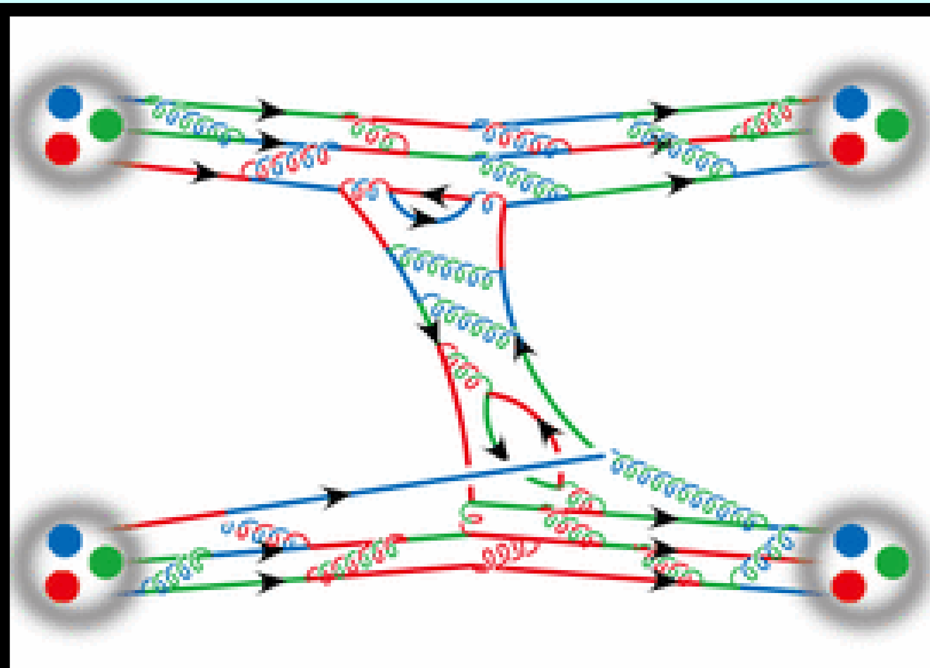
Nuclear Force from Lattice QCD

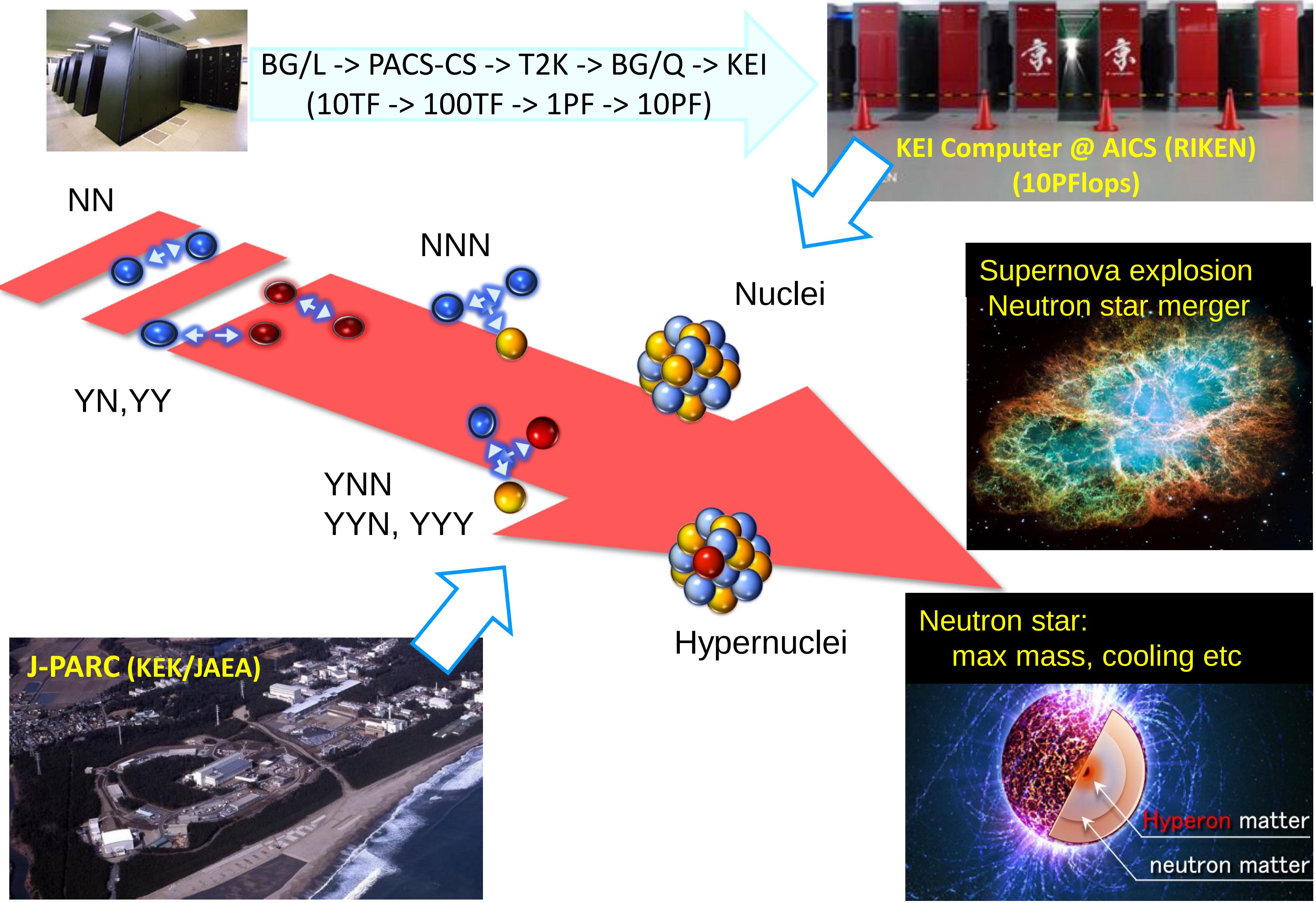


Hadrons to Atomic nuclei
HAL
from Lattice QCD

Univ. Tsukuba	S. Aoki, N. Ishii, H. Nemura, M. Yamada, K. Sasaki
RIKEN	K. Murano, T. Doi, Y. Ikeda, T. Hatsuda
Nihon Univ.	T. Inoue
Univ. Tokyo	B. Charron

T. Hatsuda (Nishina Center, RIKEN)

HAL QCD Strategy : From QCD to Compact stars



Advanced Institute for Computational Science (AICS), RIKEN

10 PFlops supercomputer KEI “京” (full operation started on Sep.28, 2012)

http://www.aics.riken.jp/en/

Five “strategic” programs (FY 2010-2015)

1. Life and Medicine
2. New Materials
3. Environment
4. Engineering
5. Particle, Nuclear and Astrophysics

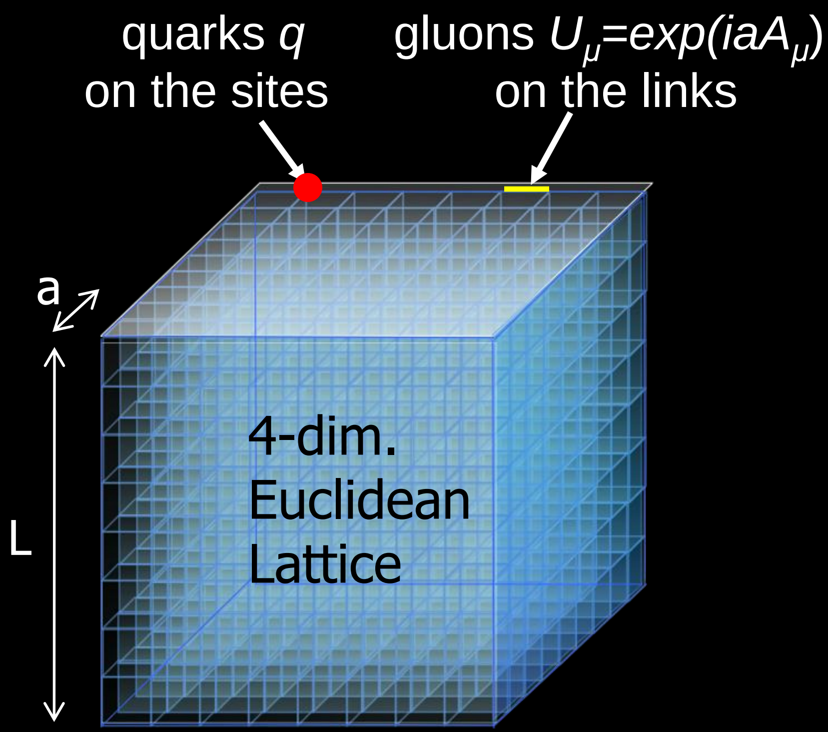
Project 1: Baryon-Baryon interaction from lattice QCD simulations at physical point
Project 2: Large scale quantum many-body calculation of nuclei and its applications
Project 3: Realistic simulation of supernova explosion and black-hole formation
Project 4: Large scale simulation of first generation of stars and galaxies

Physical point simulation started : 96^4 lattice, a=0.1fm, L=9.6fm, m_π=135MeV

Quantum Chromo Dynamics

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

Nambu (1966)



What can be done

- hadron properties & interactions
- hot plasma in equilibrium

What is difficult

- cold plasma
- phenomena far from equilibrium

$$Z = \int [dU][dq d\bar{q}] \; \exp\left[- \int d\tau d^3x \mathcal{L}_{\mathrm{E}} \right]$$

Monte Carlo method

Observable
=O(g, m, a, L)

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

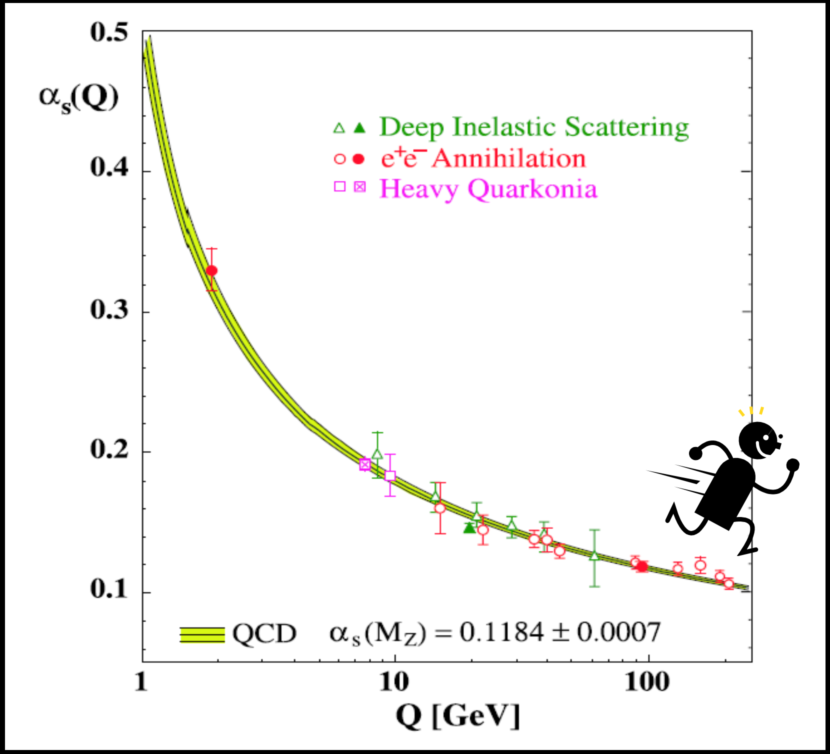
Running masses: $m_q(Q)$

quark masses (from lattice QCD)	**[MeV]** (MS-bar @ 2GeV)
m_u	2.19(15)
m_d	4.67(20)
m_s	94(3)

FLAG working group, arXiv:1011.4408 [hep-lat]

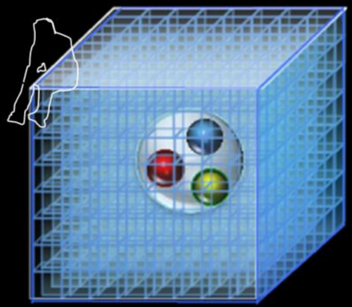
Running coupling: $\alpha_s(Q)=g^2/4\pi$

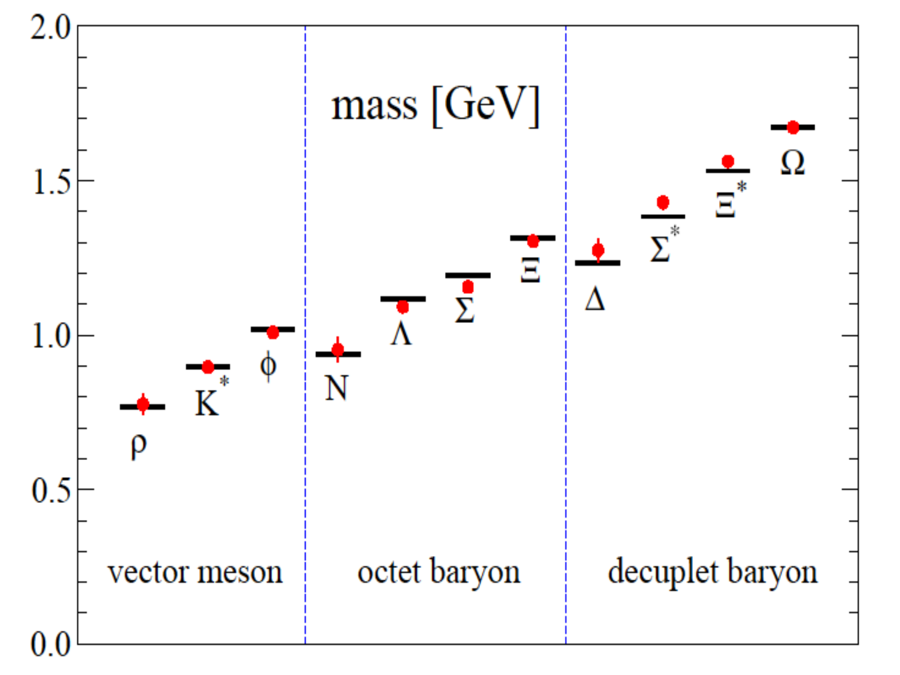
Bethke, Eur. Phys. J C(2009)64:689 →

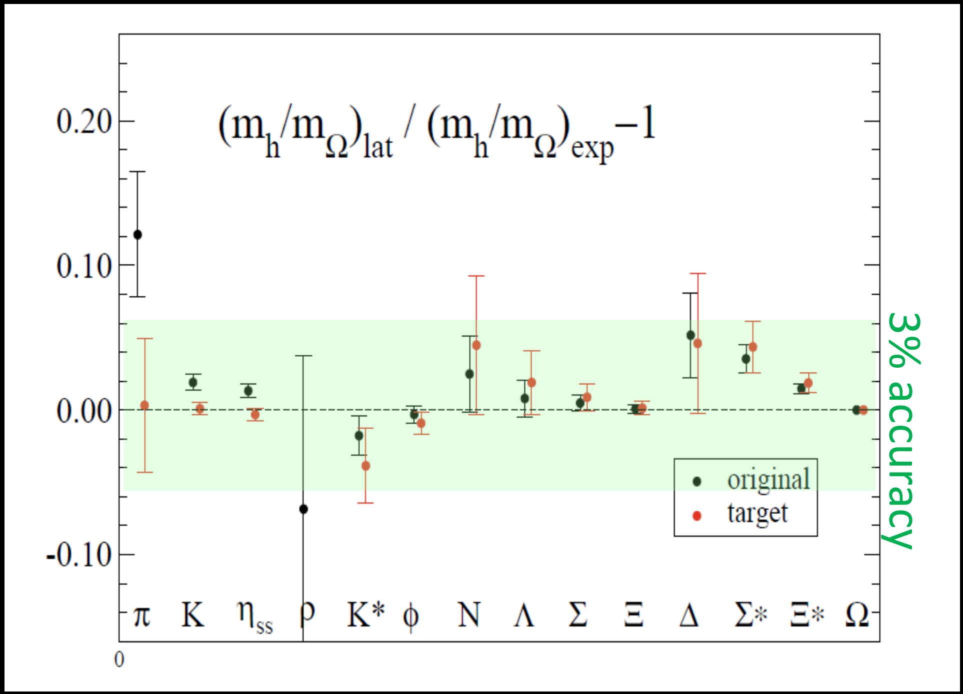


Hadron masses @ 2009
m_π > 156 MeV

Hadron masses @ 2010
m_π =135 MeV







PACS-CS Collaboration,
Phys.Rev.D79 (2009)034503

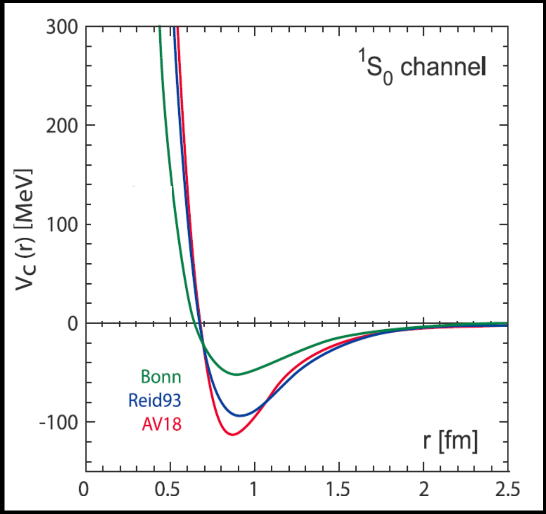
PACS-CS Coll.: Phys. Rev.D81 (2010) 074503

Baryon force: From phenomenology to 1st principle

- NN int.: about 4500 np and pp scatt. data

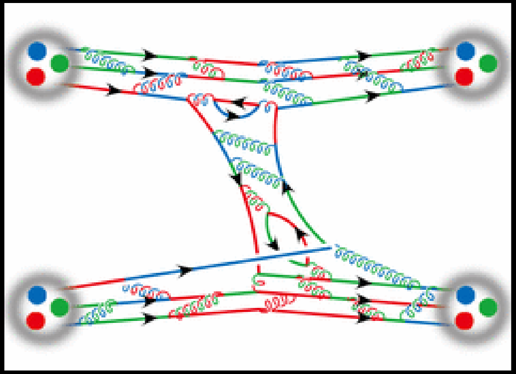
"high precision" NN interactions	# of parameters
CD Bonn (p space)	38
AV18 (r space)	40
EFT in N^3LO ($n\pi$+contact)	24

R. Machleidt, arXiv:0704.0807 [nucl-th]

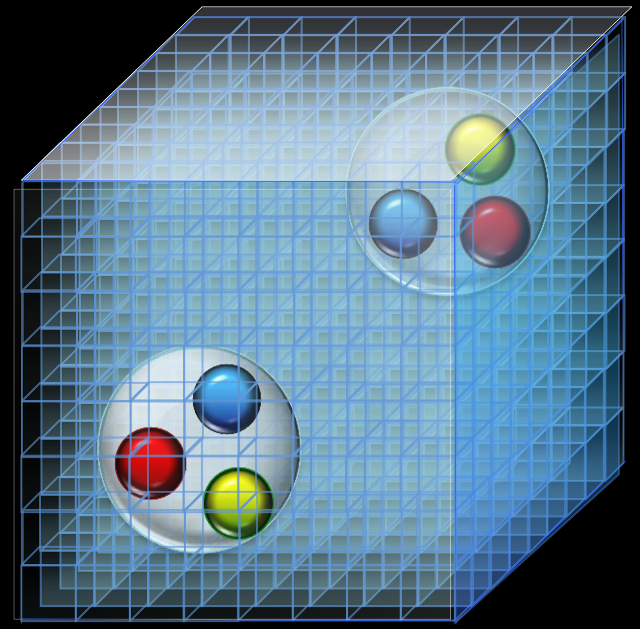


- NNN, YN, YY : data very limited
- YNN, YYN, YYY : none

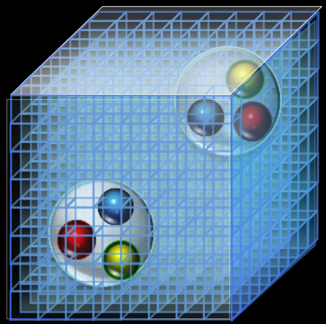
QCD has only four parameters :
m_u, m_d, m_s, Λ_{QCD}



Multi-hadrons on the Lattice



Methods to extract NN interaction from LQCD



[1] Temporal correlation:

$E_{NN}(L)$ → NN phase shift, binding energy

$$\frac{2 Z_{00}(1,q)}{L\pi^{1/2}} = k \cot \delta_0(k)$$

Luscher, Nucl. Phys. B354 (1991) 531

- quenched QCD: CP-PACS Coll. (1995)
- full QCD: NPLQCD Coll. (2006-)
- bound nuclei Yamazaki et al., (2010-)

[2] Spatial correlation :

BS wave function → NN potential → observables

$$(E - H_0)\phi(\mathbf{r}) = \int U(\mathbf{r}, \mathbf{r}')\phi(\mathbf{r}')d\mathbf{r}'$$

← half off-shell T-matrix

- NN system (quenched QCD) : Ishii, Aoki & T.H., PRL 99, 022001 (2007).
- NN, YN systems (full QCD): HAL QCD Coll. (2008-)
- Space-time hybrid method (full QCD): HAL QCD Coll. , PLB (2012)

HAL QCD Method

Ishii, Aoki & T.H., (2007, 2008, 2010)

(i) Take your favorite interpolating operator

e.g. $$N(x) = \epsilon_{abc} q^a(x) q^b(x) q^c(x)$$

← observables do not depend on the choice   Haag, Nishijima, Zimmermann (1958)

(ii) Calculate the equal-time BS amplitude

$$\phi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 6q \rangle$$

(iii) Define the potential

$$(E - H_0)\phi(\vec{r}) = \int U(r, \vec{r}')\phi(\vec{r}') d^3r'$$

(iv) Derivative expansion

$$U(\vec{r}, \vec{r'}) = V(\vec{r}, \nabla)\delta^3(\vec{r} - \vec{r}')$$

$$V(\vec{r}, \nabla) = V_{\rm C}(r) + S_{12} V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\ V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots$$

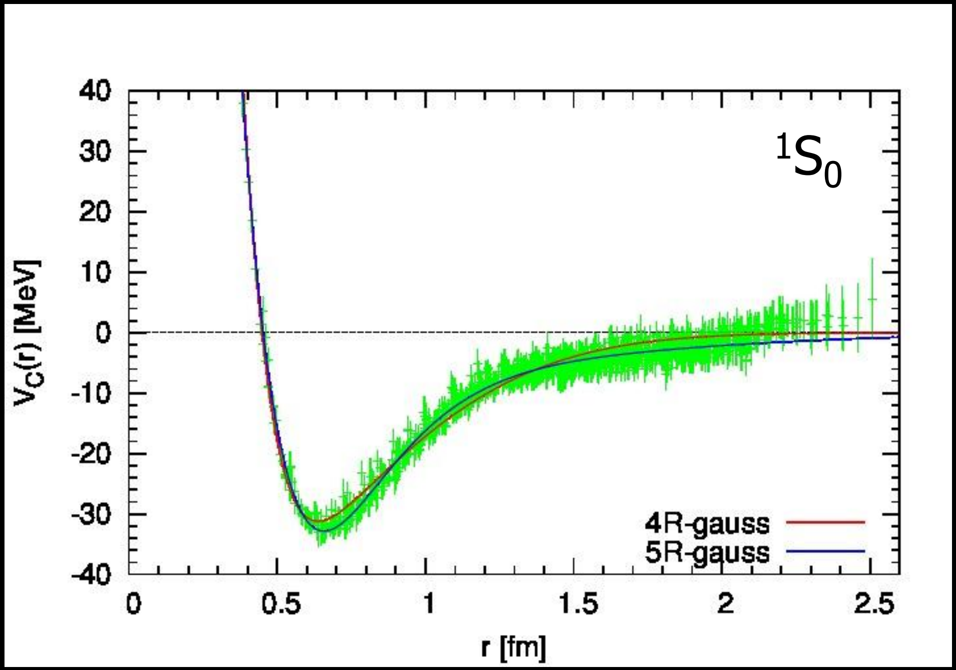
Okubo-Marshak (1958)

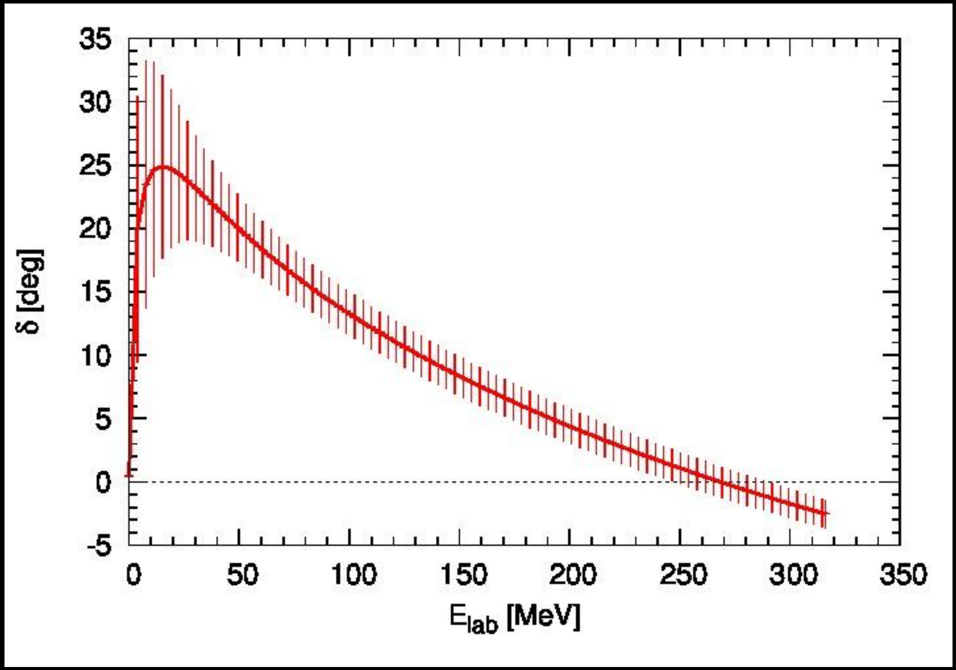
- "Potential" is <u>not an observable</u> but is a <u>nice tool</u> to calculate observables
- "Potential" is <u>volume insensitive</u> (i.e. Lattice Friendly)

Central potential in (2+1)-flavor QCD

HAL QCD Coll., Phys. Lett. B712 (2012)

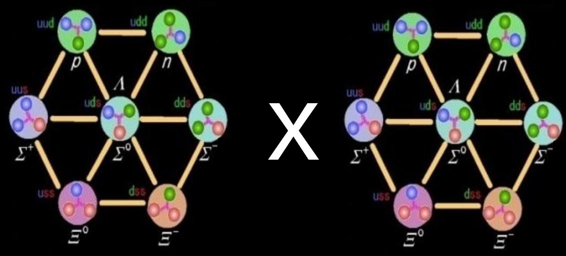
PACS-CS gauge config.
(Clover + Iwasaki)
a= 0.09 fm, L=2.9 fm
m_π= 700 MeV





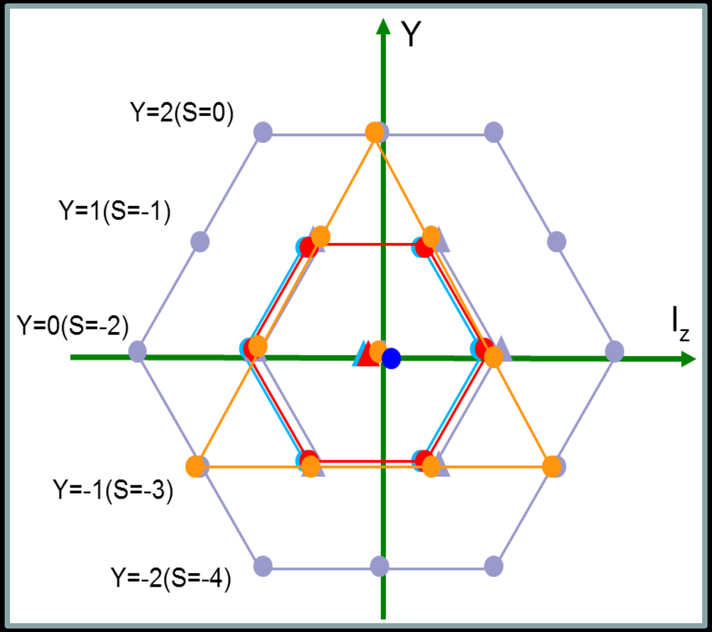
⇒ Physical point simulations (m_π=135MeV, L=9.6 fm) at KEI computer

BB forces from LQCD



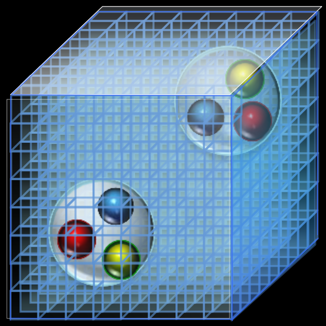
$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

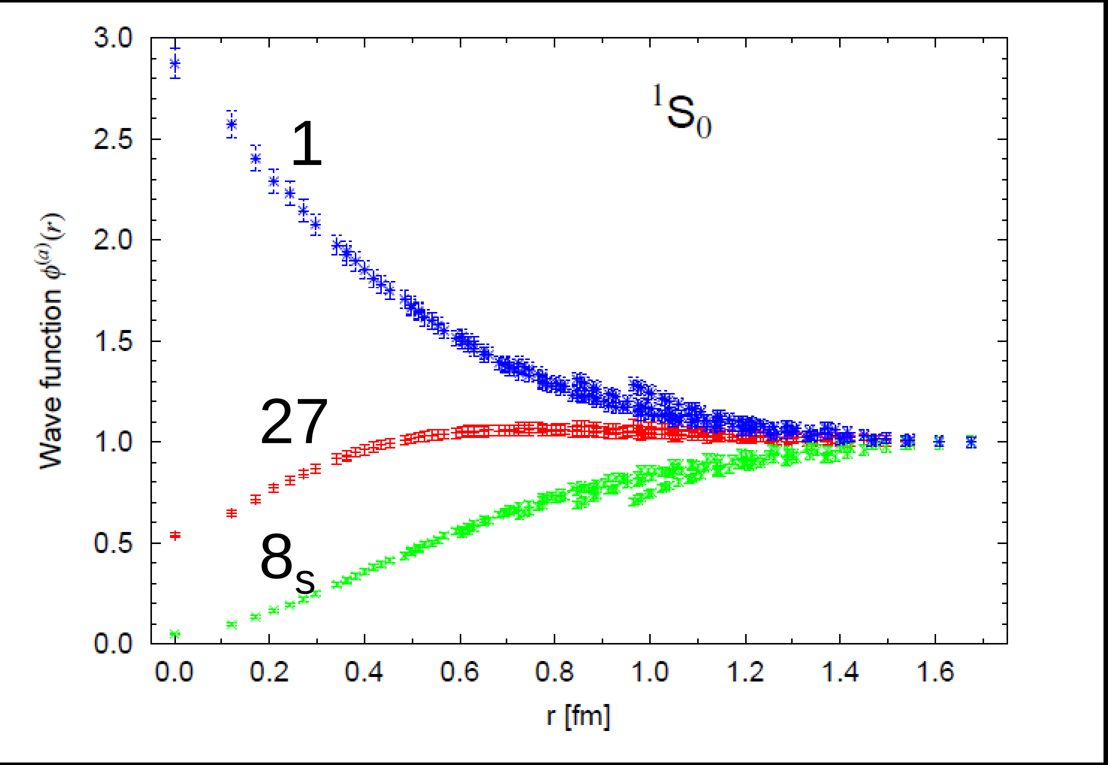
Symmetric   Anti-symmetric



- Physical origin of the short distance NN repulsion ?
- Fate of the H-dibaryon ?
- Effect of the SU(3) breaking?

Lattice BB wave functions (flavor SU(3) limit)





Iwasaki + clover
(CP-PACS/JLQCD config.)
L=1.9 fm, a=0.12 fm, 16^3x32
m_π=835 MeV, m_B=1752 MeV

HAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

Short range BB int. ⇔ Quark Pauli principle

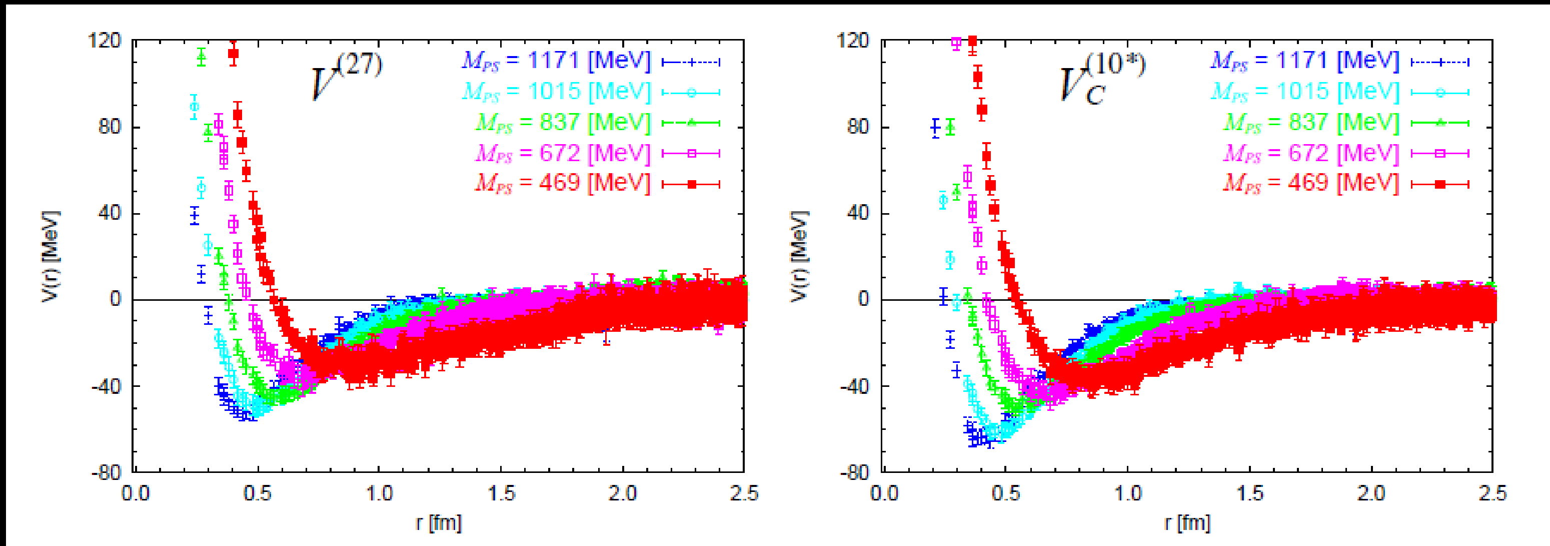
1 : allowed, 27 : partially blocked, 8_s : blocked

(Oka, Yazaki, Toki, ...)

BB potentials (flavor SU(3) limit)

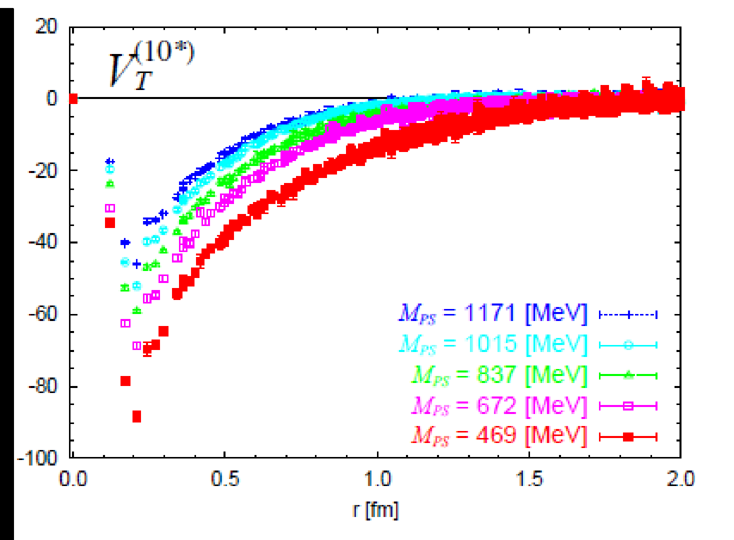
IHAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

Repulsive core in NN channel



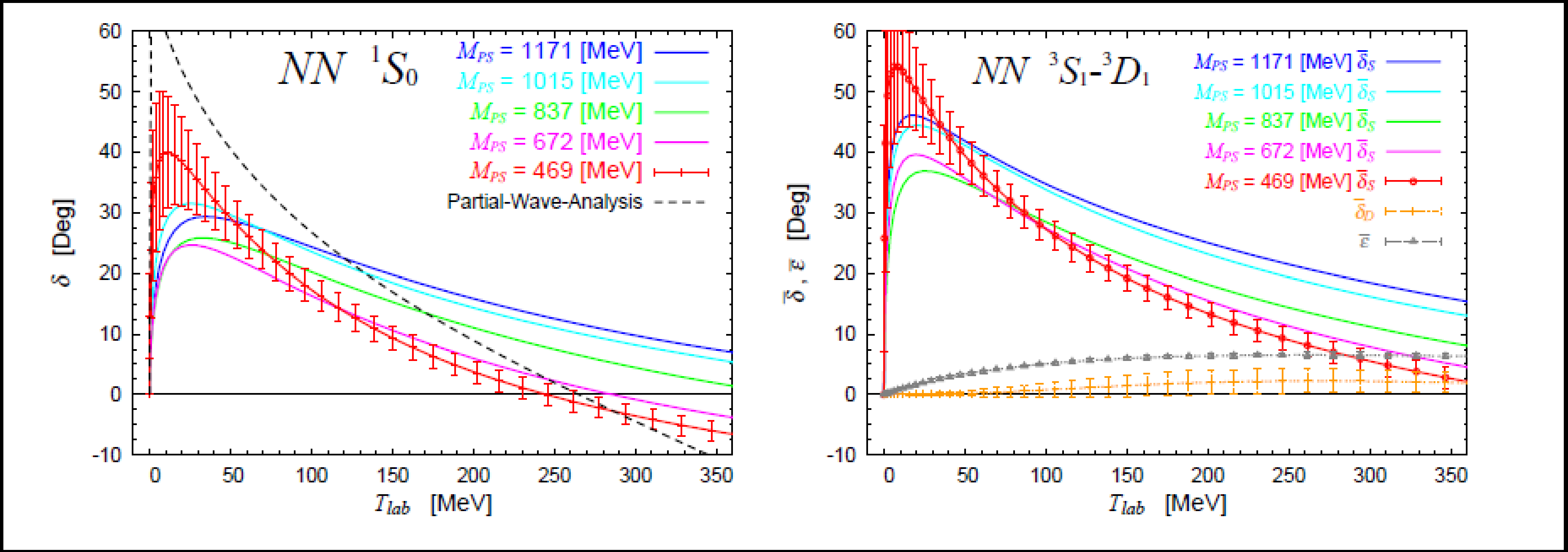
$^1S_0$ channel

$^3S_1 - {}^3D_1$ channel



Growing NN tensor force

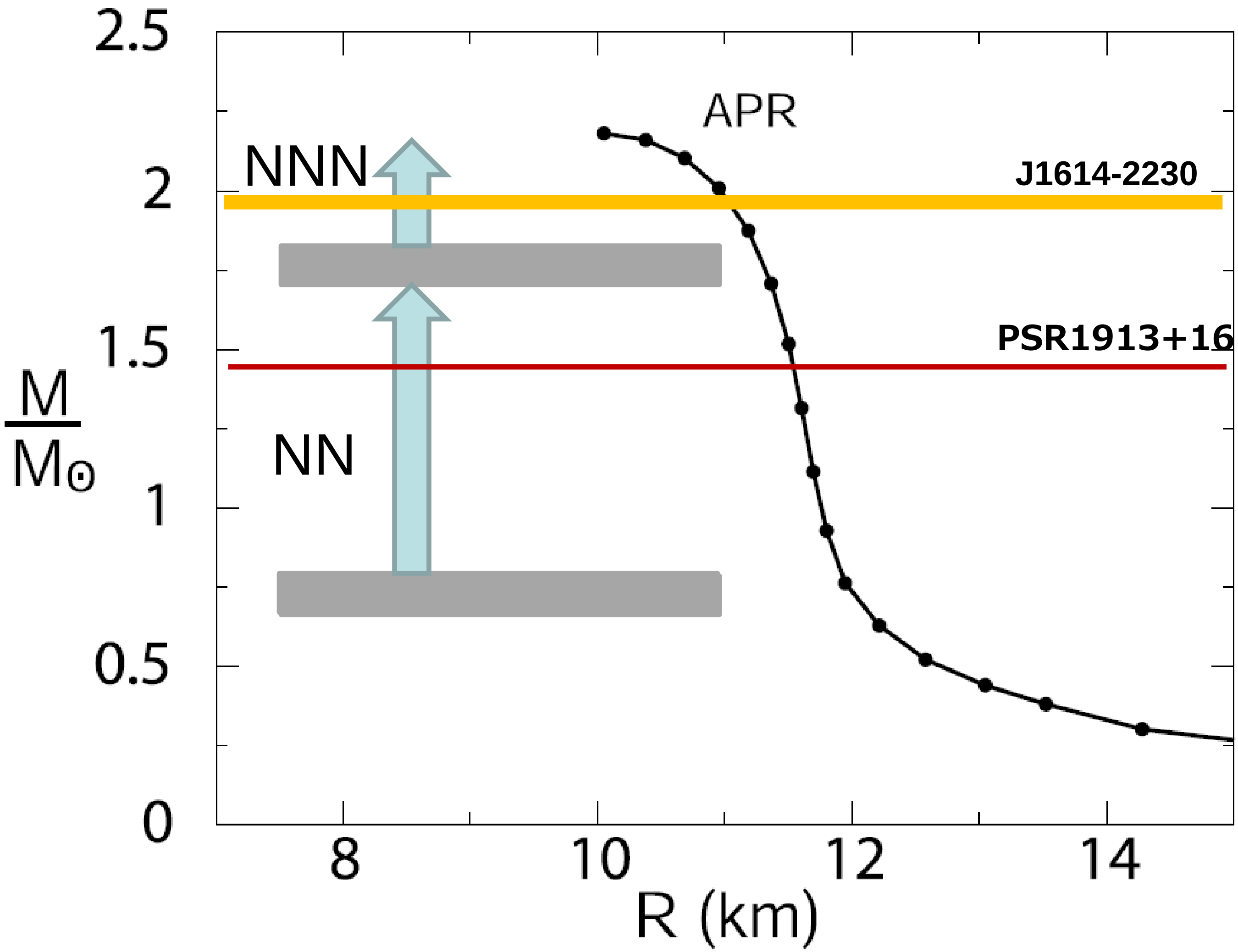
NN phase shifts in the SU(3) symmetric world



Stronger attraction in the deuteron channel

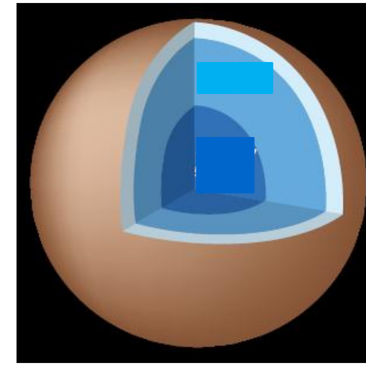
HAL QCD Coll.,
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

Mass-Radium Relation of Neutron Star

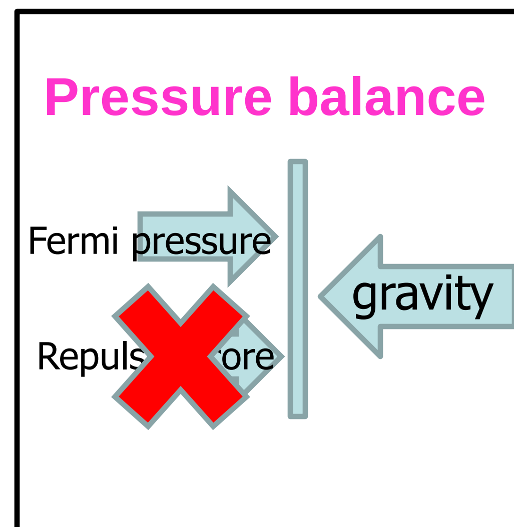


NS – WD binary

NS - NS binary

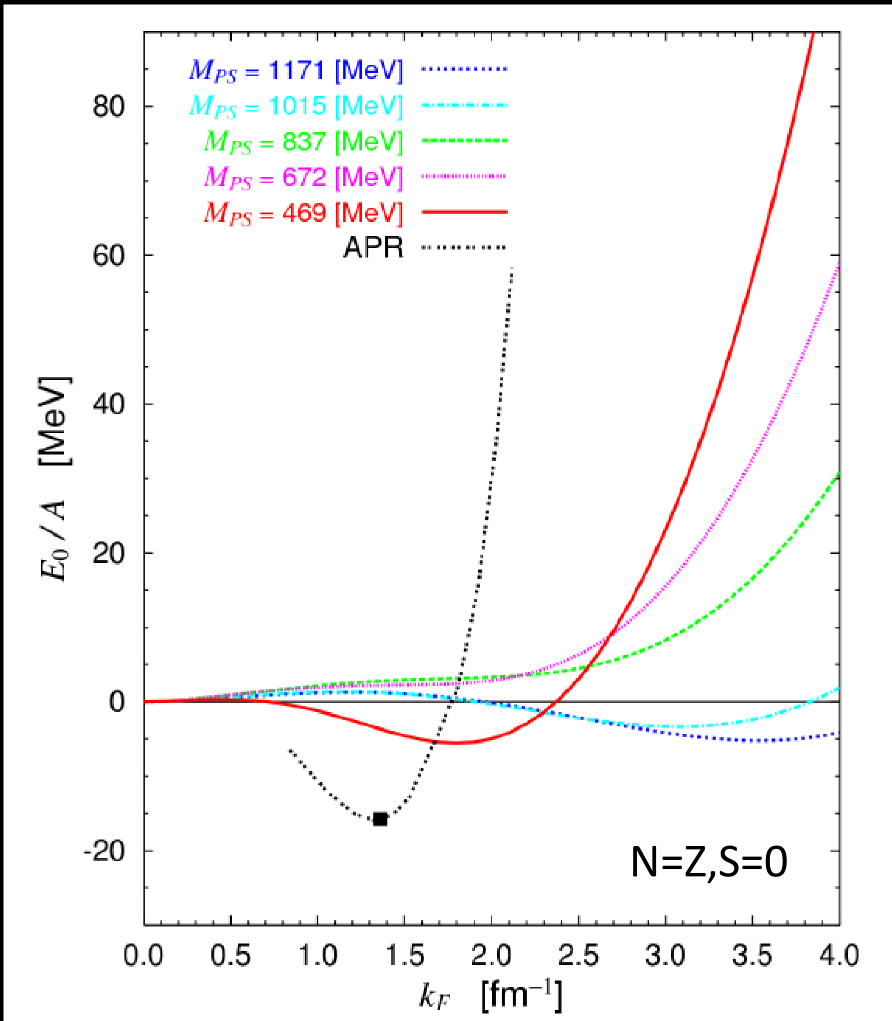


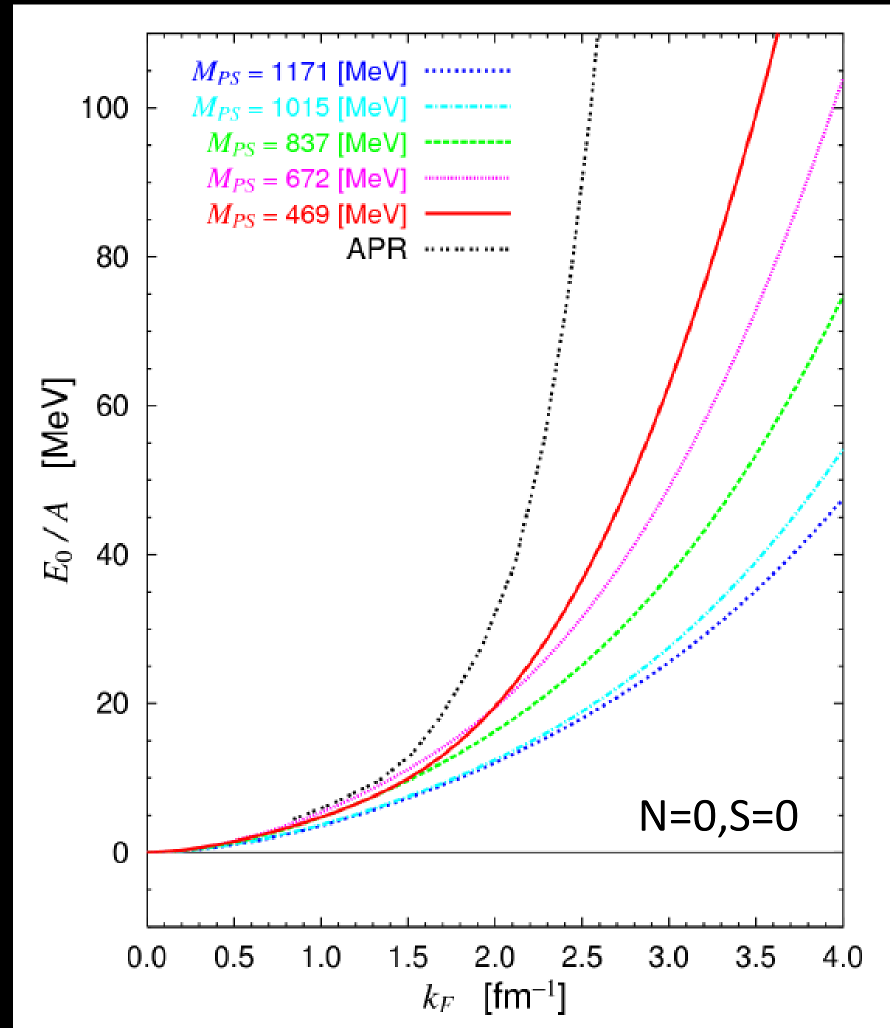
(ρ_{max} ~ $6\rho_0$)



Just for fun: Neutron star from NN potential in flavor SU(3) limit

EOS with Lattice NN force by BHF calculation → M-R relation by TOV equation

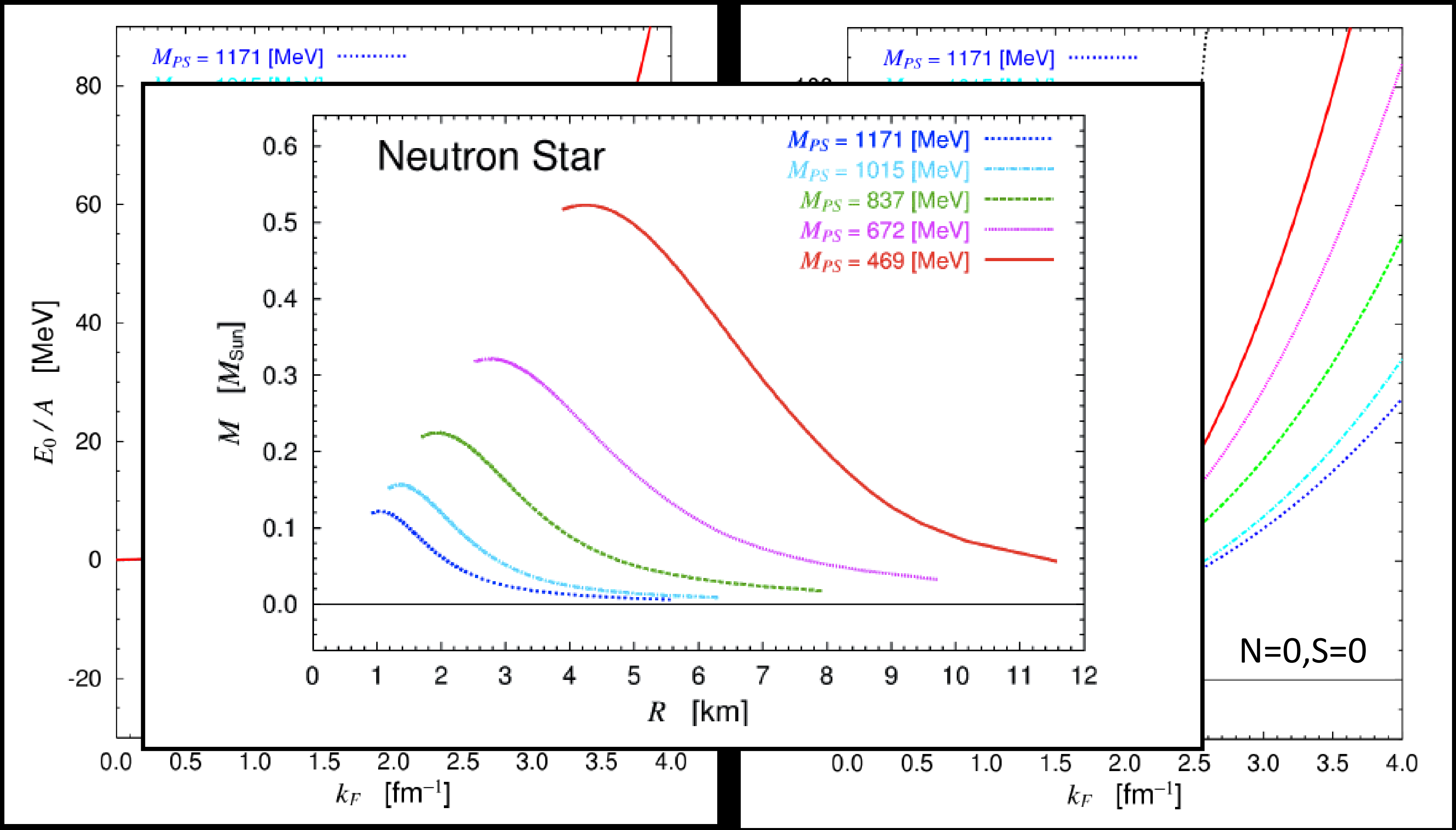




Inoue et al. [HAL QCD Coll.] (2012)

Just for fun: Neutron star from NN potential in flavor SU(3) limit

EOS with Lattice NN force by BHF calculation → M-R relation by TOV equation

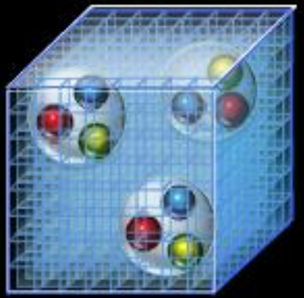


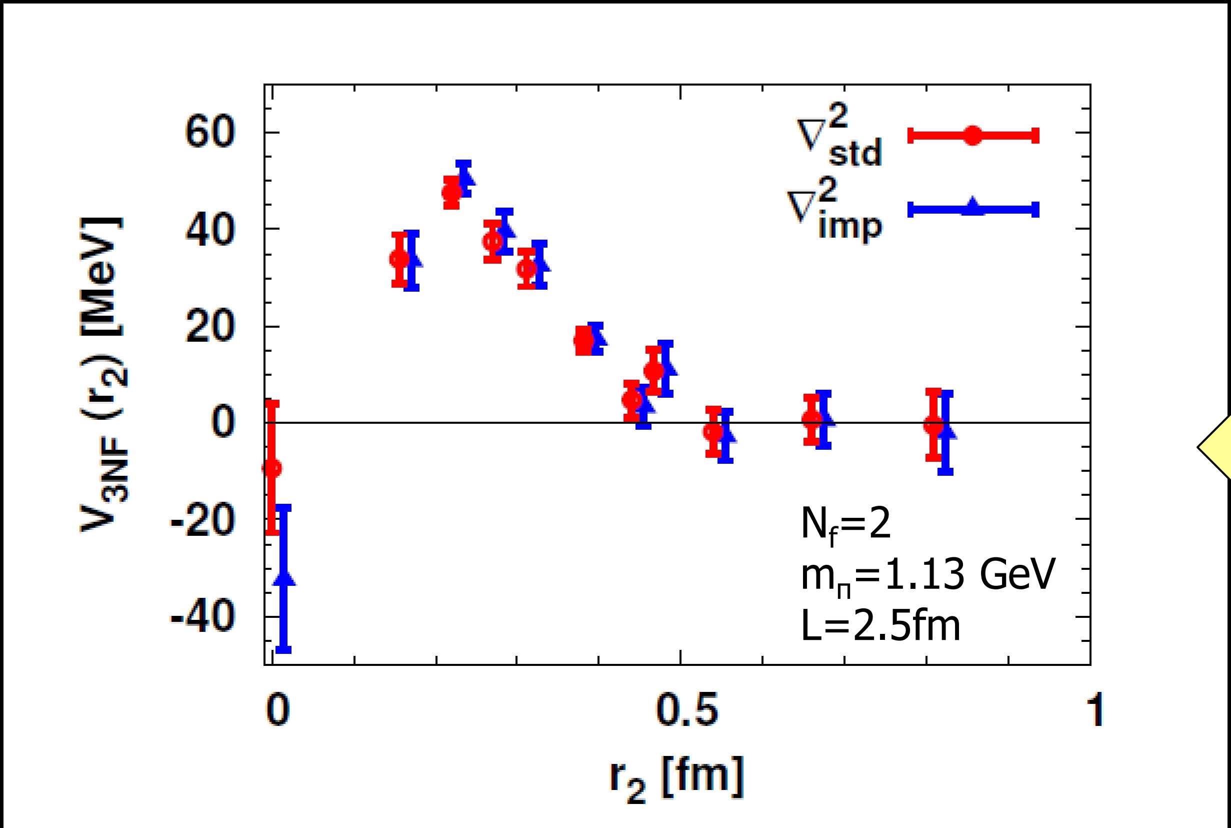
Inoue et al. [HAL QCD Coll.] (2012)

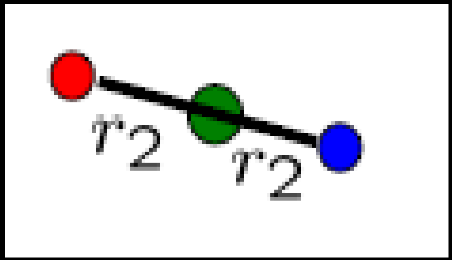
3N force (spin-isospin independent part) from LQCD

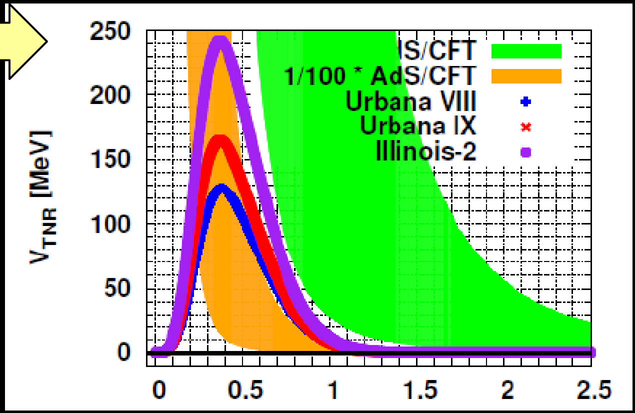
$$\psi_{3N}(\vec{r},\vec{\rho}) \equiv \langle 0|N(\vec{x}_1)N(\vec{x}_2)N(\vec{x}_3)|E_{3N}\rangle,$$

$$\left[-\frac{1}{2\mu_r}\nabla_r^2 - \frac{1}{2\mu_\rho}\nabla_\rho^2 + \sum_{i<j} V_{2N}(\vec{r}_{ij}) + V_{3NF}(\vec{r},\vec{\rho})\right]\psi_{3N}(\vec{r},\vec{\rho}) = E_{3N}\psi_{3N}(\vec{r},\vec{\rho}),$$







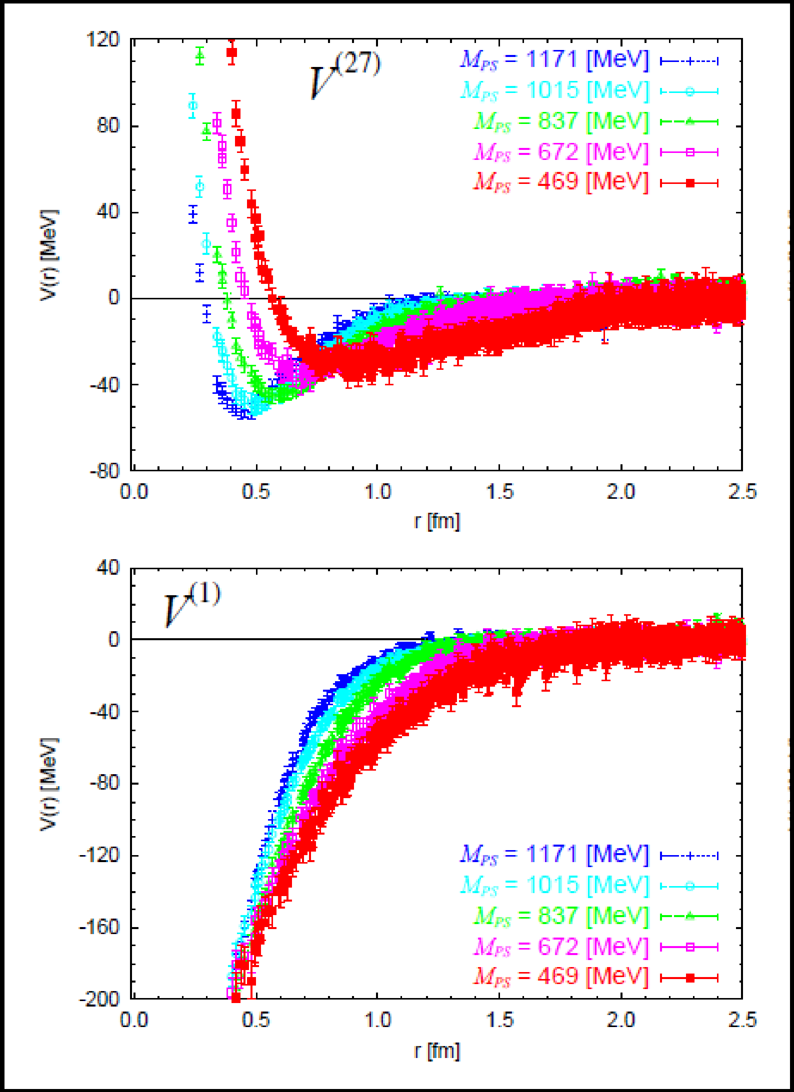


r_2 [fm]

T. Doi et al [HAL QCD Coll.], arXiv:1106.2276[hep-lat]

BB potentials (flavor SU(3) limit)

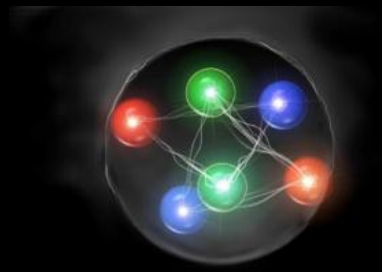
IHAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

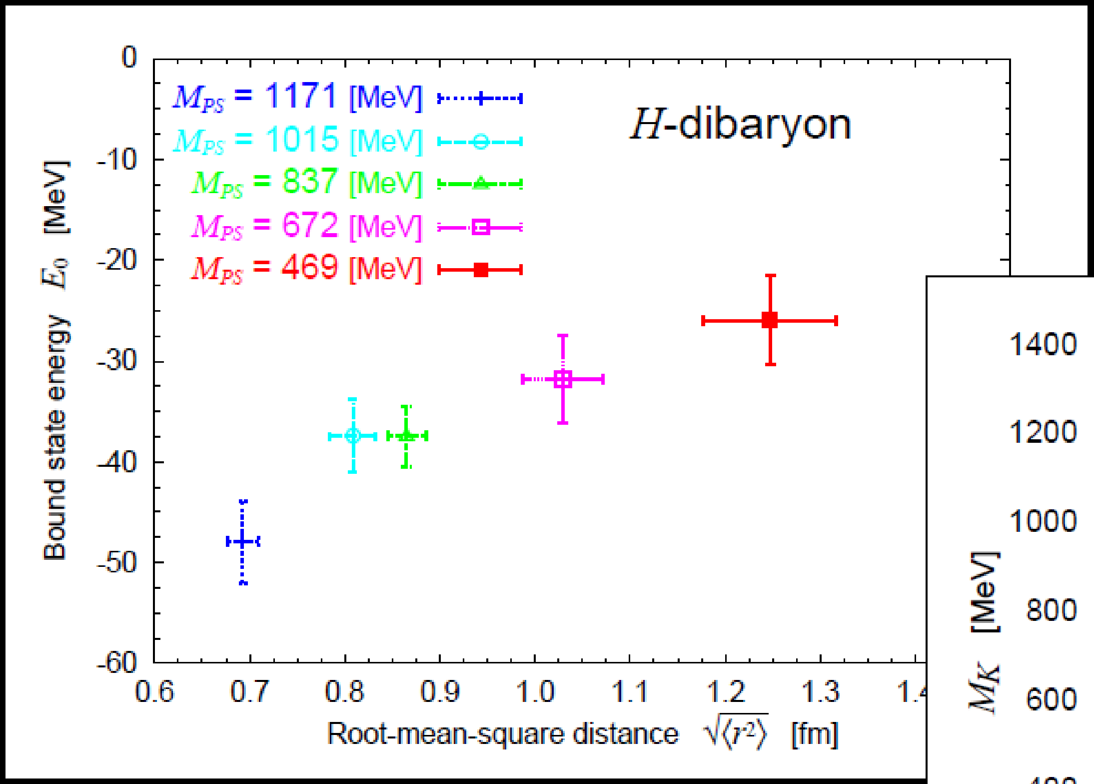


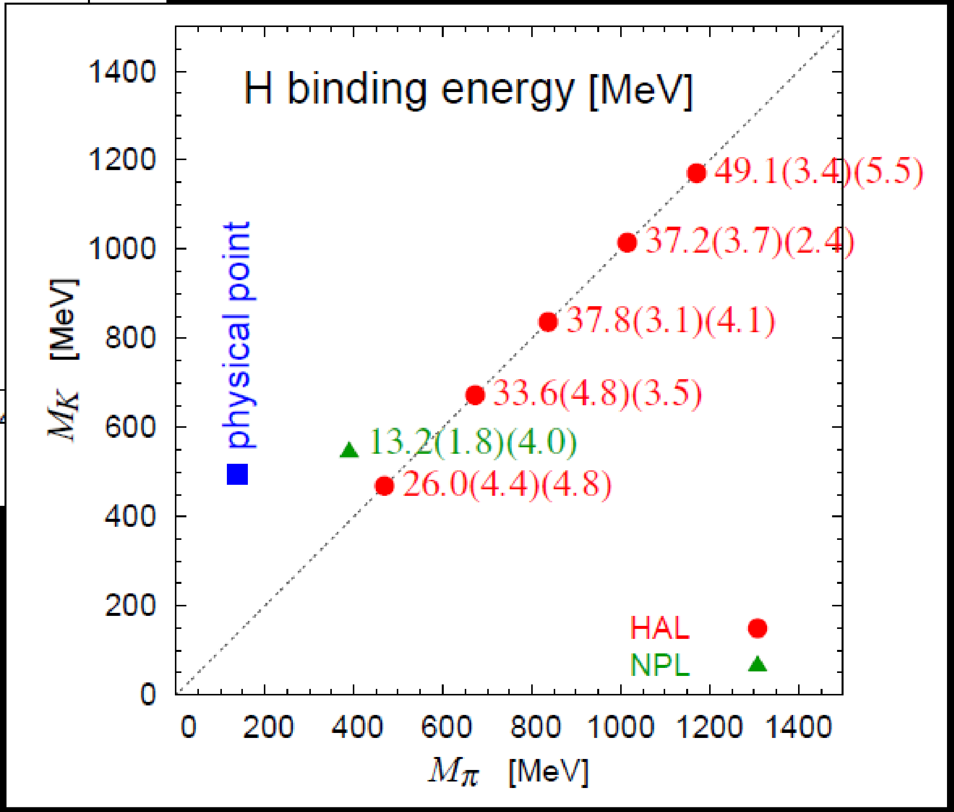
Repulsive core in NN channel

Attractive core in H channel

H-dibaryon (flavor SU(3) limit)







At physical point:

$M_{\Lambda\Lambda} < M_H < M_{\Xi N}$?

IHAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

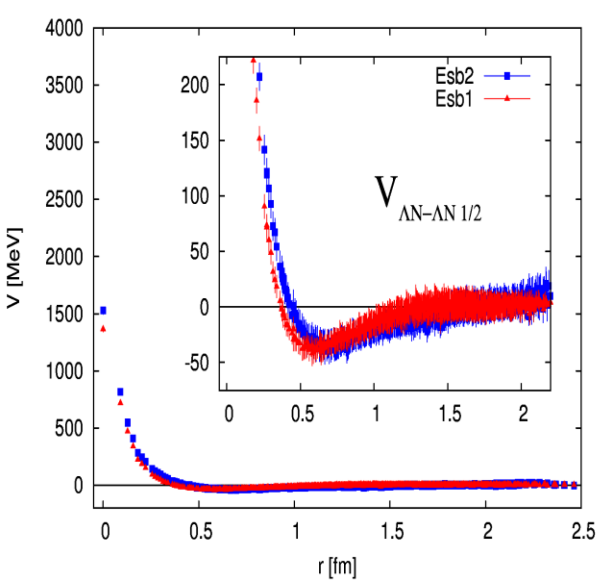
SU(3) breaking: coupled channel LQCD

Sasaki et al.
[HAL QCD Coll.] (2012)

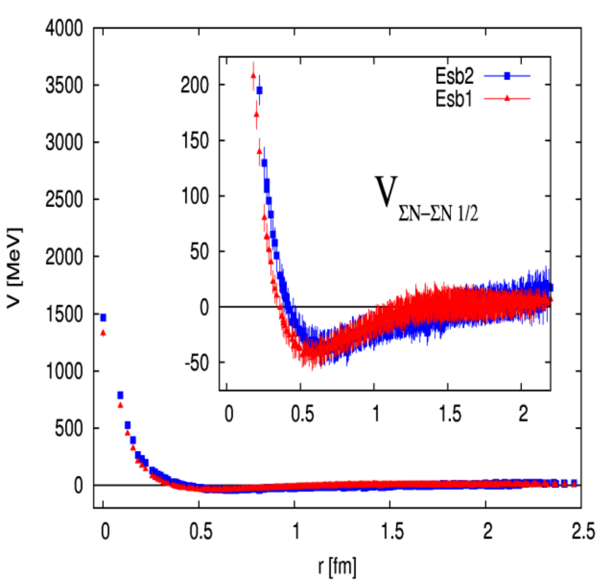
$$(k_n^2 + \nabla^2)\, \phi_n^{\alpha}(\vec{r}, t) = \int U(\vec{r}, \vec{r}')^{\alpha\beta} \phi_n^{\beta}(\vec{r}', t) d^3 r'$$

Example: S=-1, $^3S_1$, I=1/2 (m_π/m_K=0.89, 0.8)

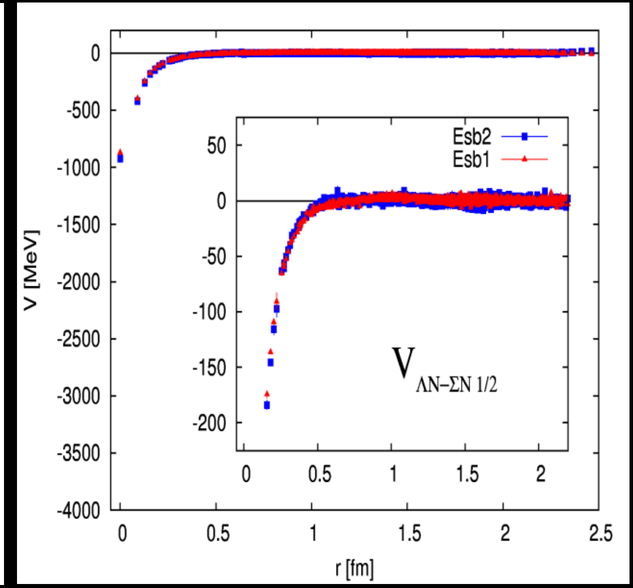
ΛN-ΛN



ΣN-ΣN

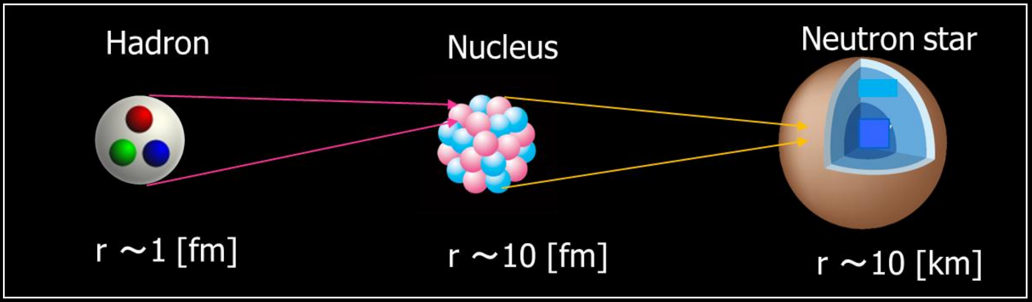


ΛN-ΣN



PACS-CS (2+1)-flavor config. L=2.9 fm

"Summary"

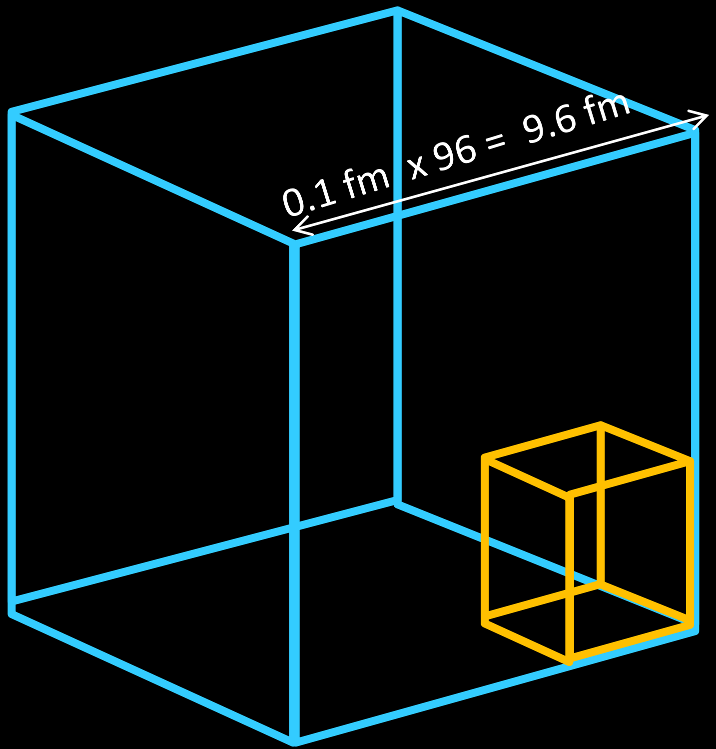


1. LQCD would replace phenomenological interactions in nuclear physics by 1st principle interactions

2. LQCD results together with nuclear many-body techniques would provide us with a firm basis of nuclear physics from QCD

3. Physical point simulations with a large volume (L=9.6 fm) is started at KEI computer

4. Direct comparison of YKU (Yamazaki-Kuramashi-Ukawa) approach and HAL QCD would become soon possible !



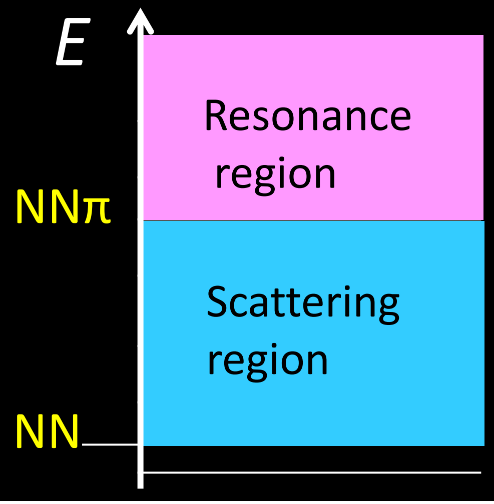
Latest Reviews

“Lattice Quantum Chromodynamical Approach to Nuclear Physics”
[HAL QCD Collaboration]
Progress of Theoretical and Experimental Physics, (2012) 01A105
http://www.oxfordjournals.org/our_journals/ptep/special_issue_a.html

- basic concepts of the non-local potential
- central, tensor, LS forces from lattice QCD
- coupled-channel YN, YY forces
- three-body force
- kaon-nucleon interaction
- going beyond the pion threshold

END

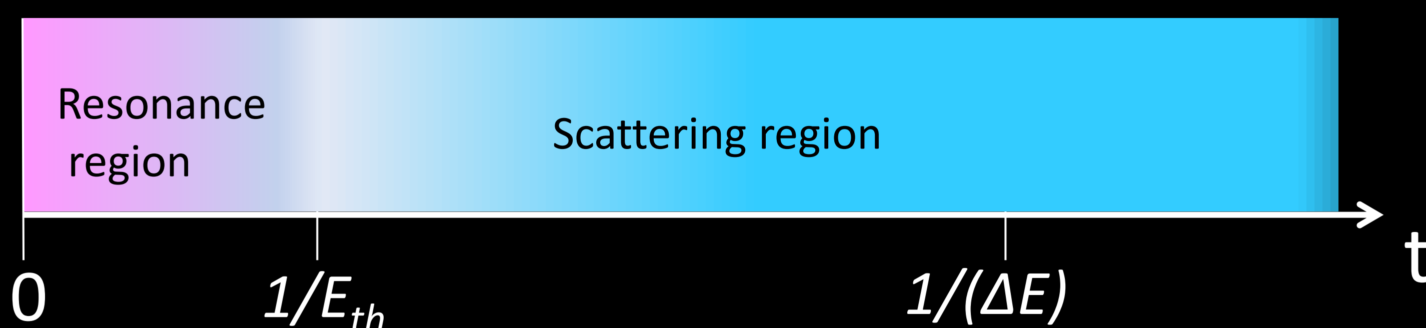
Multi-hadron Dilemma



$E_{th} \sim 135$ MeV

$\Delta E \sim 16\,(3.2/L)^2$ MeV

$$e^{-Ht} = \sum_n e^{-E_n t} |n\rangle\langle n|$$



$1/E_{th} \sim 1.5$ fm

$1/(\Delta E) \sim 12\,(L/3.2)^2$ fm

$$\Leftrightarrow \frac{\mathcal{S}}{\mathcal{N}} \sim \sqrt{N}\ \exp[-2(m_N - 1.5 m_\pi)t]$$

Solution of the Dilemma : Interaction kernel (=non-local potential)

$$\phi_n(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | n \rangle$$

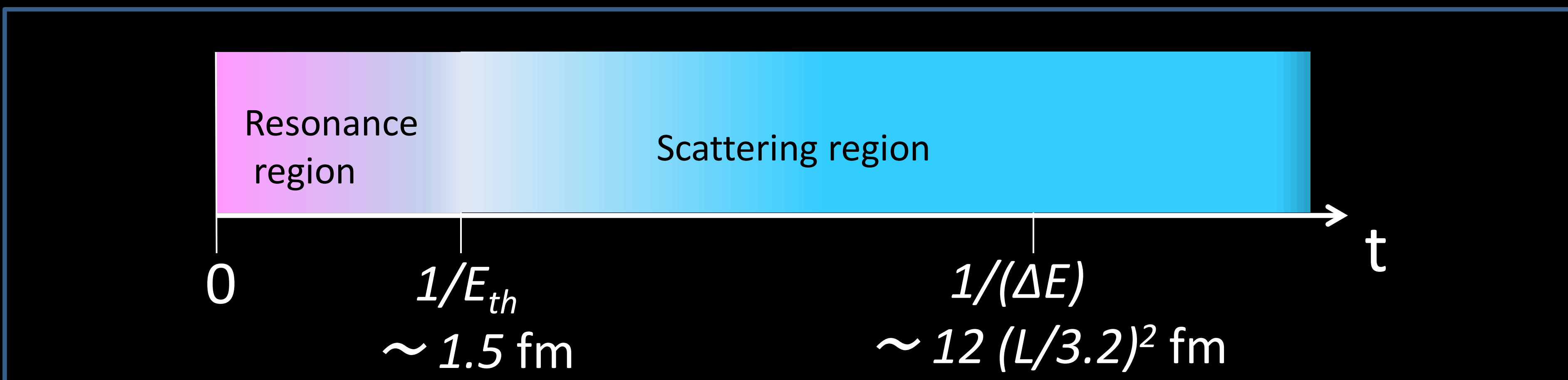
$$(k_n^2 + \nabla^2)\phi_n(\vec{r}) = \int U(\vec{r}, \vec{r}')\phi_n(\vec{r}') d^3 r'$$

Ishii, Aoki & Hatsuda,
PRL 99 (2007) 022001
PTP 123 (2010) 89

$$\phi(\vec{r}, t) = \sum_{n \le n_{\rm th}} \phi_n(\vec{r}) e^{-E_n t}$$

$$\left(-(\frac{1}{2}\partial_t)^2 - m_N^2 + \nabla^2 \right) \phi(\vec{r}, t) = \int U(\vec{r}, \vec{r}')\phi(\vec{r}', t) d^3 r'$$

Ishii et al. [HAL QCD Coll.],
Phys.Lett. B712 (2012) 437



- "Potential" is not an observable but is a nice tool to calculate observables
- "Potential" is volume insensitive (i.e. Lattice Friendly)