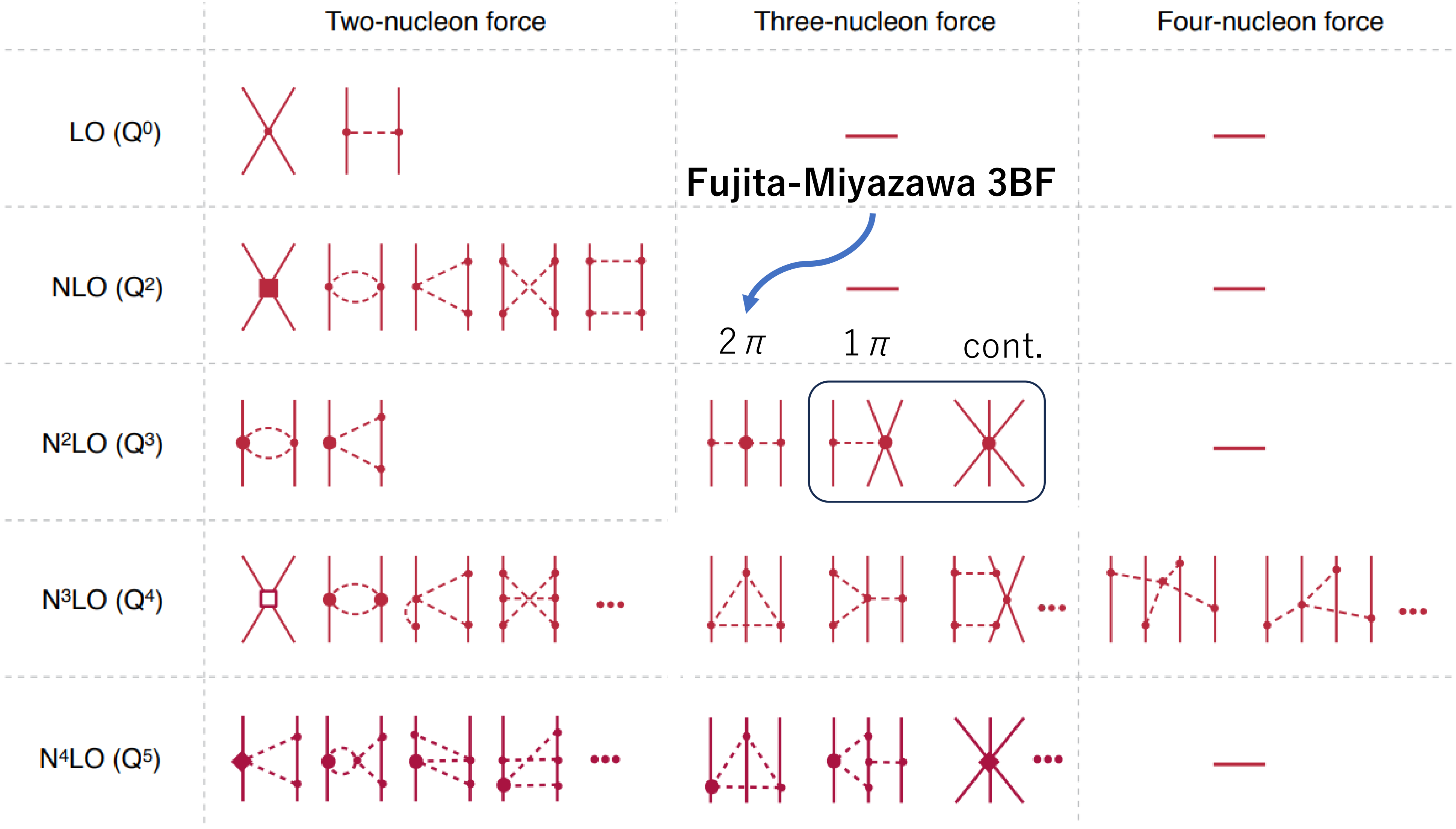
} ΛN **Potential**

NN **Potential**

N^4LO+

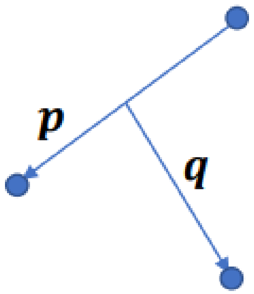
P. Reinert, and H. Krebs, and E. Epelbaum, Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order, Eur. Phys. J. A **54**, 86 (2018).

3 N　System



Two-nucleon force
Three-nucleon force
Four-nucleon force
LO (Q⁰)
NLO (Q²)
N²LO (Q³)
N³LO (Q⁴)
N⁴LO (Q⁵)
Fujita-Miyazawa 3BF
2π
1π
cont.

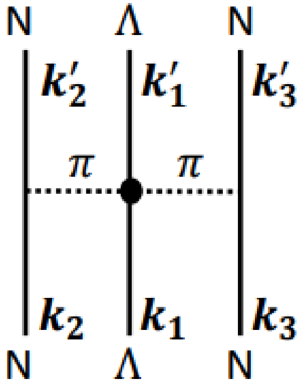
3-body force (YNN): 2π exchange type



$$V_{TPE}^{\Lambda NN} = \frac{g_A^2}{3f_0^4}(\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)\frac{(\boldsymbol{\sigma}_3 \cdot \boldsymbol{q}_{3d})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}_{2d})}{(\boldsymbol{q}_{3d}^2 + m_\pi^2)(\boldsymbol{q}_{2d}^2 + m_\pi^2)}\{-Am_\pi^2 + B\boldsymbol{q}_{3d} \cdot \boldsymbol{q}_{2d}\}$$

$$\boldsymbol{q}_{2d} = \boldsymbol{k}_2' - \boldsymbol{k}_2 = \boldsymbol{p}_1' - \boldsymbol{p}_1 - \frac{1}{2}(\boldsymbol{q}_1' - \boldsymbol{q}_1) \equiv \boldsymbol{p} - \frac{1}{2}\boldsymbol{q}$$

$$\boldsymbol{q}_{3d} = \boldsymbol{k}_3' - \boldsymbol{k}_3 = -(\boldsymbol{p}_1' - \boldsymbol{p}_1) - \frac{1}{2}(\boldsymbol{q}_1' - \boldsymbol{q}_1) \equiv -\boldsymbol{p} - \frac{1}{2}\boldsymbol{q}$$

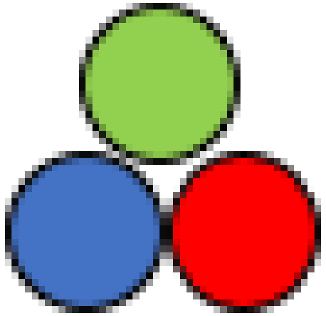


- The two parameters A and B should be determined by the NNLO LEC, but here they were estimated by the Isobar-saturation model.

[Petschauer *et al.*, Phys. Rev. C 93, 014001 (2016)] $A = 0, B = -3.0$ GeV^{-1}

NNLO fit [Haidenbauer et al., Eur. Phys. J. A (2024) 6:3] A=1.485, B=-3.01GeV^{-1}

Hypertriton　（np Λ ）

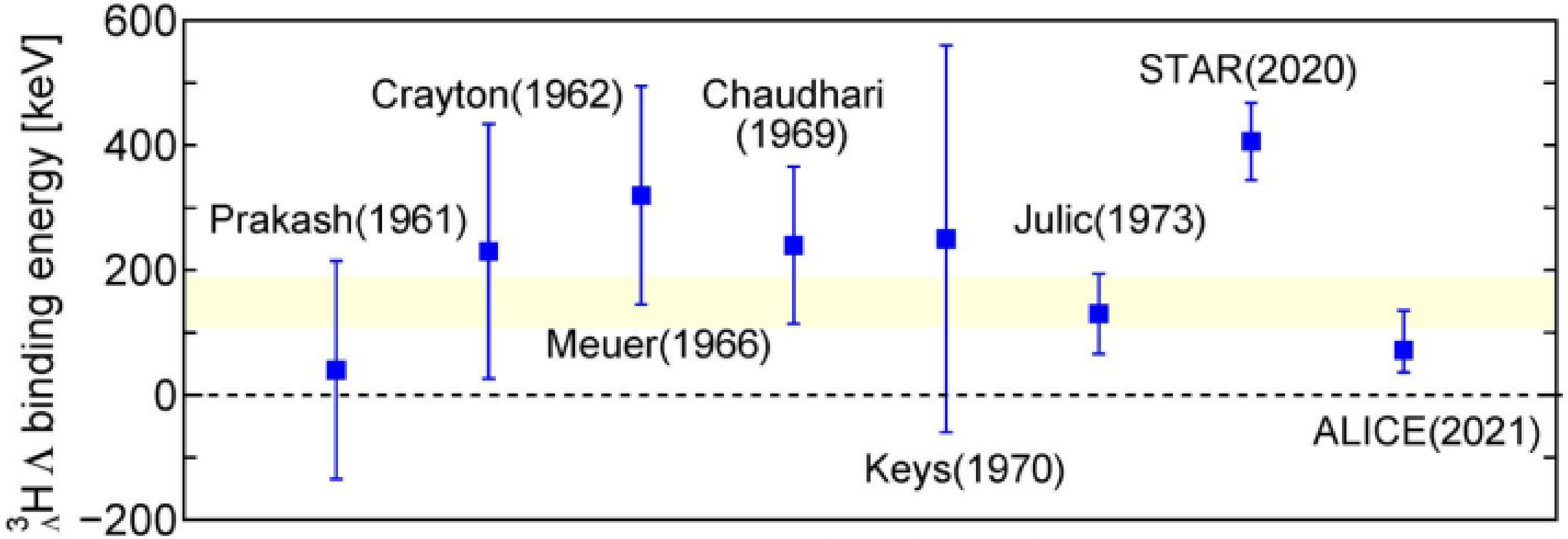


hypertriton ($^{3}_{\Lambda}H$)

The first bound hypernucleus

- **Average bond energy**

  162 ± 44 keV



exp. data [https://hypernuclei.kph.uni-mainz.de]

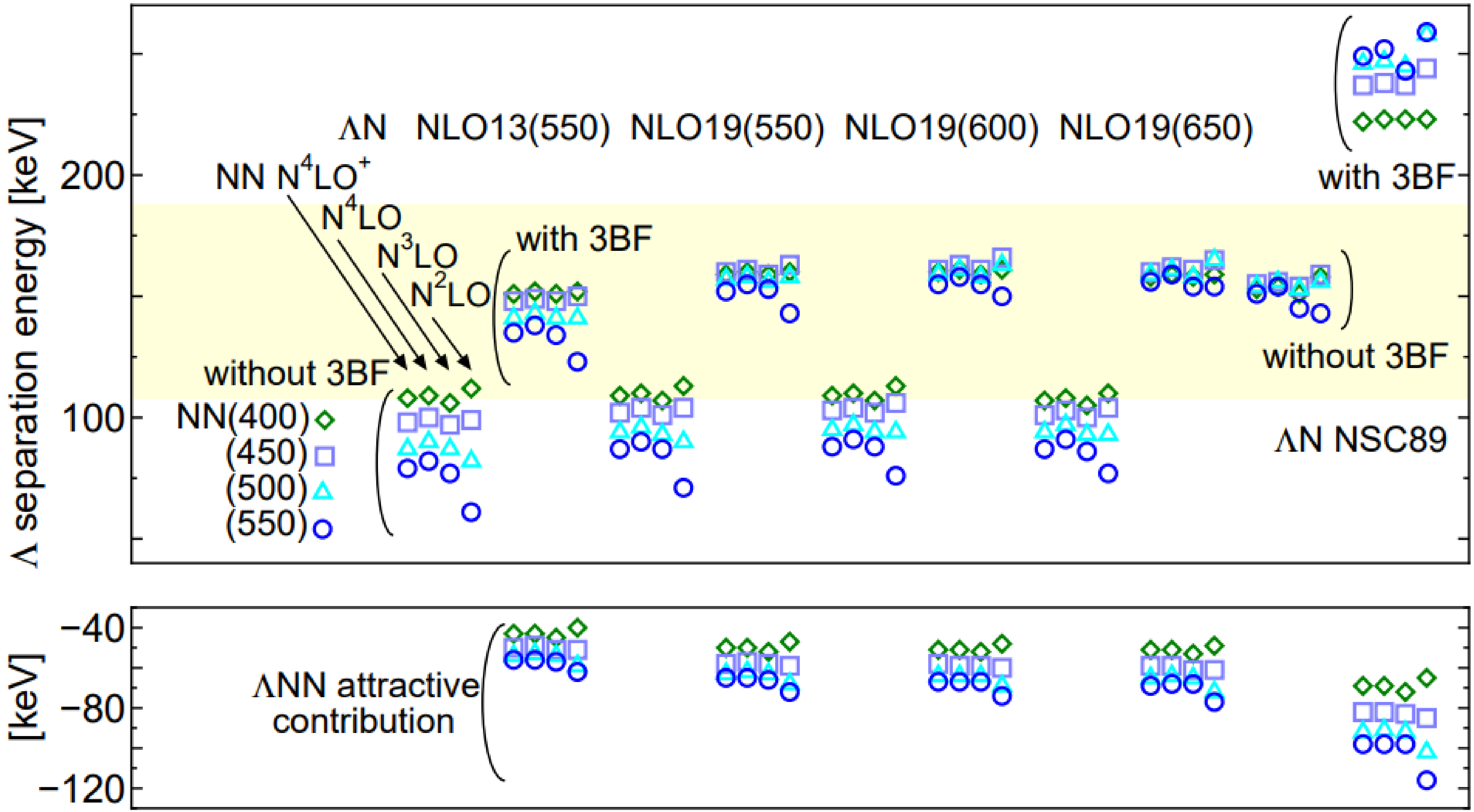
TABLE II. Hypertriton separation energies without 3BFs. Entries are in keV. The numbers in parentheses for NN and YN chiral potentials indicate the cutoff scale Λ_c in MeV.

NN potential (Λ_c)	YN potential (Λ_c) NLO13 (550)	NLO19 (550)	NLO19 (600)	NLO19 (650)	NSC89
CDBonn [23]	82	90	90	88	153
Nijmegen 93 [32]	58	69	70	70	141
Nijmegen I [32]	63	75	75	74	145
N^4LO+ (550)	79	87	88	87	151
N^4LO+ (500)	87	94	95	94	154
N^4LO+ (450)	98	102	103	101	155
N^4LO+ (400)	108	109	109	107	153
N^4LO (550)	82	90	91	91	154
N^4LO (500)	90	96	97	97	156
N^4LO (450)	100	104	104	103	156
N^4LO (400)	109	110	110	108	154
N^3LO (550)	77	87	88	86	145
N^3LO (500)	87	93	94	93	153
N^3LO (450)	97	101	102	100	154
N^3LO (400)	106	107	107	105	151
N^2LO (550)	61	71	76	77	143
N^2LO (500)	82	90	94	93	156
N^2LO (450)	99	104	106	104	159
N^2LO (400)	112	113	113	110	158

Table I: Hypertriton separation energies that include 2π-exchange ΛNN 3BF with $3b_0 + b_D = 0$ and $2b_2 + 3b_4 = -3.0$ GeV^{-1}. Entries are in keV. The numbers in parentheses for NN and YN chiral potentials indicate the cutoff scale Λ_c in MeV.

NN potential (Λ_c)	YN potential (Λ_c) NLO13 (550)	NLO19 (550)	NLO19 (600)	NLO19 (650)	NSC89
CDBonn [9]	135	151	151	151	244
Nijmegen 93 [10]	114	135	138	140	249
Nijmegen I [10]	120	142	144	145	252
N^4LO+ (550)	135	152	155	156	249
N^4LO+ (500)	141	157	159	159	246
N^4LO+ (450)	148	160	161	160	237
N^4LO+ (400)	151	159	160	158	222
N^4LO (550)	138	155	158	159	252
N^4LO (500)	143	158	161	161	247
N^4LO (450)	149	161	163	162	238
N^4LO (400)	152	160	161	159	223
N^3LO (550)	134	153	155	154	243
N^3LO (500)	141	156	158	158	245
N^3LO (450)	148	159	161	161	237
N^3LO (400)	151	159	159	158	223
N^2LO (550)	123	143	150	154	259
N^2LO (500)	141	158	163	165	258
N^2LO (450)	150	163	166	165	244
N^2LO (400)	152	160	161	159	223

[Kamada, Kohno, Miyagawa: PHYSICAL REVIEW C 108, 024004 (2023)]



[Kamada, Kohno, Miyagawa: PHYSICAL REVIEW C 108, 024004 (2023)]

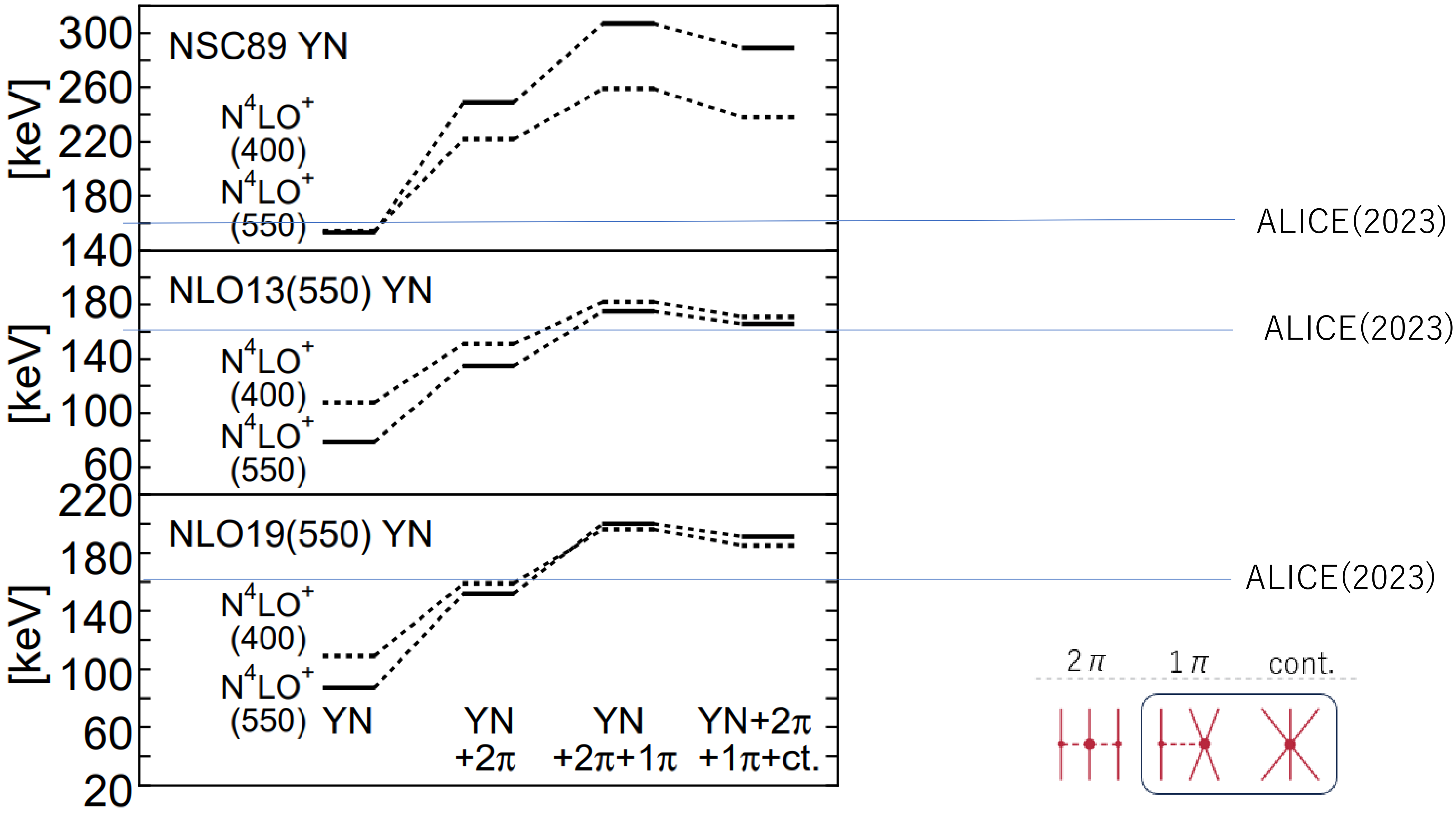
$+2\pi+1\pi$

NN potential (Λ_c)	YN potential (Λ_c) NLO13 (550)	NLO19 (550)	NLO19 (600)	NLO19 (650)	NSC89
CDBonn [23]	172	194	196	196	317
Nijmegen 93 [32]	114	183	187	191	313
Nijmegen I [32]	120	191	194	196	315
N^4LO+ (550)	175	200	204	205	307
N^4LO+ (500)	180	202	205	206	299
N^4LO+ (450)	183	201	204	203	283
N^4LO+ (400)	182	196	197	196	259
N^4LO (550)	178	201	206	208	309
N^4LO (500)	182	203	207	208	300
N^4LO (450)	184	202	205	205	283
N^4LO (400)	183	196	198	197	259
N^3LO (550)	175	200	204	204	300
N^3LO (500)	180	201	205	206	298
N^3LO (450)	184	202	205	205	284
N^3LO (400)	182	196	198	197	261
N^2LO (550)	167	195	205	210	330
N^2LO (500)	183	207	214	216	318
N^2LO (450)	187	206	210	210	292
N^2LO (400)	182	195	197	194	257

$+2\pi+1\pi+$ cont.

NN potential (Λ_c)	YN potential (Λ_c) NLO13 (550)	NLO19 (550)	NLO19 (600)	NLO19 (650)	NSC89
CDBonn [23]	163	185	187	187	300
Nijmegen 93 [32]	148	176	180	183	296
Nijmegen I [32]	154	183	186	187	298
N^4LO+ (550)	166	191	195	196	289
N^4LO+ (500)	170	192	196	196	280
N^4LO+ (450)	173	191	194	193	263
N^4LO+ (400)	171	185	187	184	238
N^4LO (550)	169	192	197	206	290
N^4LO (500)	172	193	197	198	280
N^4LO (450)	174	192	195	194	263
N^4LO (400)	172	185	187	185	239
N^3LO (550)	166	191	195	195	281
N^3LO (500)	171	192	195	196	279
N^3LO (450)	173	191	194	194	264
N^3LO (400)	172	186	187	185	241
N^2LO (550)	188	188	198	203	314
N^2LO (500)	198	198	207	207	300
N^2LO (450)	195	195	200	199	271
N^2LO (400)	184	184	186	182	236

[Kohno, Kamada, Miyagawa: PHYSICAL REVIEW C 109, 024003 (2024)]



[Kohno, Kamada, Miyagawa: PHYSICAL REVIEW C 109, 024003 (2024)]

Eur. Phys. J. A (2018) 54: 86
DOI 10.1140/epja/i2018-12516-4

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order

P. Reinert[a], H. Krebs[b], and E. Epelbaum[c]

Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

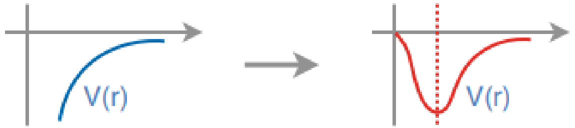
$$V(\vec{p}\,', \vec{p}) \longrightarrow V_\Lambda(\vec{p}\,', \vec{p}) = V(\vec{p}\,', \vec{p}) \exp\left[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}\right]$$

$$V_\pi(\vec{r}) \longrightarrow V_{\pi,R}(\vec{r}) = V_\pi(\vec{r}) \left[1 - \exp(-r^2/R^2)\right]^n$$

Semilocal **M**omentum-**S**pace regularized （*from Evgeny's note*）

2014: The SFR removes the unphysical short-range components but one also distorts some good long-range physics… Can one do better?

⇒ local regularization in r-space EE, Krebs, Meißner, EPJA 51 (15) 53
PRL 115 (15) 122301



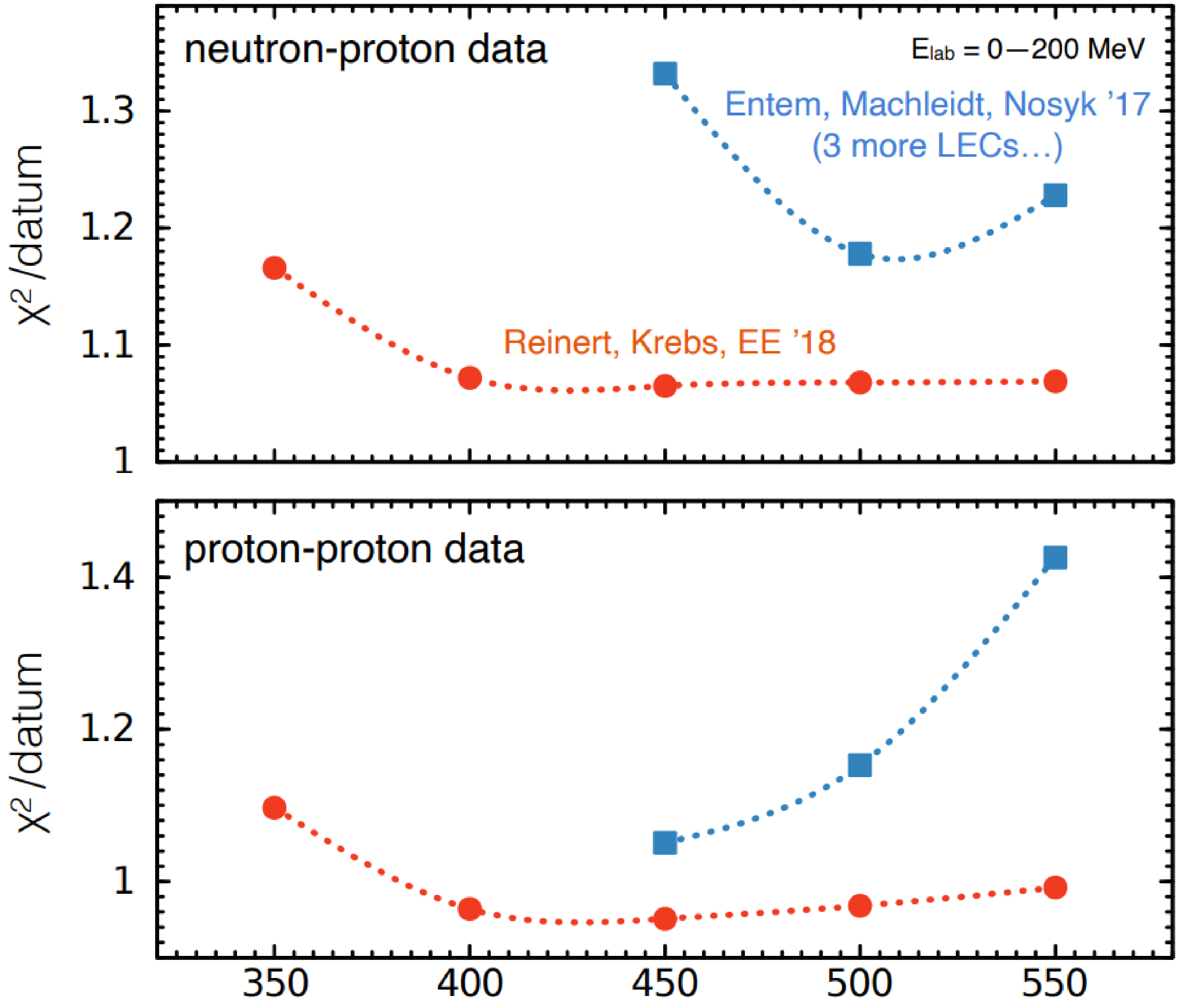
2018: The r-space regulator turned out to be inconvenient for 3N forces and currents

⇒ local regularization in momentum space Reinert, Krebs, EE, EPJA (18) 85

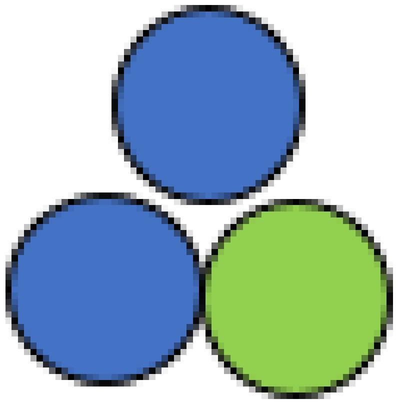
$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\, e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction},$$

$$V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\, \mu\, \frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2}\, e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

Semilocal **M**omentum-**S**pace regularized （*from Evgeny's note*)



$^{3}_{\Lambda}n$ $(nn\Lambda)$

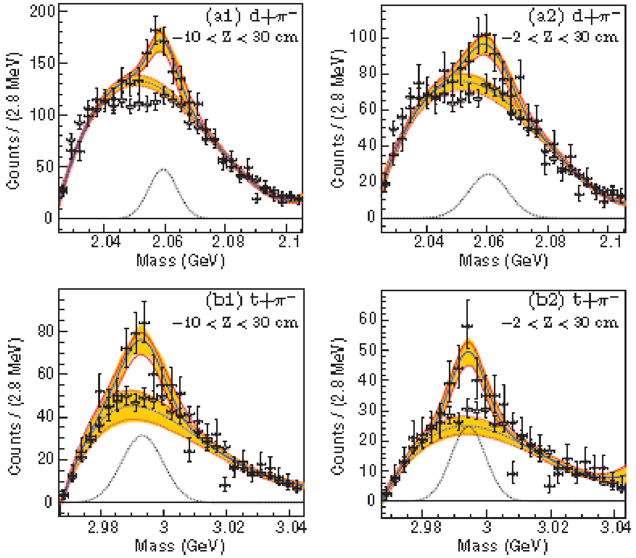


PHYSICAL REVIEW C **88**, 041001(R) (2013)

Search for evidence of ${}^{3}_{\Lambda}n$ by observing $d + \pi^-$ and $t + \pi^-$ final states in the reaction of ^{6}Li + ^{12}C at 2*A* GeV

C. Rappold,[1,2,*] E. Kim,[1,3] T. R. Saito,[1,4,5,†] O. Bertini,[1,4] S. Bianchin,[1] V. Bozkurt,[1,6] M. Kavatsyuk,[7] Y. Ma,[1,4] F. Maas,[1,4,5] S. Minami,[1] D. Nakajima,[1,8] B. Özel-Tashenov,[1] K. Yoshida,[1,5,9] P. Achenbach,[4] S. Ajimura,[10] T. Aumann,[1,11] C. Ayerbe Gayoso,[4] H. C. Bhang,[3] C. Caesar,[1,11] S. Erturk,[6] T. Fukuda,[12] B. Göküzüm,[1,6] E. Guliev,[7] J. Hoffmann,[1] G. Ickert,[1] Z. S. Ketenci,[6] D. Khaneft,[1,4] M. Kim,[3] S. Kim,[3] K. Koch,[1] N. Kurz,[1] A. Le Fèvre,[1,13] Y. Mizoi,[12] L. Nungesser,[4] W. Ott,[1] J. Pochodzalla,[4] A. Sakaguchi,[9] C. J. Schmidt,[1] M. Sekimoto,[14] H. Simon,[1] T. Takahashi,[14] G. J. Tambave,[7] H. Tamura,[15] W. Trautmann,[1] S. Voltz,[1] and C. J. Yoon[3]
(HypHI Collaboration)

[1] *GSI Helmholtz Centre for Heavy Ion Research, Planckstrasse 1, 64291 Darmstadt, Germany*



$$ {}^{3}_{\Lambda}n \rightarrow p + n + n + \pi^- \quad \text{and} \quad {}^{3}_{\Lambda}n \rightarrow t + \pi^- $$

$$\underline{\psi}^{(12)} = \frac{1}{E - \underline{H}_0} \underline{T}_{12}(1 - P_{12})\underline{\psi}^{(13)} ,$$

$$\underline{\psi}^{(13)} = \frac{1}{E - \underline{H}_0} \underline{T}_{13}(\underline{\psi}^{(12)} - P_{12}\underline{\psi}^{(13)}) .$$

$$\eta(E)\underline{\psi} = \underline{K}(E)\underline{\psi} ,$$

Search for E with η (E)=1.

先行研究

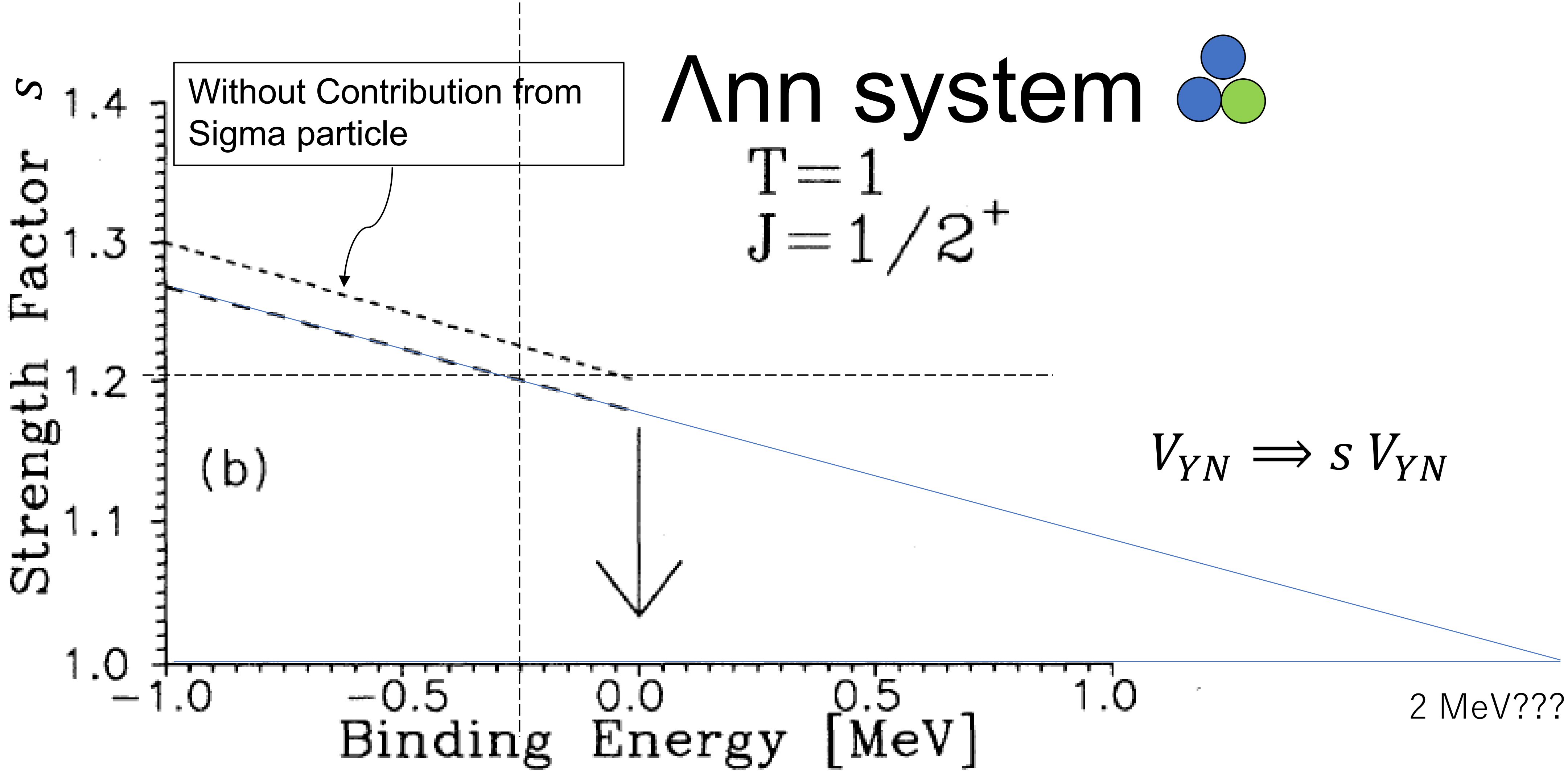
K. Miyagawa, H. Kamada, W. Glöckle, and V. Stoks, Phys. Rev. C 51, 2905 (1995).

Λnn system

T=1

$J=1/2^{+}$

$V_{YN} \Longrightarrow s\, V_{YN}$



$(1, \frac{1}{2}^{+})$. The short dashed lines result if the transition t matrix elements $t_{N\Lambda,N\Sigma}$ are switched off (see text).

先行研究

PHYSICAL REVIEW C **92**, 054608 (2015)

Resonances in the Λnn system

Iraj R. Afnan*
School of Chemical and Physical Sciences, Flinders University, GPO Box 2100, Adelaide 5001, Australia

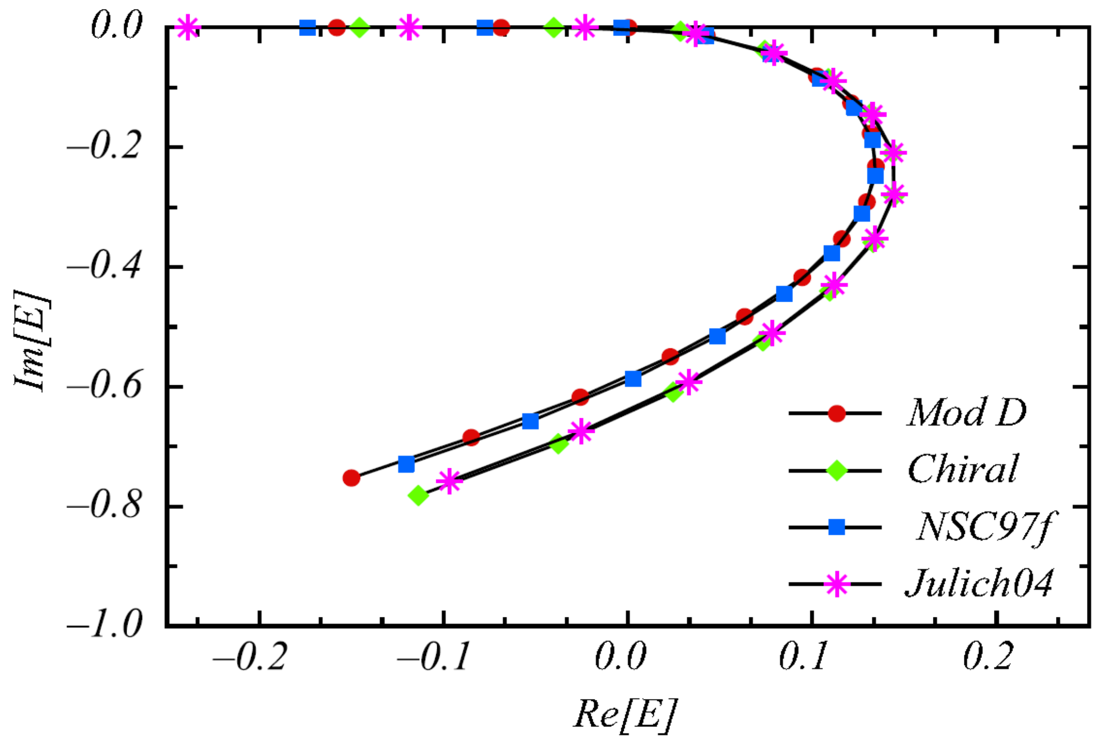
Benjamin F. Gibson†
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
(Received 3 September 2015; published 11 November 2015)

Yamaguchi Separable potential

$$V = -\lambda_Y \frac{1}{p^2 + \beta_Y^2} \frac{1}{p'^2 + \beta_Y^2}$$

$$\beta_Y = \frac{3 + \sqrt{9 - 16\frac{r}{a}}}{2r}, \lambda_Y = \frac{4\beta_Y^3}{\pi\mu(r\beta_Y - 1)},$$

a = Scattering length, r = Effective range



Complex Energy Method

H. Kamada, Y. Koike, W. Glöckle,
Progress of Theoretical Physics **109**, No.5, pp.869-874 (2003)

Lippmann-Sc[illegible]er equation:

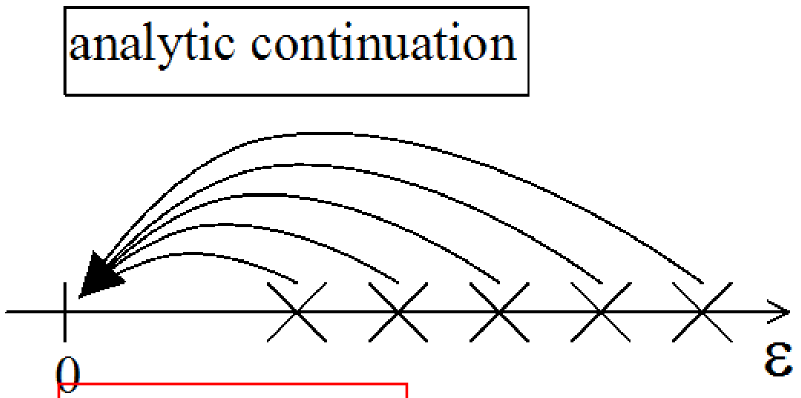
ϵ = *infinitesimal*

$$t = v + vG_0 t,$$

$$t(p, p'; E) = v(p, p') + \int v(p, p'') \frac{1}{E - p''^2/m + i\epsilon} t(p'', p'; E) dp''$$

$$\int_a^b \frac{f(x)}{x_0 - x + i\epsilon} dx = \int_a^b \frac{f(x) - f(x_0)}{x_0 - x} + f(x_0) \int_a^b \frac{dx}{x_0 - x + i\epsilon}$$

$$= A + f(x_0) \left(\log \left(\frac{|a - x_0|}{|b - x_0|} \right) - i\pi \right).$$



Point Method

$$C_N(x) = \cfrac{f(x_1)}{1 + \cfrac{a_1(x - x_1)}{1 + \cfrac{a_2(x - x_2)}{1 + \cdots \cfrac{a_N(x - x_N)}{1}}}}$$

the coefficients a_l may be determined recursively from the formula

$$a_l(x_l - x_{l+1}) = 1 + \cfrac{a_{l-1}(x_{l+1} - x_{l-1})}{1 + \cfrac{a_{l-2}(x_{l+1} - x_{l-2})}{1 + \cdots \cfrac{a_1(x_{l+1} - x_1)}{1 - [f(x_1)/f(x_{l+1})]}}}$$

and

$$a_1 = \{[f(x_1)/f(x_2)] - 1\}/(x_2 - x_1)$$

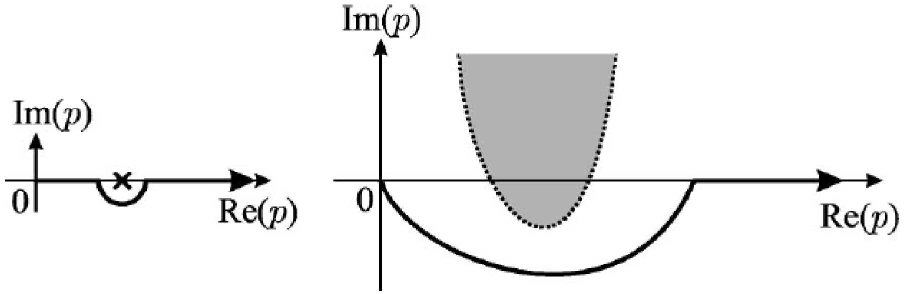
• Complex Energy Method

Application to 4N scattering Problem

- E. Uzu, H. Kamada, and Y. Koike, Phys. Rev. C 68, 061001 (2003), arXiv:nucl-th/0310001.
- A. Deltuva and A. C. Fonseca, Phys. Rev. C 86, 011001 (2012), arXiv:1206.4574 [nucl-th].
- A. Deltuva and A. C. Fonseca, Phys. Rev. C 87, 014002 (2013), arXiv:1301.1905 [nucl-th].
- A. Deltuva and A. C. Fonseca, Phys. Rev. C 87, 054002 (2013), arXiv:1304.5410 [nucl-th].
- A. Deltuva and A. C. Fonseca, Phys. Rev. C 90, 044002 (2014), arXiv:1409.7318 [nucl-th].

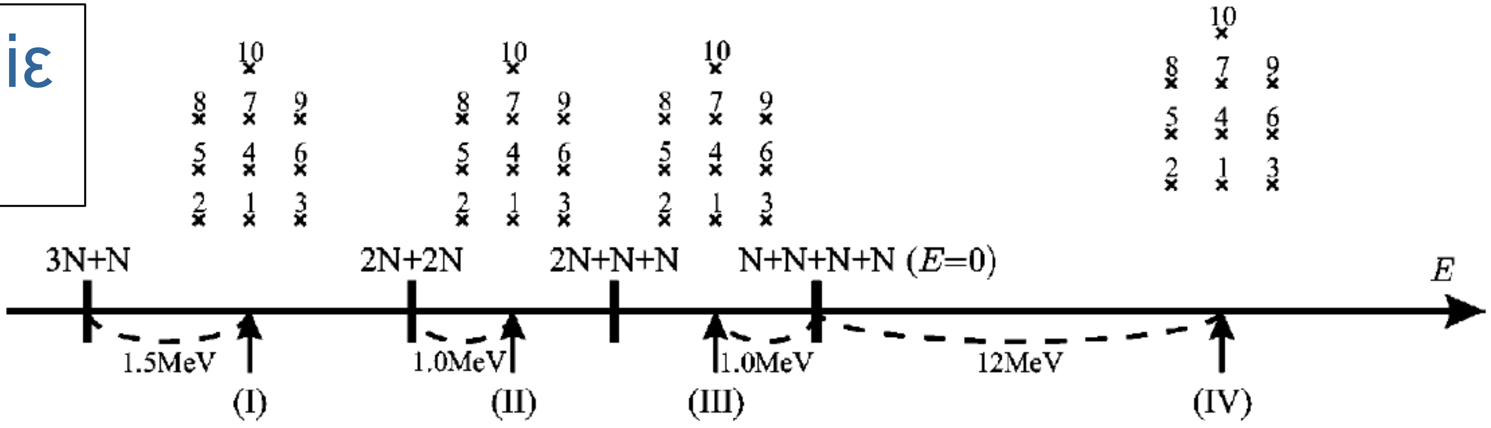
$E=E_r - i\Gamma/2$

$\Gamma > 0$



$E=E_r + i\varepsilon$

$\varepsilon > 0$



Results from EPJ Web Conf. **113**, 07004 (2016)

Jmax=4

factor	Resonance energy $E_r - i\Gamma/2$ [MeV]	Hiyama *et al.* [1]
1.00	0.25 -0.40i	-
1.05	0.15 -0.20i	-
1.10	0.08 -0.15i	-
1.20	-0.243 (bound)	-0.054 (bound)

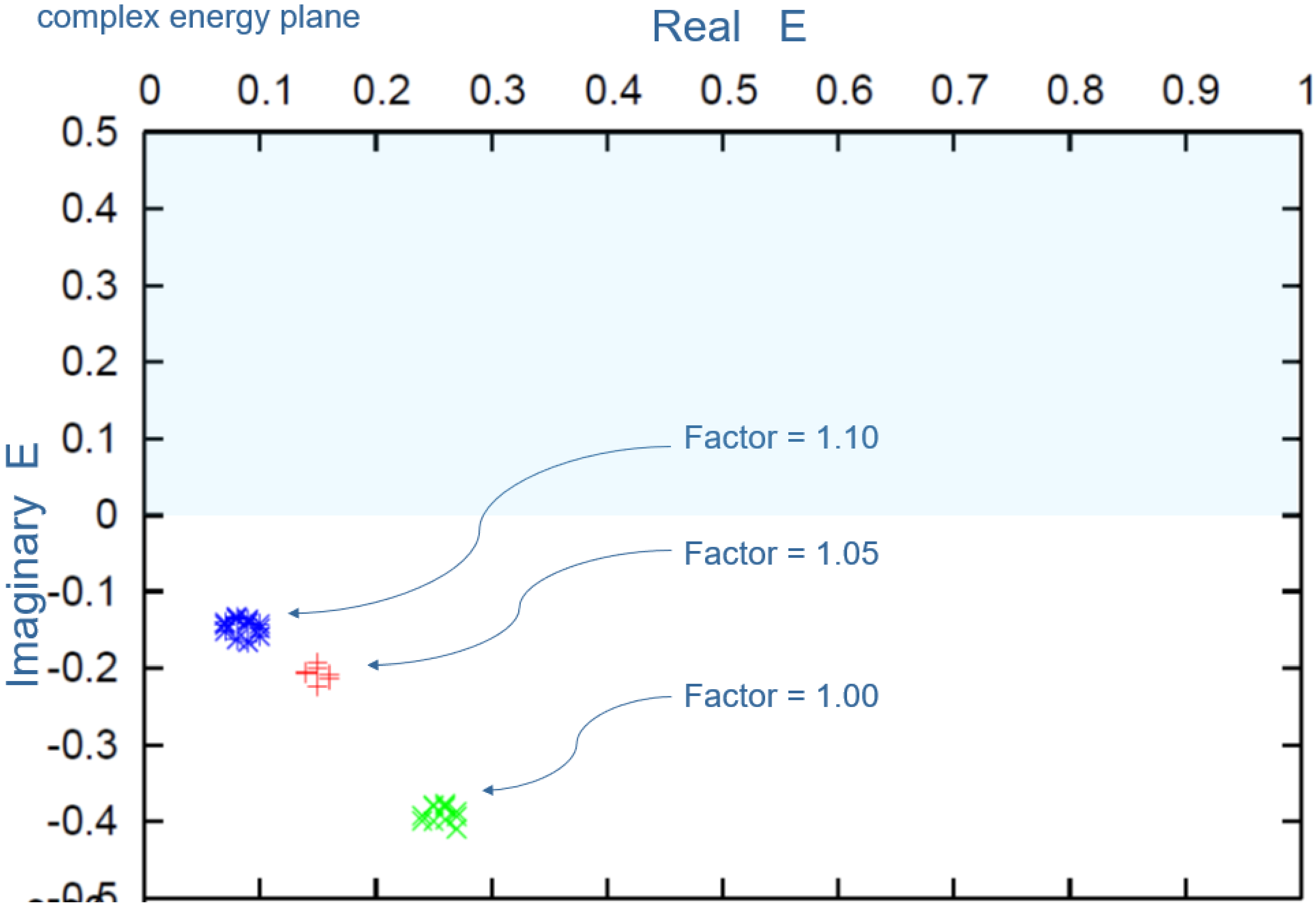
- **NNpot. : Nijm93**

V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, and J.J. de Swart, Phys. Rev. C **49**, 2950 (1994).

- **NYpot. : Nijmegen YN (89)**

P.M.M. Maessen, Th.A. Rijken, and J.J. de Swart, Phys. Rev. C **40**, 2226 (1989).

Position of Resonance pole in the complex energy plane

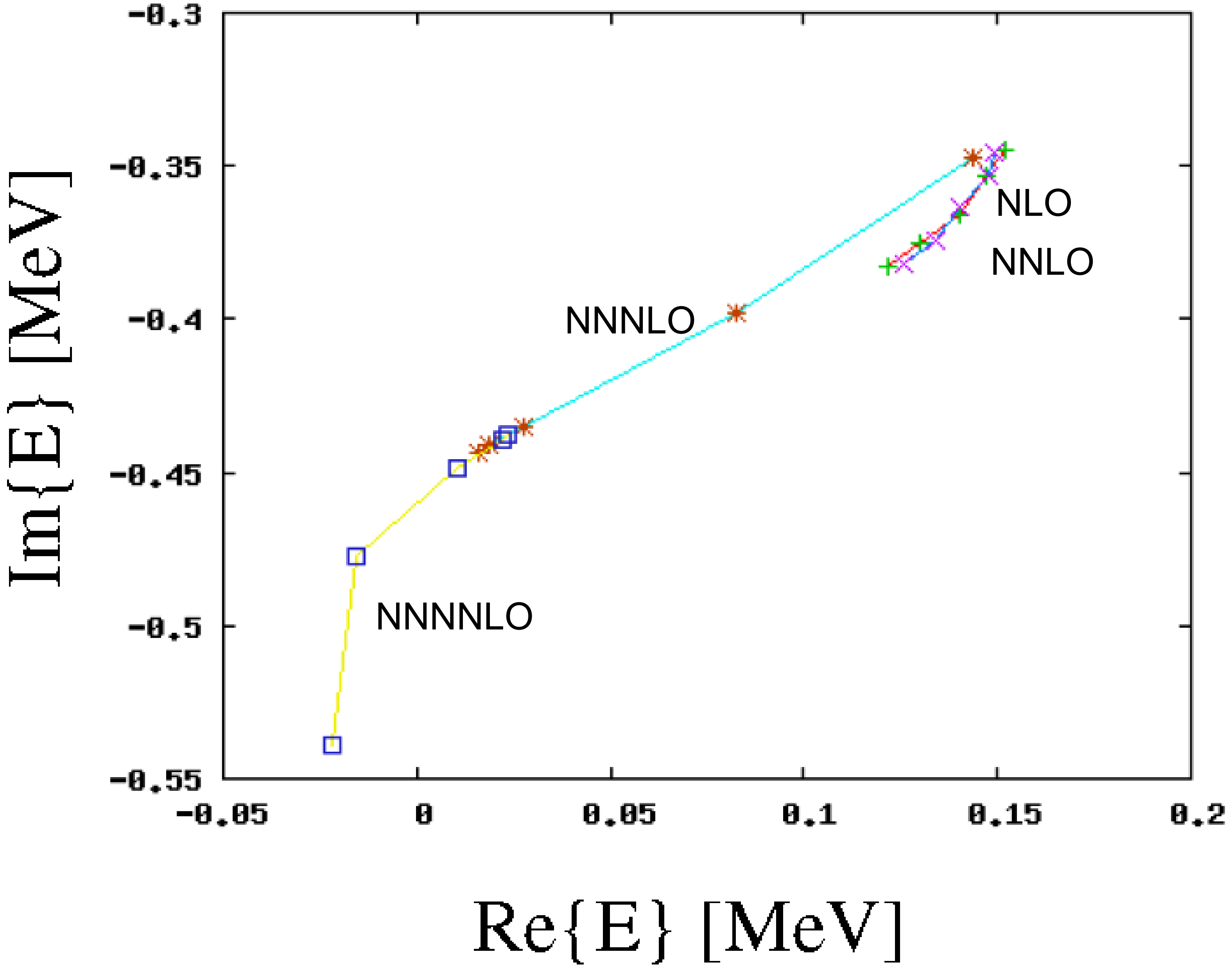


$J=1/2$, $T=1$, V_{NN}(nn)

Cut off R	NNNNLO	NNNLO	NNLO	NLO
0.8	-0.0219-i0.539	0.144-i0.347	0.149-i0.346	0.152-i0.345
0.9	-0.0155-i0.477	0.0823-i0.398	0.148-i0.353	0.147-i0.353
1.0	0.0105-i0.449	0.0275-i0.435	0.140-i0.363	0.140-i0.366
1.1	0.0219-i0.439	0.0188-i0.441	0.134-i0.374	0.130-i0.375
1.2	0.0233-i0.438	0.0158-i0.444	0.126-i0.382	0.122-i0.383

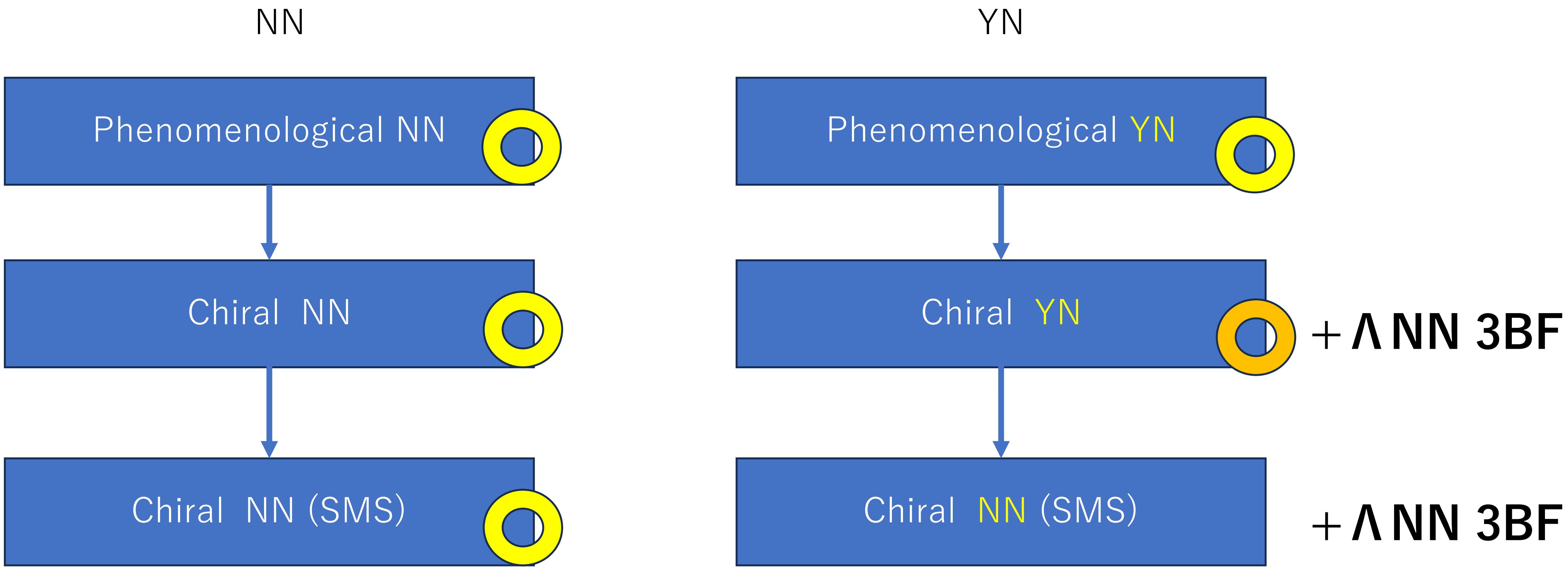
CDBonn: 0.466-i0.567

J=1/2, T=1, V_{NN}(nn)



Im{E} [MeV]
Re{E} [MeV]
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
-0.05
0
0.05
0.1
0.15
0.2
NLO
NNLO
NNNLO
NNNNLO

New Chiral YN potential



INPUT

- 2body force (NN) : Chiral forces N^4LO+ [Cut off scale; 400,450,500,550 MeV]
- 2body force (YN): Chiral forces (Juelich) NLO13,NLO19 [Cut off 550, 600, 650 MeV]
- 3body force (YNN): 2pi exchange type

J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, and W. Weise, Hyperon-nucleon interaction at next-to-leading order in chiral effective field theory, Nucl. Phys. A **915**, 24 (2013).

NLO13(550)

J. Haidenbauer, U.-G. Meißner, and A. Nogga, Hyperon-nucleon interaction within chiral effective field theory revisited, Eur. Phys. J. A **56**, 91 (2020).

NLO19(550),NLO19(600), NLO19(650)

P. M. M. Maessen, Th. A. Rijken, and J. J. de Swart, Soft-core baryon-baryon one-boson-exchange models. II. Hyperon-nucleon potential, Phys. Rev. C **40**, 2226 (1989).

NSC89

} ΛN Potential

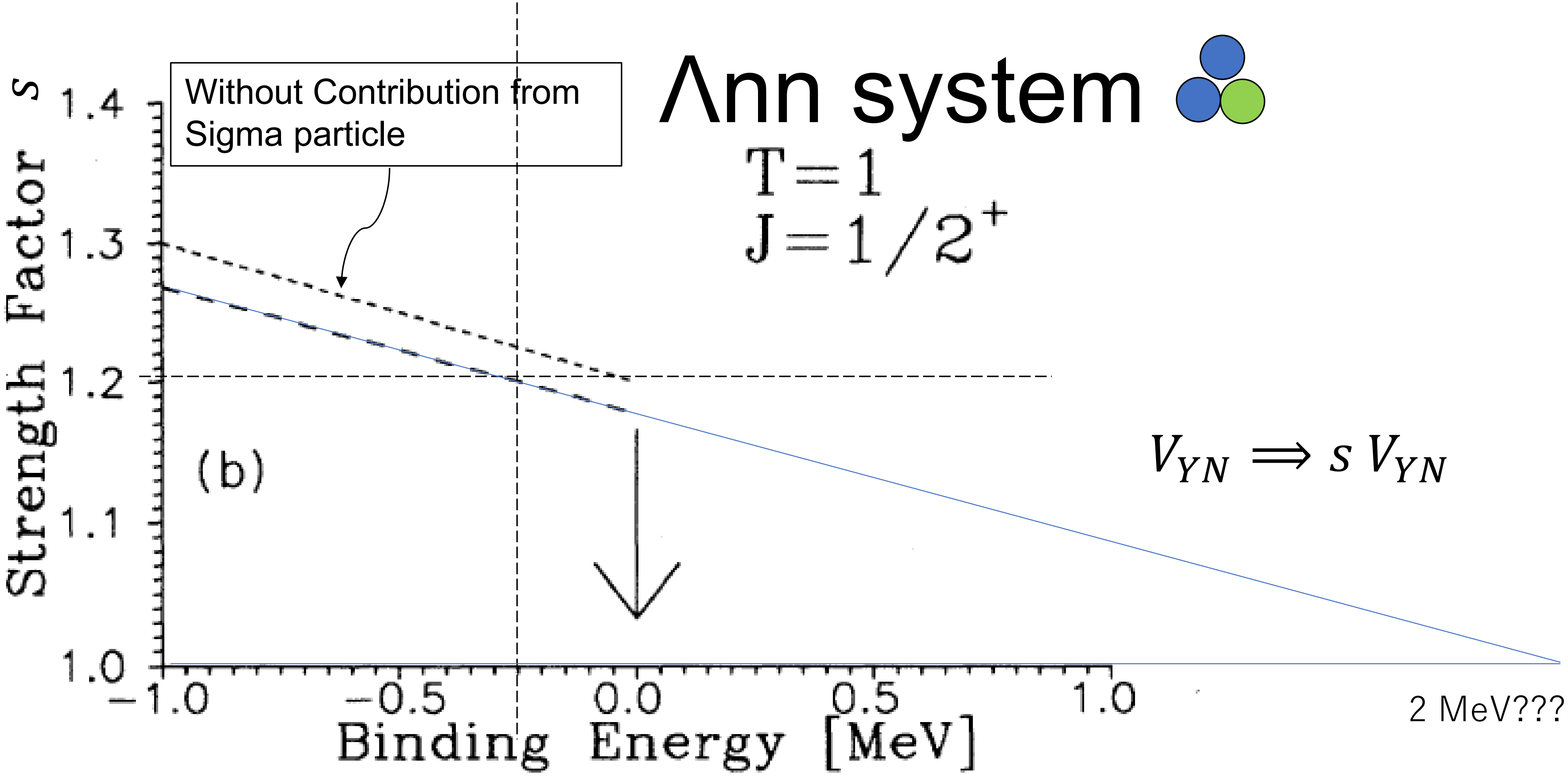
NN Potential

N^4LO+

P. Reinert, and H. Krebs, and E. Epelbaum, Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order, Eur. Phys. J. A **54**, 86 (2018).

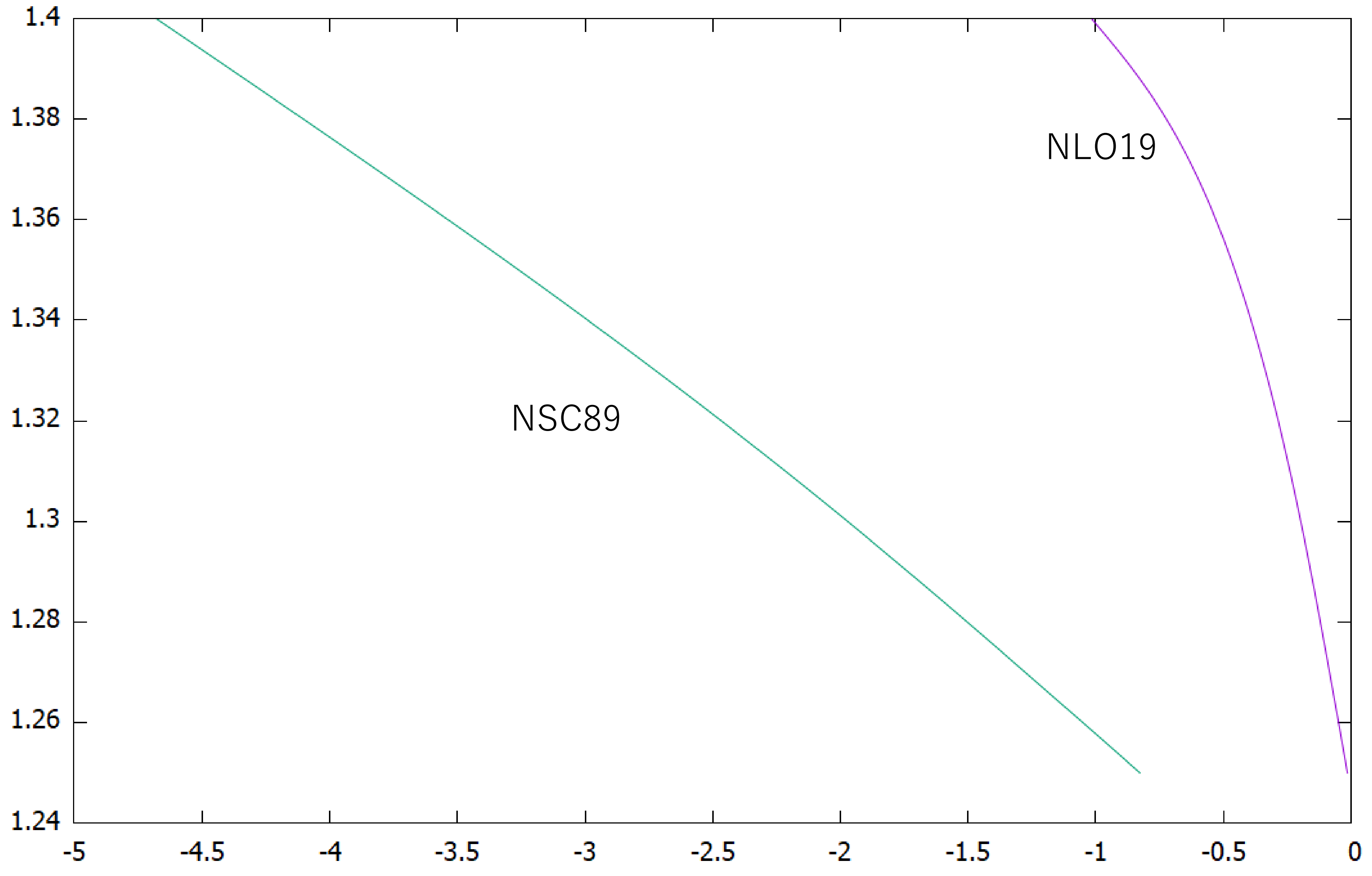
先行研究

K. Miyagawa, H. Kamada, W. Glöckle, and V. Stoks, Phys. Rev. C 51, 2905 (1995).



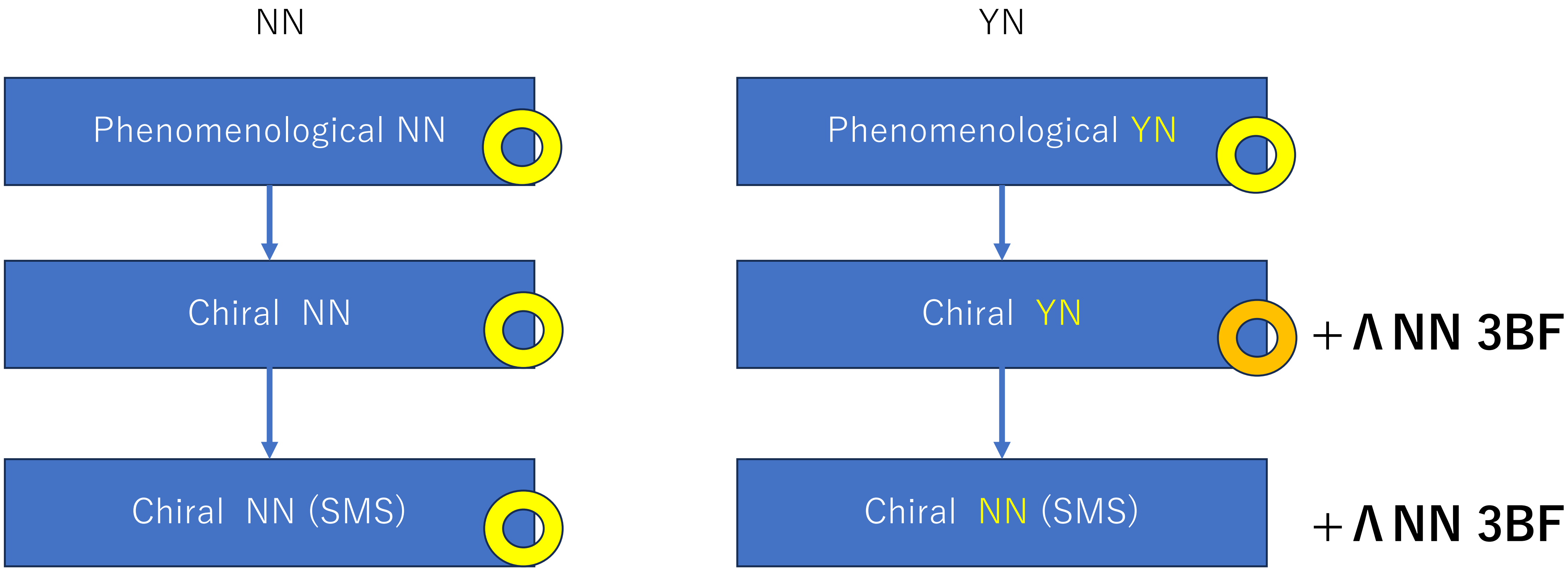
$V_{YN} \Longrightarrow s\, V_{YN}$

$(1, \frac{1}{2}^{+})$. The short dashed lines result if the transition t matrix elements $t_{N\Lambda,N\Sigma}$ are switched off (see text).

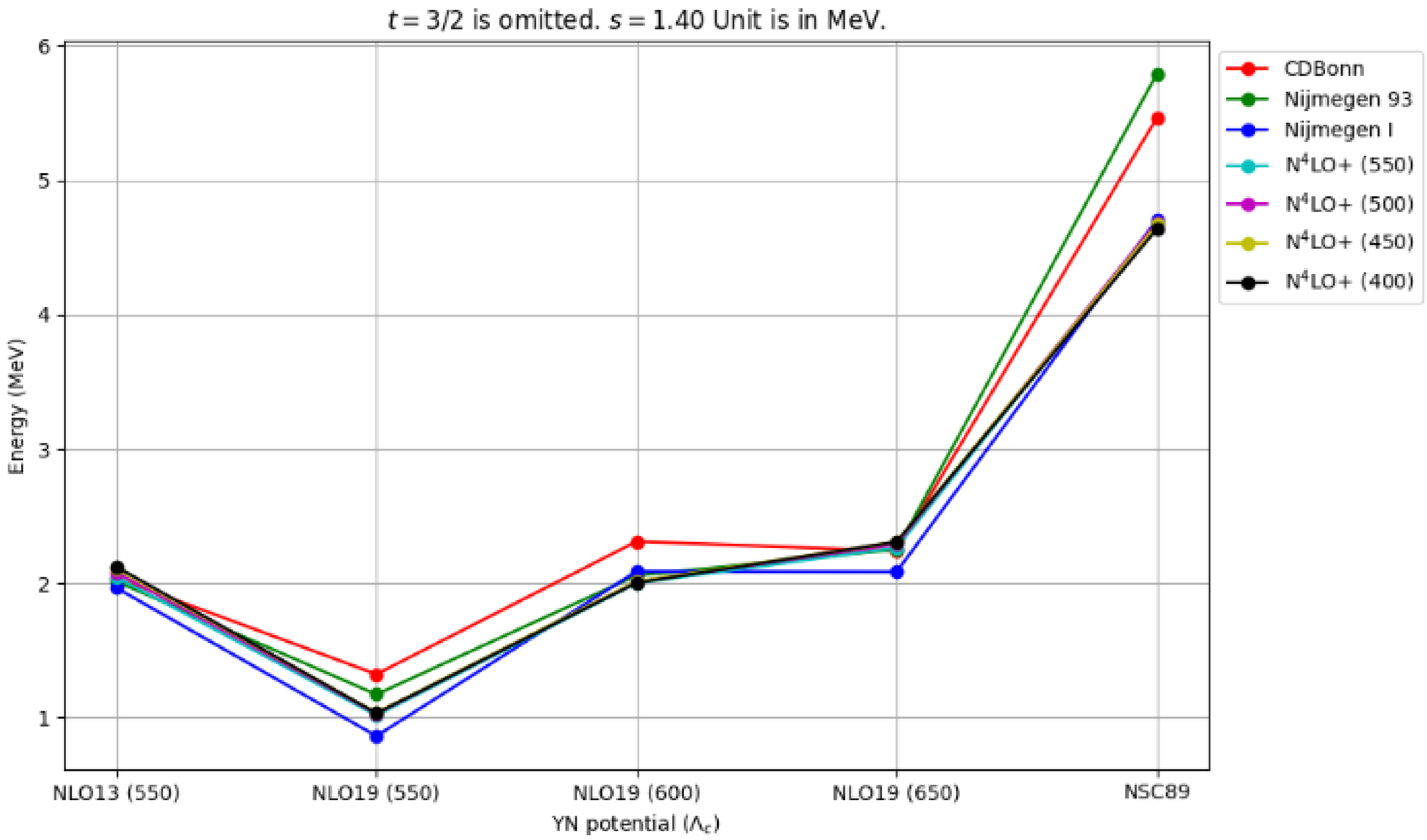


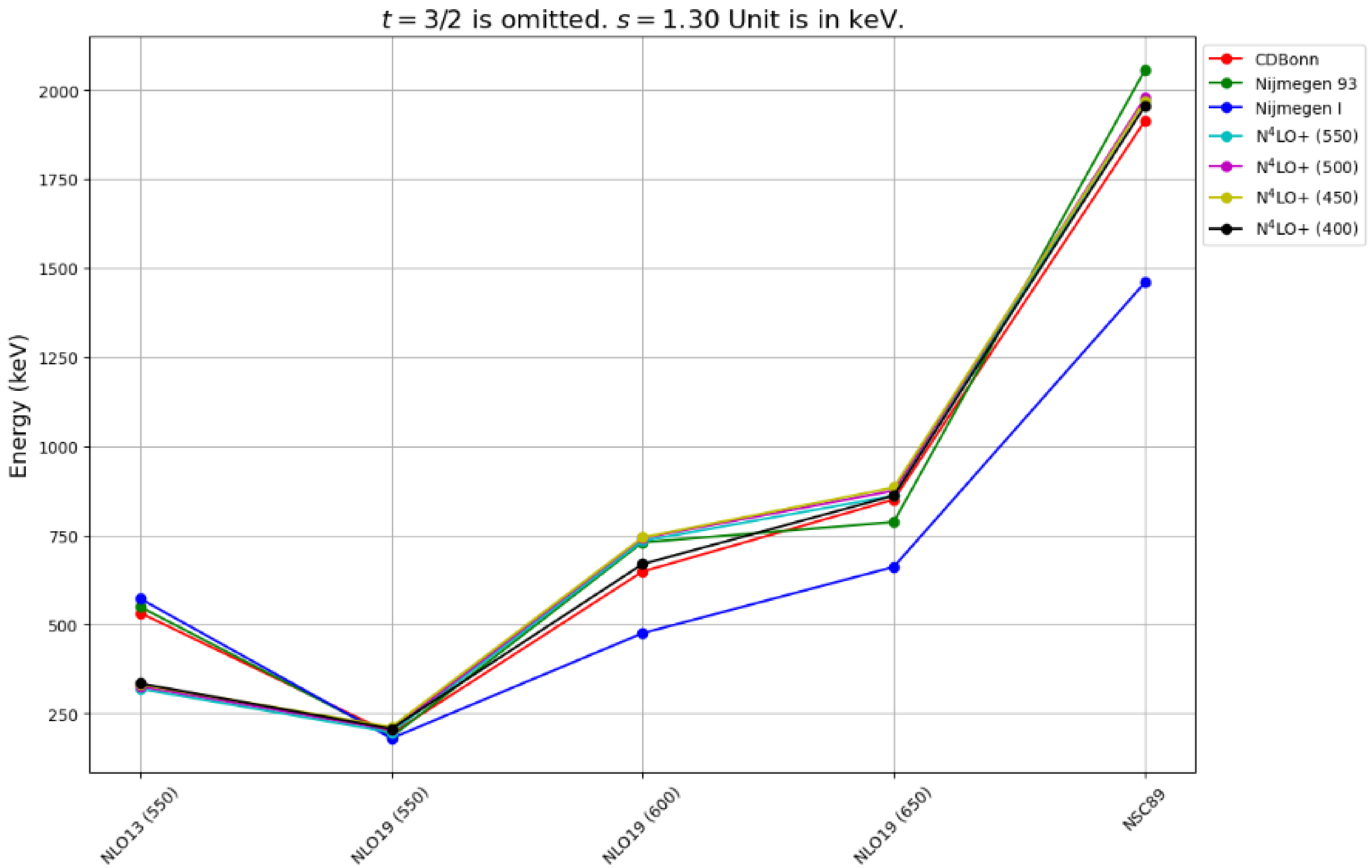
1.4
1.38
1.36
1.34
1.32
1.3
1.28
1.26
1.24
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
NLO19
NSC89

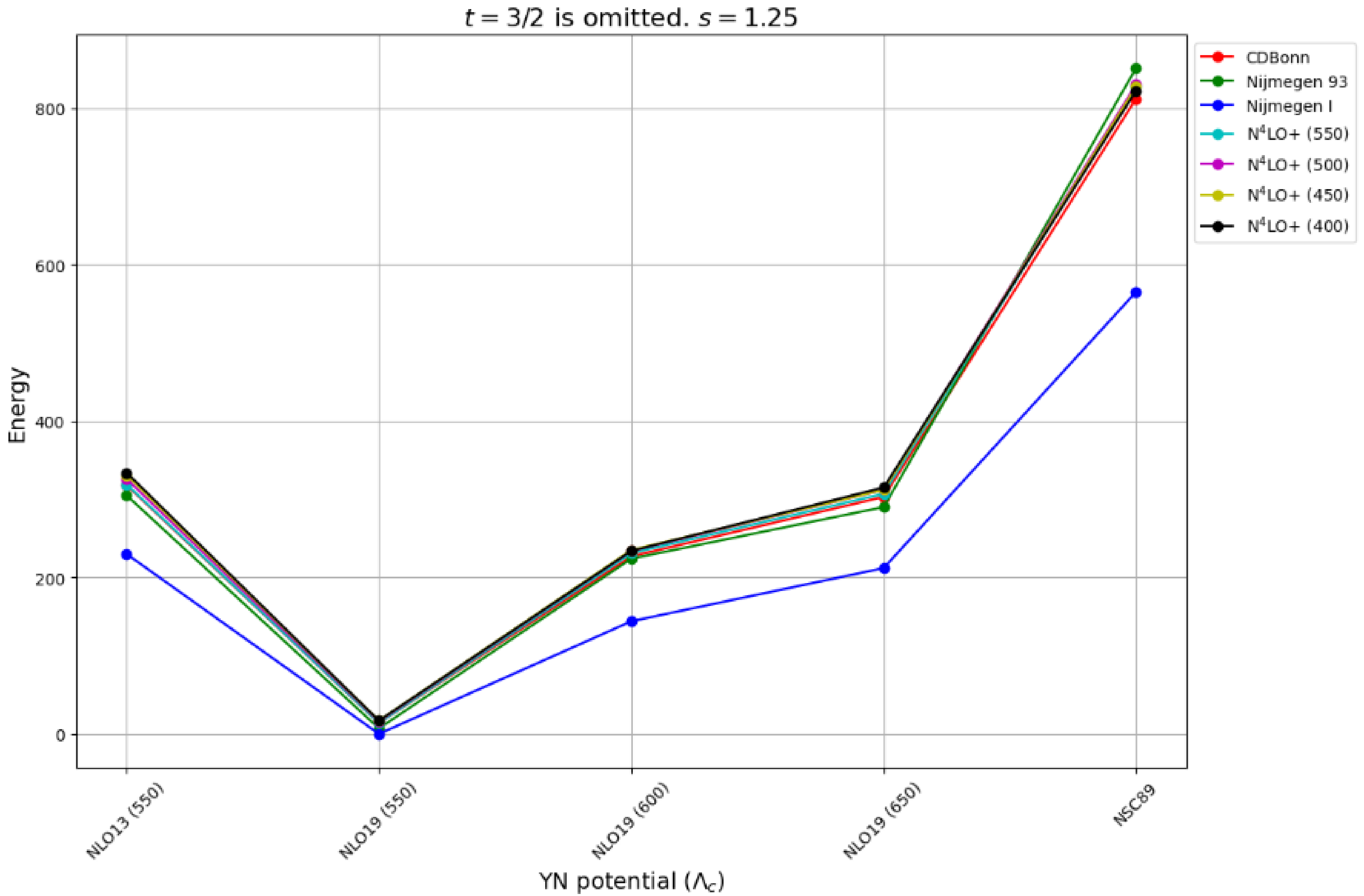
New Chiral YN potential



Thank You for your attention.

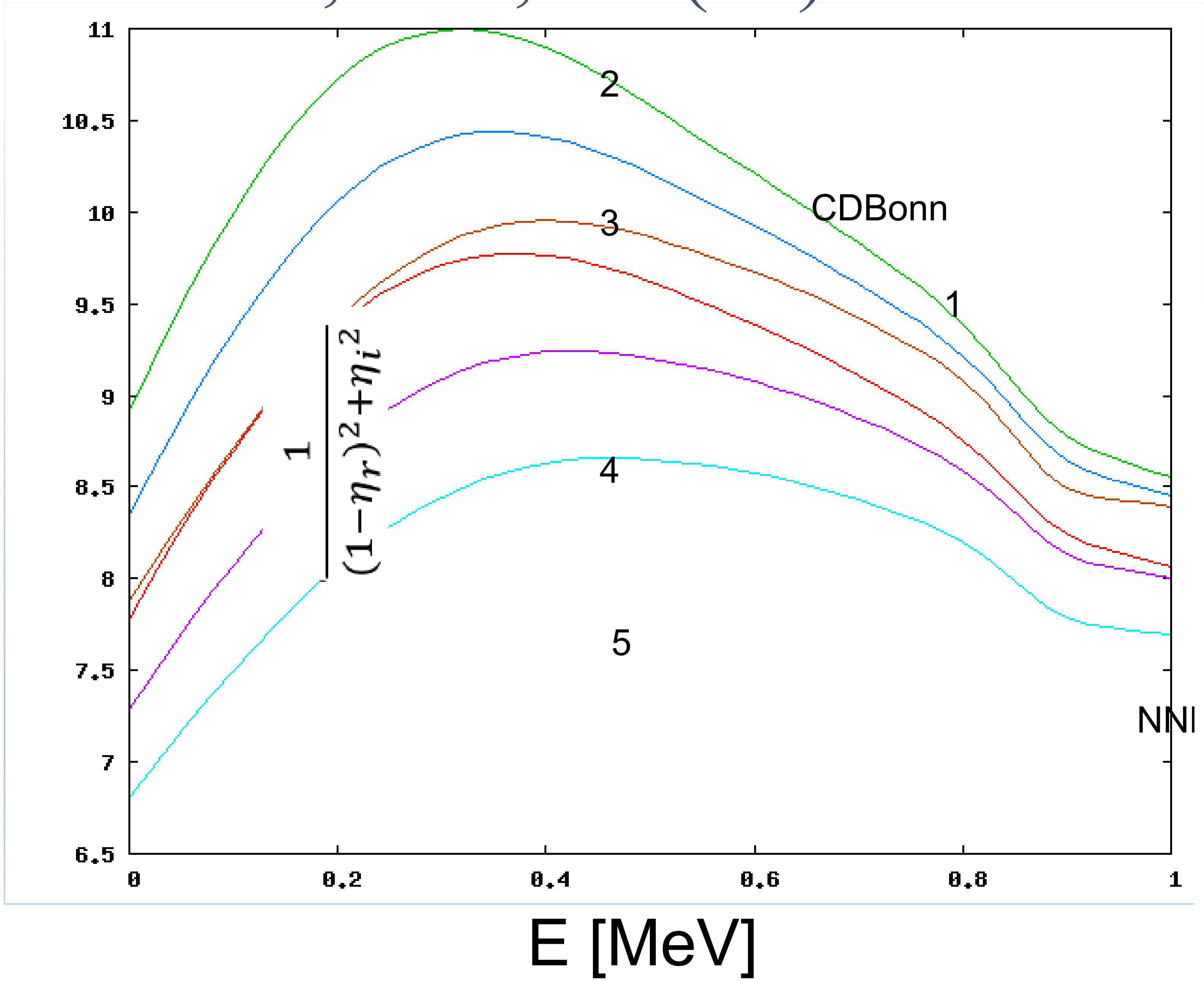

$t = 3/2$ is omitted. $s = 1.40$ Unit is in MeV.
Energy (MeV)
YN potential (Λ_c)
NLO13 (550)
NLO19 (550)
NLO19 (600)
NLO19 (650)
NSC89
CDBonn
Nijmegen 93
Nijmegen I
N^{4}LO+ (550)
N^{4}LO+ (500)
N^{4}LO+ (450)
N^{4}LO+ (400)
1
2
3
4
5
6


$t = 3/2$ is omitted. $s = 1.30$ Unit is in keV.
Energy (keV)
2000
1750
1500
1250
1000
750
500
250
NLO13 (550)
NLO19 (550)
NLO19 (600)
NLO19 (650)
NSC89
CDBonn
Nijmegen 93
Nijmegen I
N4LO+ (550)
N4LO+ (500)
N4LO+ (450)
N4LO+ (400)

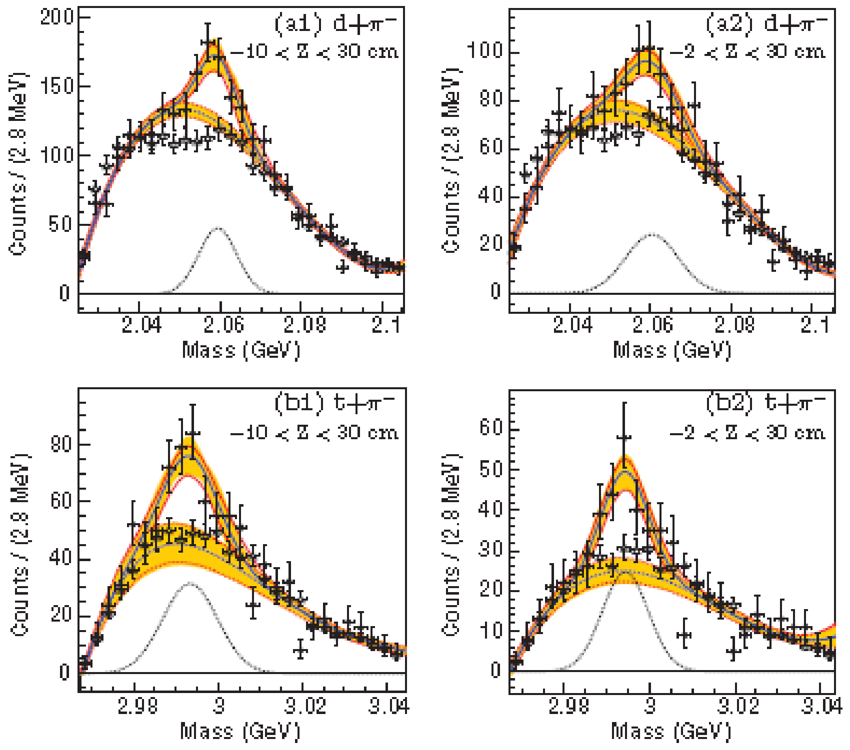


$t = 3/2$ is omitted. $s = 1.25$
Energy
YN potential (Λ_c)
0
200
400
600
800
NLO13 (550)
NLO19 (550)
NLO19 (600)
NLO19 (650)
NSC89
CDBonn
Nijmegen 93
Nijmegen I
N^4LO+ (550)
N^4LO+ (500)
N^4LO+ (450)
N^4LO+ (400)

J=1/2, T=1, V_{NN}(nn)



11
10.5
10
9.5
9
8.5
8
7.5
7
6.5
0
0.2
0.4
0.6
0.8
1
2
CDBonn
3
1
4
5
NNI
$\frac{1}{(1-\eta_r)^2+\eta_i^2}$
E [MeV]



(a1) d+π⁻
−10 < Z < 30 cm
(a2) d+π⁻
−2 < Z < 30 cm
(b1) t+π⁻
−10 < Z < 30 cm
(b2) t+π⁻
−2 < Z < 30 cm
Counts / (2.8 MeV)
Mass (GeV)

Channel numbers (J=1/2, T= 1)

2 body j_max	Λ	Σ	N	Sum
1	4	10	10 *2	34
2	10	18	18 *2	64
3	12	26	26 *2	90
4	18	34	34 *2	120
5	20	42	42 *2	146

$$|pq\alpha r\rangle \equiv |pq(ls)j(\lambda\tfrac{1}{2})I(jI)J(tt_r)T\rangle$$

or

NΣ : **t=3/2** is also involved !

$$|pq\beta r\rangle \equiv |pq(ls)j(\lambda\tfrac{1}{2})I(jI)J[(t_r\tfrac{1}{2})t\tfrac{1}{2}]T\rangle \ .$$

(Possibility of Bound state)

- **Juelich YN model**

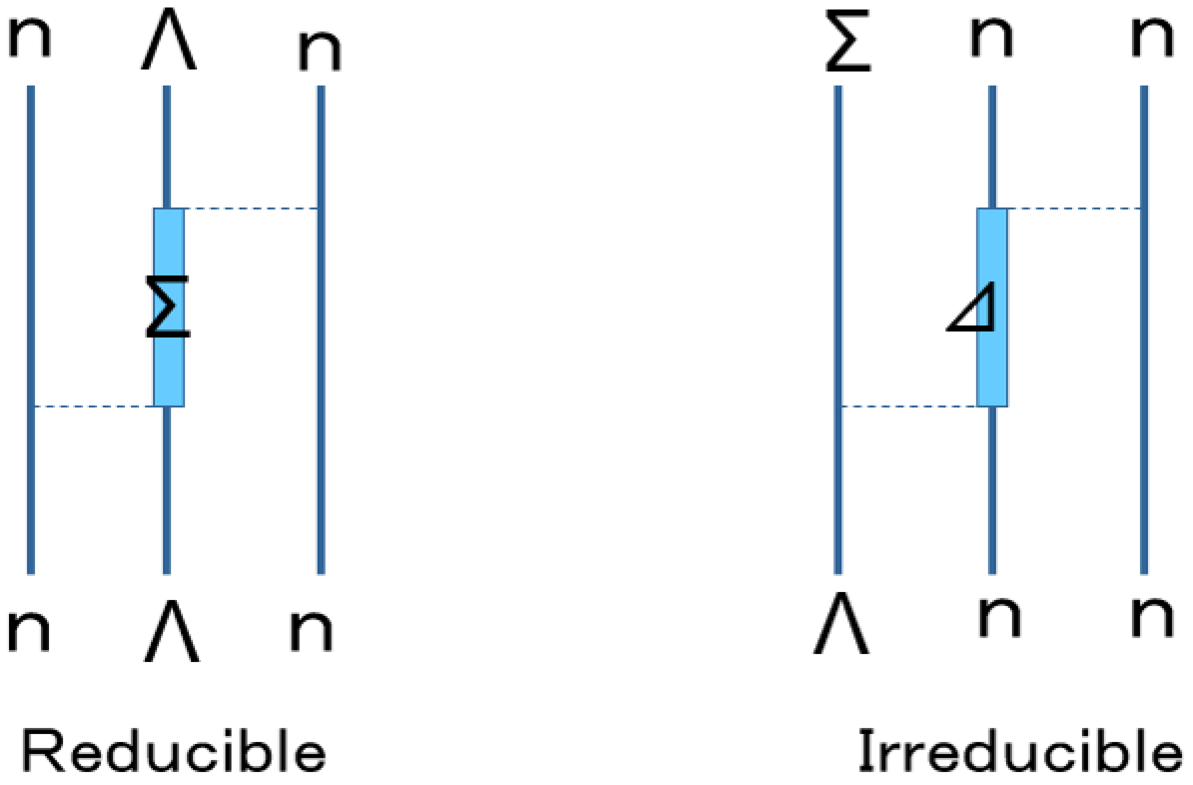
A.G. Reuber, K. Holinde, and J. Speth, Czech. J. Phys. **42**, 1115 (1992).
J. Haidenbauer et al, PRC72, 044005 (2005)

- **Chiral Effective Field Theory version YN interaction**

J. Haidenbauer, S. Petschauer, N. Kaiser, U-G. Meissner, A. Nogga, W. Weise, Nucl. Phys. A915, 24 (2013).

(Possibility of Bound state)

- **3-body force with ⊿ particle**



Next pages are for discussion after presentation.

Table 4 Scattering lengths (a) and effective ranges (r) for singlet (s) and triplet (t) S-waves (in fm), for ΛN, ΣN with isospin $I = 1/2,\ 3/2$, and for $\Sigma^+ p$ with inclusion of the Coulomb interaction

Λ [MeV]	SMS NLO			SMS N^{2}LO			NLO13	NLO19
	500	550	600	500	550	600	600	600
$a_s^{\Lambda N}$	−2.80	−2.79	−2.79	−2.80	−2.79	−2.80	−2.91	−2.91
$r_s^{\Lambda N}$	2.87	2.72	2.63	2.82	2.89	2.68	2.78	2.78
$a_t^{\Lambda N}$	−1.59	−1.57	−1.56	−1.56	−1.58	−1.56	−1.54	−1.41
$r_t^{\Lambda N}$	3.10	2.99	3.00	3.16	3.09	3.17	2.72	2.53
Re $a_s^{\Sigma N\ (I=1/2)}$	1.14	1.15	1.10	1.03	1.12	1.06	0.90	0.90
Im $a_s^{\Sigma N}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Re $a_t^{\Sigma N\ (I=1/2)}$	2.58	2.42	2.31	2.60	2.38	2.53	2.27	2.29
Im $a_t^{\Sigma N}$	−2.60	−2.95	−3.09	−2.56	−3.26	−2.64	−3.29	−3.39
$a_s^{\Sigma N\ (I=3/2)}$	−4.21	−4.05	−4.11	−4.37	−4.19	−4.03	−4.45	−4.55
$r_s^{\Sigma N}$	3.93	3.89	3.75	3.73	3.89	3.74	3.68	3.65
$a_t^{\Sigma N\ (I=3/2)}$	0.46	0.47	0.47	0.38	0.44	0.41	0.44	0.43
$r_t^{\Sigma N}$	−5.08	−4.74	−4.82	−5.70	−4.96	−5.72	−4.59	−5.27
$a_s^{\Sigma^+ p}$	−3.41	−3.30	−3.44	−3.47	−3.39	−3.25	−3.56	−3.62
$r_s^{\Sigma^+ p}$	3.75	3.73	3.59	3.61	3.73	3.65	3.54	3.50
$a_t^{\Sigma^+ p}$	0.51	0.52	0.52	0.41	0.48	0.45	0.49	0.47
$r_t^{\Sigma^+ p}$	−5.46	−5.12	−5.19	−6.74	−5.50	−6.41	−5.08	−5.77

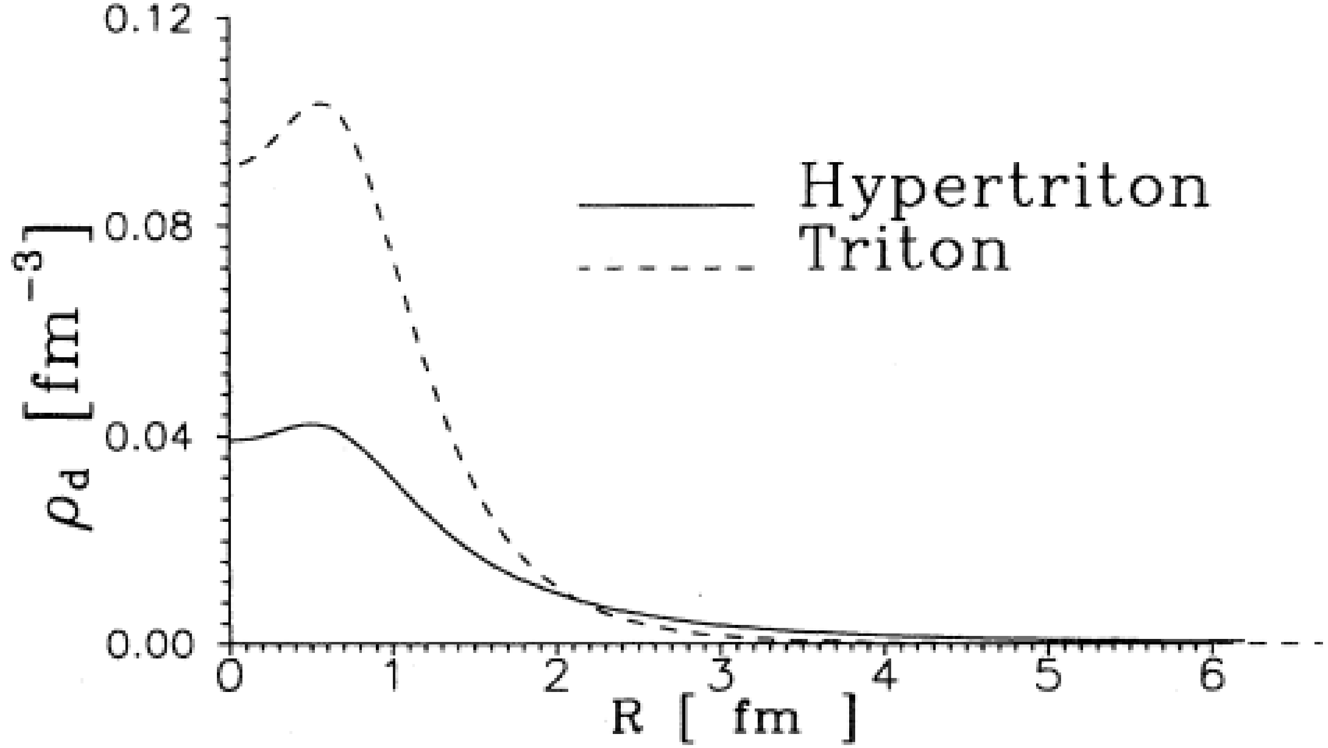
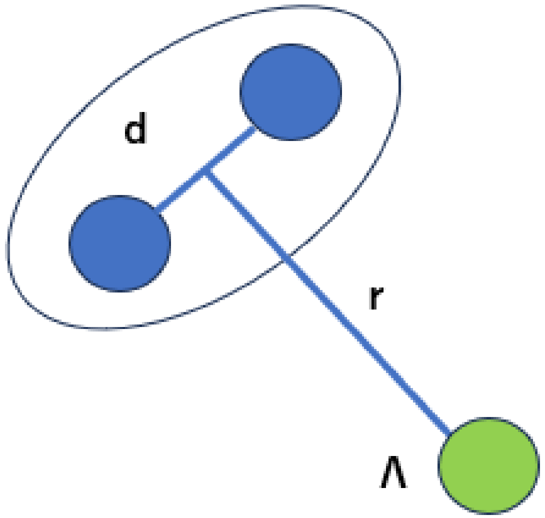


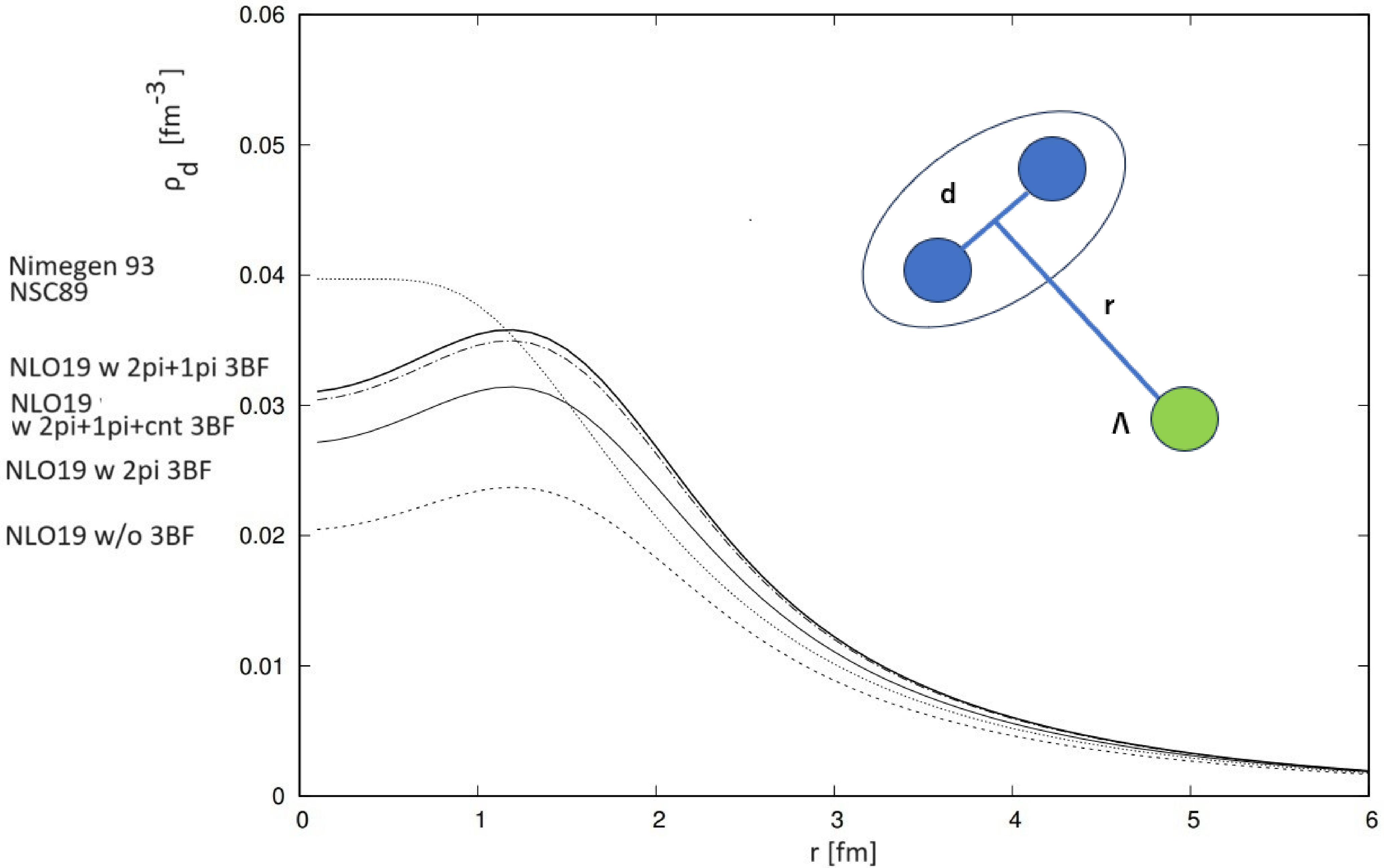
FIG. 11. Comparison of the deuteron overlap functions $\rho_d(r)$ for the triton and the hypertriton.



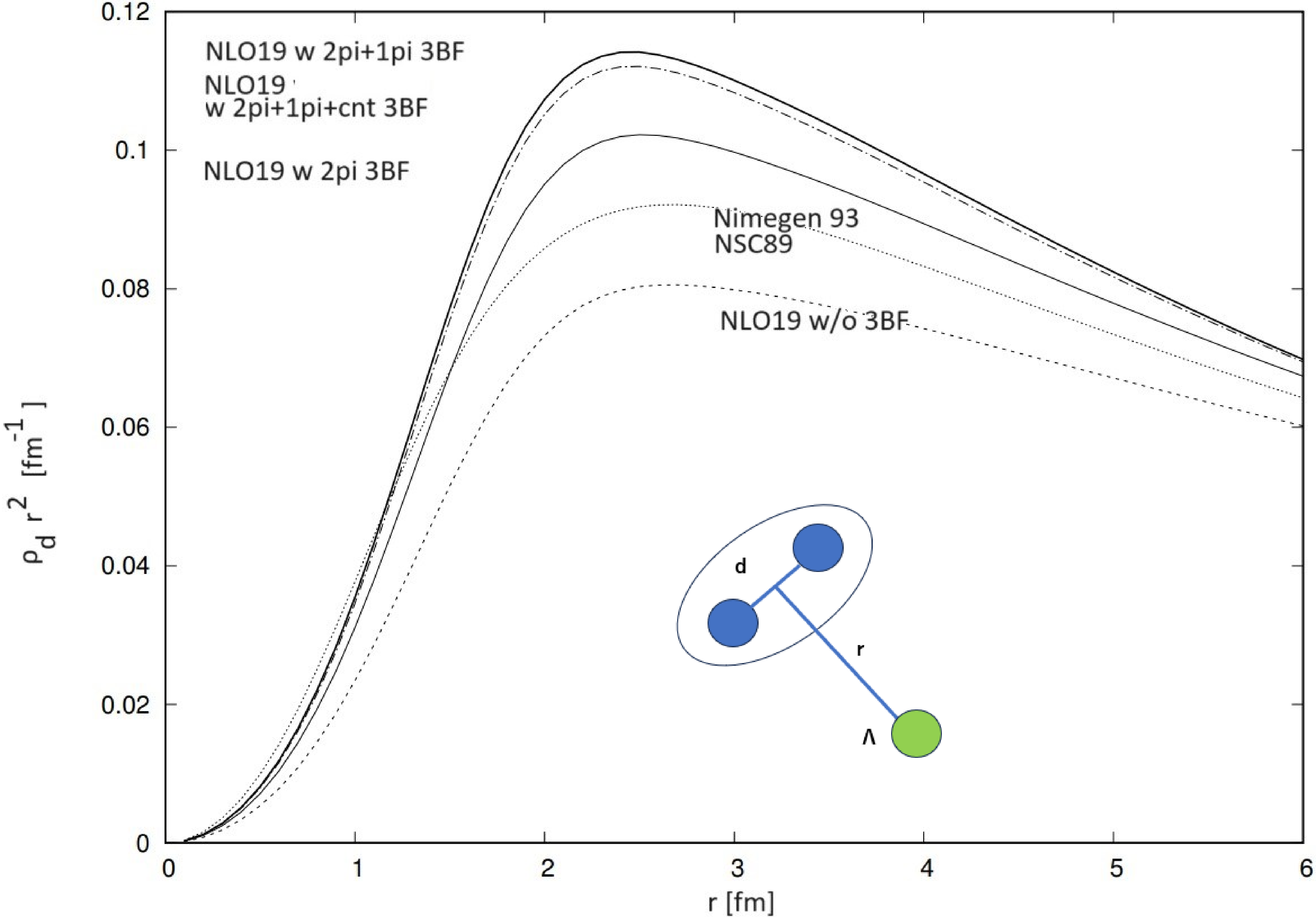
$$P_d \equiv \int_0^\infty dr r^2 \rho_d(r) , \qquad (13)$$

we find $P_d = 0.448$ for the triton and $P_d = 0.987$ for the hypertriton

[K. Miyagawa, H. Kamada, W. Glöckle, and V. Stoks, Phys. Rev. C 51, 2905 (1995)]



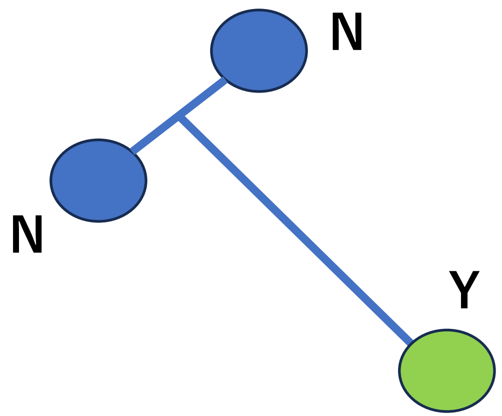
Nimegen 93
NSC89
NLO19 w 2pi+1pi 3BF
NLO19
w 2pi+1pi+cnt 3BF
NLO19 w 2pi 3BF
NLO19 w/o 3BF
ρ_d [fm^-3]
r [fm]
d
r
Λ
0.06
0.05
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5
6



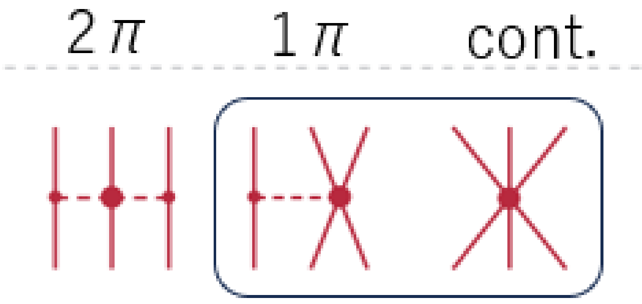
NLO19 w 2pi+1pi 3BF
NLO19
w 2pi+1pi+cnt 3BF
NLO19 w 2pi 3BF
Nimegen 93
NSC89
NLO19 w/o 3BF
d
r
Λ
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
$\rho_d r^2$ [fm^{-1}]
r [fm]

Expectation values of the Kinetic energies in Hypertriton (Unit in MeV)

	Nijm93 NSC89	N^4LO+ NLO19	N^4LO+ NLO19 2pi 3BF	N^4LO+ NLO19 2pi+1pi 3BF	N^4LO+ NLO19 2pi+1pi+cnt. 3BF
N-N relative	20.27 (0.23)	16.46 (0.12)	16.85 (0.17)	17.13 (0.20)	17.07 (0.19)
(NN)-Y relative	2.20 (0.80)	1.87 (0.40)	2.50 (0.72)	2.87 (0.65)	2.80 (0.63)
Total	23.51	18.85	20.07	20.83	20.70



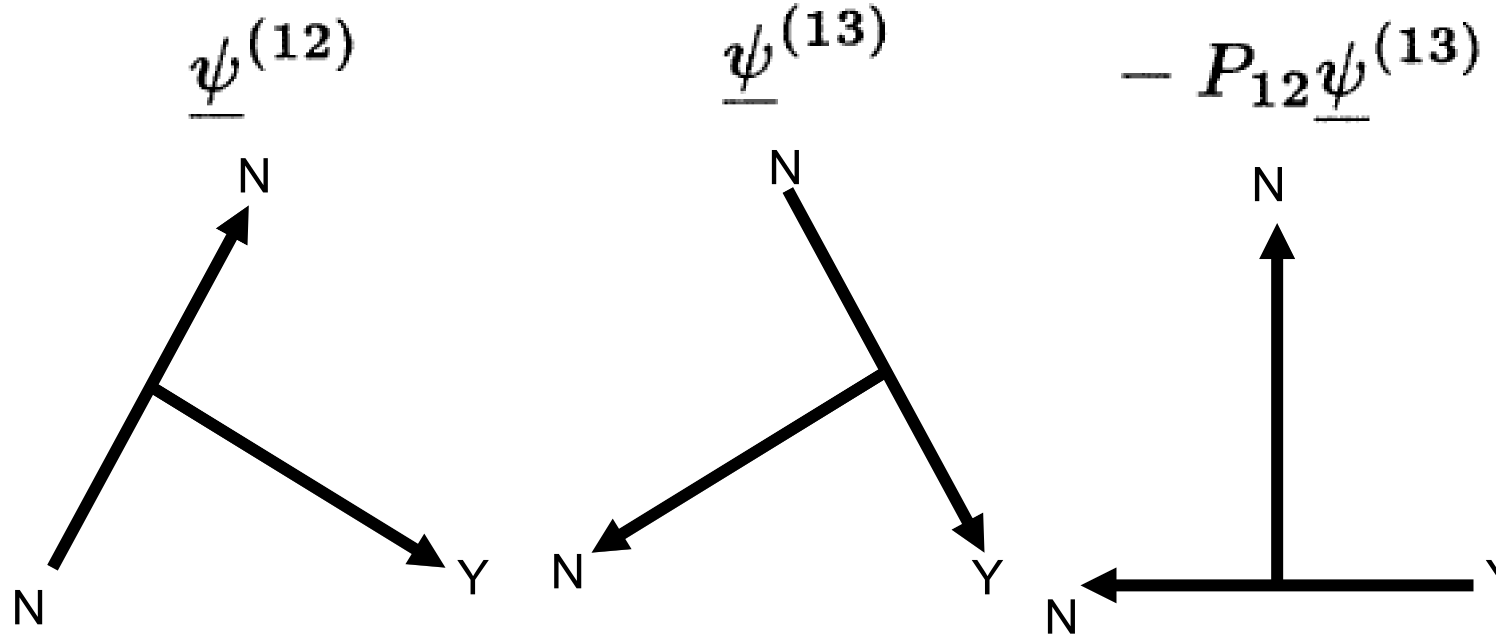
Inside of ( ) is the contribution from Σ particle.



Faddeev Components

$$\Psi_{NN\Lambda} = \psi_{\Lambda}^{(12)} + (1 - P_{12})\, \psi_{\Lambda}^{(13)} \ ,$$

$$\Psi_{NN\Sigma} = \psi_{\Sigma}^{(12)} + (1 - P_{12})\, \psi_{\Sigma}^{(13)} \ .$$



Conclusion and Outlook

- **For the first time**, we performed Faddeev calculations of Hypertriton including three-body forces.
- The YNN 3 body force had a **non-negligible** effect on the binding energy.

- Since the YN potential of **SMS** is NNLO, it is now possible to include 2π-type three-body forces **consistently**.
- (It is now possible to include three-body forces in the NCSM of medium-mass hypernuclei.)

This work is supported by Japan Society for the Promotion of Science (JSPS) KAKENHI Grants No. JP19K03849 and No. JP22K03597.