Few-Nucleon Systems Interacting via Non-local Quark-Model Baryon-Baryon Interaction

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1. **Introduction**
2. **Quark-model baryon-baryon interaction fss2**
3. **Three-cluster Faddeev formalism using two-cluster RGM kernels**
4. **3*N* and 4*N* bound-state problems**
5. ***nd* and *pd* elastic scattering**
6. **Breakup cross sections and analyzing powers → skip**
7. **Summary**

Motivation

Comprehensive understanding of few-baryon systems using quark-model baryon-baryon interaction fss2

Analysis of the off-shell effect related to the non-locality of the quark-model baryon-baryon interaction, based on the naive 3-quark structure of baryons

essential difference from the meson-exchange potentials in the description of the short-range repulsion:

… non-local kernel by the quark-exchange *vs.* local hard core

(through the color-magnetic *quark-quark* interaction) (phenomenological)

2-body – 3-body – 4-body …

from bound-states to scattering and breakup reactions

***nd* scattering and *pd* scattering … effect of the Coulomb force**

few-baryon systems including the strangeness: hyper-triton, $\Sigma^{-}d$ scattering

${}^{4}_{\Lambda}$H , ${}^{4}_{\Lambda}$He , ${}^{4}_{\Lambda\Lambda}$H , …

Renormalized RGM : elimination of energy-dependence in RGM kernels

H. Matsumura, M. Orabi, Y. Suzuki, Y. Fujiwara, D. Baye, P. Descouvemont, M. Theeten
3-cluster semi-microscopic calculations using 2-cluster non-local RGM kernels:
Phys. Lett. B659 (2008) 160; Phys. Rev. C76, 054003 (2007)

Three-body systems for composite particles

Three-cluster equation using two-cluster RGM kernel: Y. Fujiwara, H. Nemura, Y. Suzuki, K. Miyagawa, and M. Kohno, Prog. Theor. Phys. 107, 745 (2002); Y. Fujiwara, Y. Suzuki, K. Miyagawa, M. Kohno, and H. Nemura, Prog. Theor. Phys. 107, 993 (2002).

3-types of singularities for scattering calculations

1. **deuteron singularity of 2-body T-matrix → Noyes-Kowalski method**
   t = real at $0 < E_{inc} < |\varepsilon_d|$ — **Prog. Theor. Phys. 124 (2010) 433 ;**
   t = complex at $E_{tot} = E_{inc} + \varepsilon_d > 0$ — **125 (2011)729; 125(2011)957, 979**
2. **moving singularity of 3-body free Green function → spline interpolation and subtraction method (by Bochum-Krakow group)**
3. **Coulomb singularity → extension of the Vincent-Phatak method for the screened Coulomb force**

A practical method to solve cut-off Coulomb problems in the momentum space --- Application to the Lippmann-Schwinger Resonating-Group Method and the *pd* elastic scattering --- : Y. Fujiwara and K. Fukukawa, Prog. Theor. Phys. 128 (2012) 301

B_8B_8 interactions by fss2

Y. F., C. Nakamaoto, Y. Suzuki, M. Kohno
PRC64 (2001) 054001
PRC65 (2002) 014001

A natural and accurate description of *NN*, *YN*, *YY* interactions in terms of (*3q*)-(*3q*) RGM

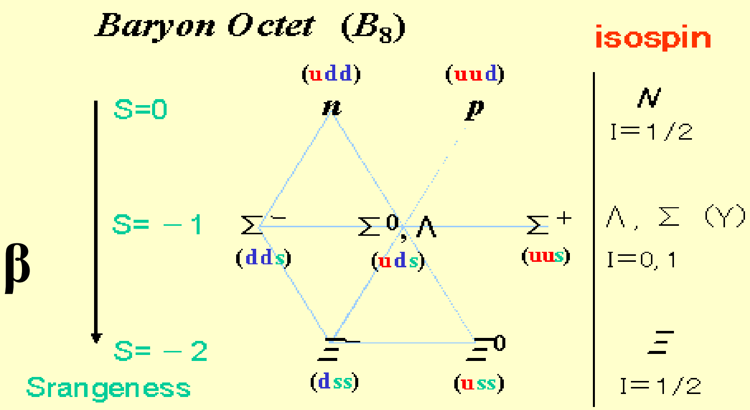
Number of parameters less than 20

- Short-range repulsion and *LS* by quarks
- Medium-attraction and long-rang tensor by S, PS and V meson exchange potentials (fss2) (*Cf*. FSS without V)

Y. F. ,C. Nakamoto, Y. Suzuki, PRC54 (1996) 2180

Model Hamiltonian

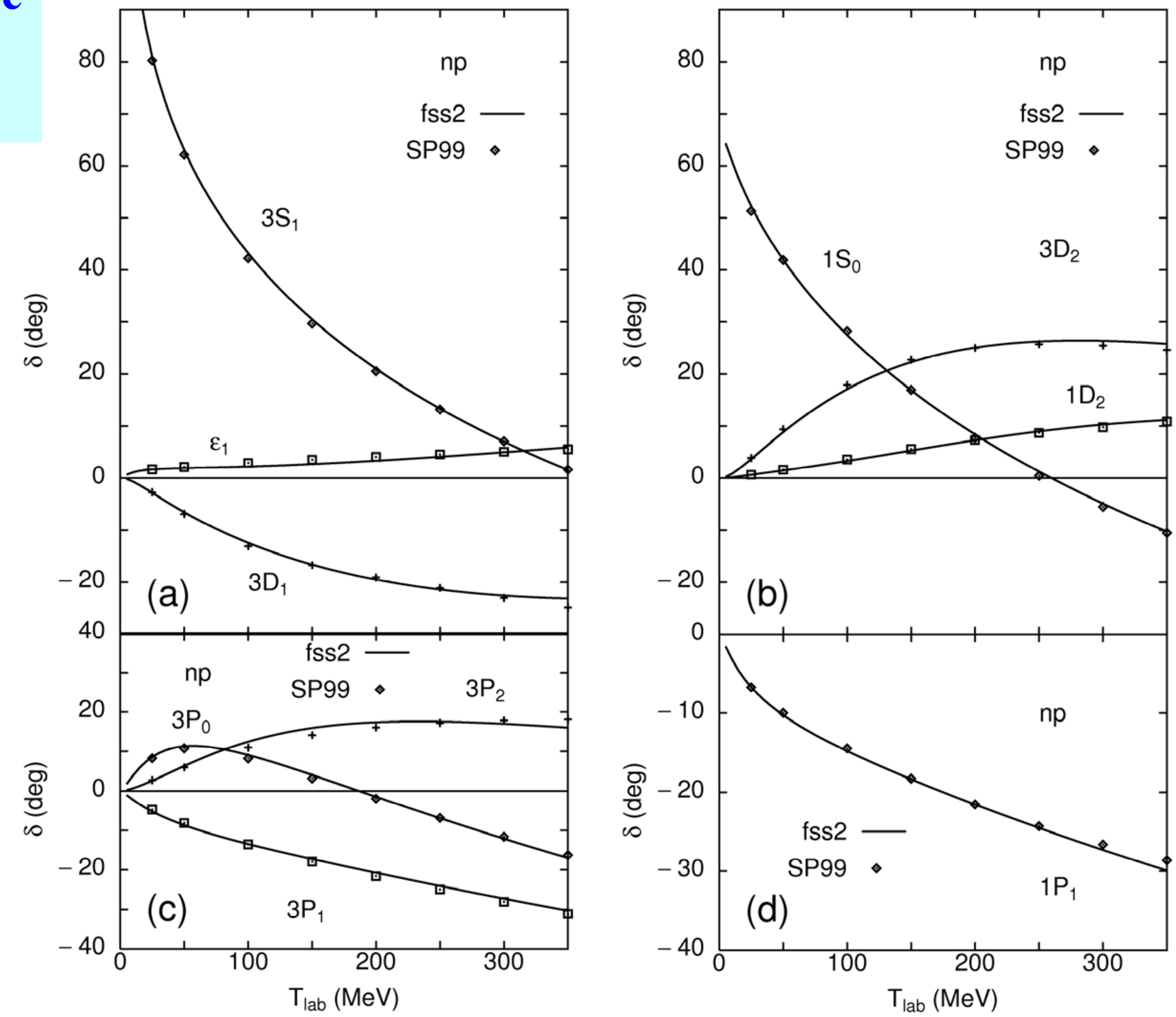
$$H = \sum_{i=1}^{6} (m_i + p_i^2/2m_i) + \sum_{i<j}^{6} (U_{ij}^{\mathrm{Conf}} + U_{ij}^{\mathrm{FB}} + \sum_\beta U_{ij}^{\mathrm{S}\beta} + \sum_\beta U_{ij}^{\mathrm{PS}\beta} + \sum_\beta U_{ij}^{\mathrm{V}\beta})$$



$$\langle \phi(3q)\phi(3q) | E - H | \mathcal{A} \{\phi(3q)\phi(3q)\chi(\mathbf{r})\} \rangle = 0$$

QMPACK homepage http://qmpack.homelinux.com/~qmpack/php

NN phase shifts by fss2



I ≤ 2

Three-cluster Faddeev formalism using the two-cluster RGM kernels

Removal of the energy dependence by the renormalized RGM
Matsumura, Orabi, Suzuki, Fujiwara, Baye, Descouvemont, Theeten
3-cluster semi-microscopic calculations using 2-cluster non-local RGM kernels:
Phys. Lett. B659 (2008) 160; Phys. Rev. C76, 054003 (2007)

$N = 1 - K$

$[\varepsilon - H_0 - v_{\mathrm{RGM}}(\varepsilon)]\, \chi = 0$ **with** $v_{\mathrm{RGM}}(\varepsilon) = V_{\mathrm{D}} + G + \varepsilon K$ ***εK method***

$\Downarrow$ $(\varepsilon = E - E_{\mathrm{int}})$

$\Lambda[\varepsilon - H_0 - v_{\mathrm{RGM}}]\, \Lambda \psi = 0$ **with** $v_{\mathrm{RGM}} = V_{\mathrm{D}} + G + W$

$$W = \Lambda\left[\frac{1}{\sqrt{N}}(H_0 + V_{\mathrm{D}} + G)\frac{1}{\sqrt{N}} - (H_0 + V_{\mathrm{D}} + G)\right]\Lambda$$

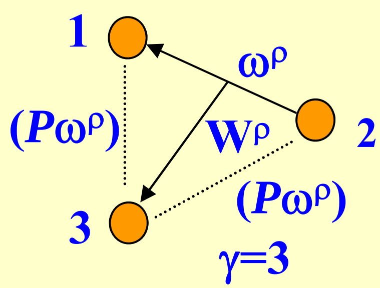
non-local kernel

Properties of RGM kernels

1) non-locality
2) energy-dependence → eliminated
3) Pauli-forbidden states in $\Lambda N - \Sigma N$ (I=1/2), $\Lambda\Lambda - \Xi N - \Sigma\Sigma$ (I=0), $\Xi\Lambda - \Xi\Sigma$ (I=1/2) $^1S_0$: i.e. SU_3 $(11)_s$ → RGM T-matrix

Prog. Theor. Phys. 107 (2002) 745; 993

2010.5.20 SOTANCP2



Coulomb problem for 3-body *pd* scattering

Step 1. 2-body *t*-matrix (sharp cut-off at quark level)

$$t^\rho = (v_{\mathrm{RGM}} + \omega^\rho) + (v_{\mathrm{RGM}} + \omega^\rho) G_0 t^\rho$$

ω^ρ: error function Coulomb

isospin formalism → only for I=1 pair with factor 2/3

Step 2. AGS (Alt-Grassberger-Sandhas) equation

$$U^\rho |\phi\rangle = G_0^{-1} P |\phi\rangle + P t^\rho G_0 U^\rho |\phi\rangle$$

with $|\phi\rangle = |\boldsymbol{q}_{0,}\psi_d\rangle$ and $P = P_{(123)} + P_{(123)}{}^2$

Step 3. 2-potential formula for AGS equations

$$U^\rho = T_\omega^\rho + (1 + T_\omega^\rho G_0) \tilde{U}_\omega^\rho (1 + G_0 T_\omega^\rho)$$

$$\langle\phi | U^\rho | \phi\rangle = \langle\phi | T_\omega^\rho | \phi\rangle + \langle\psi^{\rho(-)} | \tilde{U}_\omega^\rho | \psi^{\rho(+)}\rangle$$

$\rho \to \infty$ ↓ (Solutions of the Coulomb-modified AGS equation)

$$\langle\phi | U | \phi\rangle \equiv \langle\phi | T_C | \phi\rangle + e^{i\zeta^\rho} [\langle\phi | U^\rho | \phi\rangle - \langle\phi | T_\omega^\rho | \phi\rangle] e^{i\zeta^\rho}$$

phase factor $\zeta^\rho \to \sigma_\ell - \delta_\ell^\rho$

ρ = 8 fm (elastic) , 16 – 20 fm (breakup)

our method： **C.M. Vincent and S.C. Phatak, Phys. Rev. C10, 391 (1974）**

Connection condition for the *K*-matrix (–*k* cot *δ*) from Coulomb distorted total wave function (with finite *ρ*)

$$\tilde{K}_\ell^\rho \{W[\tilde{F}_\ell^\rho, u_\ell]_{R_{out}} K_\ell^\rho - kW[\tilde{F}_\ell^\rho, \upsilon_\ell]_{R_{out}}\}$$
$$= k\{W[\tilde{G}_\ell^\rho, u_\ell]_{R_{out}} K_\ell^\rho - kW[\tilde{G}_\ell^\rho, \upsilon_\ell]_{R_{out}}\}$$

K_ℓ^ρ by solving AGS euation for $\upsilon + \omega_C^\rho$

$$\lim_{\rho\to\infty} \tilde{K}_\ell^\rho = K_\ell^N = -k \cot \delta_\ell^N$$

Step 4. elastic differential cross sections

$$\frac{d\sigma}{d\Omega} = |\langle \phi | U | \phi \rangle|^2$$

Step 5. breakup cross sections

$$\langle \boldsymbol{p}, \boldsymbol{q} | \Sigma_\gamma (v_{RGM} + \omega^\rho)_\gamma | \Psi^{\rho(+)} \rangle = \langle \boldsymbol{p}, \boldsymbol{q} | (1+P) t^\rho G_0 U^\rho | \phi \rangle = \langle \boldsymbol{p}, \boldsymbol{q} | U_0^\rho | \phi \rangle$$

$$\langle \boldsymbol{p}, \boldsymbol{q} | U_0 | \phi \rangle = \lim_{\rho\to\infty} e^{iz_\rho(p)} \langle \boldsymbol{q}, \psi_{\boldsymbol{p}}^{\rho(-)} | \tilde{U}_0^\rho | \psi^{\rho(+)} \rangle e^{i\zeta_\rho(q_0)} = \langle \boldsymbol{q}, \psi_{\boldsymbol{p}}^{(-)} | \tilde{U}_0 | \psi^{(+)} \rangle$$

common phase factors

$$\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS} = w \left| \sum_{\alpha=1}^{3} \langle \boldsymbol{p}_\alpha, \boldsymbol{q}_\alpha | t^\rho G_0 U^\rho | \phi \rangle \right|^2$$

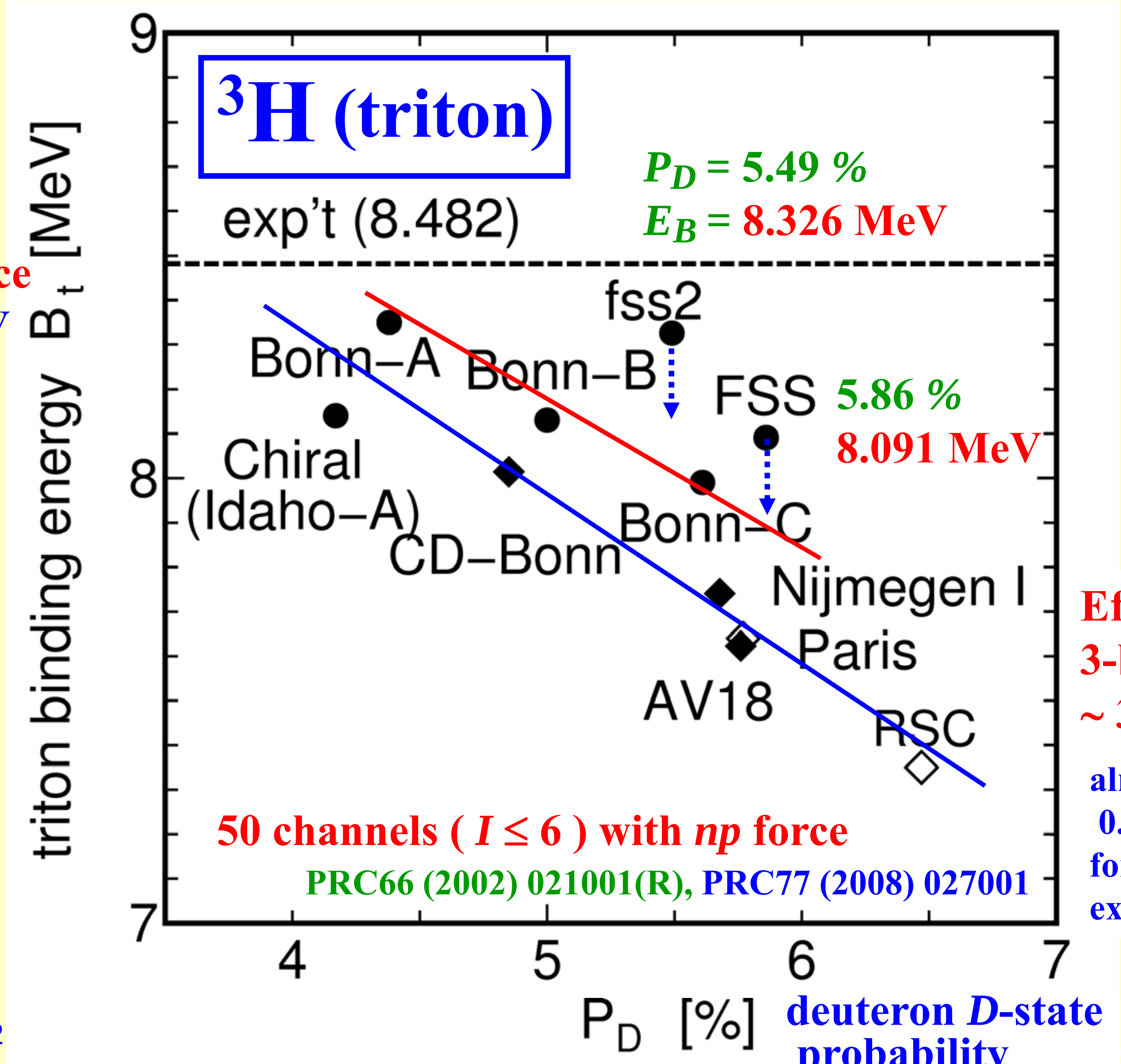
(*ρ* : should be large enough)

***pp* and *np* half off-shell *t*-matrices**

phase space factor

$$E = E_{inc} + \varepsilon_d = \frac{\hbar^2}{M}(\boldsymbol{p}_\alpha{}^2 + \frac{3}{4}\boldsymbol{q}_\alpha{}^2) \text{ with } E_{inc} = \frac{3\hbar^2}{4M}\boldsymbol{q}_0{}^2$$

effect of
charge
dependence
~190 keV



deuteron *D*-state probability

Effect of
3-body force
~ 350 keV ?

almost half of
0.5 – 1 MeV
for the meson
exchange models

SHOA: Suzuki-Horiuchi-Orabi-Arai

Deuteron properties

	AV18	AV8'	SHOA	fss2
ε_d (MeV)	2.2246	2.244	2.242	2.225
P_D (%)	5.76	5.78	5.77	5.49
rms (fm)	1.967	1.961	1.961	1.960
Q_d (fm2)	0.270	0.269		0.270
η	0.0250	0.0252		0.0253
μ_d (μ_0)	0.847	0.847		0.849
KE (MeV)	19.814	19.891	19.881	

Benchmark calculations for AV8' by H. Kamada et. al., Phys. Rev. C64, 044001 (2001)

no CIB, no Coulomb

^{3}H	AV8'	rms (fm)		fss2	rms (fm)	
I_{max}	E (MeV)	^{3}H	^{3}He	E (MeV)	^{3}H	^{3}He
$S+D$	–7.442	1.84	2.02	–8.261	1.76	1.92
2	–7.610	1.81	2.01	–8.228	1.75	1.93
4	–7.754	1.80	1.99	–8.322	1.75	1.92
5	–7.763	1.80	1.99	–8.326	1.75	1.92
6	–7.765	1.80	1.99	–8.326	1.75	1.92
SVM	–7.76 –7.767	1.75 (m)	1.75 (m)	exp.	1.755±0.086	1.959±0.030 1.9642±0.011

Faddeev-Yakubovsky equation for 4 identical Fermions

$$\psi = G_0 t P[(1-P_{(34)})\psi + \varphi]$$

$$\varphi = G_0 t \tilde{P}[(1-P_{(34)})\psi + \varphi]$$

with $\quad t = V^{RGM} + V^{RGM} G_0 t,$

$$P = P_{(12)}P_{(23)} + P_{(13)}P_{(23)}, \quad \tilde{P} = P_{(13)}P_{(24)}$$

Total wave function

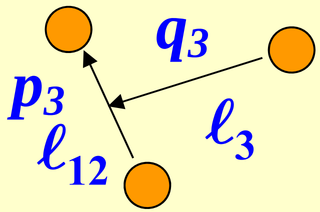
$$\Psi = (1+P)\{[1-P_{(34)}(1+P)]\psi + (1+\tilde{P})\varphi\}$$

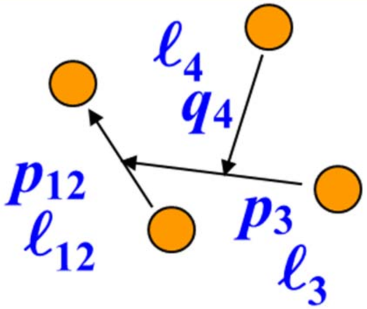
(3 body case)

$$\psi = G_0 t P \psi$$

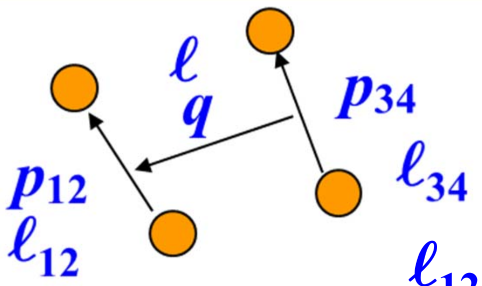
Total wave function

$$\Psi = (1+P)\psi$$





Y-type



H-type

$(\ell_{12} s_{12}) I_{12} \le I_{max} = 6$

$\ell_{12}+\ell_3+\ell_4, \ \ell_{12}+\ell_{34}+\ell \le (\ell^{sum})_{max}$

by A. Nogga

6	6	6	3 points

0   1.2 fm^{-1} 3 fm^{-1} 16 fm^{-1}

AV8'

$\ell_{\rm sum}^{\rm max}$	n_1-n_2-n_3=6-6-3			n_1-n_2-n_3=10-10-5		
	$-E_B(\alpha)$ (MeV)	K.E. (MeV)	R_α (fm)	$E_B(\alpha)$ (MeV)	K.E. (MeV)	R_α (fm)
2	−21.53	83.24	1.605	−21.46	83.10	1.607
4	−24.94	97.52	1.509	−24.88	97.32	1.512
6	−25.60	101.13	1.490			
8	−25.97	102.61	1.482			(1.485)
10	−25.99			(−25.93)		
SVM				−25.92	102.35	1.486

fss2

$\ell_{\rm sum}^{\rm max}$	n_1-n_2-n_3=6-6-3			n_1-n_2-n_3=10-10-5		
	$-E_B(\alpha)$ (MeV)	K.E. (MeV)	R_α (fm)	$E_B(\alpha)$ (MeV)	K.E. (MeV)	R_α (fm)
2	−24.76	76.36	1.496	−24.73	76.29	1.498
4	−27.35	85.67	1.439	−27.32	85.46	1.443
6	−27.78	87.90	1.428	−27.76		
8	−27.95	88.44	1.425			(1.429)
10	−27.96			(−27.93)		

Comparison of the 3*N* and 4*N* bound-state energies with other calculations

A. Nogga, H. Kamada and W. Glöckle, Phys. Rev. Lett. 85, 944 (2000)

Potential	P_d (%)	^{3}H (MeV)	^{4}He (MeV)
fss2	**5.490**	**–8.326**	**–27.9**
CD-Bonn	4.833	–8.012	–26.26
AV18	5.760	–7.623	–24.28
Nijm I	5.678	–7.736	–24.98
Nijm II	5.652	–7.654	–24.56
Exp.		**–8.482**	**–28.30**

fss2: neglects the charge dependence and Coulomb

Average –24.77 MeV
3.5 MeV missing

Effect of charge dependence of the *NN* force : ~ 190 × 2 ~ 400 keV

Ours: – 27.5 MeV + 0.8 MeV (Coulomb) = – 26.7 MeV
1.6 MeV missing → almost half of above

Characteristics of the *nd* and *pd* scattering systems

- **Channel-spin formalism with $S_c=(I_d=1)\times(s_N=1/2)=3/2+1/2$ is convenient for elastic scattering.**
  $S_c=3/2$ "Pauli principle" $\rightarrow$ weak distortion effect
  $S_c=1/2$ strong distortion effect of the deuteron
- **Breakup process is important ($\varepsilon_d=2.224$ MeV), since it influences the elastic scattering. *NN* singularity and the moving singularity should be properly treated at $E_n>3$ MeV.**
- **Enough partial waves $(I_{max}=4)$ should be included even for $E_n<10$ MeV, since the deuteron is widely spread.**
  If Coulomb is added, $I_{max}=4$ is still not sufficiently large.

Many challenging problems still remain

3-body force, spin-doublet scattering length ${}^2a$, A_y-puzzle, Coulomb effect, relativistic effect, breakup cross sections, …

nd and *pd* eigenphase shifts at $E_N = 3$ MeV

	nd		*pd* (nuclear)		
model	fss2	AV18 (+UR3N)	fss2	AV18 (+UR3N)	PSA
$^2S_{1/2}$	149.2	144.7 (149.2)	152.8	147.8 (152.2)	155.15 ±0.23
$^4S_{3/2}$	–69.6	–69.9 (–69.7)	–62.5	–63.1 (–63.1)	–63.80 ±0.11

- The cutoff Coulomb radius is $\rho = 9$ fm. (in degree)
- AV18 (+UR3N), PSA : A. Kievsky, S. Rosati, W. Tornow and M. Viviani, Nucl. Phys. A607 (1996) 402

fss2 predictions are similar not to AV18, but to AV18+UR3N

nd scattering length: ^2a and ^4a

TM99: Tucson-Melbourne 3-body force

	$E_B(^3H)$ (MeV)		$^2a_{nd}$ (fm)		$^4a_{nd}$ (fm)
model	*NN*	*NN*+TM99	*NN*	*NN*+TM99	*NN* (+TM99)
fss2	8.311	–	0.66	–	6.30
CD-Bonn 2000	8.005	8.482	0.925	0.569	6.347
AV18	7.628	8.482	1.248	0.587	6.346
Nijm I	7.742	8.485	1.158	0.594	6.342
exp.	8.482		0.65 ± 0.04		6.35 ± 0.02

Other calculations by H. Witala et al., Phys. Rev. C68, 034002 (2003) , with $I_{max} = 5$

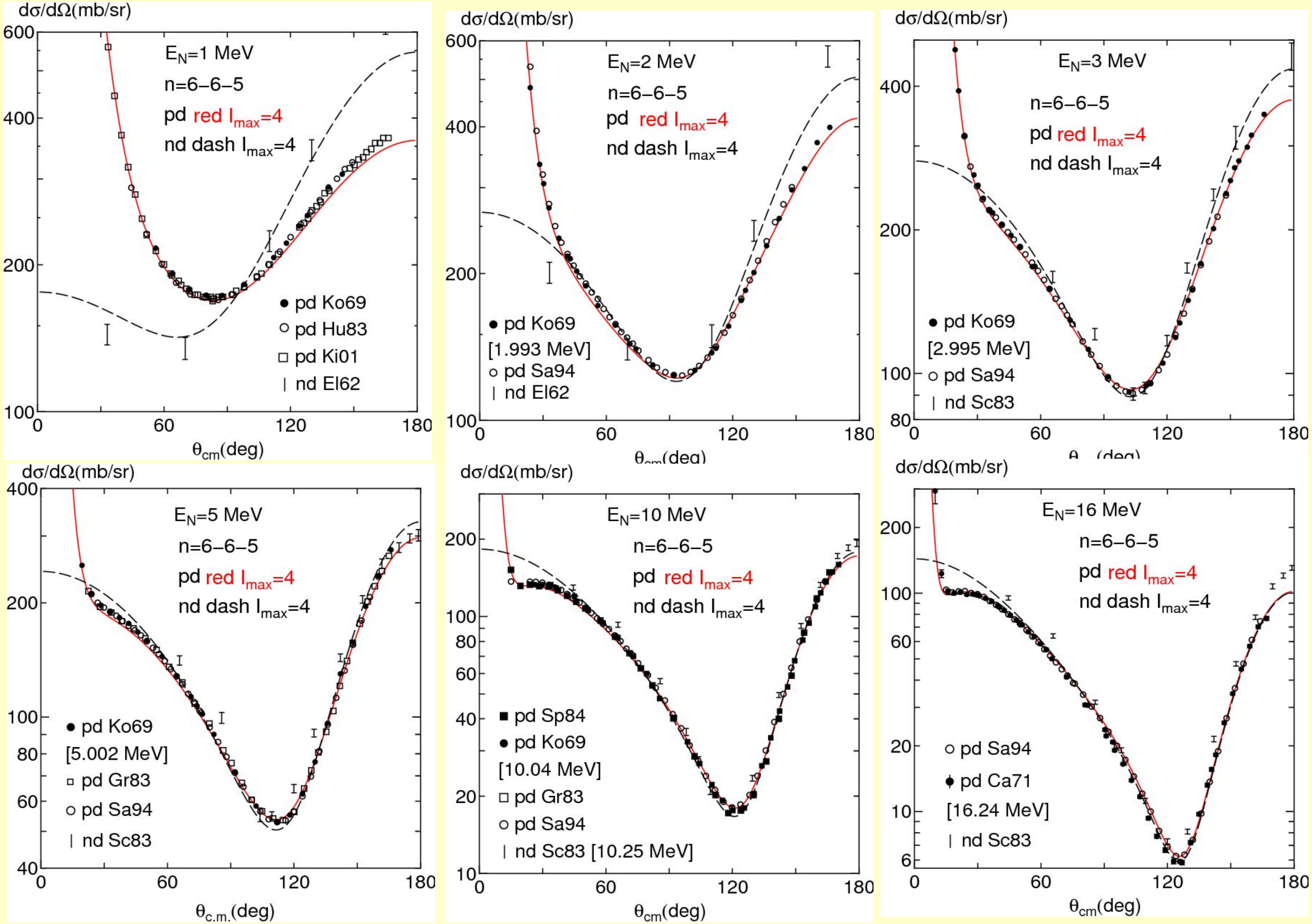
The charge dependence of the *NN* force is neglected in fss2.

Non-local Gaussian potentials are practically used with $I_{max} = 4$ (up to *G*-wave) .

Experimental data are almost reproduced without the three-body force.

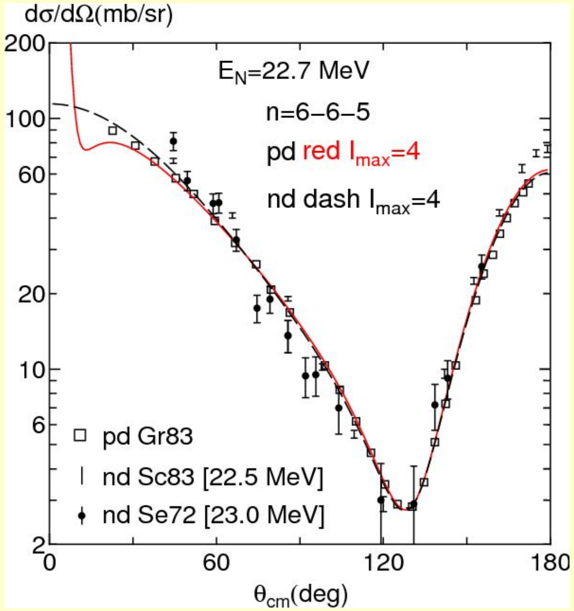
Differential cross sections (1 – 16 MeV)

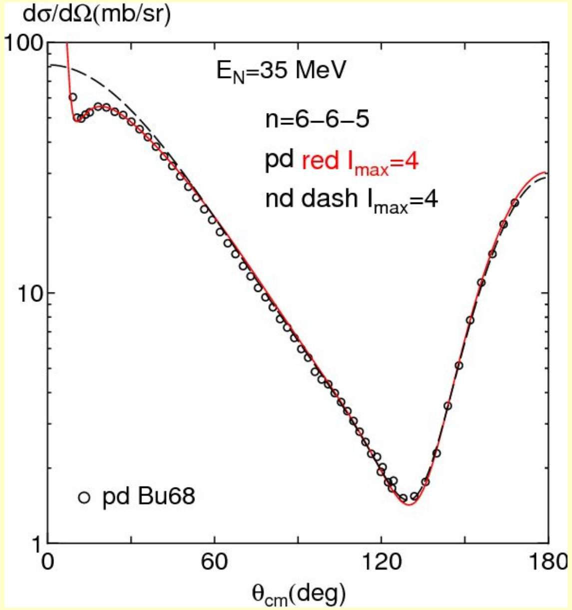
bars: *nd* **dots or circles: *pd***
dash: no Coul. **red: with Coul.**

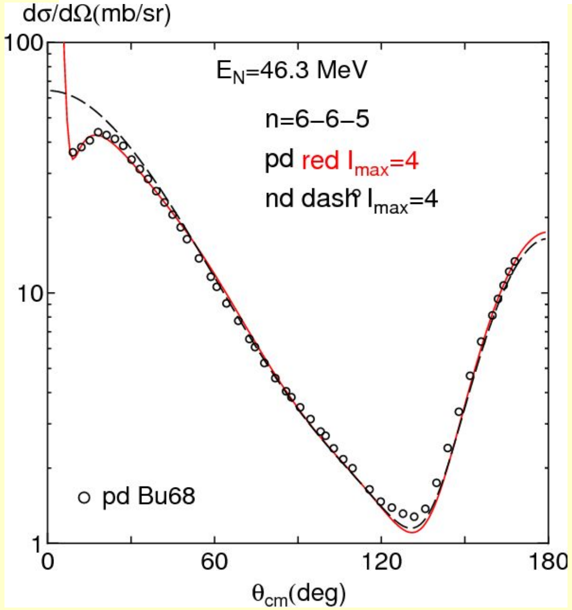


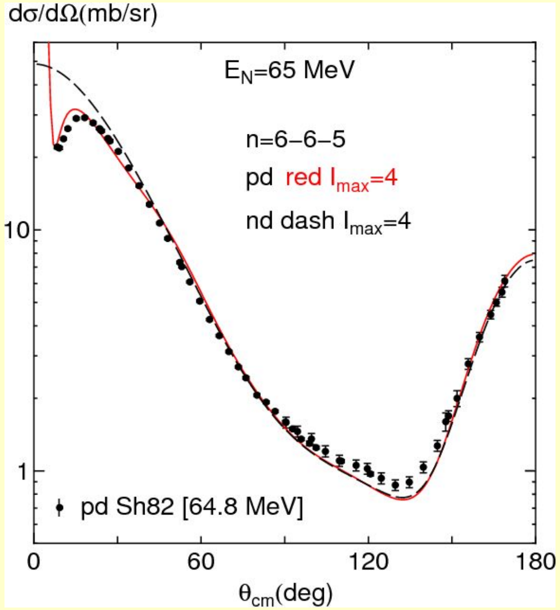
Differential cross sections (22.7 – 95 MeV)

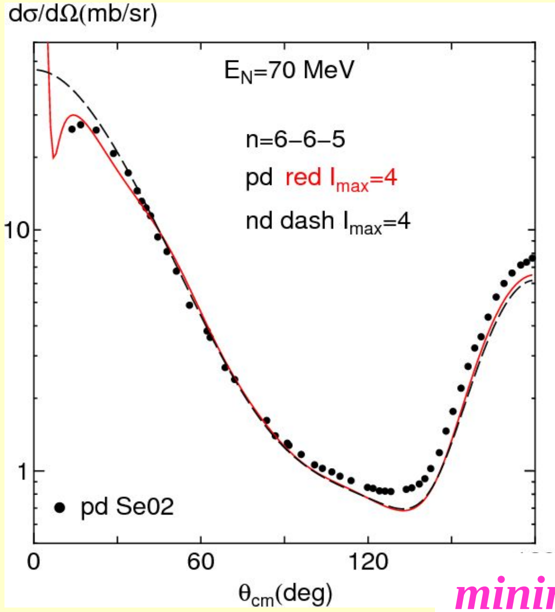
bars: *nd*   dots or circles: *pd*
dash: no Coul. red: with Coul.

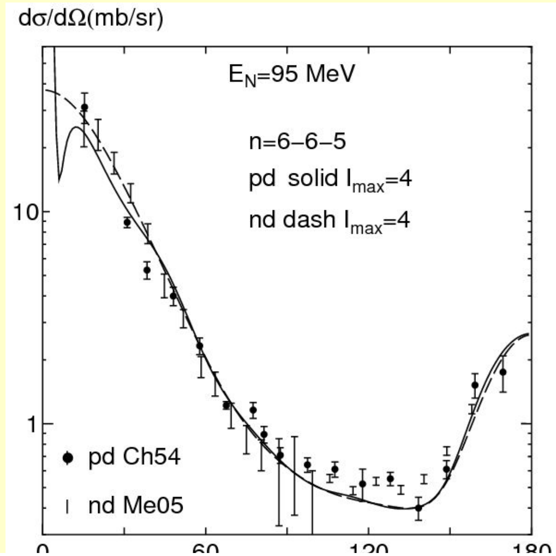








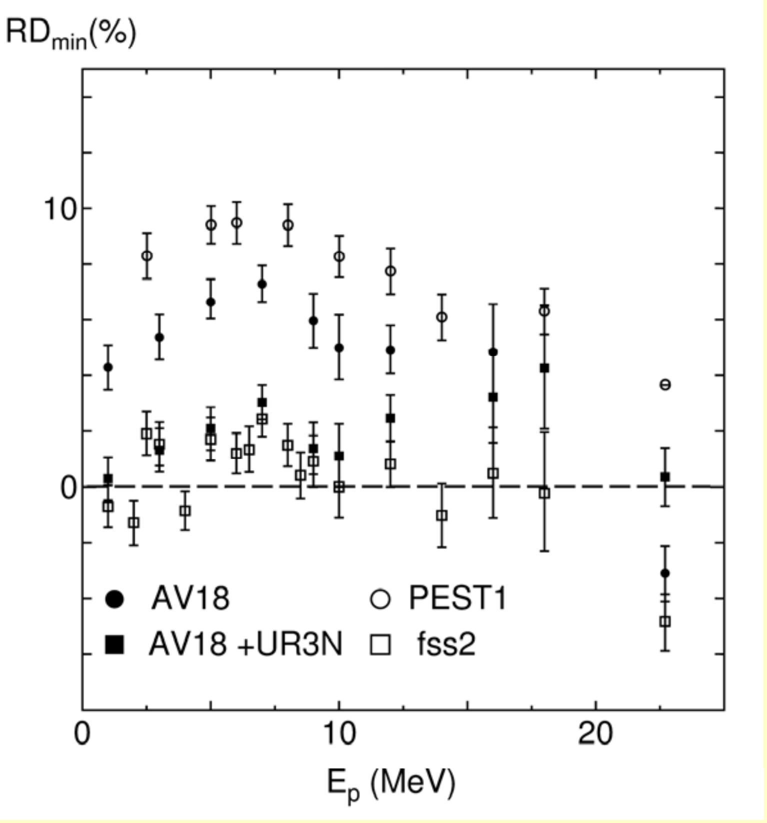


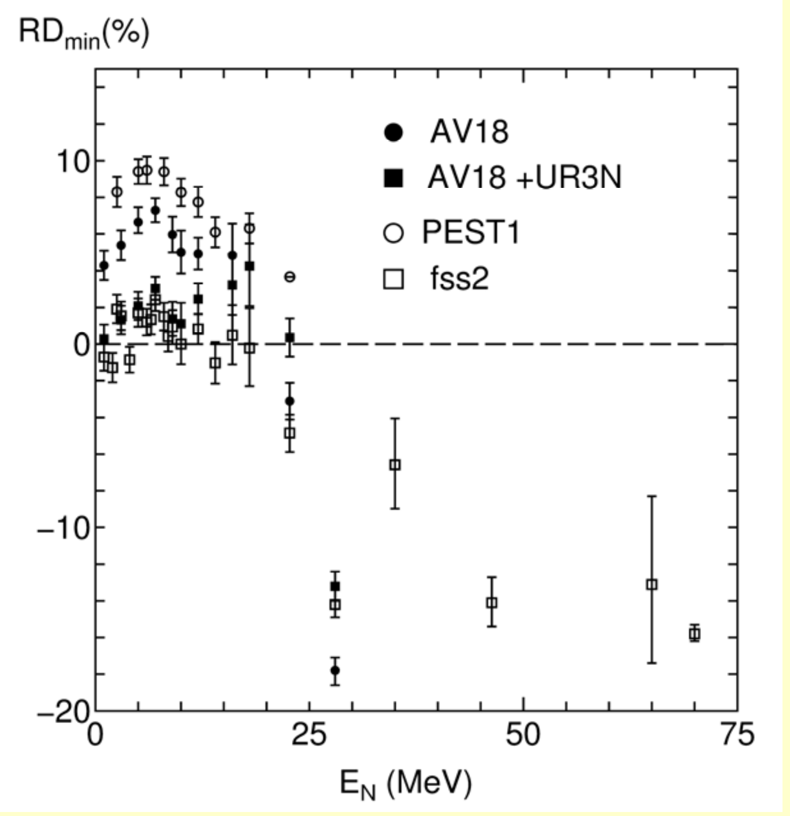


minimum points are below the data

Deviation of the diffraction minima from experiment measured by

$$RD_{\min} = [(d\sigma / d\Omega)_{\min}^{cal} - (d\sigma / d\Omega)_{\min}^{\exp}]/(d\sigma / d\Omega)_{\min}^{\exp}$$

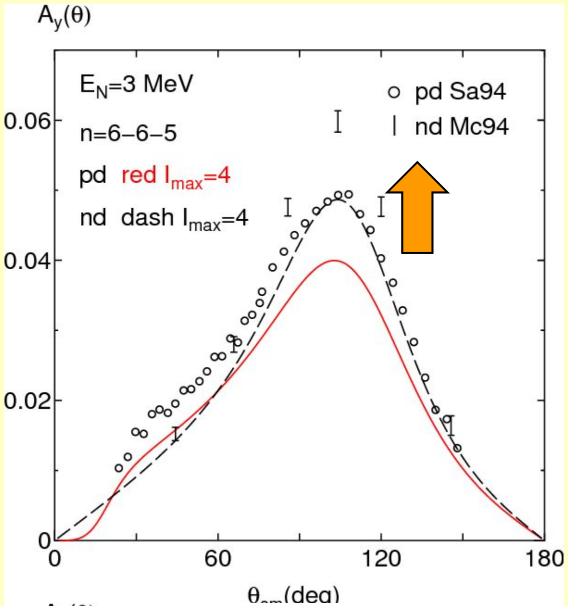


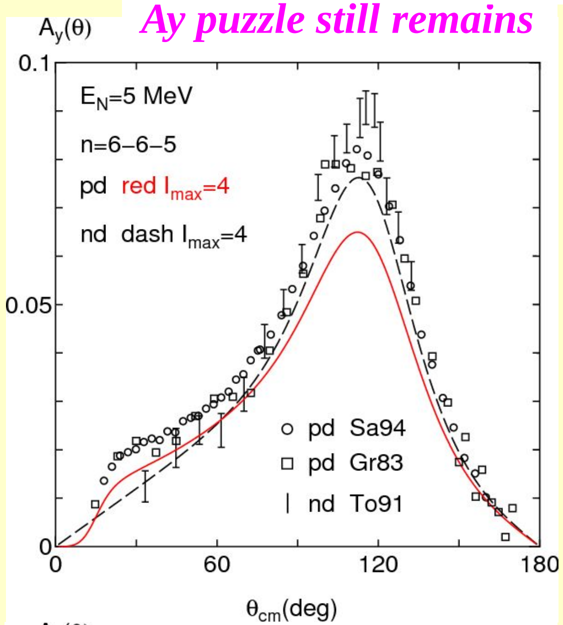


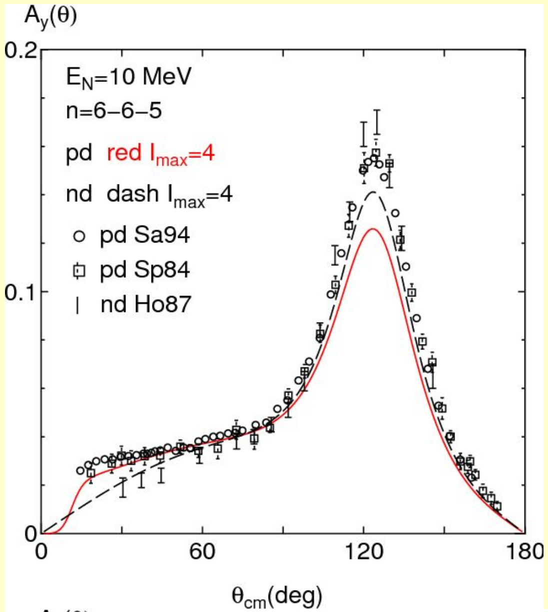
Nucleon analyzing power $A_y(\theta)$

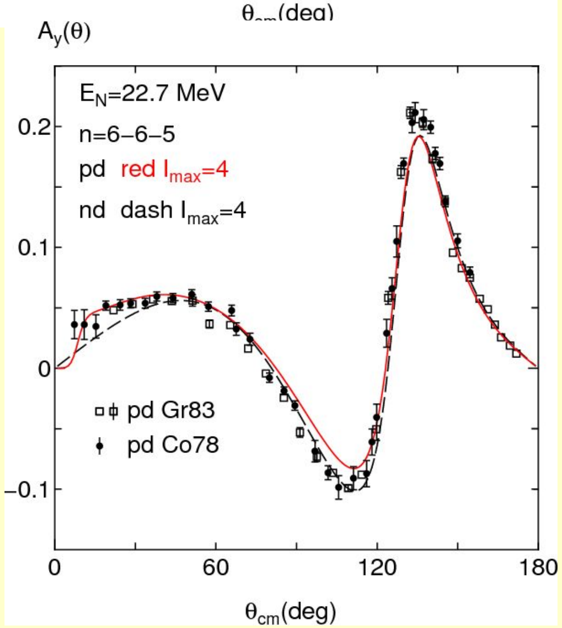
bars: ***nd*** **dots or circles:** ***pd***
dash: no Coul. red: with Coul.

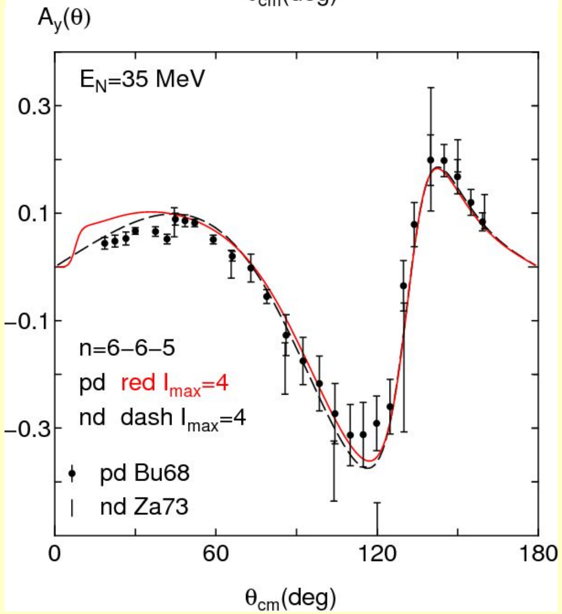
Ay puzzle still remains

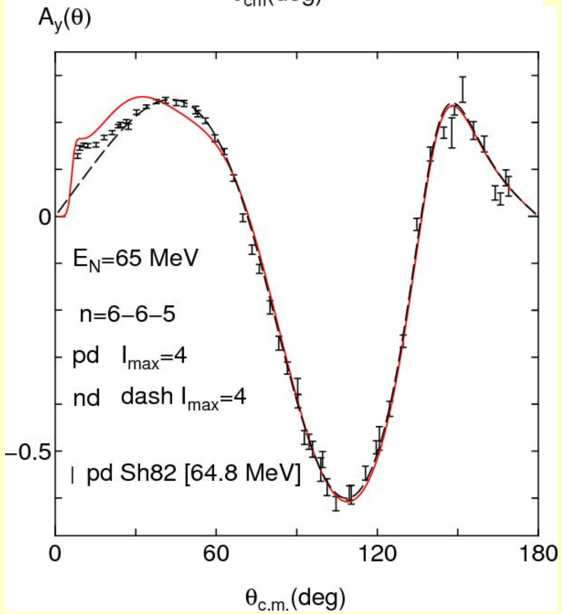






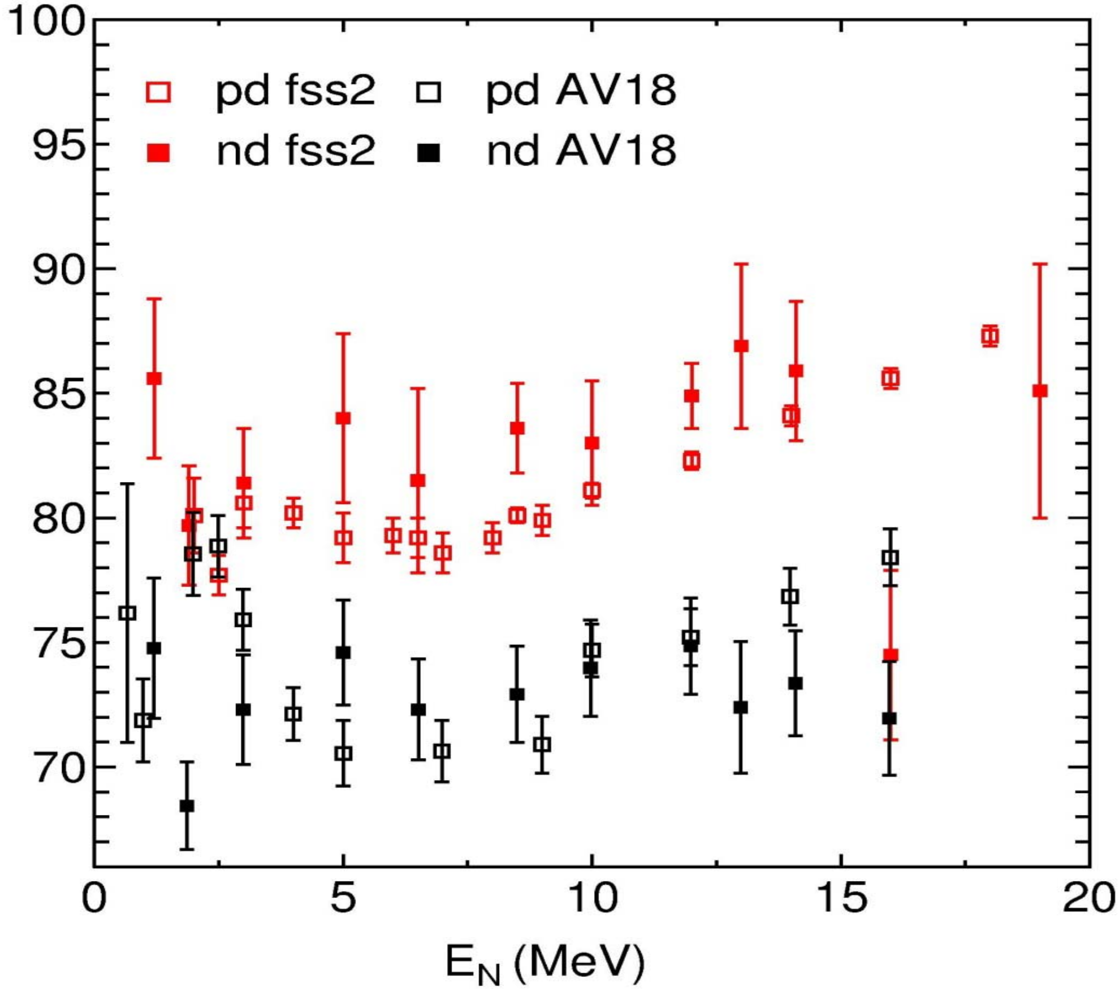






The Energy dependence of the A_y puzzle

Theory to experimental ratio of $A_y(\theta)$ at the maximum point for $E_N \leq 19$ MeV



fss2 **80-85%**

AV18 **70-80%**

Improved from the results of AV18 calculations

Effect by the 3*N* force is small

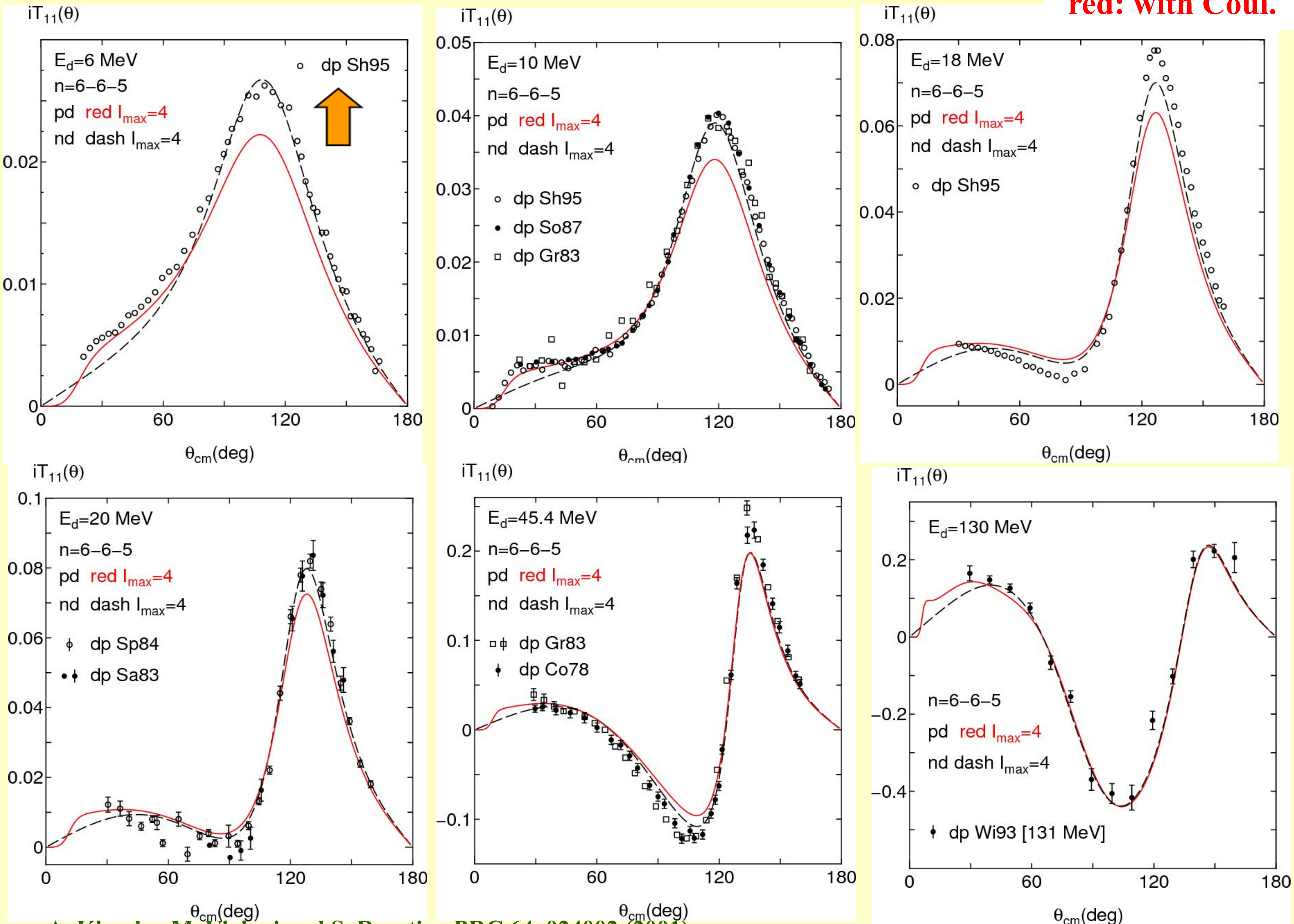
W. Tornow, J. Phys. G: Nucl. Part. Phys. 35 (2008), 125104

Deuteron analyzing power $iT_{11}(\theta)$ (vector-type)

$\vec{d} + p$

dash: no Coul.

red: with Coul.



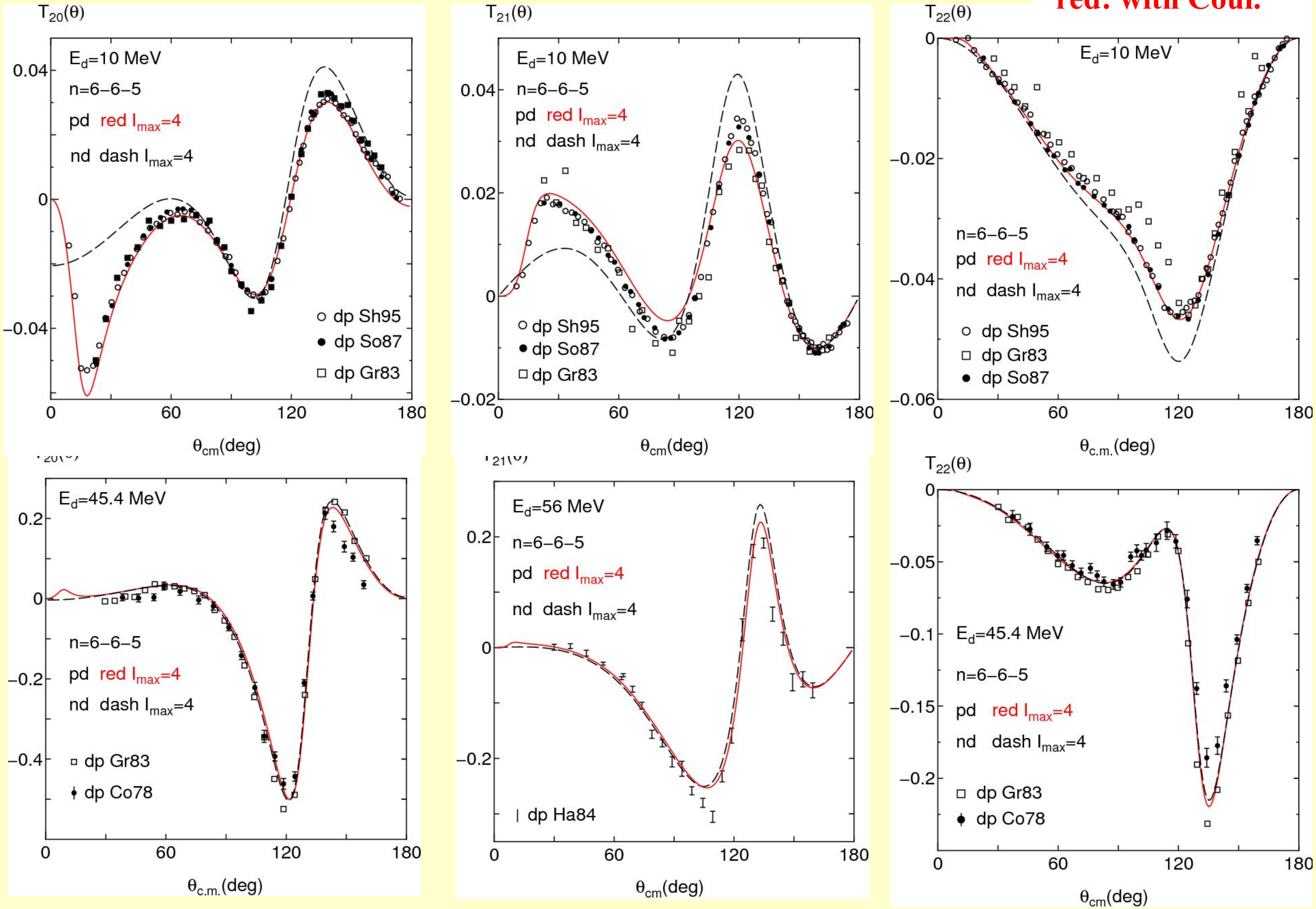
A. Kievsky, M. Viviani and S. Rosati, PRC 64, 024002 (2001).

Deuteron analyzing power $T_{2m}(\theta)$ (tensor-type)

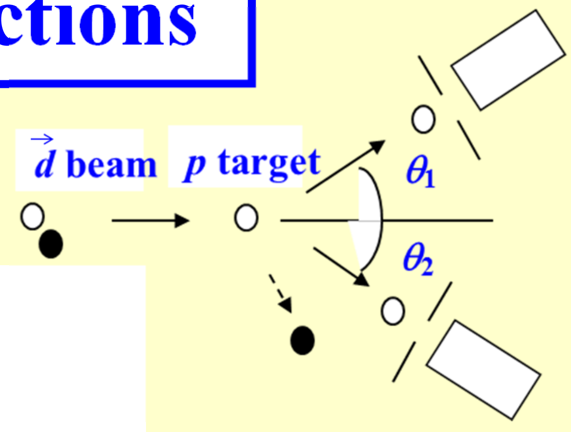
$\vec{d} + p$

dash: no Coul.

red: with Coul.

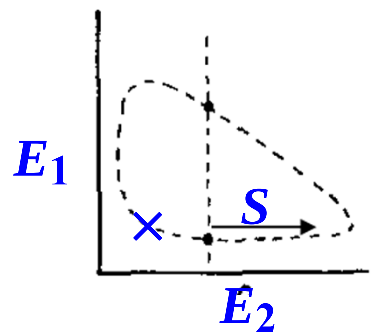


breakup differential cross sections



(Various breakup configurations)

- QFS (quasi-free scattering) $k_\alpha=0$
- FSI (final-state interaction) $p_\alpha=0$
- COLL (collinear) $q_\alpha=0$
- SS (standard space star) 120° perpendicular
- COP, CST (coplanar star) 120° coplanar
- SCRE (Symmetric constant relative energy)
- non-standard: other non-specific configurations



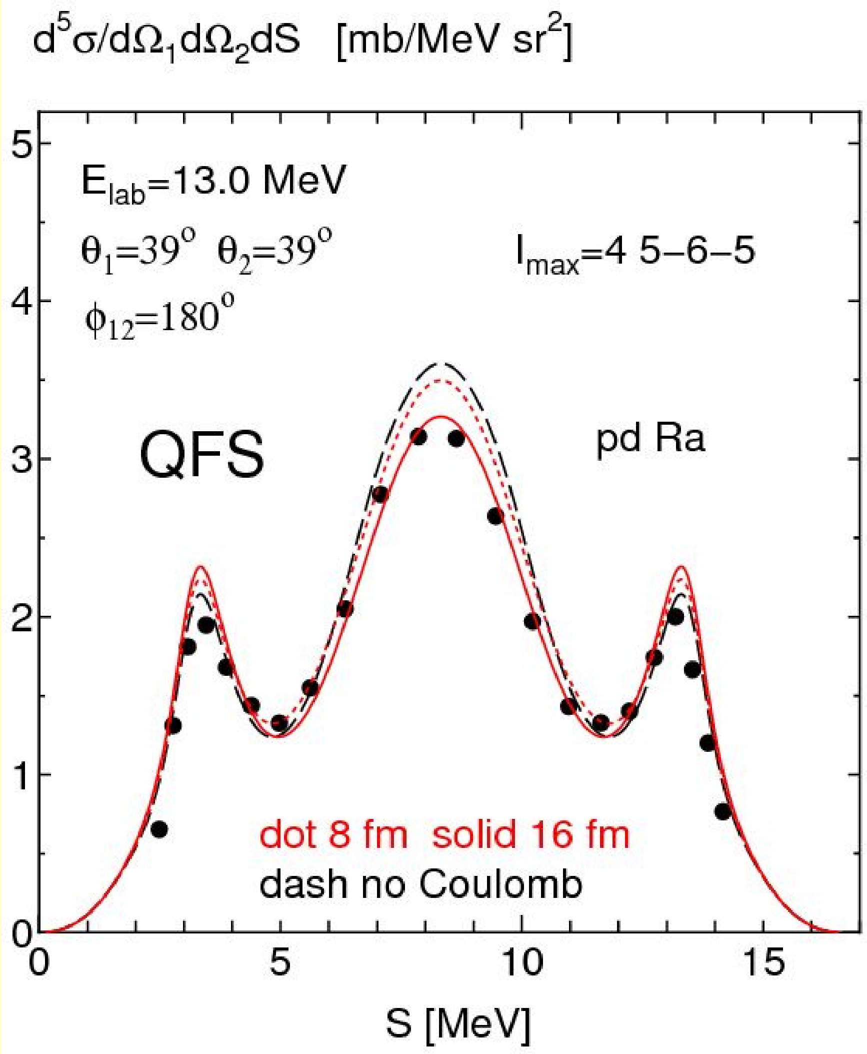
- QFS ($E_p \leq 65$ MeV)
- SCRE $\vec{d}+p$ ($E_d = 19$ MeV)
- COLL $\vec{d}+p$ ($E_d = 16$ MeV)
- KVI data $\vec{d}+p$ ($E_d = 130$ MeV)

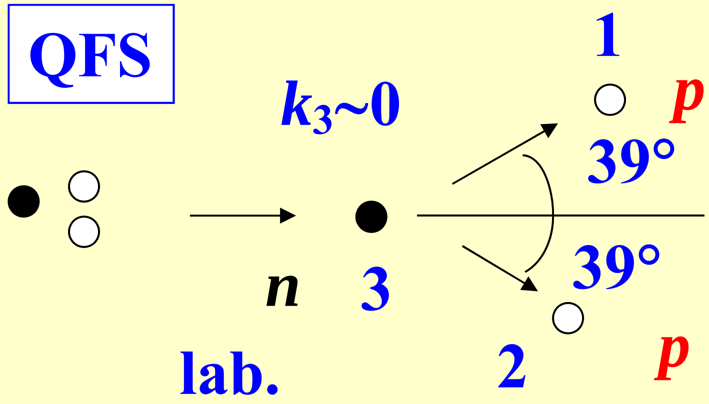
Experimental data

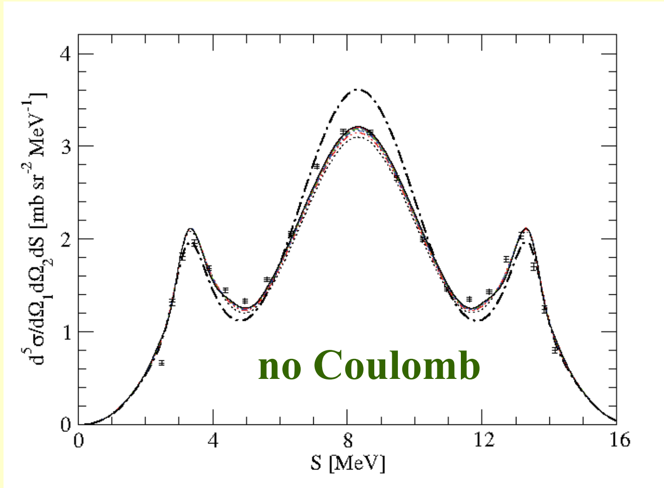
S. Kimura et al., contributions to apfb2011 at Korea
J. Ley et al., Phys. Rev. C73, 064001 (2006)
F. D. Correll et al., Nucl. Phys. A475 (1987) 407
S. T. Kistryn et. al., Phys. Rev. C72, 044006 (2005); Phys. Lett. B641 (2006) 23
E. Stephan et. al., Phys. Rev. C82, 014003 (2006)

also comparison with meson-exchange predictions

Breakup differential cross sections at E_p=13 MeV



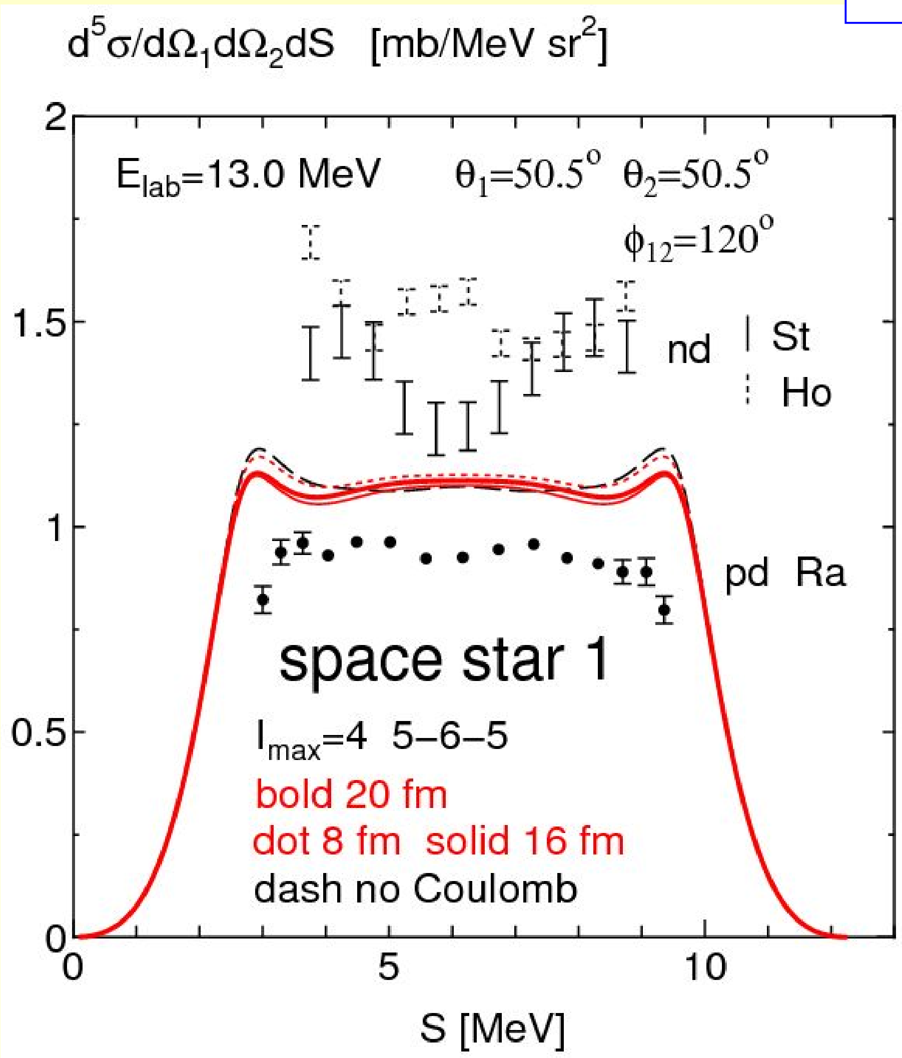


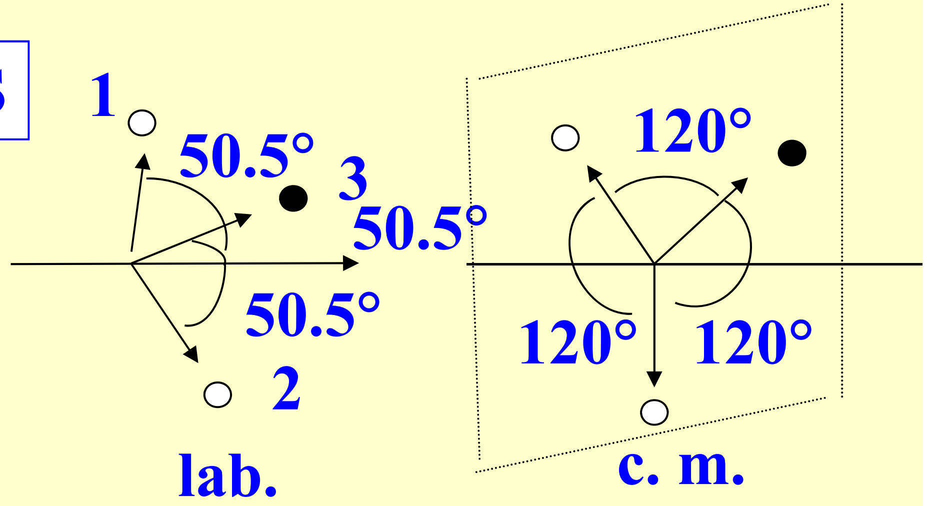


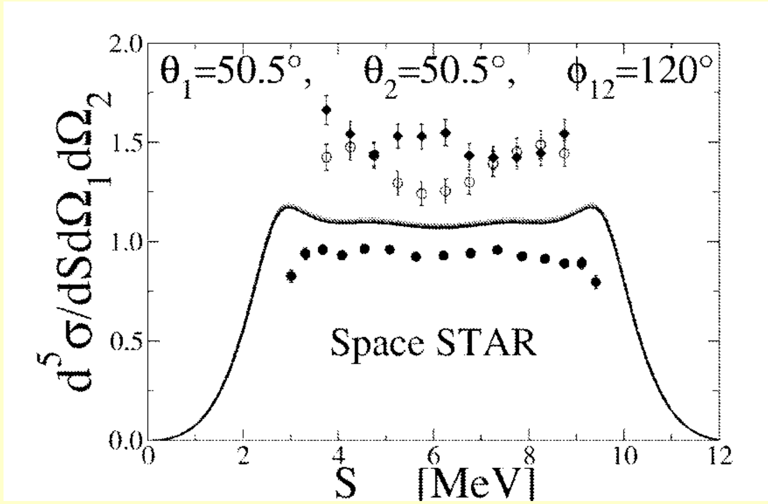
red with Coulomb

H. Witala et al., Eur. Phys. J. A41, 385 (2009)

SS





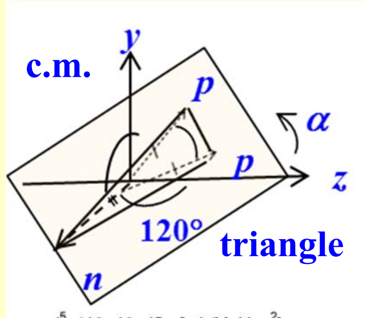


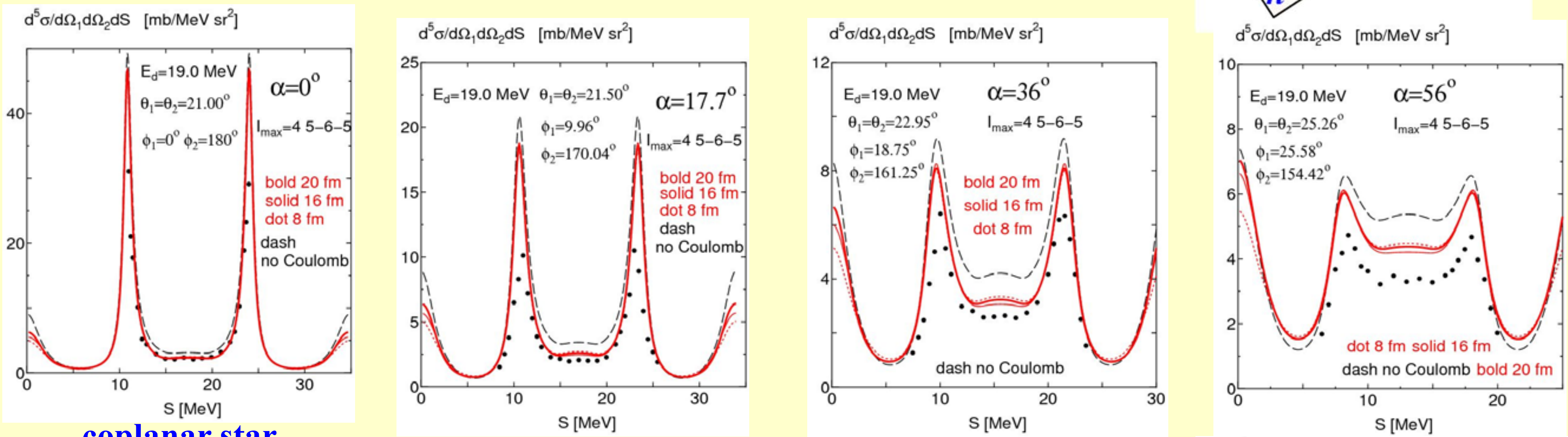
J. Kuroś-Żolnierczuk et al. (2002)

serious problem !

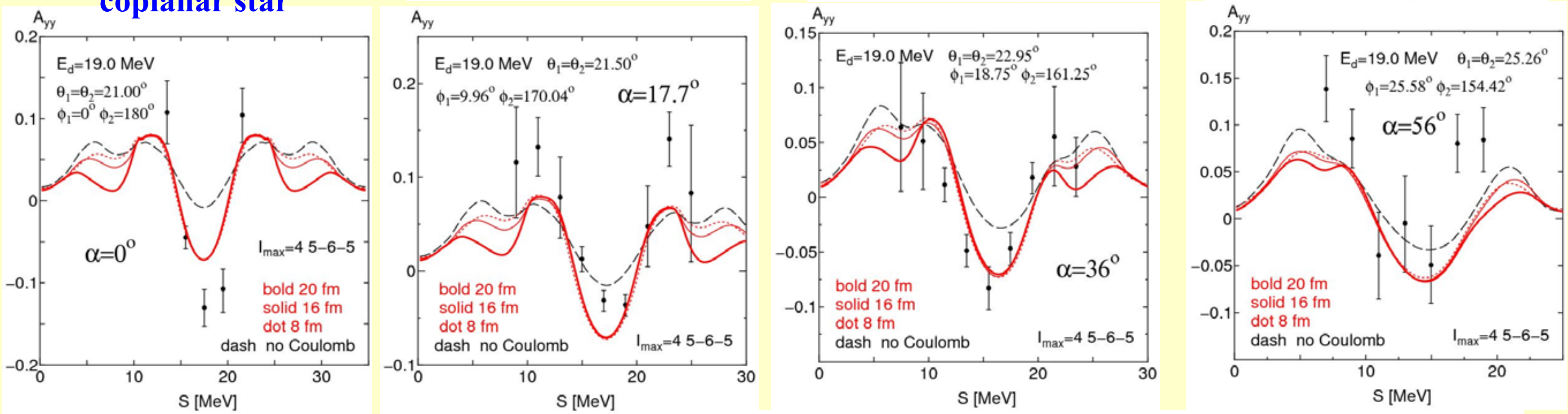
SCRE $\vec{d} + p$ ($E_d = 19$ MeV)

Symmetric constant relative Energy geometry





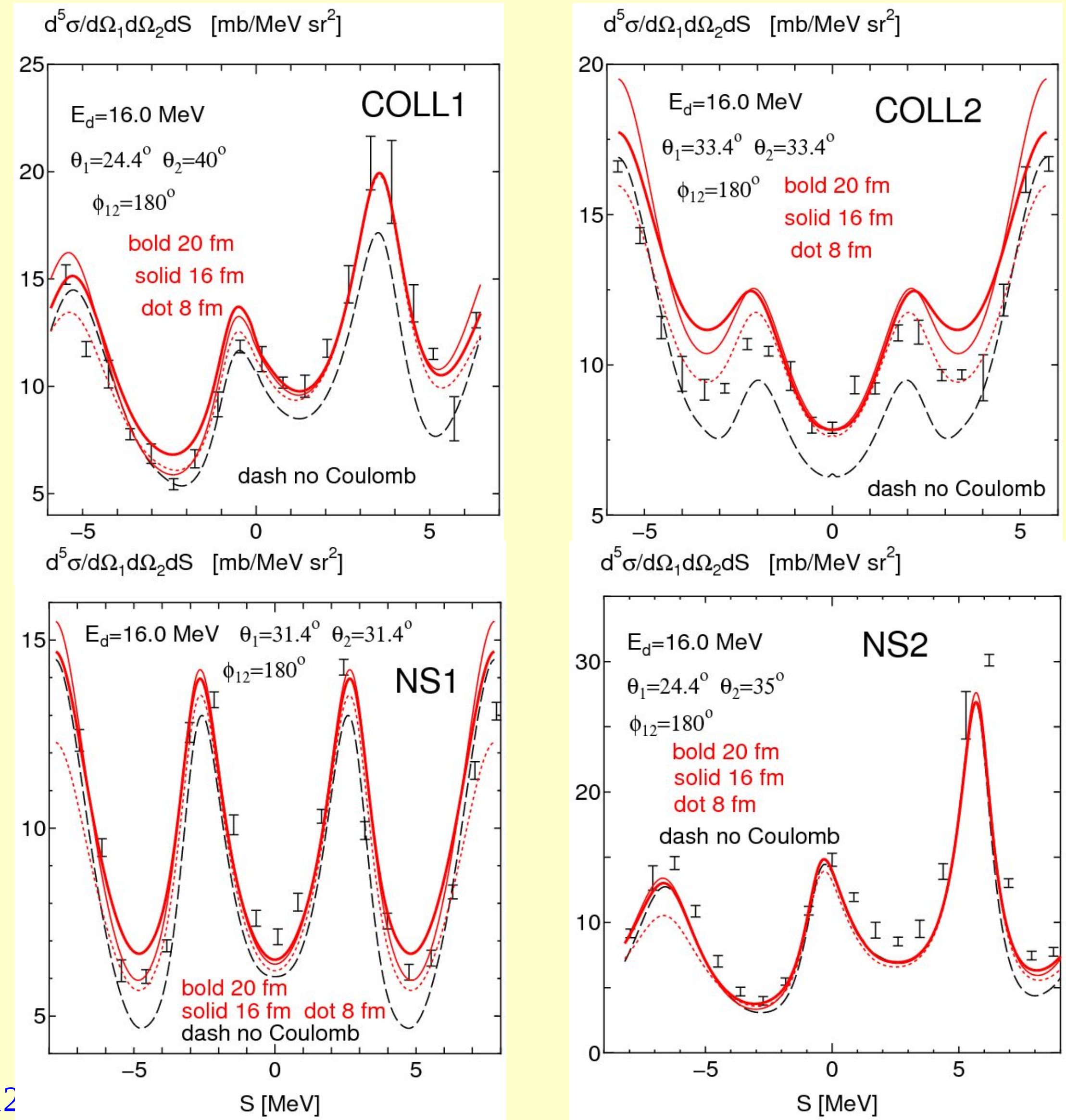
coplanar star

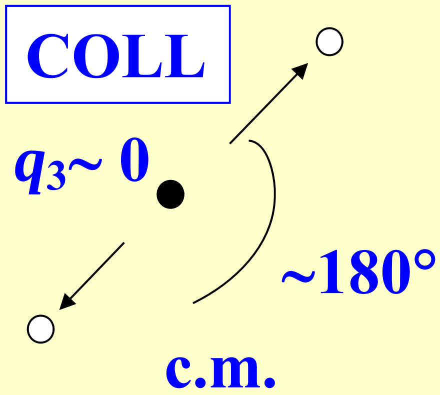


Köln data J. Ley et al., Phys. Rev. C73, 064001 (2006)

Cf. **new KUTL data by K. Ishibashi et al. in apfb2011 proceedings**

$E_d = 16$ MeV deuteron incident $\vec{d} + p$ ($E_p = 8$ MeV)



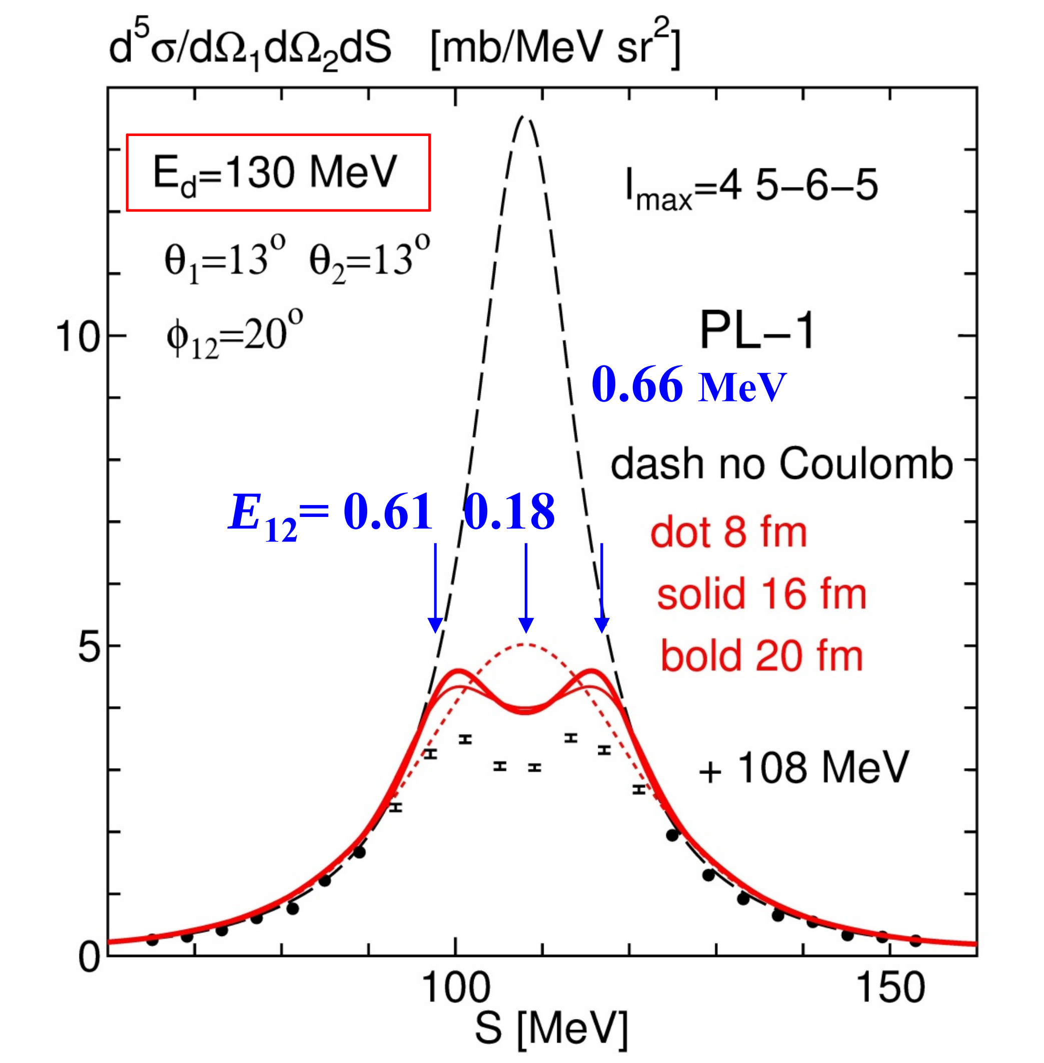


some improvement by Coulomb !

Experimental data:
F. D. Correll et al., Nucl. Phys. A475 (1987) 407

2012.12

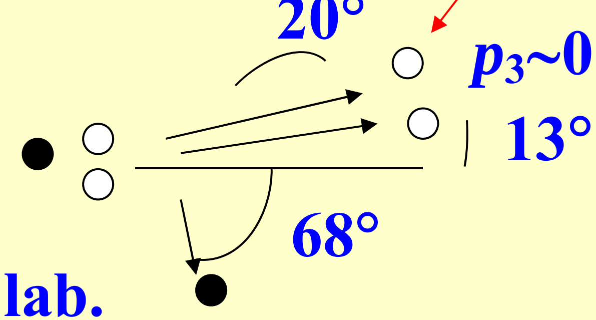
Comparison with KVI data

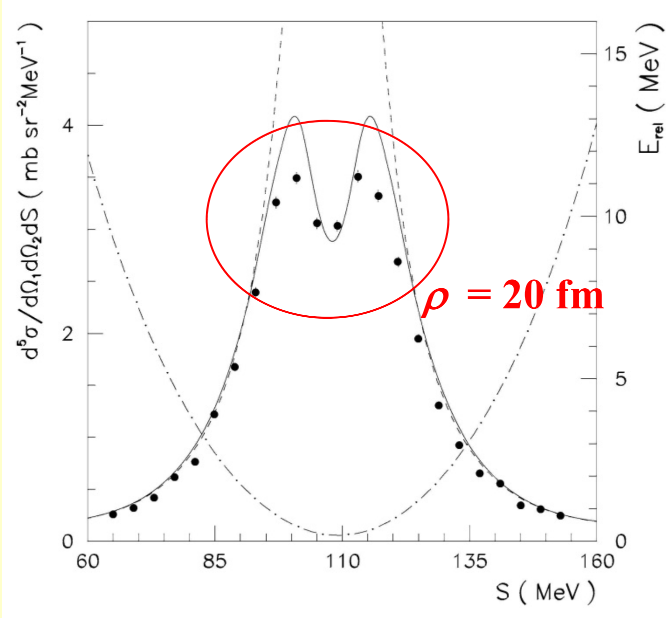


Coulomb force is very important !

pp FSI

$\phi_{12} = \phi_1 - \phi_2$



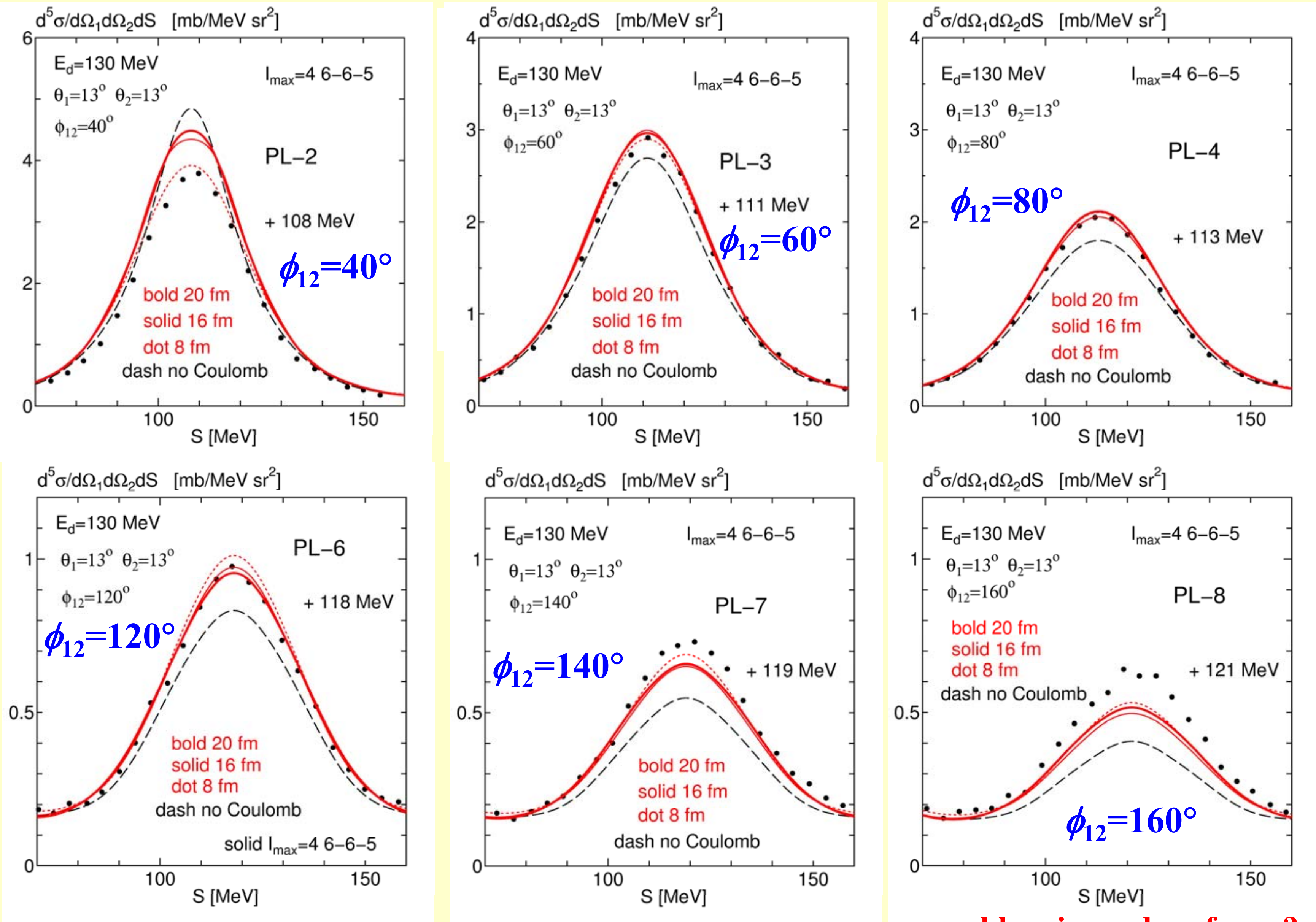


CD Bonn + △

St. Kistryn et al., Phys. Rev.C72, 044006 (2005); Phys. Lett. B641 (2006) 23

by Deltuva

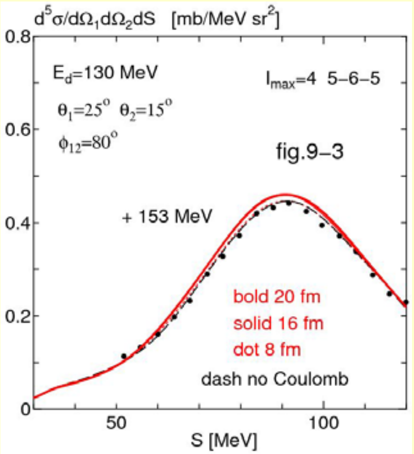
$\theta_1 = \theta_2 = 13°$ $\phi_{12} = \phi_1 - \phi_2$

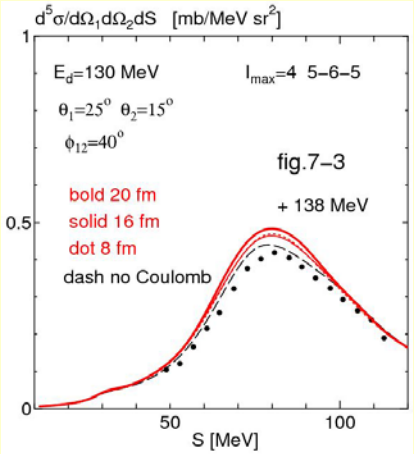


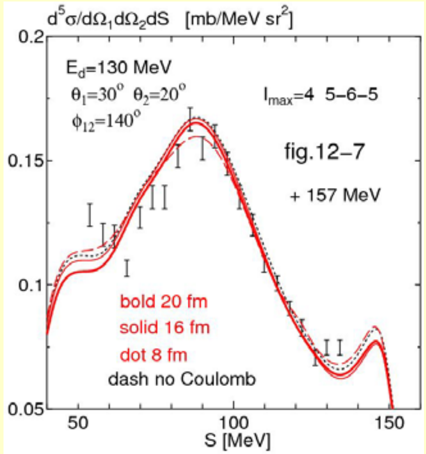
problem in nuclear force ?

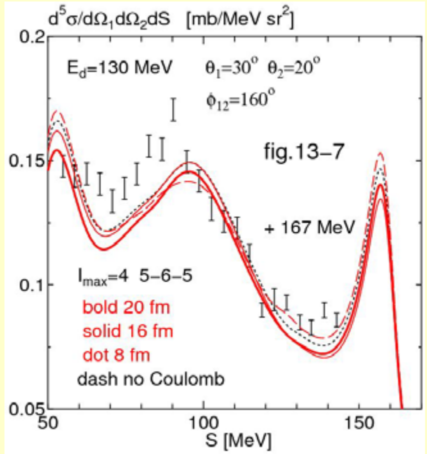
Comparison with KVI data

cross sections at E_d=130 MeV

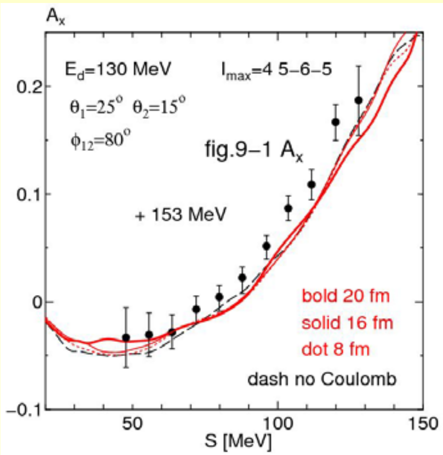


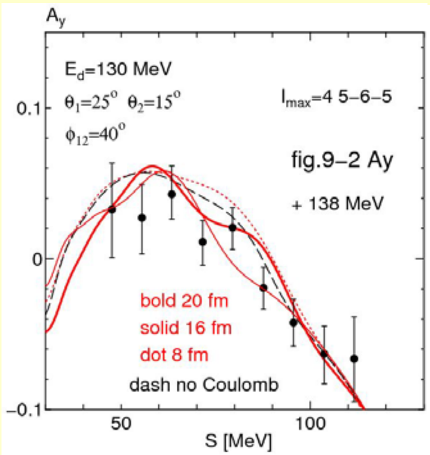


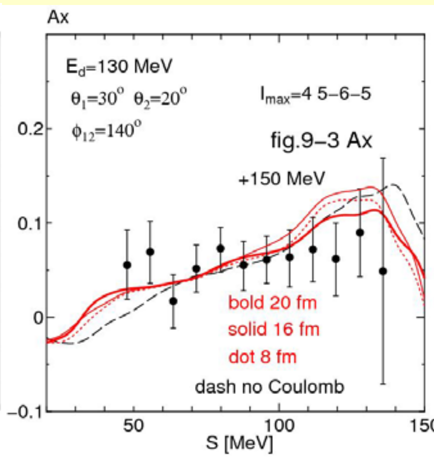


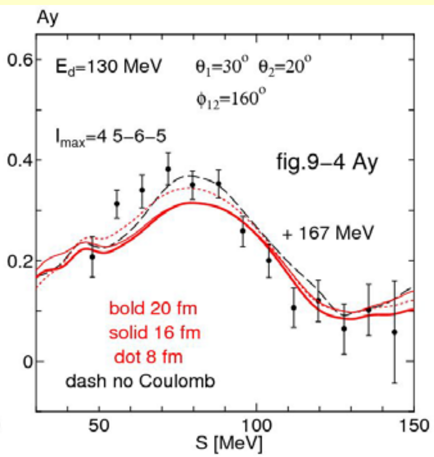


vector analyzing powers of the deuteron in the corresponding geometry









Summary

Three-cluster Faddeev formalism using the quark-model *NN* interaction fss2 can reproduce overall characteristics of the *nd* and *pd* scattering below $E_N \leq 65$ MeV without the 3*N* force, as long as the energy-dependence of the RGM kernel is properly treated.

- **charge rms radii and the binding energies of the triton and alpha**
- **scattering lengths : ^2a and ^4a for the *nd* elastic scattering**
- **improvement of the A_y puzzle, but still about 20% difference. A similar problem exists in the vector-type deuteron analyzing power iT_{11}.**
- **diffraction minima of the differential cross sections are well reproduced for $E_N \leq$ 35 MeV, but slightly underestimated for higher energies.**
- **many breakup differential cross sections and deuteron analyzing power, but severe discrepancy exists in 13 MeV symmetric space star configuration and some non-standard configurations**
- **a large Coulomb cut-off radius ρ such as 16 – 20 fm involves problems**

It is important to deal with the energy dependence of the RGM kernel properly.

Many improvements of the low-energy elastic scattering is related to the sufficiently attractive nature of fss2 in the $^2S_{1/2}$ state, in which the deuteron distortion effect is very important.

The non-local off-shell effect of the quark-model *NN* interaction can replace a part (about half) of the 3-body force needed in the meson-exchange potentials.

Conclusion for the off-shell effect

Once the on-shell properties are correctly reproduced, the off-shell effect of the baryon-baryon interaction to the *Nd* scattering is relatively small.
Cf. **big difference from the situation in the 3α bound state.**

Future problems

- **application to ($\Sigma^- d$) - (Λd) scattering problem**
  " short-range repulsion by the quark Pauli principle"
- **0^+ and 1^+ states of $^4{}_{\Lambda}$H , $^4{}_{\Lambda}$He , and $^4{}_{\Lambda\Lambda}$H , …**