# Electroweak strength functions and tensor correlations in $^4$He

One day workshop on
ab initio study of nuclear structure and reaction
Icho Kaikan meeting room
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Wataru Horiuchi (Hokkaido Univ.)

Collaborators: Yasuyuki Suzuki (Niigata Univ., RIKEN Nishina Center)
Koji Arai (Nagaoka Nat'l College of Tech.)

# Tensor correlations in nuclear systems

- Tensor force : main component of the nuclear force
  → Important role for nuclear structures and reactions
  - Need wave function including explicit tensor correlations
    → *Ab initio* method
  - Observable?

- Contents
  - Correlated basis approach+ Complex scaling method
    Unifying bound- and unbound states in a single scheme
  - Tensor correlations in $^4$He: Strength functions
    - Photoabsorption reaction: E1 strength function
    - Spin-dipole (SD) strength functions
    - Sum rules ⇔ ground state correlations
  - Summary

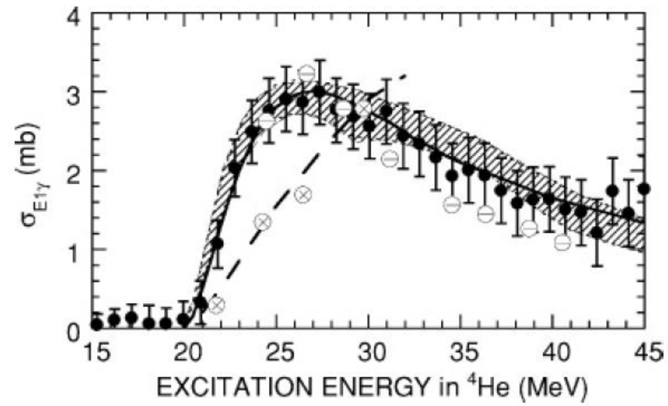
# Electroweak excitations of $^4$He

- Strength (response) function in nuclei
  - Resonant and continuum structures
- Low-energy photoabsorption reaction of $^4$He
  - Electric dipole excitation
  - Recent measurements
    - Peak ～27MeV

      S. Nakayama et al., PRC 76, 021305 (2007).

    - Peak ～30 MeV

      T. Shima et al., PRC 72, 044004 (2005).




Taken from S. Nakayama et al.
PRC 76, 021305 (2007).

- Excitation of light nuclei induced by the weak interaction
  - Neutrino-nucleus reaction (Gamow-Teller, Spin-dipole, etc.)

Reliable model is needed → *Ab initio* study

# Electroweak operators and spectrum

## Electric dipole (E1) $J^\pi T = 0^+0 \rightarrow 1^-1$

$$\mathcal{M}_{1\mu} = \sum_{i=1}^{4} \frac{e}{2}(1-\tau_{3_i})(\boldsymbol{r}_i - \boldsymbol{x}_4)_\mu$$

## Spin-dipole (SD) $J^\pi T = 0^+0 \rightarrow \lambda^-0, \lambda^-1$

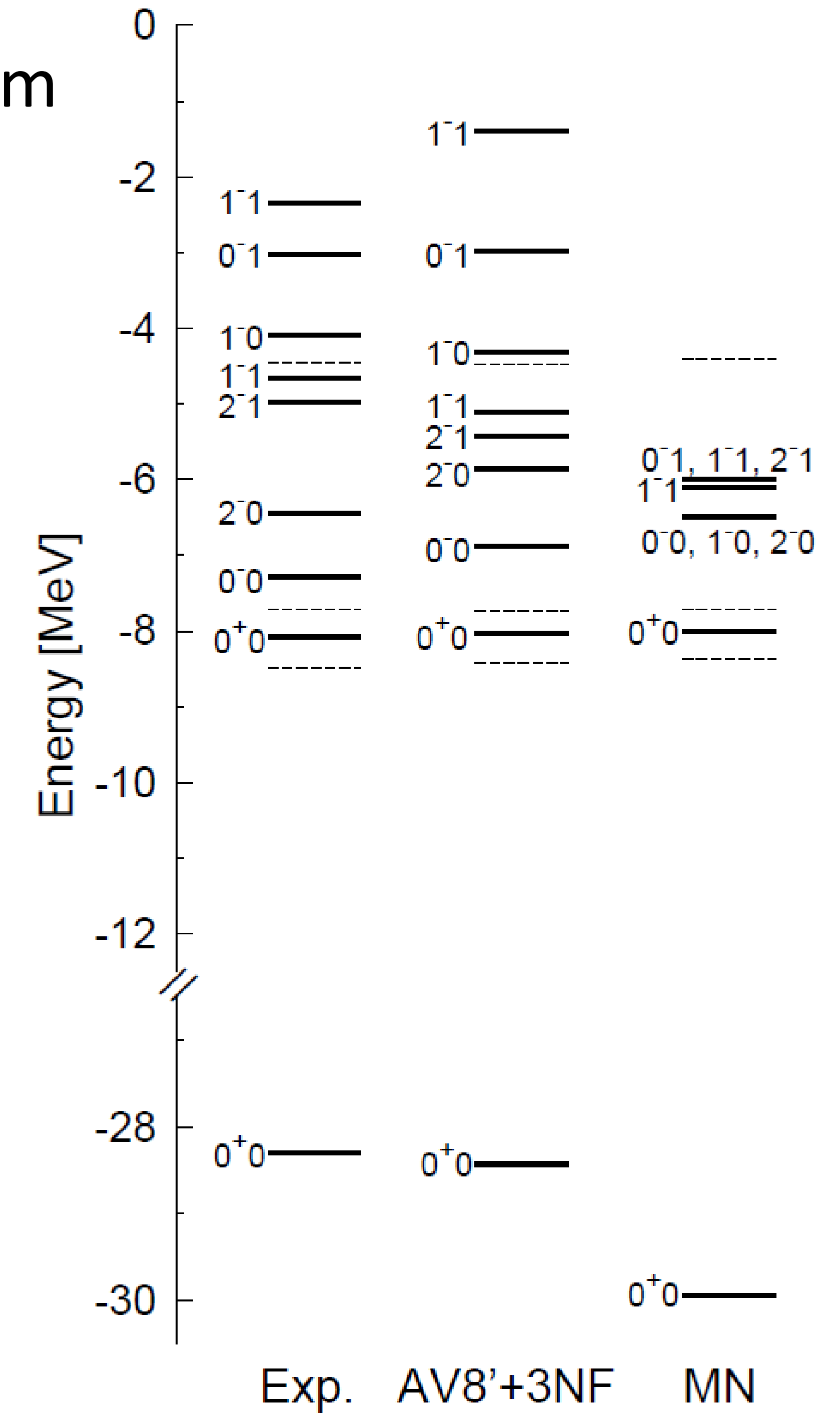
$$\mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{4} [(\boldsymbol{r}_i - \boldsymbol{x}_4)\times \boldsymbol{\sigma}_i]_{\lambda\mu}\, T_i^p$$

$$T_i^{\mathrm{IS}} = 1, \quad T_i^{\mathrm{IV0}} = \tau_z(i), \quad T_i^{\mathrm{IV}\pm} = t_\pm(i)$$

- $^4$He: seven negative parity states

  $J^\pi T = 0^-1, 1^-1, 2^-1, 0^-0, 1^-0, 2^-0$
- Degenerate levels only with central force

  → Non-central force is necessary



# Variational calculation for many-body system

Hamiltonian
$$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} v_{ijk}$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r) P_{1\pi} P_{2\pi} + V_t(r) S_{12} + V_b(r) \boldsymbol{L}\cdot\boldsymbol{S}$$

- Argonne v8 type interactions (AV8', G3RS); "bare" interaction
  central, tensor, spin-orbit
- Three-nucleon force (3NF) E. Hiyama et al. PRC70, 031001(R) (2002)
  → reproduce inelastic form factor of the first excited state of $^4$He.

Basis function

$$\Psi_{(LS)JM_JTM_T} = \mathcal{A}\left\{ \left[ \psi_L^{(\rm space)} \psi_S^{(\rm spin)} \right]_{JM_J} \psi_{TM_T}^{(\rm isospin)} \right\}$$

$$\psi_{SM_S}^{(\rm spin)} = \left| [\cdots[[[\tfrac{1}{2}\tfrac{1}{2}]_{S_{12}}\tfrac{1}{2}]_{S_{123}}]\cdots]_{SM_S} \right\rangle$$

$\psi_{LM}^{(\rm space)}$ : correlated Gaussian combined with two global vectors

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$F_{(L_1L_2)LM}(u_1, u_2, A, \boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}} A \boldsymbol{x}\right) [\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x}) \mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

# Correlated basis approach: global vector representation

Correlated gaussian combined with two global vectors Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$\phi^{\pi}_{(L_1L_2)LM_L}(A, u_1, u_2) = \exp(-\tilde{x}Ax)[\mathcal{Y}_{L_1}(\tilde{u}_1 x)\mathcal{Y}_{L_2}(\tilde{u}_2 x)]_{LM_L}$$

**x**: any relative coordinates (cf. Jacobi) $\mathcal{Y}_\ell(\boldsymbol{r}) = r^\ell Y_\ell(\hat{\boldsymbol{r}})$

$$\tilde{x}Ax = \sum_{i,j=1}^{N-1} A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j$$

$$\tilde{u}_i x = \sum_{k=1}^{N-1}(u_i)_k \boldsymbol{x}_k$$

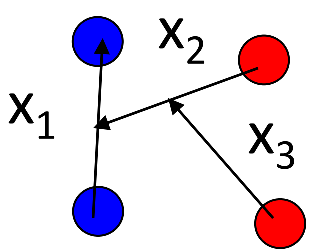
"bare" interaction can be used

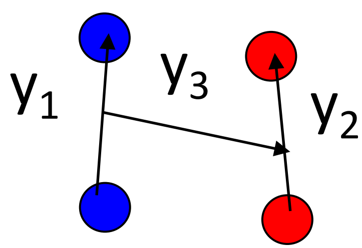
Some advantages

Formulation for *N* particle system

Analytical expression for matrix elements

No change of the form under any coordinate transformations




$$y = Tx \implies \tilde{y}By = \tilde{x}\tilde{T}BTx$$

$$\tilde{v}y = \widetilde{\tilde{T}v}x$$

Variational parameters *A, u* → Stochastic variational method

K. Varga and Y. Suzuki, PRC52, 2885 (1995).

The ground state energy of $^4$He reproduces the benchmark cal.

H. Kamada et al., PRC64, 044001 (2001)

# Photoabsorption cross section by the complex scaling method (CSM)

Photoabsorption cross section $\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$

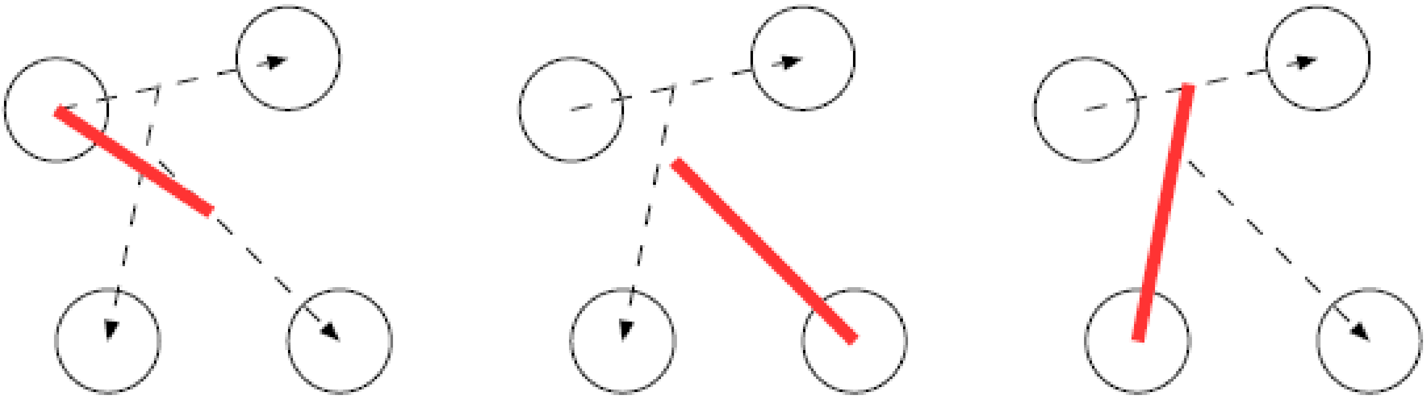
Strength function
$$S(E) = \mathcal{S}_{\mu f} |\langle \Psi_f | \mathcal{M}_{1\mu} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$
$$= -\frac{1}{\pi} \mathrm{Im} \sum_\mu \langle \Psi_0 | \mathcal{M}_{1\mu}^\dagger \frac{1}{E - H + E_0 + i\epsilon} \mathcal{M}_{1\mu} | \Psi_0 \rangle,$$

Complex rotation $U(\theta): \quad \boldsymbol{r}_j \to \boldsymbol{r}_j \mathrm{e}^{i\theta}, \quad \boldsymbol{p}_j \to \boldsymbol{p}_j \mathrm{e}^{-i\theta}$ Outgoing-wave B.C.

Expanded in $L^2$ basis
$$\Psi_\lambda^{JM\pi}(\theta) = \sum_i C_i^\lambda(\theta) \Phi_i(\boldsymbol{x})$$
$$H(\theta)\Psi_\lambda^{JM\pi}(\theta) = E_\lambda(\theta)\Psi_\lambda^{JM\pi}(\theta)$$

Strength function
$$S(E) = -\frac{1}{\pi} \sum_{\mu\lambda} \mathrm{Im} \frac{\widetilde{\mathcal{D}}_\mu^\lambda(\theta) \mathcal{D}_\mu^\lambda(\theta)}{E - E_\lambda(\theta) + i\epsilon}$$
$$\mathcal{D}_\mu^\lambda(\theta) = \left\langle (\Psi_\lambda^{JM\pi}(\theta))^* \right| \mathcal{M}_{1\mu}(\theta) \left| U(\theta)\Psi_0 \right\rangle$$
$$\widetilde{\mathcal{D}}_\mu^\lambda(\theta) = \left\langle (U(\theta)\Psi_0)^* \right| \widetilde{\mathcal{M}}_{1\mu}(\theta) \left| \Psi_\lambda^{JM\pi}(\theta) \right\rangle$$

# Configuration for final state




(i) $\Psi_f^{\rm sp} = \mathcal{A}\left[\Phi_0^{(4)}(i)\mathcal{Y}_1(r_1 - x_4)\right]_{1M}\eta^{(4)}_{T_{12}T_{123}10}$

- The ground state combined with $Y_1(r_1-x_4)$
- Complete set of isospin wave function(T=1)
- Basis and possible angular momentum couplings are included independently.

(ii)

3N: three-body cal.

3N-N: p-wave

(iii)

2N: two-body cal.

2N-N: p-wave

3N*-N: s-wave

**Explicit correlation of 3+1 and 2+1+1**

Note: The coherent E1 state $\sum_\mu \mathcal{M}_{1\mu}|0\rangle$ exhausts 100% of the non-energy-weighted sum rule.

**<u>Non energy weighted Sum rule</u>**

99.6%

# Total photoabsorption cross section

Photoabsorption cross section

$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

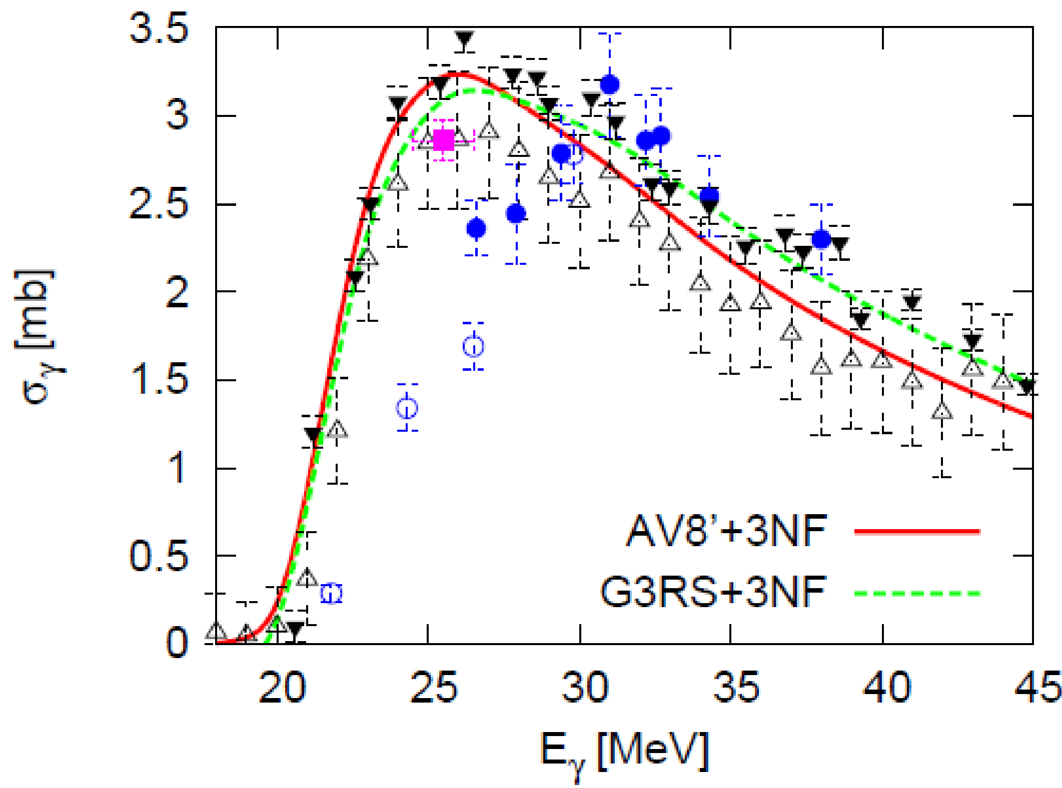
Interaction: AV8'+3NF, G3RS+3NF

3NF: E. Hiyama et al., PRC70, 031001(2004).

Comparison with the measurements

→ good agreement above 30 MeV

Disagree at the low energy with the data by Shima et al.




△ S. Nakayama et al., (2007)

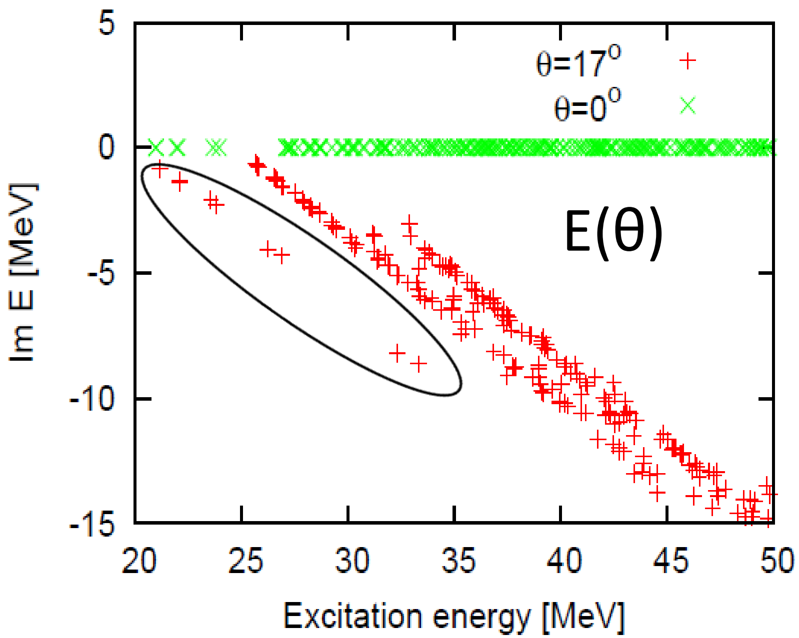
■ D.P. Wells et al. (1992)

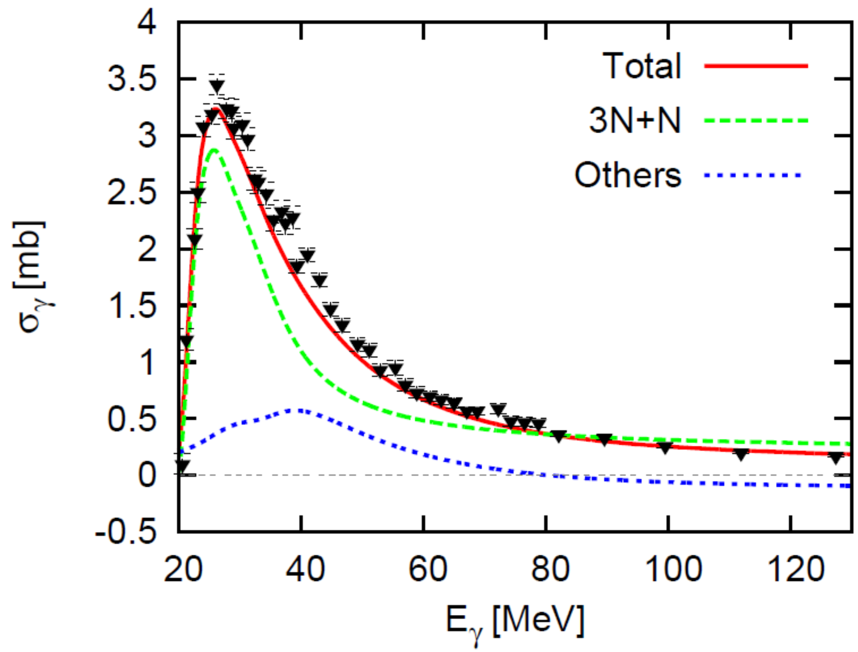
▼ Y. M. Arkatov et al.,(1974).

○ T. Shima et al., (2005).

● T. Shima et al., new measurement

# 3N+N cluster structure in the total photoabsorption cross section with CSM




▼ Y. M. Arkatov et al., Yad. Fiz. 19, 1172 (1974).

$$S(E) = \sum_{\lambda} S_\lambda = \sum_{\lambda \in 3NN} S_\lambda + \sum_{\lambda \notin 3NN} S_\lambda$$

3N+N configurations are dominant in the low-energy cross section

# Photonuclear sum rule

$$m_\kappa = \int_0^\infty E_\gamma^\kappa \sigma_\gamma(E_\gamma) dE_\gamma$$

κ=-1; Bremstrahlungs (non-energy weighted) sum rule

$$m_{-1} = \mathcal{G}\left(Z^2\langle r_p^2\rangle - \frac{Z(Z-1)}{2}\langle r_{pp}^2\rangle\right)$$

$$\mathcal{G} = 4\pi^2 e^2/3\hbar c$$

99.6%

κ=0; Thomas-Reiche-Kuhn (TRK) sum rule

$$m_0 = \mathcal{G}\frac{3NZ\hbar^2}{2Am_N}(1+K)$$

*K*: Enhancement factor

$$K = \sum_{q=1}^{8} K_q$$

$$V_q = \sum_{i<j} v^{(q)}(r_{ij})\mathcal{O}_{ij}^{(q)}$$

$$K_q = \frac{2Am_N}{3NZ\hbar^2 e^2}\frac{1}{2}\sum_\mu \langle \Psi_0 | [\mathcal{M}_{1\mu}^\dagger, [V_q, \mathcal{M}_{1\mu}]] | \Psi_0 \rangle$$

| | | AV8′+3NF | |
|---|---|---|---|
| $q$ | | $\langle V_q \rangle$ | $K_q$ |
| 1 | 1 | 17.39 | 0 |
| 2 | $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ | −9.59 | 0 |
| 3 | $\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ | −5.22 | 0.011 |
| 4 | $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ | −59.42 | 0.460 |
| | | (−12.51) | (0.187) |
| 5 | $S_{ij}$ | 0.75 | 0 |
| 6 | $S_{ij}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ | −70.93 | 0.574 |
| | | (−68.65) | (0.667) |
| 7 | $(\boldsymbol{L} \cdot \boldsymbol{S})_{ij}$ | 11.09 | 0 |
| 8 | $(\boldsymbol{L} \cdot \boldsymbol{S})_{ij}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ | −15.93 | 0.061 |
| | Total | −131.9 | 1.11 |

**Pion exchange potential is essential to account for the TRK sum rule**

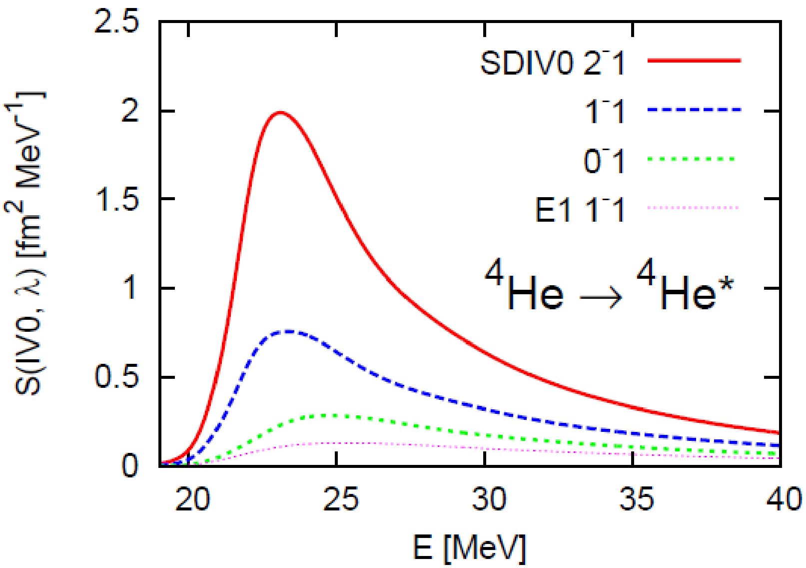
$$V_{OPEP} = \frac{1}{3}\frac{f^2}{\hbar c} m_\pi c^2 (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\left\{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) S_{12}\right\} \frac{e^{-\mu r}}{\mu r}$$

# Spin-dipole strength function

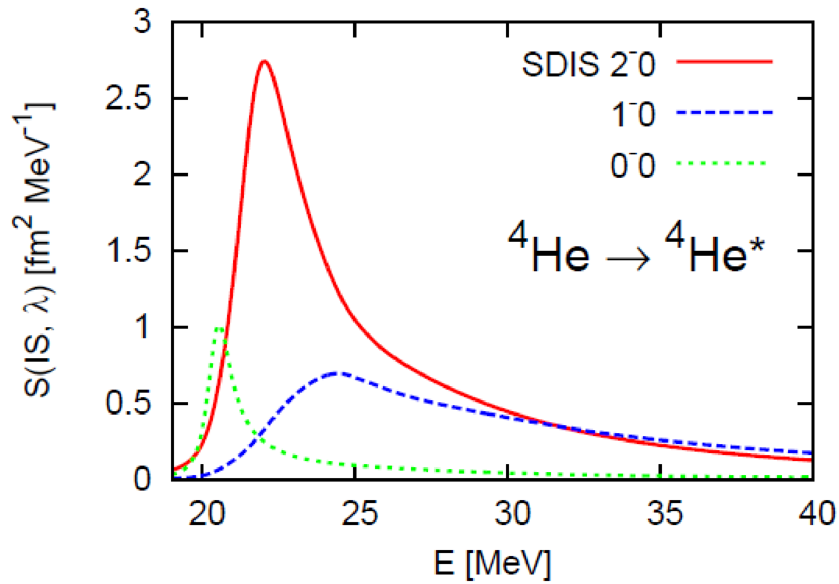
Spin-dipole operator $$\mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{4} [(\boldsymbol{r}_i - \boldsymbol{x}_4) \times \boldsymbol{\sigma}_i]_{\lambda\mu} \, \tau^p_{3i}$$

$$S(p, \lambda, E) = \mathcal{S}_{f\mu} \left| \langle \Psi_f | \, \mathcal{O}^p_{\lambda\mu} \, | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - E)$$

Isovector (IV0)




Isoscalar (IS)




- Relatively small decay widths of $0^-0$, $2^-0$ (0.84, 2.01 MeV)
- Resonant structure ⇔ Strength function

# Non energy weighted spin-dipole sum rule

$$m_0(p,\lambda) = \int_0^\infty S(p,\lambda,E)dE = \sum_{\kappa=0}^{2} U_{\lambda\kappa}\langle \mathcal{Q}^p_{(\kappa)0}\rangle$$

$$\mathcal{Q}^p_{(\kappa)0} = \sum_{i,j=1}^{A} \left([\boldsymbol{\rho}_i \times \boldsymbol{\rho}_j]_\kappa \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]_\kappa\right) T_i^{p\dagger} T_j^p,$$

$$\boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{x}_N$$

$$(U_{\lambda\kappa}) = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{5}{3} & \frac{5}{6} & \frac{1}{6} \end{pmatrix}$$

| IV0 | |
|---|---|
| $m_0(p,\lambda)$ | SR |
| 4.59 | 4.59 |
| 9.35 | 9.36 |
| 18.36 | 18.38 |

Sum rule ～100%

$(0s)^4$ : Ratio of SD SR 1 : 3 : 5

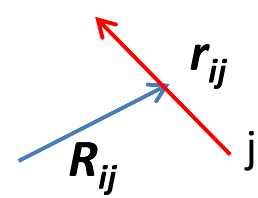
Cal (IV0):1.47 : 3 : 5.89 (κ=2 contribution)

cf. $$m_0(p,2) = \frac{5}{3}m_0(p,0) + \frac{1}{2}(5\langle \mathcal{Q}^p_{(1)0}\rangle - 3\langle \mathcal{Q}^p_{(2)0}\rangle)$$
$$\approx \frac{5}{3}m_0(p,0) - \frac{3}{2}\langle \mathcal{Q}^p_{(2)0}\rangle.$$

Strong evidence of the tensor correlation

Two nucleon relative coordinates

$$\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j, \quad \boldsymbol{R}_{ij} = \frac{\boldsymbol{r}_i + \boldsymbol{r}_j}{2} - \boldsymbol{x}_N$$




$$[\boldsymbol{\rho}_i \times \boldsymbol{\rho}_j]_{\kappa\mu} = \underline{-\frac{1}{4}[\boldsymbol{r}_{ij} \times \boldsymbol{r}_{ij}]_{\kappa\mu}} + [\boldsymbol{R}_{ij} \times \boldsymbol{R}_{ij}]_{\kappa\mu}$$
$$+ \frac{1}{2}(1-(-1)^\kappa)[\boldsymbol{r}_{ij} \times \boldsymbol{R}_{ij}]_{\kappa\mu},$$

**κ=2: tensor operator** $\left([\boldsymbol{r}_{ij} \times \boldsymbol{r}_{ij}]_2 \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]_2\right)$

From measurement

$$\langle \mathcal{Q}^p_{(\kappa)0}\rangle = \sum_{\lambda=0}^{2} U^{-1}_{\kappa\lambda} m_0(p,\lambda)$$

# Summary

- Four-body calculation for electroweak strength functions of $^4$He
- Correlated Gaussian with double global vectors

  + complex scaling method

- Electroweak (E1, SD) strength function
  - Photoabsorption reaction
  - SD strength function and spectrum

    Good agreement with experiments

    Future: Neutrino-nucleus reaction
- Sum rule
  - Photonuclear sum rules -> importance of the OPEP
  - SD sum rule -> tensor correlations in the ground state