RCNP研究会

第2回実証的原子核物理学研究会

2012年2月22日、23日

中性子過剰な炭素同位体の四重極集団性

Quadrupole Collectivity of Neutron-rich Carbon Isotopes

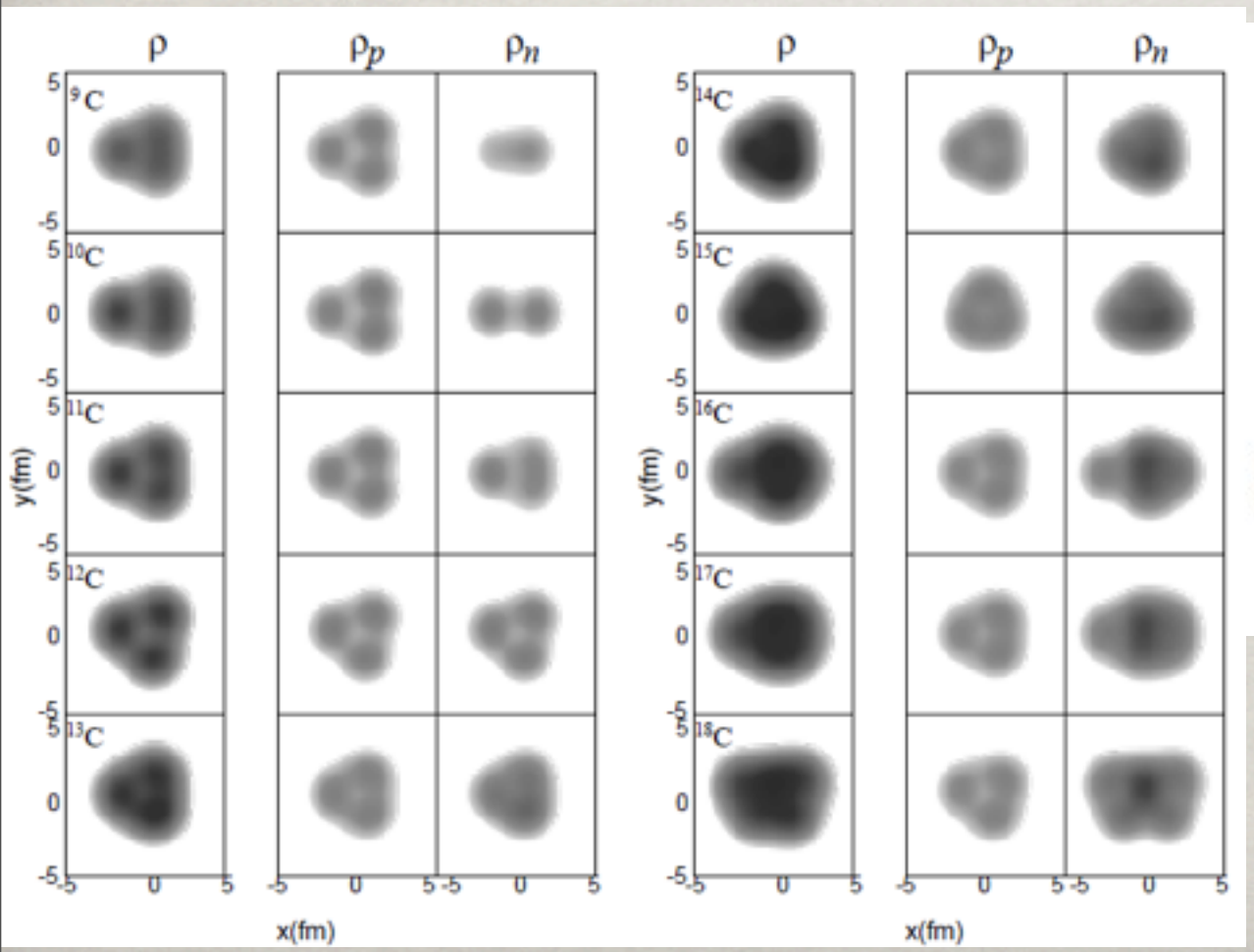
大阪大学・核物理研究センター

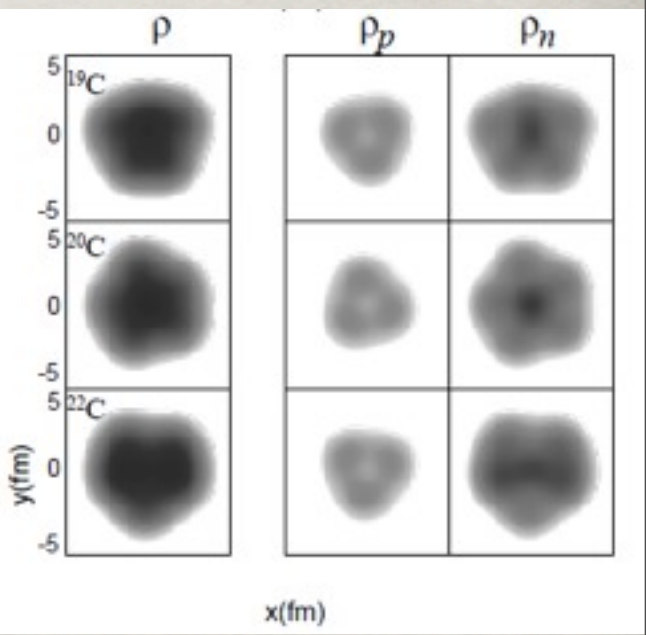
王 惠仁　ONG Hooi Jin (オン フイージン)

Progress of Theoretical Physics Supplement No. 142, 2001

Structure of Light Unstable Nuclei Studied with Antisymmetrized Molecular Dynamics

Yoshiko KANADA-EN'YO and Hisashi HORIUCHI*

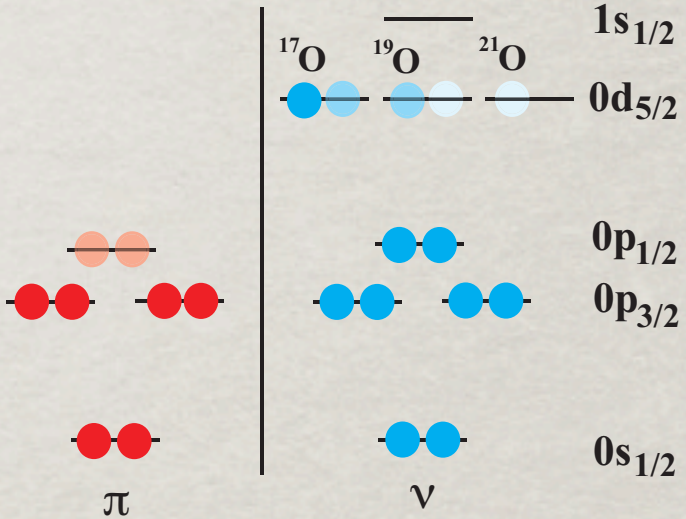




Ground-state spin-parities of even-odd Carbon and Oxygen isotopes

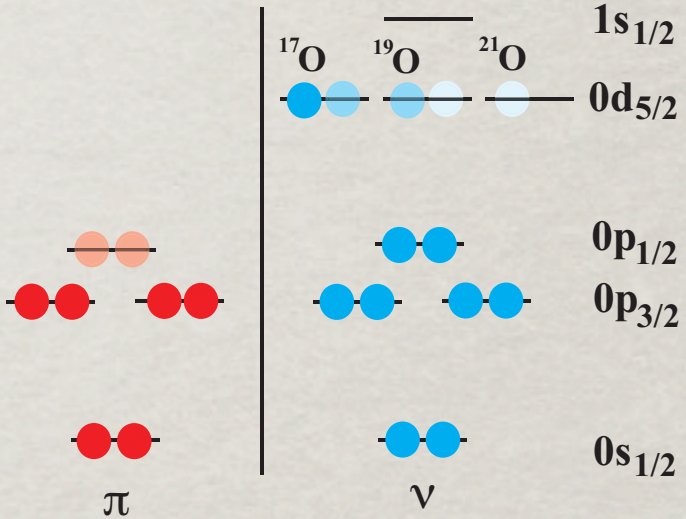
^{17}O ^{19}O ^{21}O

5/2+ 5/2+ (5/2+)

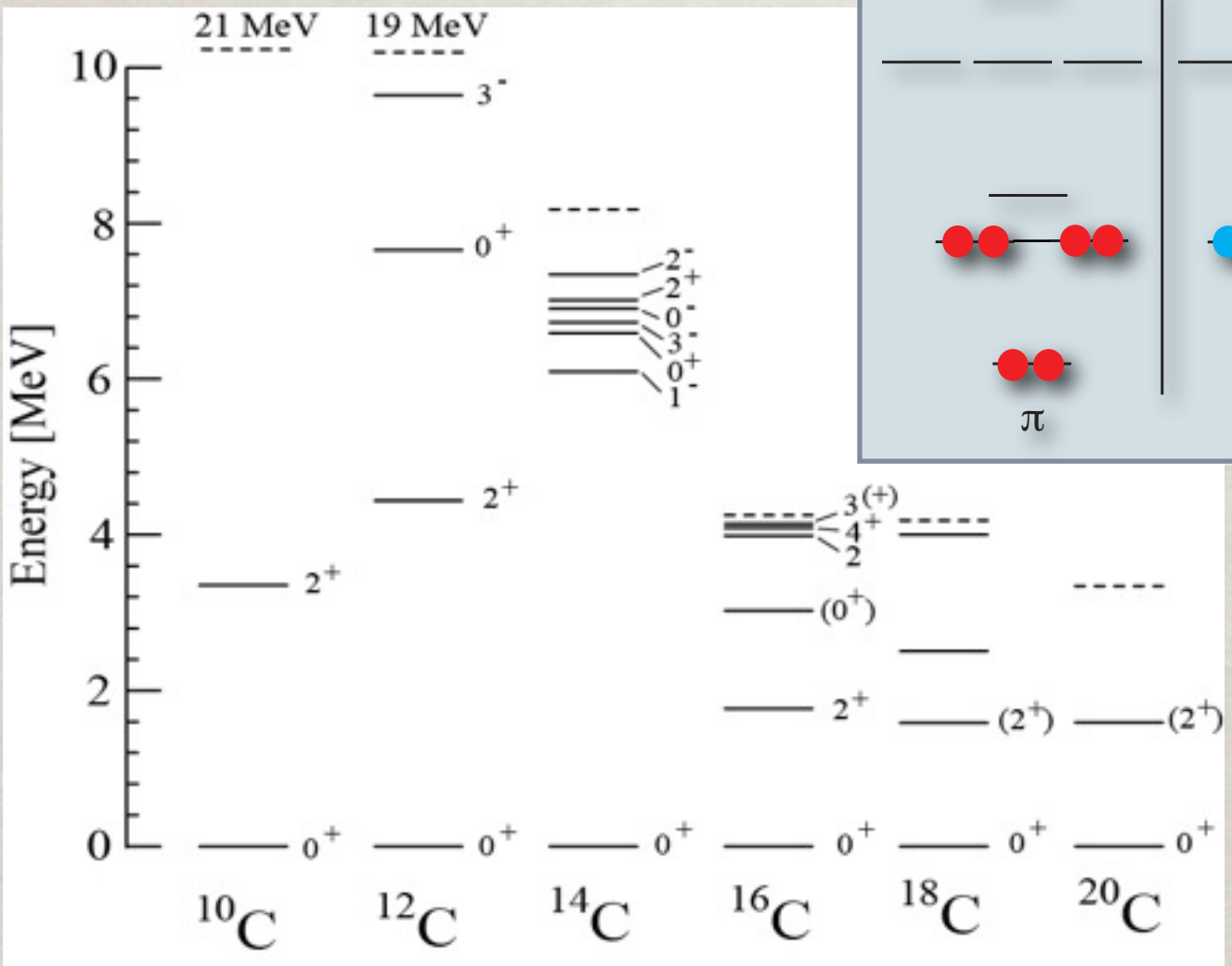


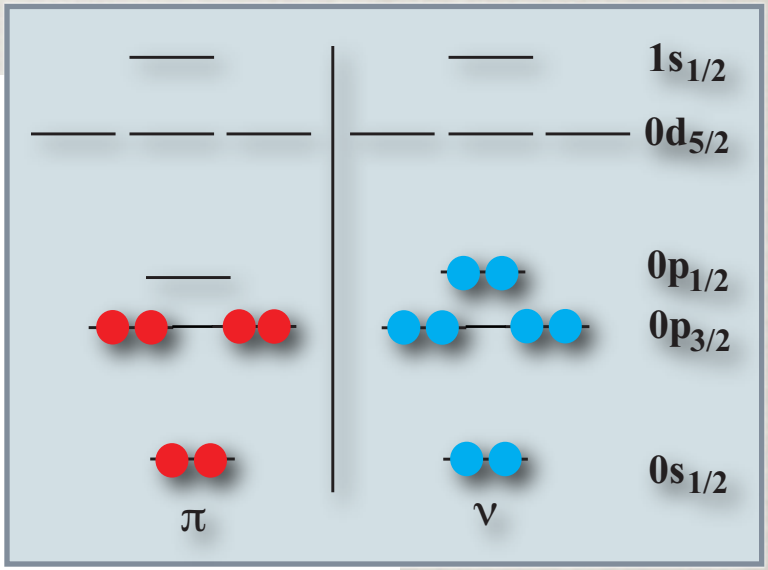
Ground-state spin-parities of even-odd Carbon and Oxygen isotopes

^{17}O	^{19}O	^{21}O
5/2+	5/2+	(5/2+)
⇕	⇕	⇕
^{15}C	^{17}C	^{19}C
1/2+	3/2+	(1/2+)



Level schemes of even-even Carbon isotopes





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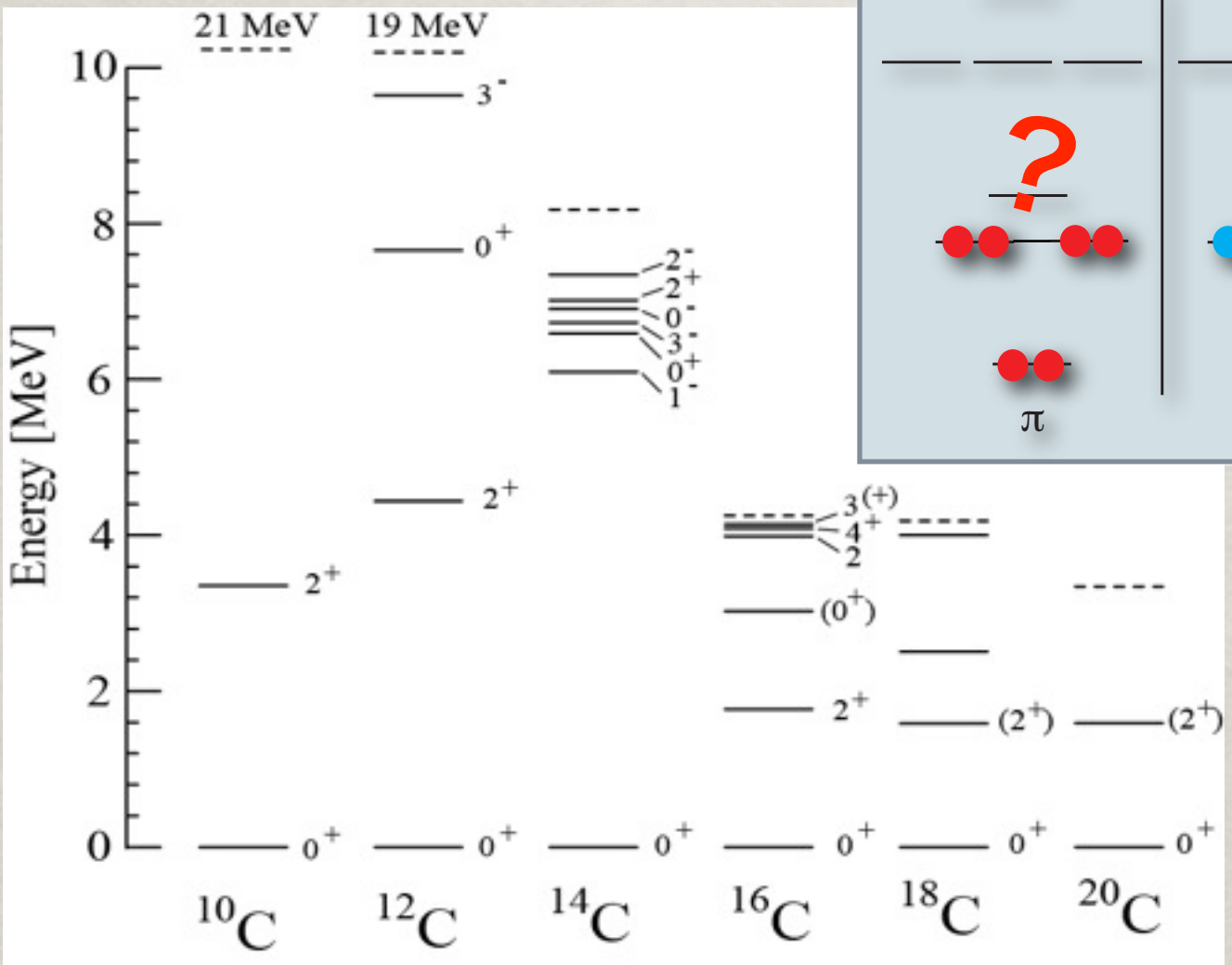

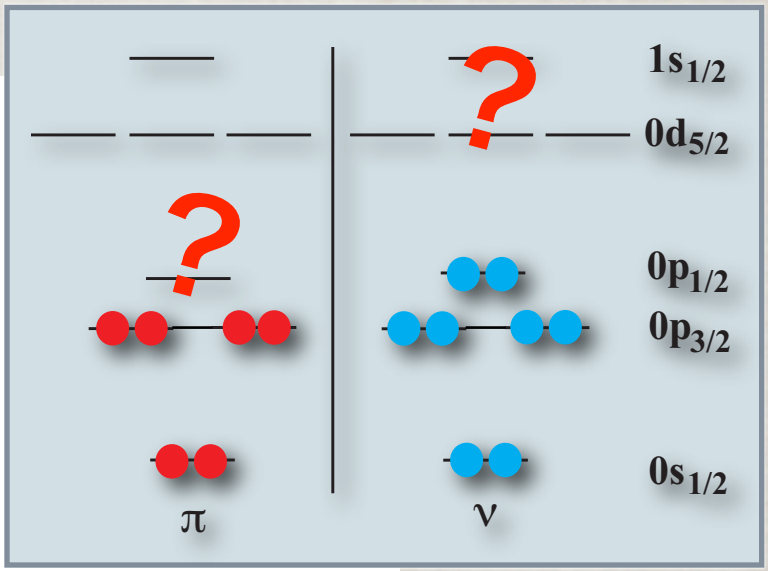

(Experiment) Papers on quadrupole collectivities of even-even Carbon isotope

- N. Imai *et al*, PRL 92, 062501 (2004)
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- M. Wiedeking *et al*, PRL 100, 152501 (2008)
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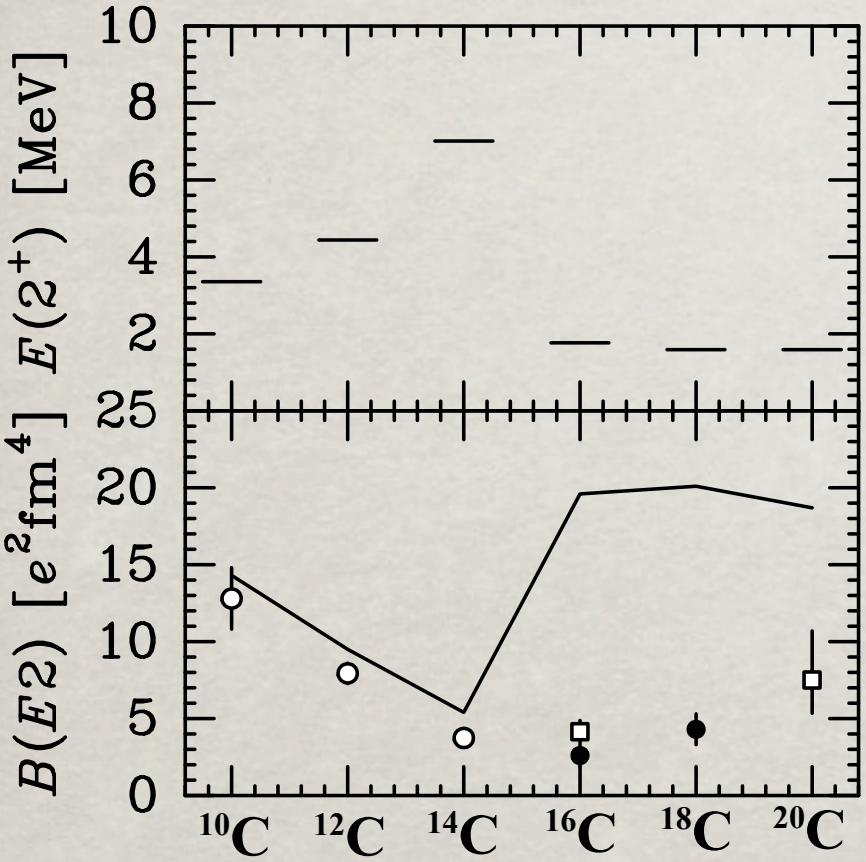

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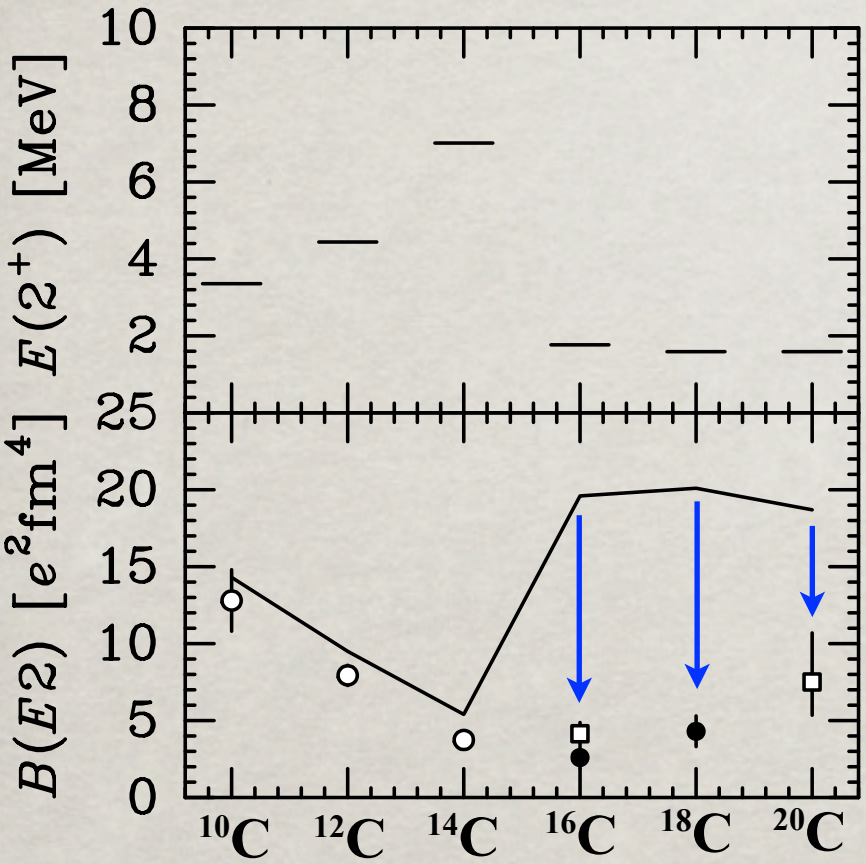

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Anomalously small *B(E2)* !!

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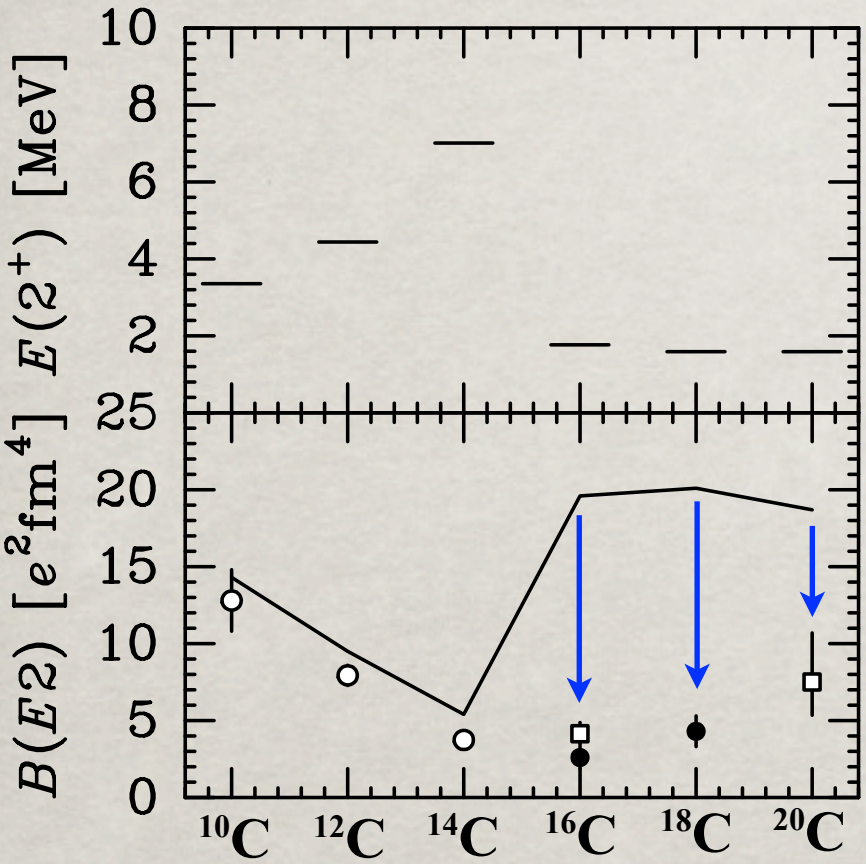

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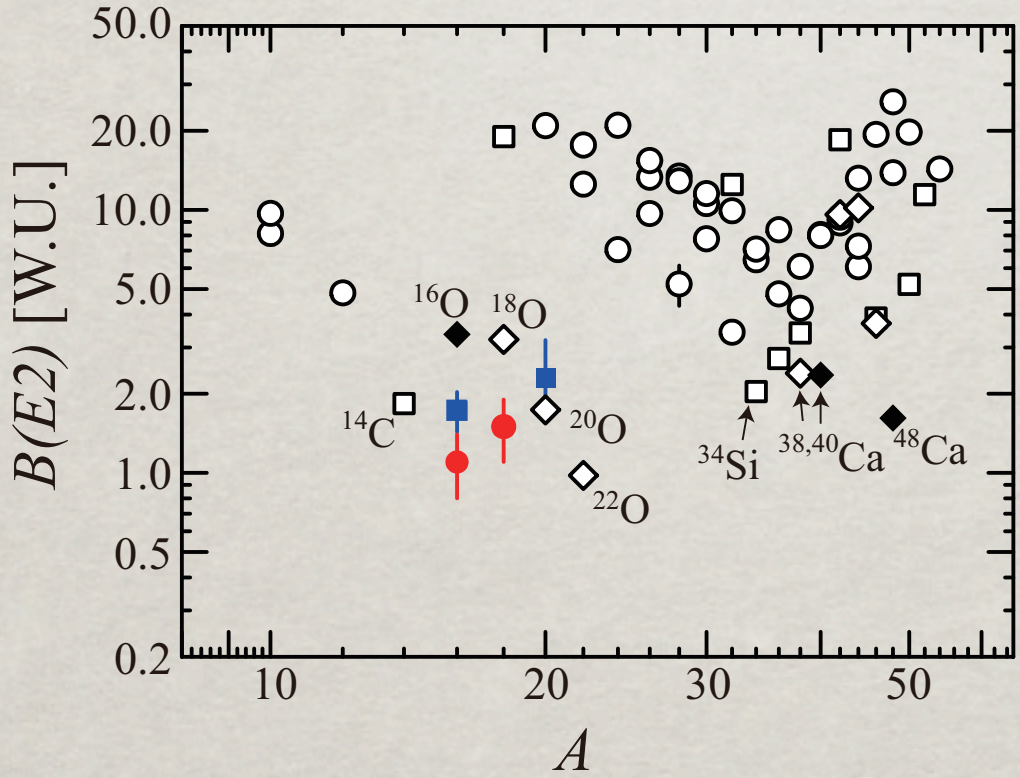
Anomalously small *B(E2)* !!

Small proton contribution towards quadrupole collectivity

Z=6 proton-closed shell?

B(*E*2) in Weisskopf Unit

$$B_{\mathrm{W}}(E2) = \frac{1}{4\pi}\left(\frac{3}{5}\right)^2 (1.2)^4 A^{4/3}$$



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Others S. Raman *et al*, ADNDT 78,1(2001)

Determination of proton/neutron contribution to quadrupole collectivities

- proton contribution <= *B*(*E2*) (Electromagnetic (EM) probe)
  (Lifetime measurement or Coulomb excitation)
- neutron contribution
  - isospin symmetry (e.g. *B*(*E2*) of mirror nucleus)
  - combination of TWO probes, e.g.
    - EM probe and (p,p')
    - EM probe and (α,α')
    - (π, π'), i.e. π^- and π^+ inelastic scattering

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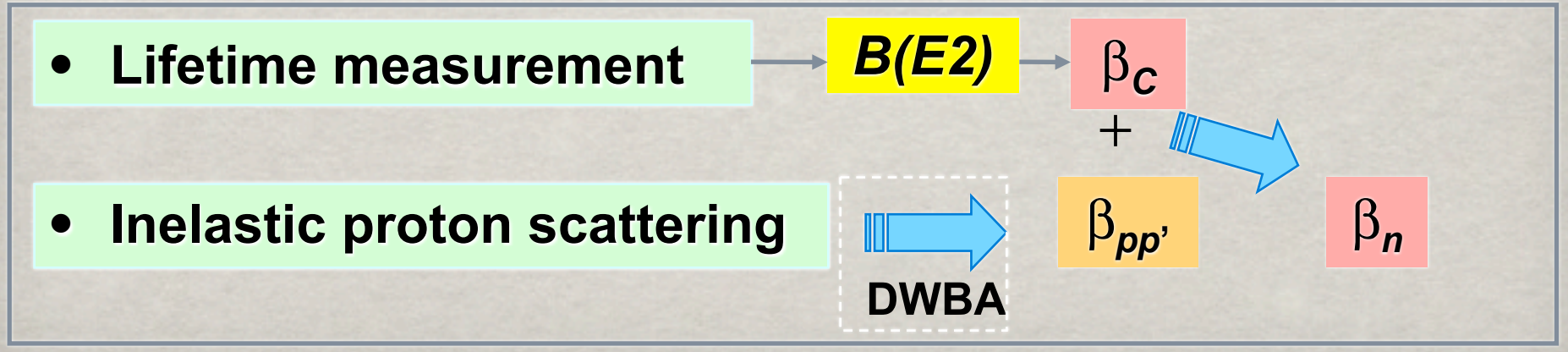


Bernstein's prescription

A. M. Bernstein et al, Phys. Letts. B 103,255(1981)

A. M. Bernstein et al, Comm. Nucl. Part. Phys. 11,203(1983)

Proton/Neutron transition matrix element (rank λ) :

$$M^{\lambda}_{n(p)}(I_i \to I_f) = \int \rho^{n(p)}_{fi}(r) r^{\lambda+2} dr$$

neutron(proton) transition density

In collective model for isoscalar excitation :

$$R = R_0(1 + \sum_M \beta_L / \sqrt{2L+1} Y^L_M(\hat{r})) \quad \text{and} \quad \rho_{fi}(r) = \frac{\beta_L R_0}{\sqrt{2L+1}} \frac{d\rho_i}{dr}$$

$$\frac{M_n}{M_p} = \frac{N}{Z}\frac{\delta_n}{\delta_p}$$

For hadron probe F,

$$\delta_F \propto \frac{(b^F_p M_p + b^F_n M_n)}{(b^F_p Z + b^F_n N)}$$

Combined with EM probe,

$$\frac{M_n}{M_p} = \frac{b^F_p}{b^F_n}\left(\frac{\delta_F}{\delta_{EM}}\left(1 + \frac{b^F_n N}{b^F_p Z}\right) - 1\right)$$

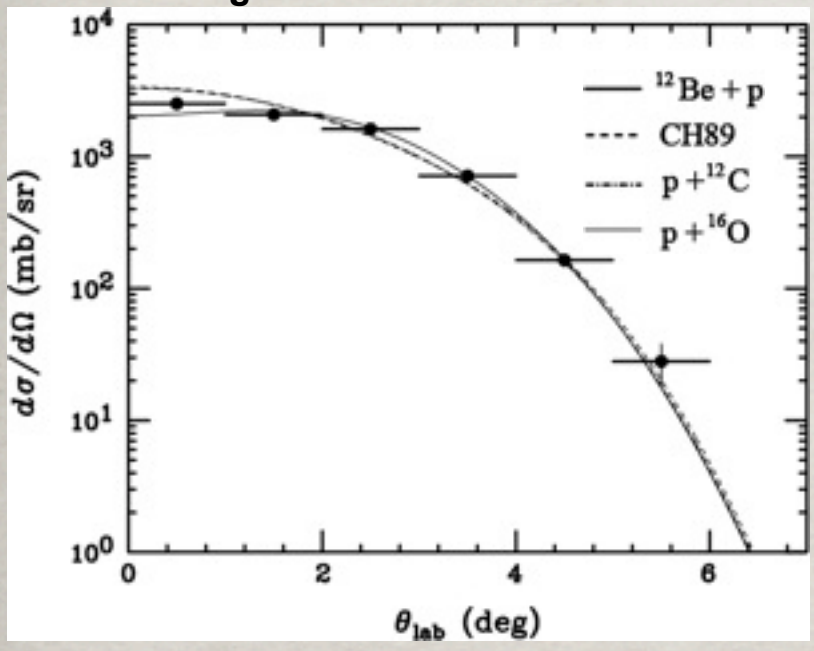
Table 1
Ratios of external parameters for various probes [see eq. (2)].

External field	Energy	b^F_n/b^F_p
EM	–	0
p_n [a]	–	∞
p	10–50 MeV	≈3 [b]
n	10–50 MeV	≈1/3
π^-	160–200 MeV	≈3
π^+	160–200 MeV	≈1/3
p	0.8 GeV	0.83 [c]
p	1 GeV	0.95 [d]
α	All	1

Determination of $\beta_{pp'}$ and $\delta_{pp'}$

from inelastic proton-scattering cross section

Angular distribution of $d\sigma/d\Omega$



H.J. Ong *et al*, PRC 73, 024610 (2006)

- **Phenomenological analysis (DWBA calculation)**

Optical Potential	$\beta_{pp'}$	$\delta_{pp'}$ (=$\beta_{pp'}R$)[fm]
CH89 [1)]	0.476(37)	1.39(11)
p+^{16}O [2)]	0.440(33)	1.26(9)
p+^{12}C [2)]	0.531(42)	1.47(12)
p+^{12}Be [3)]	0.435(32)	1.62(12)

1) R. L. Varner et al, Phys.Rep.201,57(1991)

2) C. M. Perey, F. G. Perey, At. Data Nucl. Data Tables 17,1(1976)

3) A. A. Korsheninnikov et al, PLB343,53(1995)

$\beta_{pp'}$ = 0.47(5); $\delta_{pp'}$ = 1.44(17) [fm]

Extracting neutron matrix element of ^{16}C (1)

Inelastic proton scattering on ^{16}C and ***B*(*E*2) measurement**

$$\frac{M_n}{M_p} = \frac{b_p}{b_n}\left(\frac{\delta_{pp'}}{\delta_{\mathrm{EM}}}\left(1+\frac{b_n N}{b_p Z}\right)-1\right),\ N = 10,\ Z = 6 \text{ for } {}^{16}\mathrm{C}$$

$\delta_{pp'}$ = 1.44±0.17 [fm] ; $b_n / b_p = 3$

δ_C = 0.41±0.06 [fm] ; $b_n = 0$, $b_p = 1$

N.Imai *et al*, PRL 92, 062501 (2004)

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$$\left|\left|\frac{M_n / M_p}{N / Z}\right|\right| = 4.0 \pm 0.8 \quad >> 1$$

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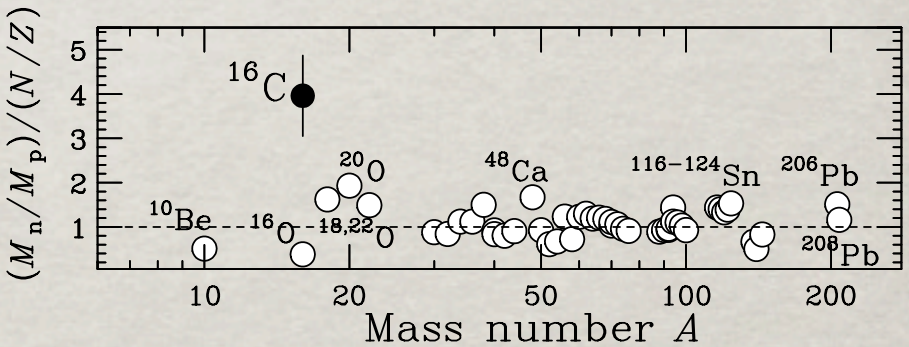

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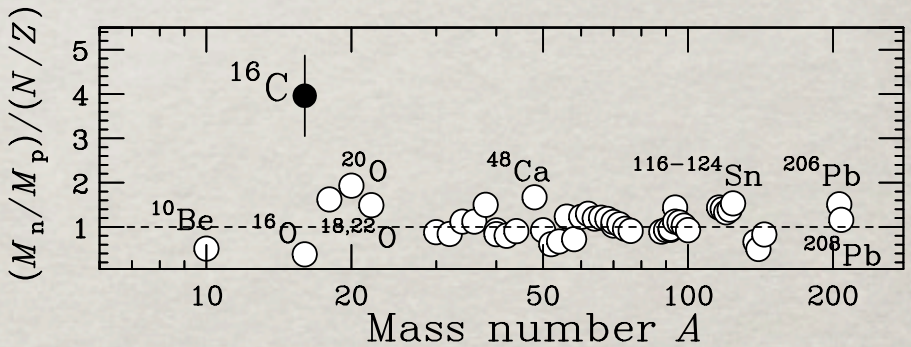
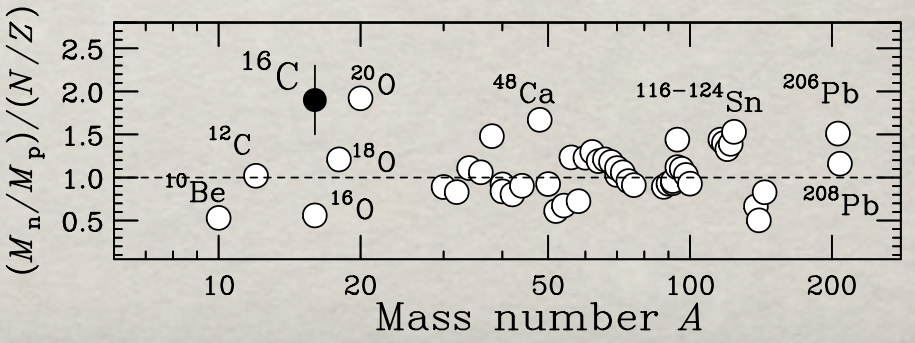


With latest *B*(*E*2) value

δ_C = 0.85±0.11 [fm]

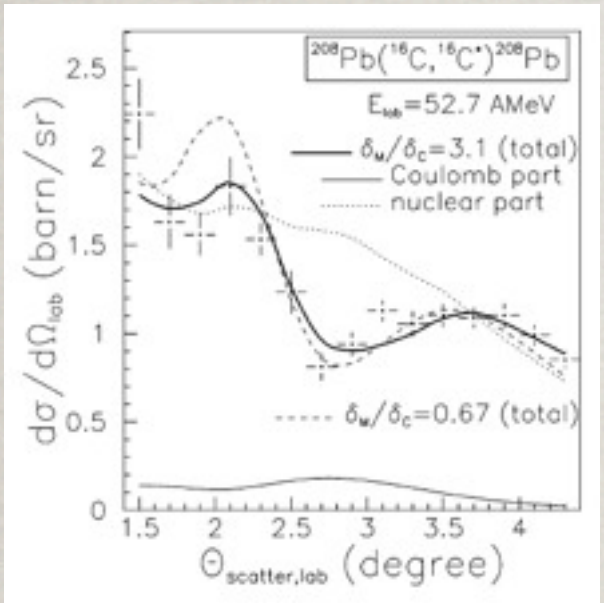
$$\left|\left|\frac{M_n / M_p}{N / Z}\right|\right| = 1.9 \pm 0.4 \quad >> 1$$

H.J. Ong *et al*, PRC 78, 014308 (2008)



Extracting neutron matrix element of ^{16}C (2)

Coulomb-nuclear interference (^{16}C+^{208}Pb)



- Determination of δ_M/δ_{EM} from inelastic-scattering cross section
- Applying Bernstein's prescription

$$\frac{M_n}{M_p} = \frac{b_p^{\mathrm{Pb}}}{b_n^{\mathrm{Pb}}}\left(\frac{\delta_{\mathrm{M}}}{\delta_{\mathrm{EM}}}\left(1+\frac{b_n^{\mathrm{Pb}}N}{b_p^{\mathrm{Pb}}Z}\right)-1\right)$$

with assumptions

$$b_n^{\mathrm{Pb}} = b_p^n\frac{Z}{A}+b_n^n\frac{N}{A} \qquad b_p^{\mathrm{Pb}} = b_p^p\frac{Z}{A}+b_n^p\frac{N}{A}$$

$$\Rightarrow \left|\left|\frac{M_n/M_p}{N/Z}\right|\right| = 4.6\pm1.0$$

Z. Elekes *et al.*, Phys. Lett. B 586, 34 (2004)

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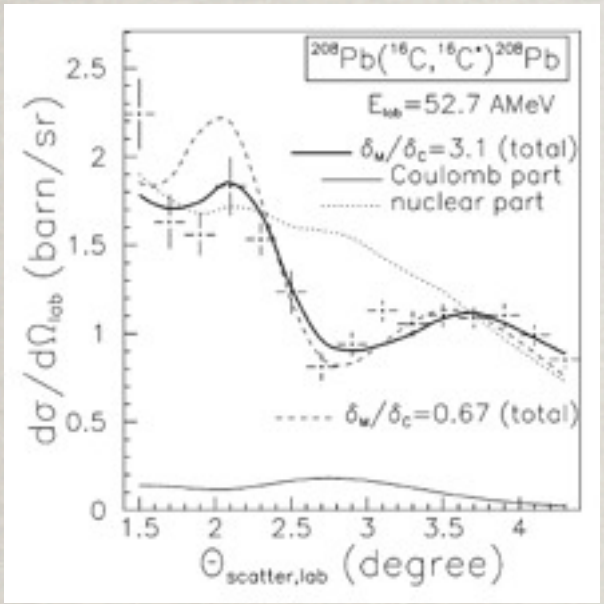

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Z. Elekes *et al.*, Phys. Lett. B 586, 34 (2004)

Revision: Coulomb-nuclear interference and (p,p')

$$\left(Z\cdot b_p^{Pb}+N\cdot b_n^{Pb}\right)\cdot\delta_M^{Pb} = N\cdot b_n^{Pb}\cdot\delta_n + Z\cdot b_p^{Pb}\cdot\delta_p, \quad (1)$$

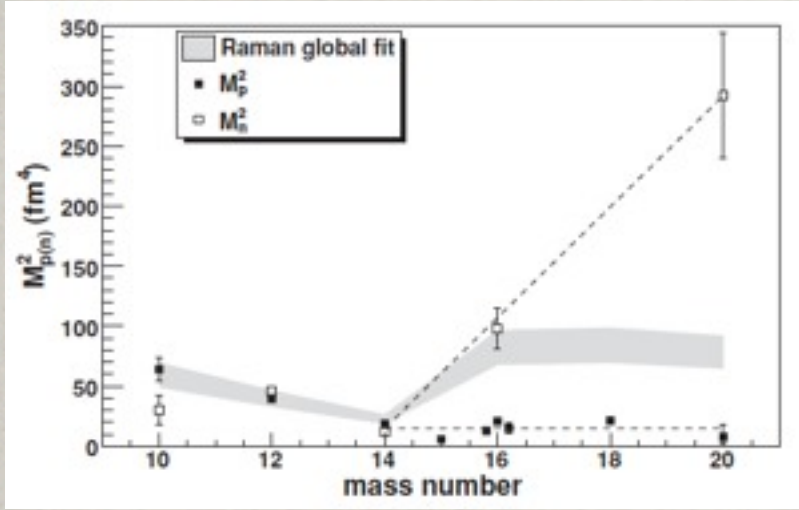
$$\left(Z\cdot b_p^{pp}+N\cdot b_n^{pp}\right)\cdot\delta_M^{pp} = N\cdot b_n^{pp}\cdot\delta_n + Z\cdot b_p^{pp}\cdot\delta_p, \quad (2)$$

$$\left|\left|\frac{M_n/M_p}{N/Z}\right|\right| = 2.5\pm0.4$$

Z. Elekes *et al.*, Phys. Rev. C 78, 027301 (2008)

Extracting proton/neutron matrix elements of ^{20}C

Inelastic proton scattering on ^{20}C and **Coulomb-nuclear interference** (^{20}C+p) (^{20}C+^{208}Pb)



$B(E2) < 3.7\ e^2\text{fm}^4$

Z. Elekes *et al.*, Phys. Rev. C 79, 011302(R) (2009)

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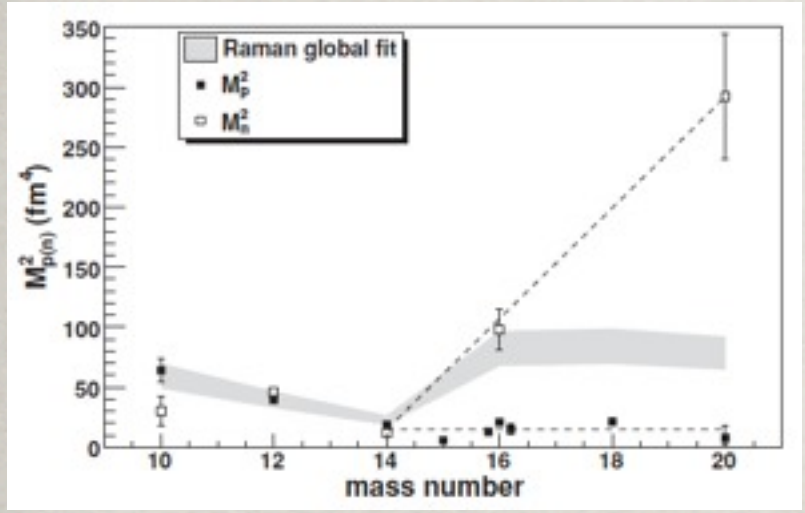
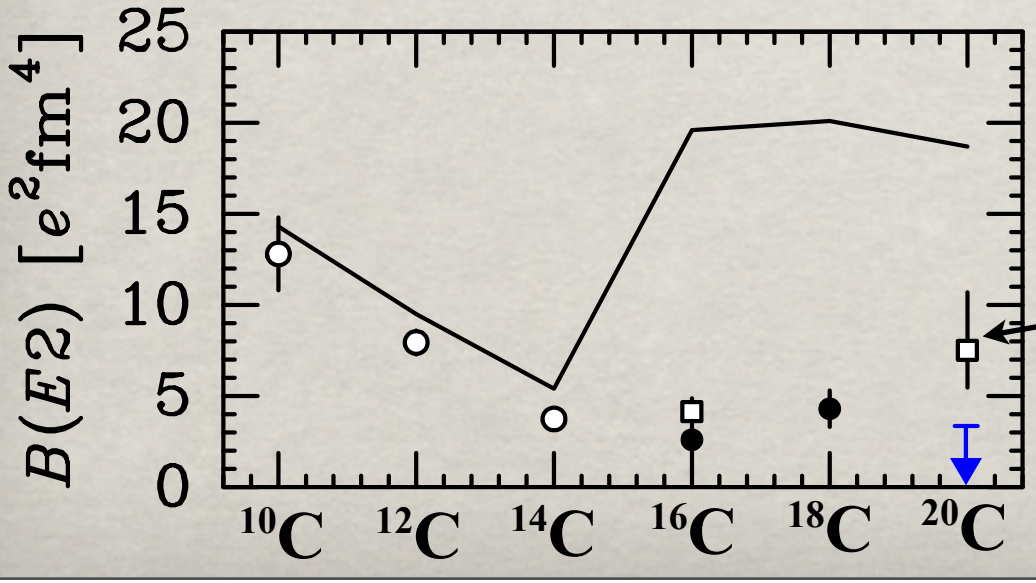


$B(E2) < 3.7\ e^2\text{fm}^4$

Z. Elekes *et al.*, Phys. Rev. C 79, 011302(R) (2009)



Inconsistent!!

lifetime measurement

M. Petri *et al*, PRL 107, 102501 (2011)

Questions (Problems)

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- Is Coulomb-nuclear interference plausible?

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- Is Coulomb-nuclear interference plausible?
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  In deriving Bernstein's prescription, the radial dependance of transition densities for neutron and proton have been assumed to be the same.
- How should we treat b_n and b_p in unstable nuclei?
- Is microscopic calculation the only way out?

Summary

- $B(E2)$ values for $2_1^+ \rightarrow 0_{gs}^+$ of even-even Carbon isotopes have been determined up to ^{20}C

- $B(E2)$ values for $^{16,18,20}C$ are small compared to the homogeneous quantum liquid-drop model

  ➡ relatively “small” proton contribution

- Determination of neutron contribution:

  ➡ many problems remaining

RIKEN-R344n Collaboration

“Neutron-dominant quadrupole collective motion in ^{16}C”

Physical Review C 73, 024610(2006)

H.J. Ong[1], N. Imai[2], N. Aoi[2], H. Sakurai[1], Zs. Dombrádi[3], A. Saito[4], Z. Elekes[3],
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M. Ishihara[2], H. Iwasaki[1], S. Kanno[4], S. Kawai[4], T. Kubo[2], K. Kurita[4],
Y. U. Matsuyama[4], S. Michimasa[6], T. Minemura[2], T. Motobayashi[2], M. Notani[6],
S. Ota[7], H. K. Sakai[4], S. Shimoura[6], E. Takeshita[4], S. Takeuchi[2], M. Tamaki[6],
Y. Togano[4], K. Yamada[4], Y. Yanagisawa[2], and K. Yoneda[2]

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[4]RIkkyo Univ. Japan,

[5]IPN, France,

[6]CNS, Univ. of Tokyo, Japan,

[7]Univ. of Kyoto

RIKEN-R376n Collaboration

“Lifetime measurements of first excited states in ^{16,18}C”

Physical Review C 78, 014308(2008)

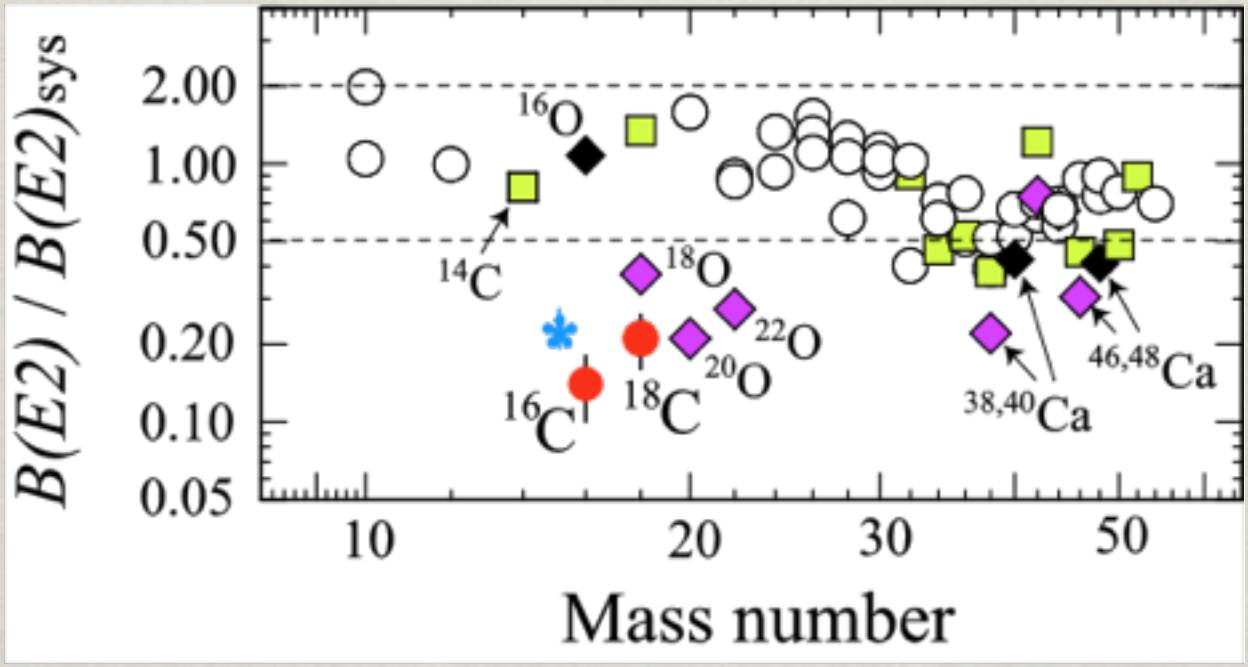
RIKEN	H. Sakurai, S. Takeuchi, N. Aoi, H. Baba, S. Bishop, M. Ishihara, T. Kubo, T. Motobayashi, Y. Yanagisawa
KEK	N. Imai
Tokyo U.	D. Suzuki, T. Nakao, H. Iwasaki, T. K. Onishi, M. K. Suzuki, Y. Ichikawa
CNS, Tokyo U.	S. Ota
Rikkyo U.	Y. Togano, K. Kurita
Titech	Y. Kondo, T. Nakamura, T. Okumura

Is homogeneous liquid-drop model valid for ^{16,18}C ?

- Global Systematics:

$$B(E2)_{\mathrm{sys}} = (5140 \pm 900) E^{-1} Z^{2} A^{-2/3}$$

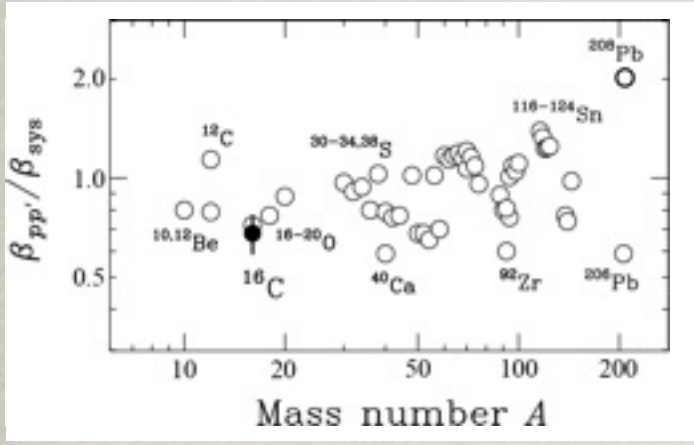
S. Raman *et al*, ADNDT 78,1(2001)



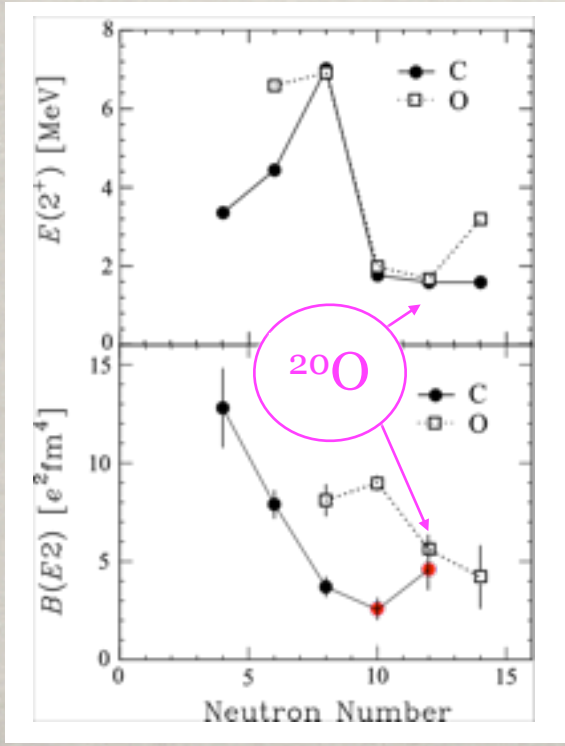
Data :

^{28}Ne	**H. Iwasaki *et al*, PLB 620,118(2005)**
* (^{16}C)	**M. Wiedeking *et al*, PRL 100, 152501 (2008)**
Others	**S. Raman *et al*, ADNDT 78,1(2001)**

Neutron-dominant?



H.J. Ong *et al*, PRC 73, 024610(2006)



- For ^{16}C
  - $\beta_{pp'}$ ⇨ "normal"
  - Small *B(E2)* ⇨ small proton contribution

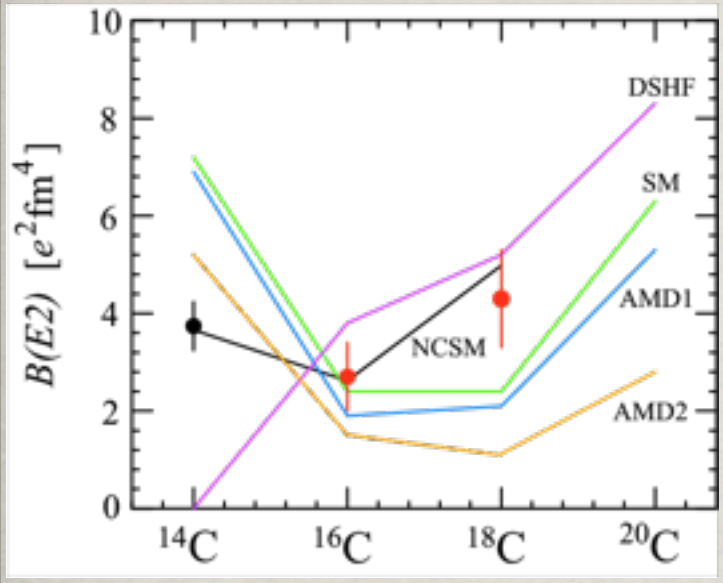
N. Imai *et al*, PRL 92,062501(2004)
H.J. Ong *et al*, PRC 73, 024610(2006)
Z. Elekes *et al*, PLB586,34(2004)

- For ^{20}O
  - Low $E(2^+)$
  - Small *B(E2)*
- $\beta_{pp'}$ + *B(E2)* ⇨ 2^+_1 state is neutron dominant

J. K. Jewell *et al*, PLB 454,181(1999);
E. Khan *et al*, PLB 490,45(2000)

Possible neutron-dominant quadrupole collective motion in ^{18}C

Microscopic Theoretical Predictions



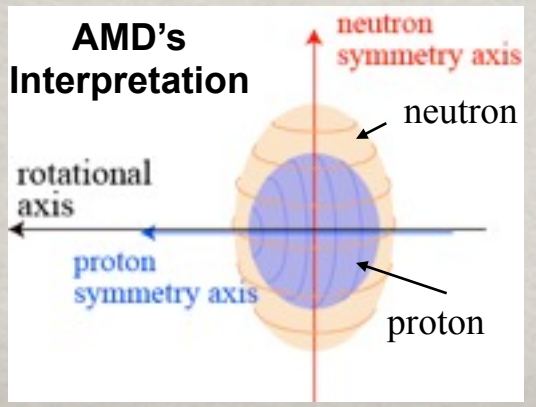
SM	R. Fujimoto, Ph. D Thesis, UT (2003)
DSHF	H. Sagawa *et al.*, PRC 70, 054316 (2004)
AMD1	Y. Kanada-En'yo, PRC 71, 014310 (2005)
AMD2	G. Thiamova *et al.*, EPJA22, 461(2004)
NCSM	S. Fujii *et al.*, PLB 650, 9 (2007)

- **Shell Model** R. Fujimoto, Ph. D Thesis, UT (2003)
- large $\pi(p_{3/2})$-$\pi(p_{1/2})$ gap
- narrowing $\nu(s_{1/2})$-$\nu(d_{5/2})$ gap with valence neutrons occupying mainly $s_{1/2}$ orbital

  ⇨small neutron effective charge
- **"No-core" Shell Model**

  S. Fujii *et al.*, PLB 650, 9 (2007)

  ⇨small neutron effective charge



- **Antisymmetrized Molecular Dynamics** Y. Kanada-En'yo, PRC 71, 014310 (2005)
  - protons/neutrons with oblate/prolate deformations