Tensor force manifestations in ab inito study of $^{2}H(d,\gamma)^{4}He$, $^{2}H(d,p)^{3}H$ and $^{2}H(d,n)^{3}He$ reactions

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○ *Ref.* *PRL23 (2011) 132502*

Introduction

- **Conventional microscopic cluster model (RGM)**

⇨ Simple cluster wave function
(*S*-wave w.fs. for the α, ^{3}H, ^{3}He, d)
Effective N-N interaction
(central + LS, no tensor, e.g. Minnesota)

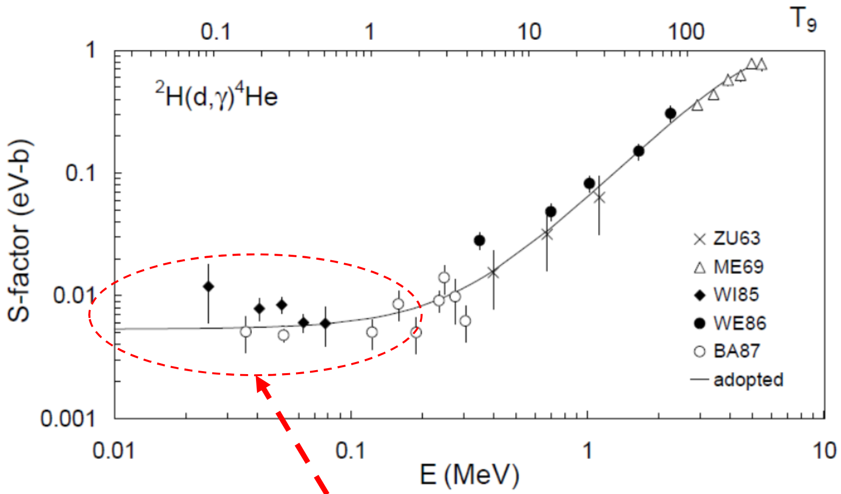
- **Extension of the microscopic cluster model**

⇨ Precise few-body w.fs. for the clusters
Realistic N-N interaction

(e.g. ^{3}He+p, *Arai et al. PRC81(2010)037301*
d+d, *Aoyama et al. FBS in press)*

➡ *Ab initio* calculation

• Role of the tensor force

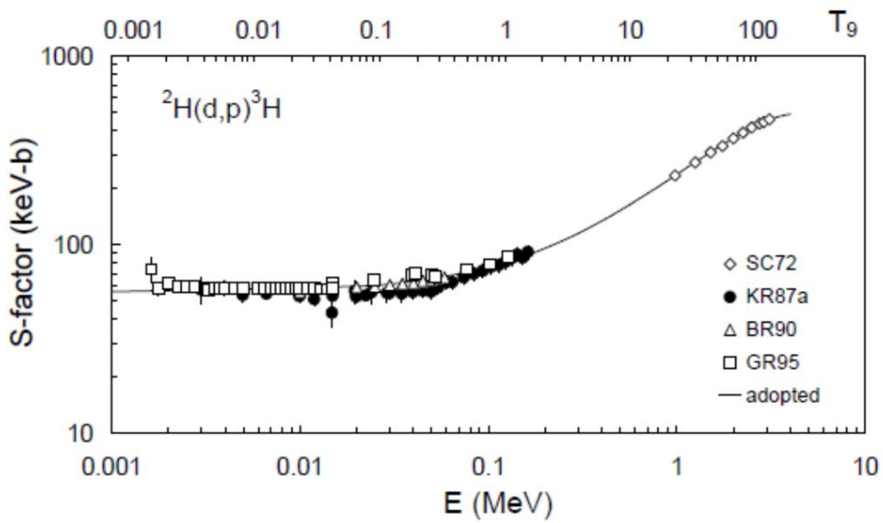


$^2H(d,\gamma)^4He$ Astrophysical S-factor

Nacre compilation (C.Angulo et al. NPA656('99)p.3)

d+d S-wave → D-state in the 0^+ g.s. of ^4He

H. J. Assenbaum and K. Langanke, PRC36('87)p.17



$^2H(d, p)^3H$, $^2H(d, n)3He$

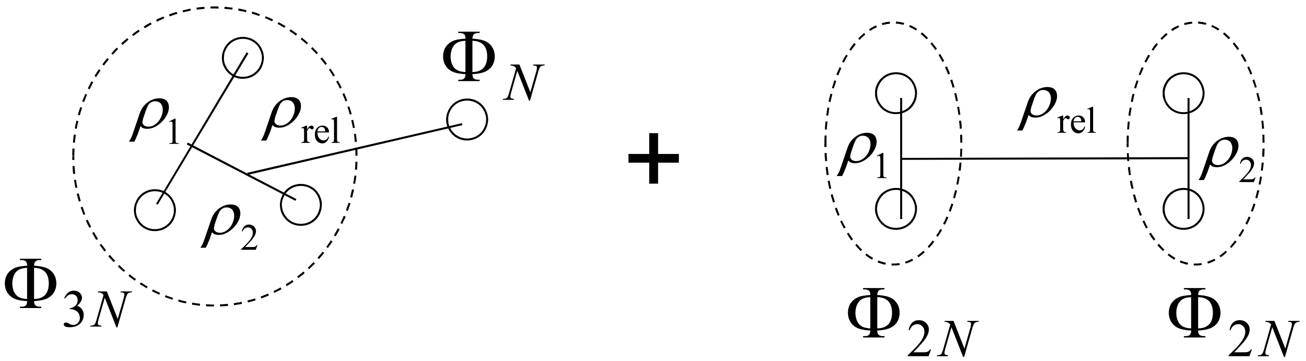
Role of the tensor force?

Microscopic Cluster Model (RGM)

[^{3}H(1/2$^+$)+p] + [^{3}He+n] + [d+d] + [pn(0$^+$) + pn(0$^+$)]
+ [2p(0$^+$) + 2n(0$^+$)] two-cluster model

Total wave function

$$\Psi = A\{\Phi_{3N}(\rho_1,\rho_2)\,\Phi_N\,\chi(\rho_{\rm rel})\} + A\{\Phi_{2N}(\rho_1)\,\Phi_{2N}(\rho_2)\,\chi(\rho_{\rm rel})\}$$



$$\ell_1,\ \ell_2,\ \ell_{\rm rel} \le 2$$

$\Phi_{3N}(\rho_1, \rho_2), \Phi_{2N}(\rho_1)$: Cluster intrinsic wave function

Precise three- and two-body wave function.

$$\Phi_{3N}(\rho_1, \rho_2) = A\{[\phi_{ST}[\chi_{\ell 1}(\rho_1)\chi_{\ell 2}(\rho_2)]_L]_J\}$$

W.f. is expanded by the Gaussian basis function.

Including the higher partial wave up to D-wave.

Basis set is selected by
the Stochastic Variational Method (SVM).

V.I. Kukulin and V. M. Krasnopol'sky, JPG3(1977) 795
K. Varga, Y. Suzuki, R. G. Lovas,NPA571(1994)447

Basis dimension	^3H, ^3He	→	N=30
	^2H	→	N=8

$\chi(\rho_{\mathrm{rel}})$: Cluster relative wave function

<u>Microscopic R-matrix method (Baye, Descouvemont)</u>

a : channel raidus

$$\begin{cases} \rho_{\mathrm{rel}} < a \text{ --- Gaussian expansion} \\ \rho_{\mathrm{rel}} > a \text{ --- Exact Coulomb function} \end{cases}$$

Microscopic *R*-matrix method

(D.Baye, et al. NPA291 ('77)230)

- Schrodinger e.q. $\left(\hat{H}+\hat{L}-E\right)\Psi^{int}=\hat{L}\,\Psi^{ext}$

- Bloch operator $\hat{L}(E)=\left(\hbar^2/2\mu r\right)\delta(r-a)\left[\frac{d}{dr}r-b\right]$

$$\begin{cases} b=0 & \text{for open channel} \\ b=2ka\,W'(2ka)/W(2ka) & \text{for closed channel} \end{cases}$$

- W.F ($r<a$)

Gaussian expansion

$$\Psi^{int}=\sum_{\alpha k} f_{\alpha k} u_{\alpha k} \varphi_{\alpha}$$

$\alpha:(\ell, I)$

$u_{\alpha k}$: Gaussian basis function

φ_{α} : Cluster internal function

$$\Longrightarrow \quad \sum_{\alpha k} f_{\alpha k} \langle u_{\alpha' k'} \varphi_{k'} | \hat{H} + \hat{L} - E | u_{\alpha k} \varphi_k \rangle = \langle u_{\alpha' k'} \varphi_{k'} | \hat{L} | \Psi^{ext} \rangle$$

$$C_{\alpha' k', \alpha k} \equiv \langle u_{\alpha' k'} \varphi_{k'} | \hat{H} + \hat{L} - E | \Psi_{\alpha k} \rangle \qquad W_{\alpha' k'} \equiv \langle u_{\alpha' k'} \varphi_{k'} | \hat{L} | \Psi^{ext} \rangle$$

$$\Longrightarrow \quad \Psi^{int} = \sum_{\alpha k} f_{\alpha k} u_{\alpha k} \varphi_\alpha = \sum_{\alpha k \alpha' k'} C^{-1}_{\alpha k, \alpha' k'} W_{\alpha' k'} u_{\alpha k} \varphi_\alpha$$

$$\Psi^{ext} = \sum_{\alpha_1} r^{-1}_{\alpha_1} \upsilon^{1/2}_{\alpha_1} C_{\alpha_1} \left\{ I_{\alpha_1} \delta_{\alpha_1 \alpha_0} - U_{\alpha_1 \alpha_0} O_{\alpha_1} \right\} \varphi_{\alpha_1} + \sum_{\alpha_2} C_{\alpha_1} W_{-\eta, \ell+1/2}(2kr)/kr \, \varphi_{\alpha_2}$$

R-matrix

$$R_{\alpha\alpha'} = \hbar^2 a/2 \left(\mu_\alpha \mu_{\alpha'}\right)^{-1/2} \left(k_\alpha / k_{\alpha'}\right)^{1/2} \sum_{kk'} u_{\alpha k}(a) C^{-1}_{\alpha k, \alpha' k'} u_{\alpha' k'}(a)$$

S-matrix

$$U = \left(Z^*\right)^{-1} Z \qquad \because \quad \Psi^{ext}(a) = \Psi^{int}(a)$$

$$Z_{\alpha\alpha'} = I_\alpha \delta_{\alpha\alpha'} - R_{\alpha\alpha'} (k_{\alpha'} a) \, I'_{\alpha'} (k_{\alpha'} a)$$

N-N interaction

- Realistic N-N pot. (*Central*+*LS*+*Tensor*)

  AV8'

  G3RS (Tamagaki, PTP39('69)91)

  \+ Phenomenological 3BF (Hiyama et al., PRC70('04))

$$\sum_{i=1}^{2} V_i \, e^{-\alpha_i (r_{12}^2 + r_{23}^2 + r_{31}^2)}$$

- Effective N-N pot. (*Central* + *Coulomb*)

  Minnesota pot. (D. R. Thompson, NPA286('77)p.53)

  → 3-range Gaussian potential which reproduces np triplet and pp single s-wave scattering length and effective range

Threshold

AV8' pot.+3BF

- d : $E = -2.18MeV$, $P_D = 5.9\%$ $E_{exp} = -2.22MeV$
- t : $E = -8.22MeV$, $P_D = 8.4\%$ $E_{exp} = -8.48MeV$
- h : $E = -7.55MeV$, $P_D = 8.3\%$ $E_{exp} = -7.72MeV$
- α : $E = -27.99MeV$, $P_D = 13.8\%$ $E_{exp} = -28.30MeV$

G3RS pot.+3BF

- d : $E = -2.13MeV$, $P_D = 5.0\%$ $E_{exp} = -2.22MeV$
- t : $E = -8.24MeV$, $P_D = 6.9\%$ $E_{exp} = -8.48MeV$
- h : $E = -7.58MeV$, $P_D = 6.9\%$ $E_{exp} = -7.72MeV$
- α : $E = -27.99MeV$, $P_D = 11.2\%$ $E_{exp} = -28.30MeV$

MN pot.

d	: E = -2.10 MeV,	E_{exp}= -2.22MeV
t	: E = -8.38 MeV,	E_{exp}= -8.48MeV
h	: E = -7.70 MeV,	E_{exp}= -7.72MeV
α	: E = -29.94 MeV,	E_{exp}= -28.30MeV

Cross section of the capture reaction

$$\sigma_\gamma^{E\lambda}(E) = \frac{2J_f+1}{(2I_1+1)(2I_2+1)} \frac{8\pi}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right) \frac{(\lambda+1)}{\lambda(2\lambda+1)!!^2}$$

$$\times \sum_{J_i I_i \ell_i} \frac{1}{(2\ell_i+1)} \left| \left\langle \Psi^{J_f \pi_f} \left\| M_\lambda^E \right\| \Psi_{\ell_i I_i}^{J_i \pi_i} \right\rangle \right|^2$$

Present cal.： E2 transition ($2^+ \rightarrow 0^+$ g.s.)

Cross section of the transfer reaction

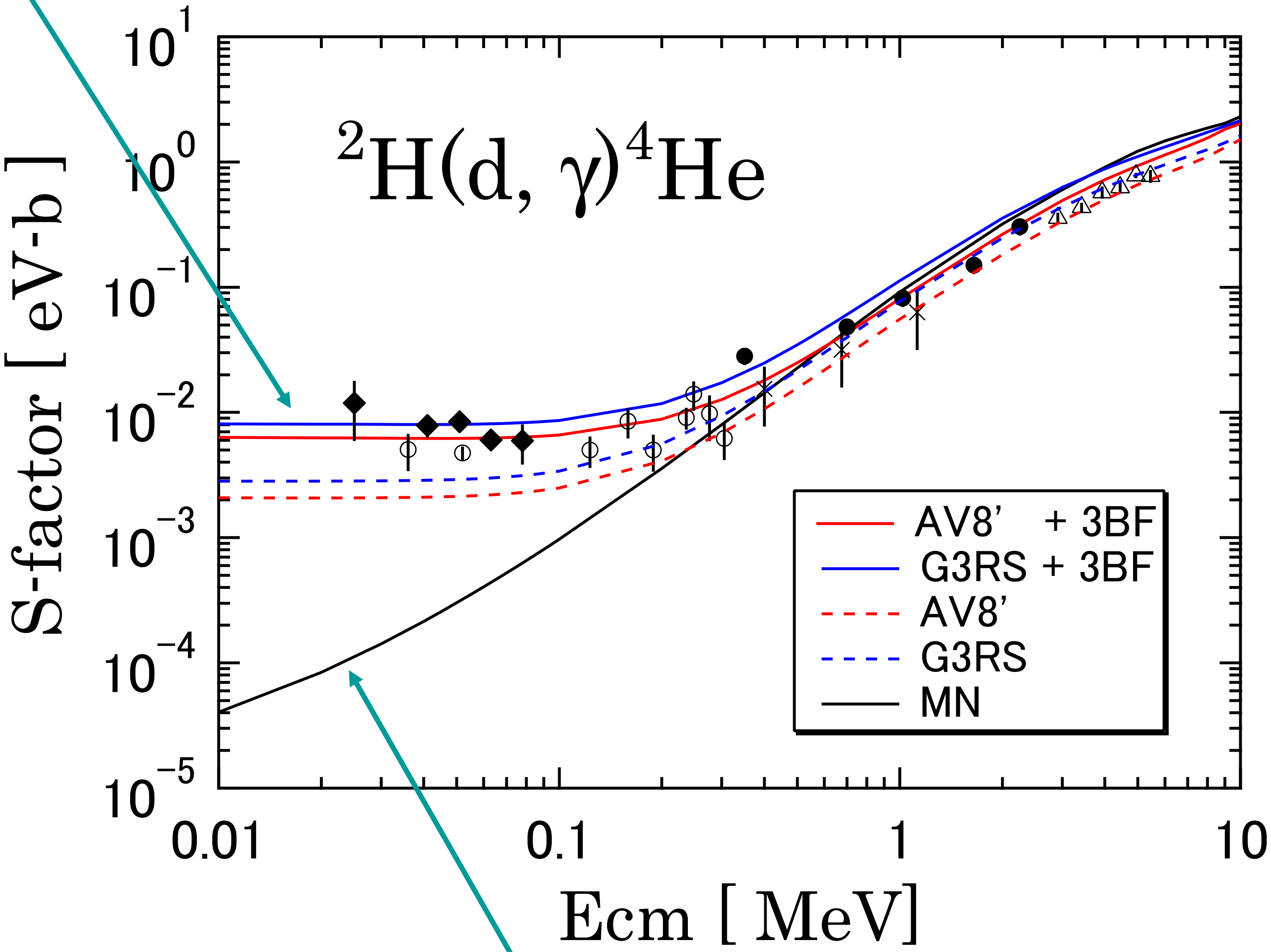
$$\sigma(E) = \frac{\pi}{k^2} \sum_{J\pi} \frac{2J+1}{(2I_1+1)(2I_2+1)} \sum_{\ell_i \ell_f I_i I_f} \left| U^{J\pi}_{i\,\ell_i I_i, f\,\ell_f I_f}(E) \right|^2$$

Presnt cal ： $J^\pi = 0^\pm, 1^\pm, 2^\pm$

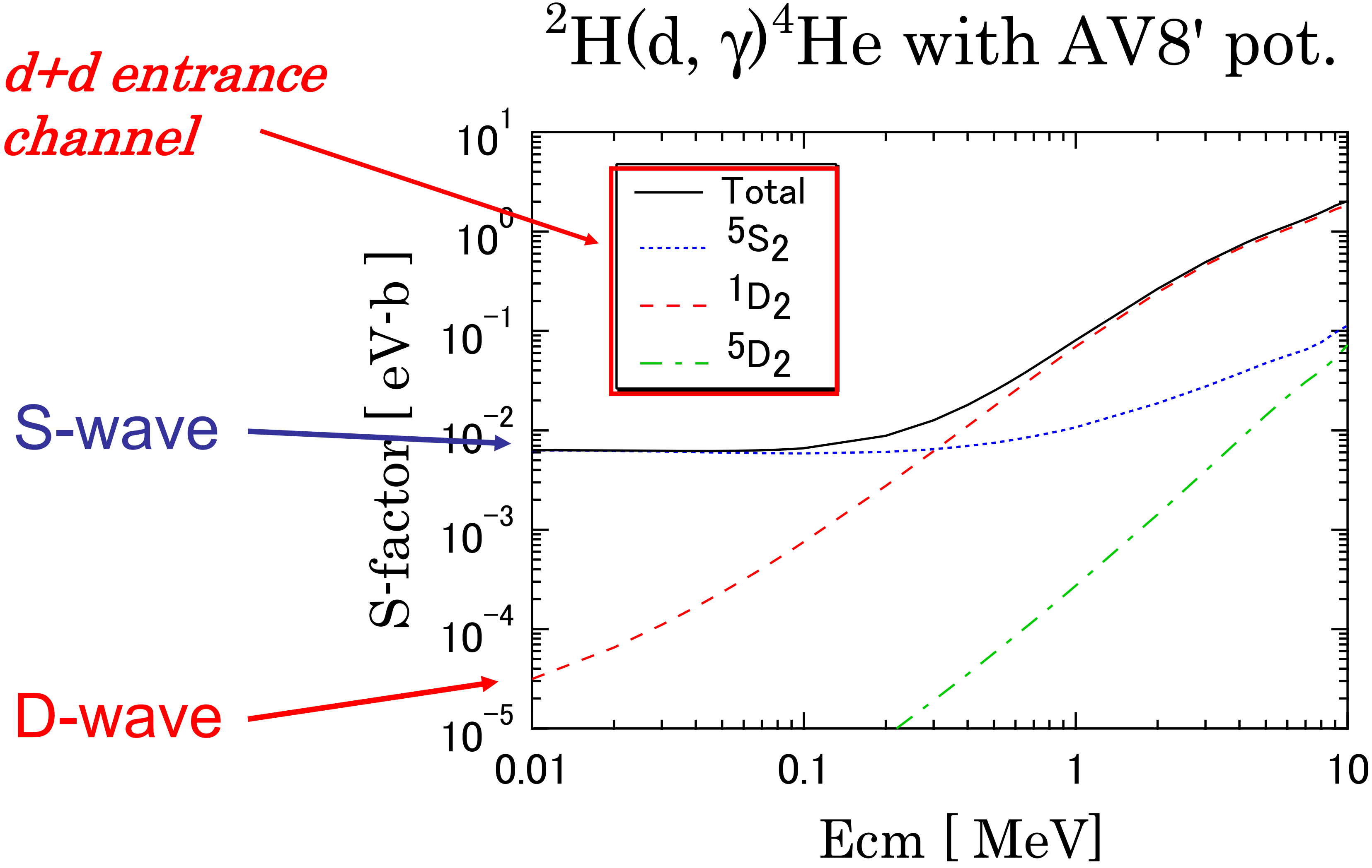
- K.Arai, D.Baye, P.Descouvemont, NPA699(02)p.963

Capture reaction

Realistic force

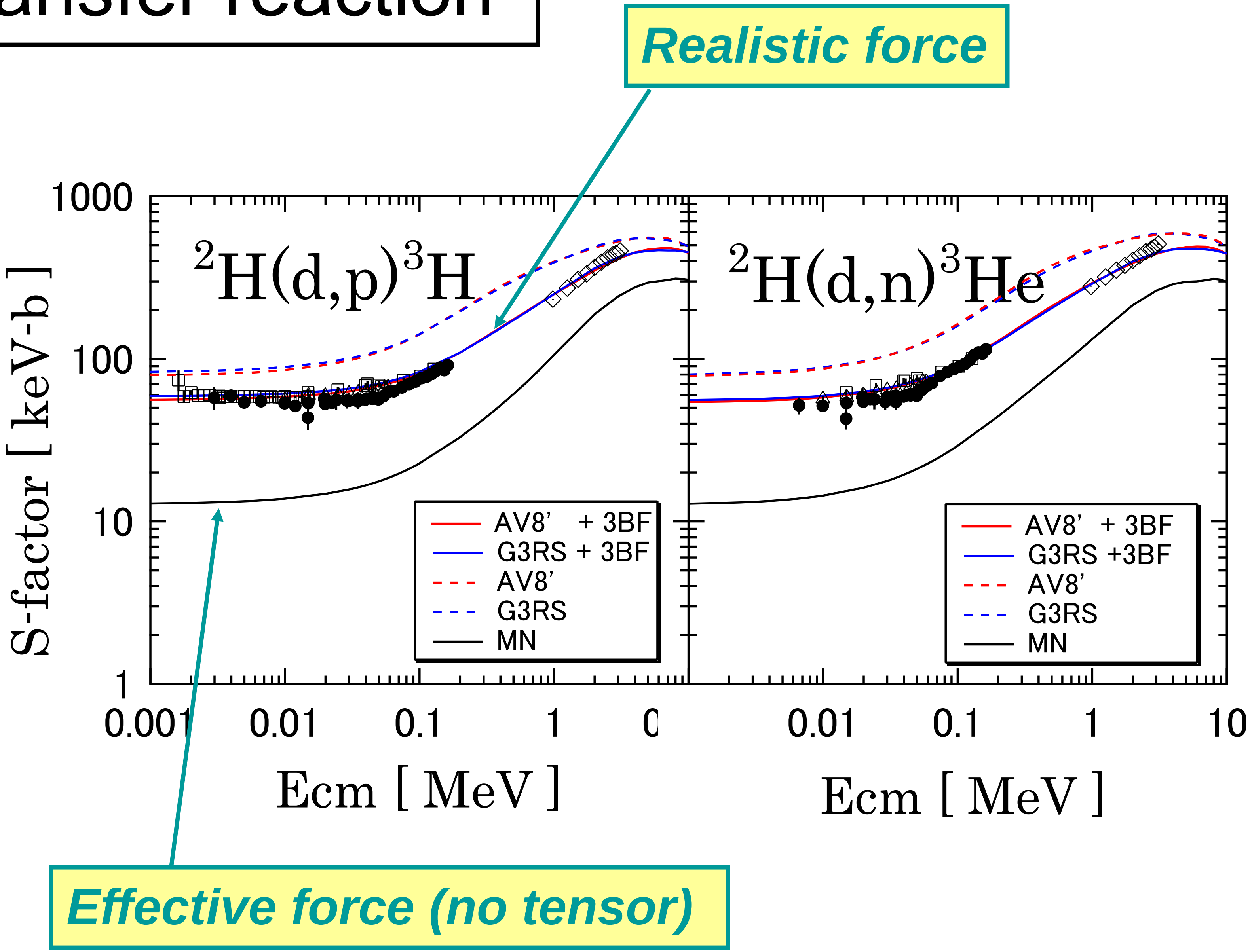


Effective force (no tensor)

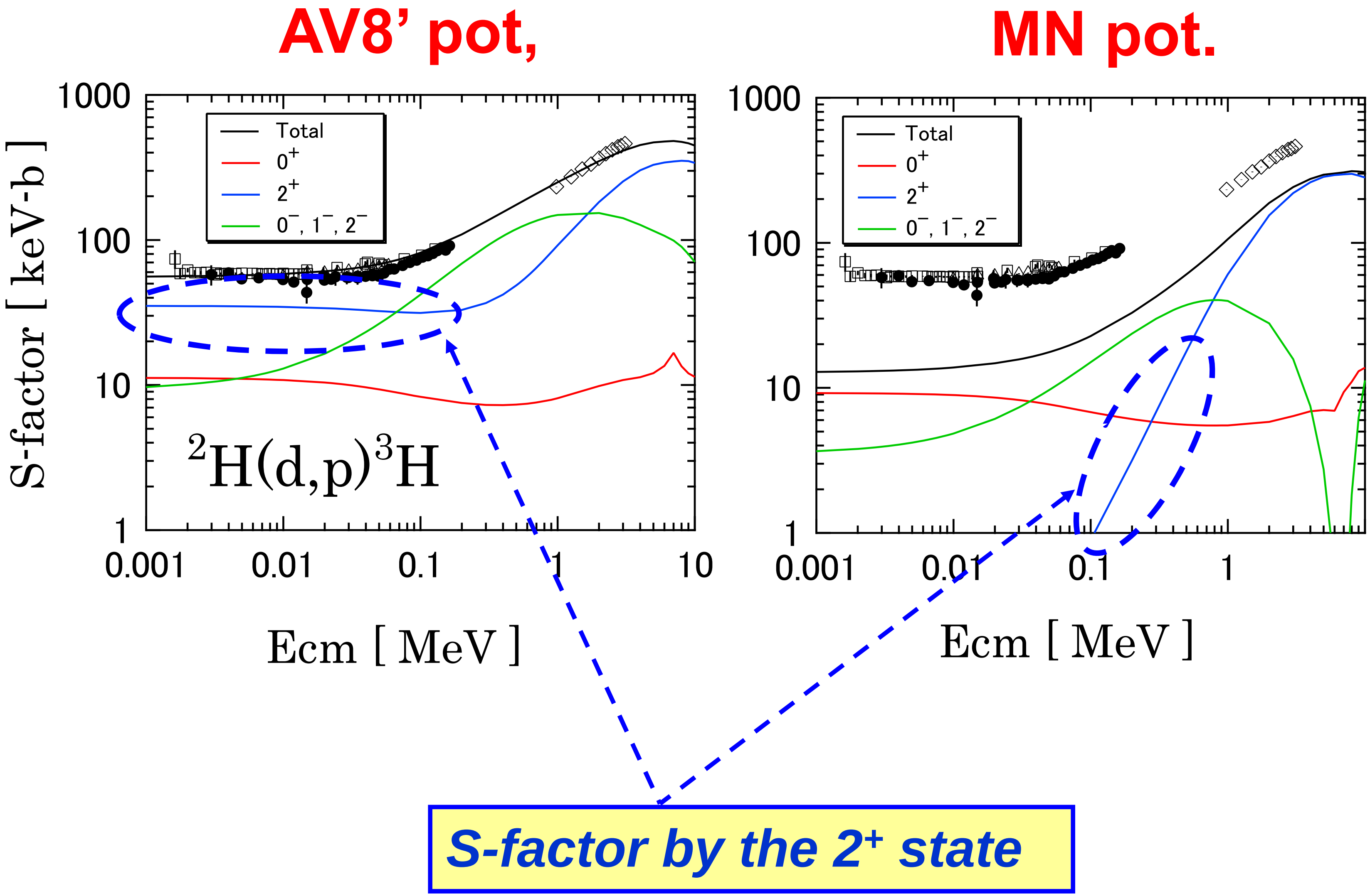


- d+d ***S-wave*** → ^4He 0^+ (***L=2, S=2***) ***D-wave*** component
- d+d D-wave → ^4He 0^+ (L=0, S=0) S-wave component

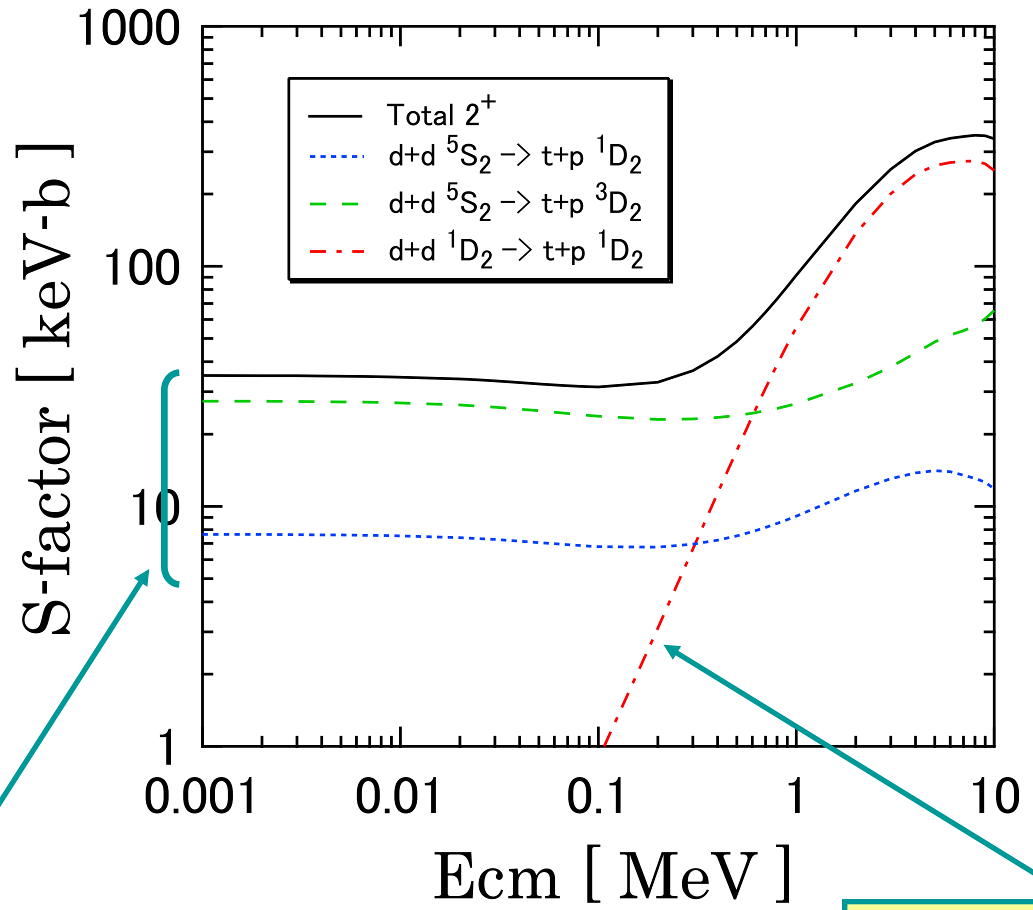
Transfer reaction



Contribution of each spin parity state in $^2H(d,p)^3H$



2^+ contribution in $^2H(d,p)^3H$ is decomposed according to the entrance & exit channel



AV8' pot.

d+d S-wave → t+p D-wave

d+d D-wave → t+p D-wave

Summary

D-wave component

- **$^{2}H(d,\gamma)^{4}He$**

$$\begin{cases} E_{cm} < 0.3\,\mathrm{MeV} & d+d \;\; S\text{-wave} \rightarrow (\mathrm{L},\mathrm{S}) = (2,2) \\ E_{cm} > 0.3\,\mathrm{MeV} & d+d \;\; D\text{-wave} \rightarrow (\mathrm{L},\mathrm{S}) = (0,0) \end{cases}$$

S-wave component

- **$^{2}H(d, p)^{3}H$, $^{2}H(d, n)^{3}He$**

$J^{\pi}=2^{+}$ contribution

Coupled by tensor force

$$\begin{cases} E_{cm} < 0.3\,\mathrm{MeV} & d+d \;\; S\text{-wave} \rightarrow t+p \; D\text{-wave} \\ E_{cm} > 0.3\,\mathrm{MeV} & d+d \;\; D\text{-wave} \rightarrow t+p \; D\text{-wave} \end{cases}$$

- Tensor force plays an essential role to reproduce the astrophysical S-factor not only in the capture reaction, $^{2}H(d,\gamma)^{4}He$, but also in the transfer reaction, $^{2}H(d,p)^{3}H$ and $^{2}H(d,n)^{3}He$.

- ***This effect of the tensor force can be seen only at very low energy !!***