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G-matrix folding potential による ハイパー核生成反応

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Development of Nijmegen interaction models

NHC-D 1977
NHC-F 1979

NHC = Nijmegen Hard Core

NSC89

NSC = Nijmegen Soft Core

Rijken & Yamamoto

NSC97

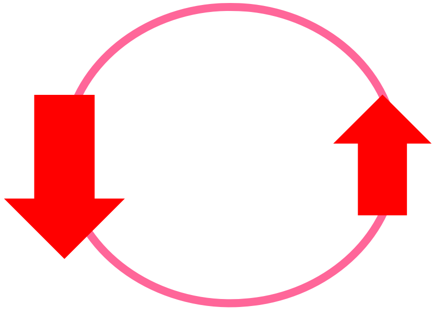
ESC04

ESC = Extended Soft Core

ESC08

実証的核物理？

多体現象（実験データ）



モデル（構造・相互作用）に基づく理解

核力も現象

核力もモデル

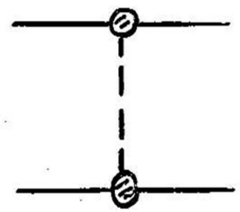
アブイニシオ核物理？　　もはやNijmegenは要らない！

Proposed by Th.A. Rijken

Extended Soft-Core Model (ESC)

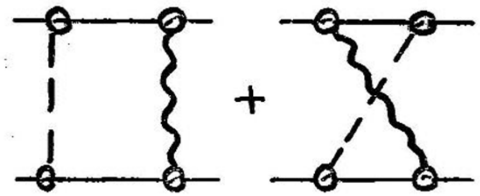
- Two-meson exchange processes are treated explicitly
- Meson-Baryon coupling constants are taken consistently with Quark-Pair Creation model

One-Boson-Exchanges:



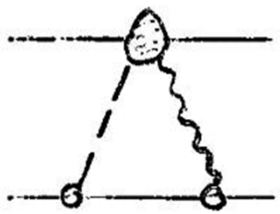
PS, S, V, AV nonets

Two-Meson-Exchanges:



PS-PS exchange

Meson-Pair-Exchanges:



(ππ),(πρ),(πω),(πη),(σσ),(πK)

Parameter fitting consistent with hypernuclear data (G-matrix)

Quark-Pauli effect in ESC08 models

ESC core = pomeron + ω

Repulsive cores are similar to each other in all channels

Assuming
"equal parts" of ESC and QM are similar to each other

Almost Pauli-forbidden states in [51] are taken into account by changing the pomeron strengths for the corresponding channels phenomenologically

$$g_P \longrightarrow \text{factor} * g_P$$

Important also in ΞN channels

by Oka-Shimizu-Yazaki

Table III. $SU(6)_{fs}$-contents of the various potentials on the isospin, spin basis.

(S, I)	$V = aV_{[51]} + bV_{[33]}$
$(0, 1)$	$V_{NN} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(1, 0)$	$V_{NN} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(0, 1/2)$	$V_{\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(1, 1/2)$	$V_{\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(0, 1/2)$	$V_{\Sigma\Sigma} = \frac{17}{18}V_{[51]} + \frac{1}{18}V_{[33]}$
$(1, 1/2)$	$V_{\Sigma\Sigma} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(0, 3/2)$	$V_{\Sigma\Sigma} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(1, 3/2)$	$V_{\Sigma\Sigma} = \frac{8}{9}V_{[51]} + \frac{1}{9}V_{[33]}$

(S, I)	$V = aV_{[51]} + bV_{[33]}$
$(0, 0)$	$V_{\Lambda\Lambda,\Lambda\Lambda} = \frac{1}{2}V_{[51]} + \frac{1}{2}V_{[33]}$
$(0, 0)$	$V_{\Xi N,\Xi N} = \frac{1}{3}V_{[51]} + \frac{2}{3}V_{[33]}$
$(0, 0)$	$V_{\Sigma\Sigma,\Sigma\Sigma} = \frac{11}{18}V_{[51]} + \frac{7}{18}V_{[33]}$
$(0, 1)$	$V_{\Xi N,\Xi N} = \frac{7}{9}V_{[51]} + \frac{2}{9}V_{[33]}$
$(0, 0)$	$V_{\Sigma\Lambda,\Sigma\Lambda} = \frac{2}{3}V_{[51]} + \frac{1}{3}V_{[33]}$
$(0, 2)$	$V_{\Sigma\Sigma,\Sigma\Sigma} = \frac{4}{9}V_{[51]} + \frac{5}{9}V_{[33]}$
$(1, 0)$	$V_{\Xi N,\Xi N} = \frac{5}{9}V_{[51]} + \frac{4}{9}V_{[33]}$
$(1, 1)$	$V_{\Xi N,\Xi N} = \frac{17}{27}V_{[51]} + \frac{10}{27}V_{[33]}$
$(1, 1)$	$V_{\Sigma\Lambda,\Sigma\Lambda} = \frac{2}{3}V_{[51]} + \frac{1}{3}V_{[33]}$
$(1, 1)$	$V_{\Sigma\Sigma,\Sigma\Sigma} = \frac{16}{27}V_{[51]} + \frac{11}{27}V_{[33]}$

Pauli-forbidden state in $V_{[51]}$ → strengthen pomeron coupling

$$\Sigma^+ p(^3S_1, T = 3/2), \Sigma N(^1S_0, T = 1/2), \text{ and } \Xi N(^1S_0, T = 1)$$

→ ESC08a/b

$$V_{BB} = \boldsymbol{a}\, V_{\text{pomeron}}$$

BB	(S,I)	$\boldsymbol{a}$
NN	(0,1) (1,0)	1.0
ΛN	(0,1/2) (1,1/2)	1.02
ΣN	(0,1/2)	1.17
	(1,1/2)	1.02
	(0,3/2)	1.0
	(1,3/2)	1.15
ΞN	(0,0)	0.96
	(0,1)	1.12
	(1,0)	1.04
	(1,1)	1.06

QM result is taken into account most faithfully

ESC08c
final version

$$\begin{aligned} V_{NN} &= (1 - a_{PB})V_P + a_{PB}V_P \\ &\equiv V(POM) + V(PB), \end{aligned}$$

$$V_{BB}(PB) = \underbrace{(w_{BB}[51]/w_{NN}[51])}_{\boldsymbol{a}} \cdot V(PB).$$

Quark-core and U_Σ / U_Ξ problem

	U_Σ	U_Ξ
Experimentally	repulsive	weakly attractive
NSC89/97	attractive	strongly repulsive
ESC04a	strongly attractive	weakly attractive
ESC04d	strongly attractive	strongly attractive
ESC08a/b	strongly repulsive	strongly attractive
ESC08c	moderately repulsive	weakly attractive

Quark-core effect

G-matrix approach to Hypernuclear systems

YN G-matrix interactions in nuclear matter

$$G_{cc_0} = v_{cc_0} + \sum_{c'} v_{cc'} \frac{Q_{y'}}{\omega - \epsilon_{B'_1} - \epsilon_{B'_2} + \Delta_{yy'}} G_{c'c_0}$$

c = (B_1B_2, T, L, S, J)

$$B_1 B_2 = \Lambda N,\ \Sigma N \text{ and } \Xi N, \text{ etc.}$$

$$\Delta_{yy'} = M_{B_1} + M_{B_2} - M_{B'_1} - M_{B'_2}$$

Coordinate representation

$$u_{c_0c_1}(k;r) = \delta_{c_0c_1} j_L(kr) + 4\pi \sum_{c_2} \int_0^\infty F_{c_1}(r,r') V_{c_1c_2}(r') u_{c_0c_2}(k;r')\, r'^2 dr'$$

$$F_c(r,r') = \frac{1}{2\pi^2} \int_0^\infty \frac{\bar{Q}_y(k_F, q, \bar{K}_{y0}) j_L(qr) j_L(qr')\, q^2 dq}{\omega - \left(\frac{\hbar^2}{2M_y}\bar{K}_{y0}^2 + \frac{\hbar^2}{2\mu_y} q^2 + U_{B_1}(\bar{k}_{B_1}) + U_{B_2}(\bar{k}_{B_2}) + \Delta_{yy0}\right)}$$

G-matrix interaction depends on k_F (or ρ)

Intermediate-state (off-shell) spectra

Continuous Choice (CON) : ⟸ our calculations

off-shell potential taken continuously from on-shell potential

Gap Choice (GAP) :

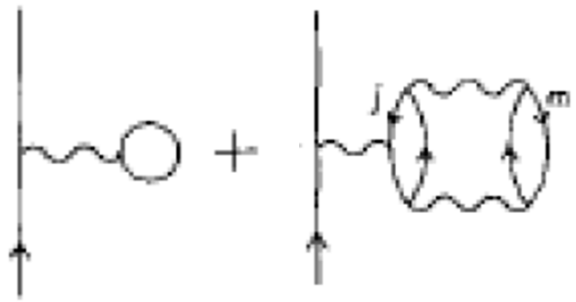
no off-shell potential

ω rearrangement effect

working repulsively

$$U_Y(k_Y) = (1-\kappa_N)\sum_{|\boldsymbol{k}_N|}\langle \boldsymbol{k}_Y\boldsymbol{k}_N \mid G_{YN}(\omega=\epsilon_Y(k_Y)+\epsilon_N(k_N)) \mid \boldsymbol{k}_Y\boldsymbol{k}_N\rangle$$

$$\kappa_N = -\sum_{N'}\langle NN' \mid \frac{\partial G_{NN}(\omega)}{\partial\omega} \mid NN' - N'N\rangle$$



Folding potentials derived from nuclear-matter G-matrix interaction

G-matrix folding potential

- N-A & A-A scattering problem
  folding of $G(r;\ \boldsymbol{\rho},\ E_{in})$ ➔ very successful
  many works including FSY papers
- Nuclear bound states with density-dependent interactions
  DDHF with $G(r;\ \boldsymbol{\rho})$ ← $G(r;\ \boldsymbol{\rho},\boldsymbol{\omega})$: $\boldsymbol{\omega}$ determined self-consistently in nuclear matter

- Y-A bound states with G-matrix folding potential
  folding of $G(r;\ \boldsymbol{\rho})$ ← $G(r;\ \boldsymbol{\rho},\ E_Y)$: E_Y determined self-consistently assuming $k_Y=0$ in nuclear matter

$$\epsilon_Y(k_y) = \frac{\hbar^2 k_Y^2}{2M_Y} + U_Y(k_Y)$$

- Toward Y-A scattering problem
  folding of $G(r;\ \boldsymbol{\rho},\ E_{in})$

Coordinate-space G-matrix interaction

Effective local interaction

Averaging for $V(r)(u_L(k;r)/j_L(kr))$:

$$G^{\mathcal{J}}_{L'S',LS}(r) = \frac{\int k^2 dk W(k;k_\Lambda) j_{L'}(kr) j_L(kr) \sum_{c_1} V c' c_1(r)\, u_{cc_1}(k;r)}{\int k^2\, dk\, W(k;k_\Lambda)\, j_{L'}(kr)\, j_L(kr)}$$

⇒ Fitted in a Gaussian form

For instance, effective LS Interaction $\mathcal{V}^{LS}$ is given by

$$\mathcal{V}^{LS}(r) = \frac{1}{2(L+1)}[-\frac{2L-1}{L} G^{L-1}_{L1,L1} - \frac{2L+1}{L(L+1)} G^{L}_{L1,L1} + \frac{2L+3}{L+1} G^{L+1}_{L1,L1}]$$

Gaussian-represented G-matrix interactions

$$G_{\Lambda N}(k_F;r) = P_+\, G^{(+)}(k_F;r) + P_-\, G^{(-)}(k_F;r)$$

$$G^{(\pm)}(k_F;r) = G_0^{(\pm)}(k_F;r) + G_{\sigma\sigma}^{(\pm)}(k_F;r)\,\mathbf{s}_\Lambda \mathbf{s}_N$$

$$+G_{LS}^{(\pm)}(r)\,\mathbf{L}(\mathbf{s}_\Lambda + \mathbf{s}_N) + G_{ALS}^{(\pm)}(r)\,\mathbf{L}(\mathbf{s}_\Lambda - \mathbf{s}_N)\} + G_T^{(\pm)}(r)\,S_{12}$$

where $P_\pm = 1/2\,(1 \pm P_r)$,
P_r being a space exchange operator
$G^{(+)}$: even-state part $\qquad$ $G^{(-)}$: odd-state part

Parametrized as (called **YNG**)

$$G_{0,\sigma\sigma}^{(\pm)}(k_F;r) = \sum_{i=1}^{3} (a_i + b_i\, k_F + c_i\, k_F^2)\, \exp(-r^2/\beta_i^2)$$

k_F dependence

k_F dependence of $G_{\sigma\sigma}^{(\pm)}$ is small, and
those of LS, ALS and tensor terms are negligible

G-matrix folding model

$$
\begin{aligned}
U_Y(\mathbf{r},\mathbf{r}') &= U_{dr} + U_{ex} \\
U_{dr} &= \delta(\mathbf{r}-\mathbf{r}') \int d\mathbf{r}'' \rho(\mathbf{r}'') V_{dr}(|\mathbf{r}-\mathbf{r}''|; \langle k_F \rangle) \\
U_{ex} &= \rho(\mathbf{r},\mathbf{r}') V_{ex}(|\mathbf{r}-\mathbf{r}'|; \langle k_F \rangle) \\
V_{dr} &= \frac{1}{2(2t_Y+1)(2s_Y+1)} \sum_{TS} (2T+1)(2S+1)[G_{TS}^{(+)} + G_{TS}^{(-)}] \\
V_{ex} &= \frac{1}{2(2t_Y+1)(2s_Y+1)} \sum_{TS} (2T+1)(2S+1)[G_{TS}^{(+)} - G_{TS}^{(-)}]
\end{aligned}
$$

G-matrix interactions $G(r;k_F)$

Averaged-k_F Approximation

$$\langle \rho \rangle = \langle \phi_Y(r) | \rho(r) | \phi_Y(r) \rangle$$

$$\langle k_F \rangle = (1.5\pi^2 \langle \rho \rangle)^{1/3}$$

A simple treatment ➡ k_F is an adjustable parameter

Mixed density $\rho(\mathbf{r}_1,\mathbf{r}_2) = \sum_j \varphi_j^*(\mathbf{r}_1)\varphi_j(\mathbf{r}_2)$ obtained from core w.f.

H.O.w.f SkHF w.f. etc.

Λ hypernuclei and ΛN interactions based on ESC08

$U_\Lambda(\rho_0)$ and partial-wave contributions

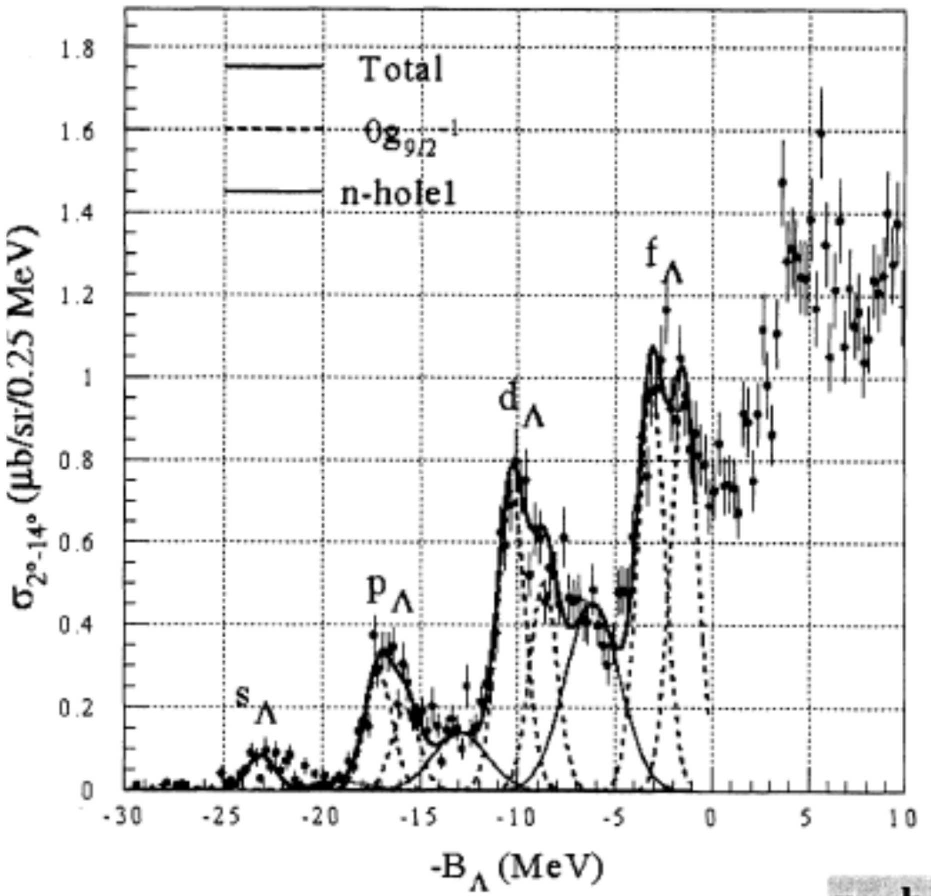
	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Λ	$U_{\sigma\sigma}$
ESC08a	−12.5	−24.1	2.4	0.0	1.2	−3.3	−1.5	−37.8	1.12
ESC08b	−12.1	−22.4	2.1	−0.2	1.3	−3.8	−1.6	−36.7	1.17
ESC08c	−12.6	−26.3	2.4	0.2	1.4	−2.7	−1.5	−39.1	0.96
NSC97e	−11.9	−26.1	1.7	0.4	2.6	−1.2	−1.1	−35.8	0.81
NSC97f	−13.2	−23.5	2.0	0.3	3.3	−0.8	−1.2	−33.1	1.36

CONr = continuous choice & ω-rearrangement

$$U_{\sigma\sigma} = (U(^3S_1) - 3U(^1S_0))/12$$

spin-spin interactions in ESC08a/b/c between NSC97e and NSC97f

$^{89}_{\Lambda}Y$



$0g_{9/2}$ Width = 1.65 MeV

	$-B_\Lambda$ (MeV)	
$\ell = 0$	-23.11 ± 0.10	
$\ell = 1$-L	-17.10 ± 0.08	$\Delta_p = 1.37 \pm 0.20$ MeV
$\ell = 1$-R	-15.73 ± 0.18	
$\ell = 2$-L	-10.32 ± 0.06	$\Delta_d = 1.63 \pm 0.14$ MeV
$\ell = 2$-R	-8.69 ± 0.13	
$\ell = 3$-L	-3.13 ± 0.07	$\Delta_f = 1.70 \pm 0.10$ MeV
$\ell = 3$-R	-1.43 ± 0.07	

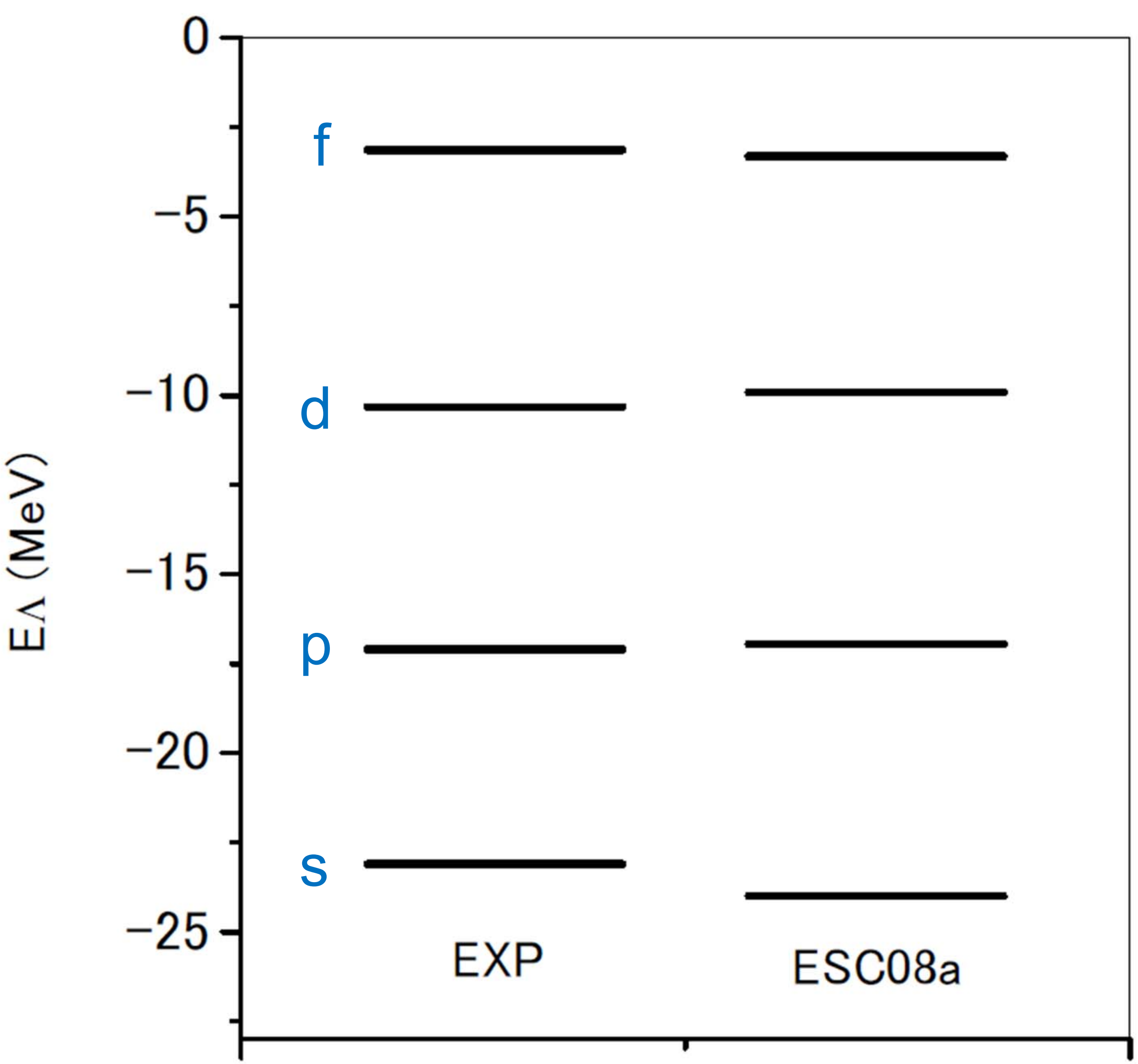
	(μb / sr)	
$\ell = 0$	0.60 ± 0.06	
$\ell = 1$-L	2.00 ± 0.22	R/L = 0.69 ± 0.12
$\ell = 1$-R	1.38 ± 0.19	
$\ell = 2$-L	5.10 ± 0.31	R/L = 0.69 ± 0.06
$\ell = 2$-R	3.52 ± 0.25	
$\ell = 3$-L	6.87 ± 0.33	R/L = 0.99 ± 0.07
$\ell = 3$-R	6.79 ± 0.31	

n – hole1 Width = 3.2 ± 0.2 MeV, $\Delta E = 4.1 \pm 0.1$ MeV

	(μb / sr)
$\ell = 0$	0.18 ± 0.06
$\ell = 1$	1.83 ± 0.14
$\ell = 2$	6.17 ± 0.28

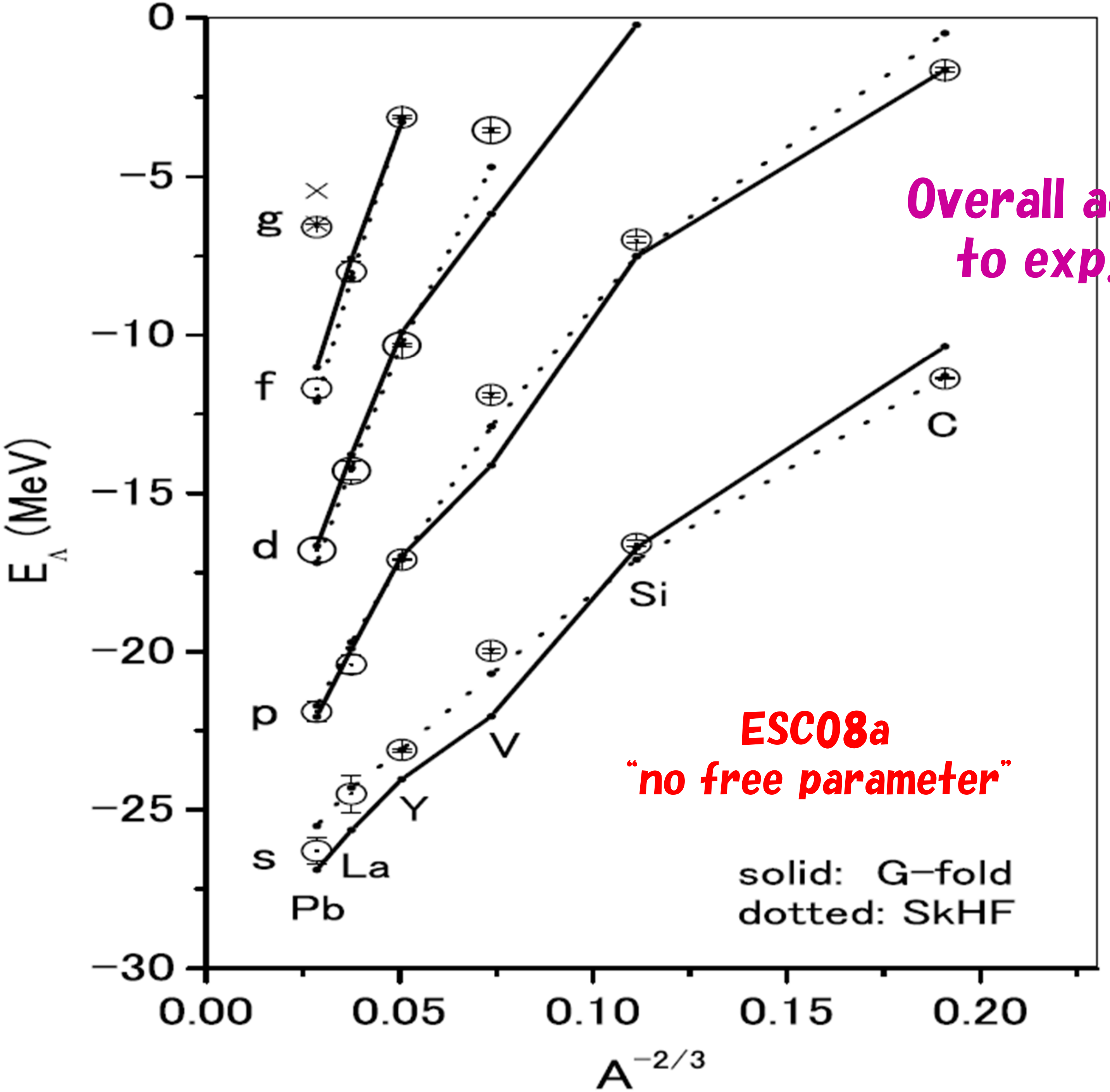
Most important data for U_Λ !

$^{89}_{\Lambda}Y$



$\langle k_F \rangle$ is determined self-consistently for each state(ADA)

In the case of taking constant $\langle k_F \rangle$, level spacing cannot be reproduced !!!



Overall agreement to exp. data

ESC08a "no free parameter"

$U_\Lambda(\rho_0) = -37$ MeV for G-matrix interactions reproducing the observed spectra of ${}^{89}_{\Lambda}Y$ differently from $U_{WS} = -30$ MeV

Self-consistent treatment of the k_F-dependence is essential in order to reproduce the spectrum

Its main origin is from ΛN-ΣN tensor coupling terms

Hypernuclear Production with GFM --- (π,K) reaction

$$\boxed{\frac{d^2\sigma}{dE_K d\Omega} = |t_{K\Lambda,\pi N}|^2 \frac{p_K E_K}{(2\pi)^2 v_\pi} \sum_f \delta(E_K + E_f - E_\pi - E_i)\, |\langle f|\hat{F}|i\rangle|^2}$$

$$\hat{F} = \int d\mathbf{r}\, F(\mathbf{r})\psi_\Lambda^+(\mathbf{r})\psi_N(\mathbf{r})$$

$$F(\mathbf{r}) = \chi_K^{(-)*}(\mathbf{r})\chi_\pi^{(+)}(\mathbf{r})$$

$\pi^+ n \rightarrow K^+ \Lambda$

$$\sum_f \delta(E_K + E_f - E_\pi - E_i)\, |\langle f|\hat{F}^+|i\rangle|^2 = -\frac{1}{\pi}\mathrm{Im}\left\langle i \middle| \hat{F}\frac{1}{E_\pi + E_i - E_K - H + i\eta}\hat{F} \middle| i \right\rangle$$

$$-\frac{1}{\pi}\mathrm{Im}\left\langle i\left|\hat{F}\frac{1}{E-H+i\eta}\hat{F}\right|i\right\rangle$$
$$=-\frac{1}{\pi}\mathrm{Im}\sum_{n'n}\int d\mathbf{r}d\mathbf{r}'\, f^*_{n'}(\mathbf{r}')\, G_{n'n}(E;\mathbf{r}',\mathbf{r})\, f_n(\mathbf{r})$$

$$\begin{aligned} f_n(\mathbf{r}) &= F(\mathbf{r})\,\langle n|\psi_N(\mathbf{r})|i\rangle = F(\mathbf{r})\,\langle A-1;n|\psi_N(\mathbf{r})|A\rangle \\ f_0(\mathbf{r}) &= \psi_N(\mathbf{r}) \quad : \quad \text{s.p.approximation} \end{aligned}$$

$$G_{n'n}(E;\mathbf{r}',\mathbf{r}) = \left\langle n'\left|\psi_\Lambda(\mathbf{r}')\frac{1}{E-H+i\eta}\psi_\Lambda(\mathbf{r})\right|n\right\rangle$$

|A-1; n> : intermediate nuclear-core state

アイコナール近似による (π, K) reaction

$$\boxed{F(\mathbf{r}) = \chi_K^{(-)*}(\mathbf{p}_K, \mathbf{r})\, \chi_\pi^{(+)}(\mathbf{p}_\pi, \mathbf{r})}$$

$$\begin{aligned}
\chi_\pi^{(+)}(\mathbf{p}_\pi, \mathbf{r}) &= \exp\left(i\mathbf{p}_\pi\mathbf{r} - \frac{1}{2}\bar{\sigma}_{\pi N}\int_\infty^z \rho(b, z')dz'\right) \\
\chi_K^{(-)*}(\mathbf{p}_K, \mathbf{r}) &= \exp\left(i\mathbf{p}_K\mathbf{r} - \frac{1}{2}\bar{\sigma}_{KN}\int_z^\infty \rho(b, z')dz'\right)
\end{aligned}$$

$$\begin{aligned}
F(\mathbf{r}) &= \exp(i\mathbf{q}\mathbf{r})\exp\left(-\frac{1}{2}\sigma T(b) - \frac{1}{2}\Delta D(b, z)\right) \equiv \exp(i\mathbf{q}\mathbf{r})\,\Gamma(r, \theta) \\
&\mathbf{q} = \mathbf{p}_\pi - \mathbf{p}_K \\
&\sigma = (\bar{\sigma}_{\pi N} + \bar{\sigma}_{KN})/2 \quad \Delta = (\bar{\sigma}_{\pi N} - \bar{\sigma}_{KN})/2 \\
&T(b) = \int_{-\infty}^\infty \rho(b, z')dz' \quad D(b) = 2\int_0^z \rho(b, z')dz' \\
&\Gamma(r, \theta) = \exp\left(-\alpha\int_a^b \rho[(r^2 sin^2\theta + z'^2)^{1/2}]dz'\right)
\end{aligned}$$

$$\frac{d^2\sigma}{dE_K d\Omega} = \frac{1}{\pi}|t_{K\Lambda,\pi N}|^2 \frac{p_K E_K}{(2\pi)^2 v_\pi} S(E)$$

$$S(E) = -\mathrm{Im}\sum_{n'n}\int d\mathbf{r}d\mathbf{r}'\, f^*_{n'}(\mathbf{r}')\, G_{n'n}(E;\mathbf{r}',\mathbf{r})\, f_n(\mathbf{r})$$

$$\left(\frac{d\sigma}{d\Omega}\right) = |t_{\pi\Lambda,KN}|^2 \frac{k_\pi E_\pi}{(2\pi)^2 v_K}$$

$$\frac{d^2\sigma}{dE_\pi d\Omega} = \frac{1}{\pi}\left(\frac{d\sigma}{d\Omega}\right) S(E)$$

$$F(\mathbf{r}) = \sum_L \sqrt{4\pi(2L+1)}\, i^L\, \tilde{j}_L(r)\, Y_{L0}(\hat{\mathbf{r}})$$

$$\exp(i\mathbf{q}\mathbf{r}) = \sum_l i^l\, j_l(qr)\, Y_{l0}(\hat{\mathbf{r}})$$

$$\tilde{j}_L(r) = i^{-L}(2L+1)^{-1} \sum_{ll'} (2l+1)(2l'+1)\langle l0l'0|L0\rangle^2\, i^l\, j_l(qr)\, \tilde{\Gamma}_{l'}(r)$$

$$\tilde{\Gamma}_l(r) = \frac{1}{2}\int_{-1}^{1} dt P_l(t)\, \exp\left(-\frac{1}{2}\sigma \int_{-\infty}^{\infty} \rho((r^2(1-t^2)+z'^2)^{1/2})dz'\right) \times \exp\left(-\Delta \int_0^{rt} \rho((r^2(1-t^2)+z'^2)^{1/2})dz'\right)$$

$$\hat{F} = \int d\mathbf{r}\, F(\mathbf{r})\, \psi_\Lambda^+(\mathbf{r})\psi_N(\mathbf{r})$$

$$S(E) = -\frac{1}{\pi}\mathrm{Im}\sum_{n'n}\int d\mathbf{r}' d\mathbf{r}\, f_{n'}^*(\mathbf{r}')\, G_{n'n}(E;\mathbf{r}',\mathbf{r})\, f_n(\mathbf{r})$$

$$G_{n'n}(E;\mathbf{r}',\mathbf{r}) = \left\langle n' \middle| \psi_\Lambda(\mathbf{r}') \frac{1}{E-H+i\eta} \psi_\Lambda^+(\mathbf{r}) \middle| n \right\rangle$$

Single particle approximation

$\alpha = nlm \;\; : \;\; \langle \alpha | \psi_N(\mathbf{r}) | i \rangle \to \phi_{nl}(r)\, Y_{lm}(\hat{\mathbf{r}})$

usual in GFM calculations

$f_\alpha(\mathbf{r}) = F(\mathbf{r})\, \phi_{nl}(r)\, Y_{lm}(\hat{\mathbf{r}})$

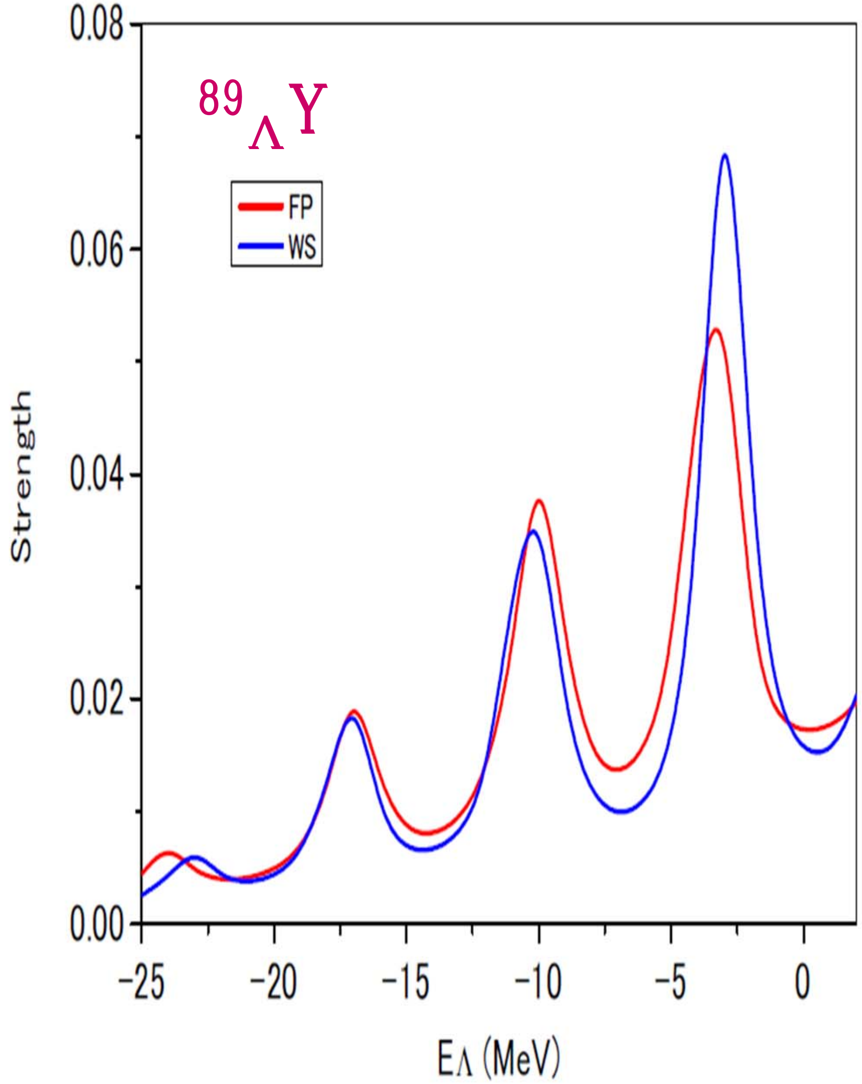
$$F(\mathbf{r})\, Y_{lm}(\hat{\mathbf{r}}) = \sum_{L'} \sqrt{4\pi(2L'+1)}\, i^{L'}\, \tilde{j}_{L'}(r)\, Y_{L'0}(\hat{\mathbf{r}})\, Y_{lm}(\hat{\mathbf{r}})$$

$$= \sum_{L'\lambda'} (2L'+1) \sqrt{\frac{2l+1}{2\lambda'+1}}\, i^{L'}\, \tilde{j}_{L'}(r)\, \langle L'0l0|\lambda'0\rangle \langle L'0lm|\lambda'm\rangle\, Y_{\lambda m}(\hat{\mathbf{r}})$$

$$F^+(\mathbf{r})\, Y^*_{lm}(\hat{\mathbf{r}}) = \text{ similarly}$$

$$\boxed{\begin{aligned} S(E) &= -\frac{1}{\pi} \mathrm{Im} \sum_{nlm} \langle i | \hat{F}^+ G \hat{F} | i \rangle \\ &= -\frac{1}{\pi} \mathrm{Im} \sum_{nl} P_{nl} \sum_{L'L} (2L'+1)(2L+1) \langle L'0l0|\lambda'0\rangle^2 \\ &\times \int_0^\infty r^2 dr \int_0^\infty r'^2 dr'\, \phi^*_{nl}(r')\, \tilde{j}^*_{L'}(r')\, G^L(E-\epsilon_{nl}; r', r)\, \phi_{nl}(r)\, \tilde{j}_{L'}(r) \end{aligned}}$$

$p_{\pi+}$=1.04 GeV/c $\theta_{\pi+}$=0°



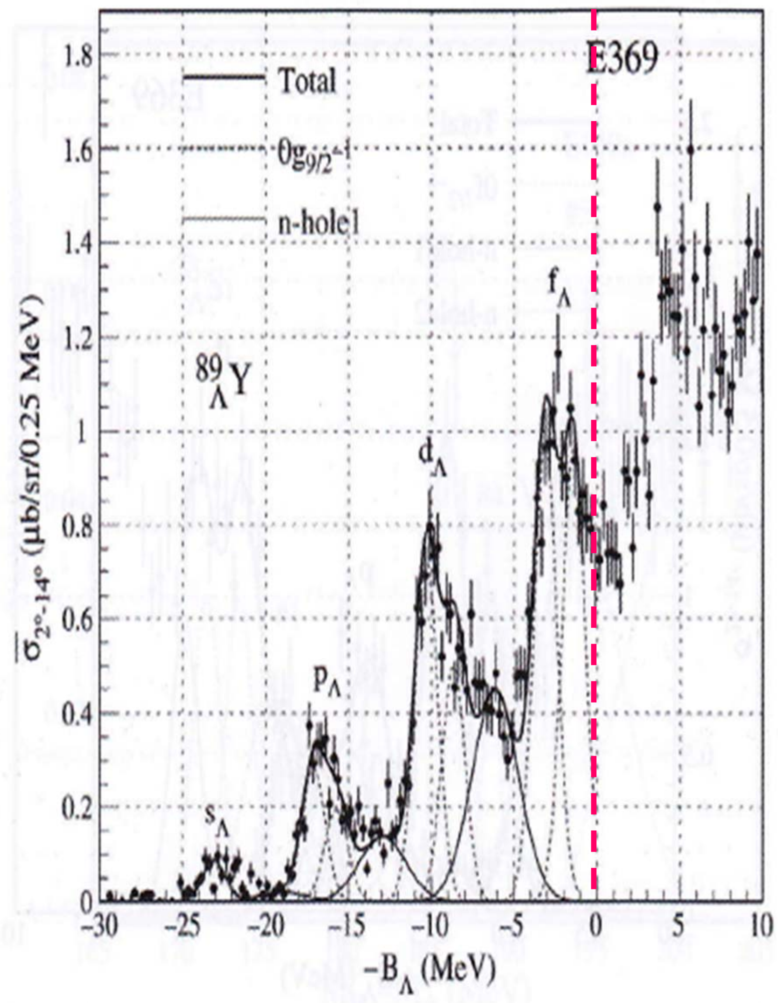
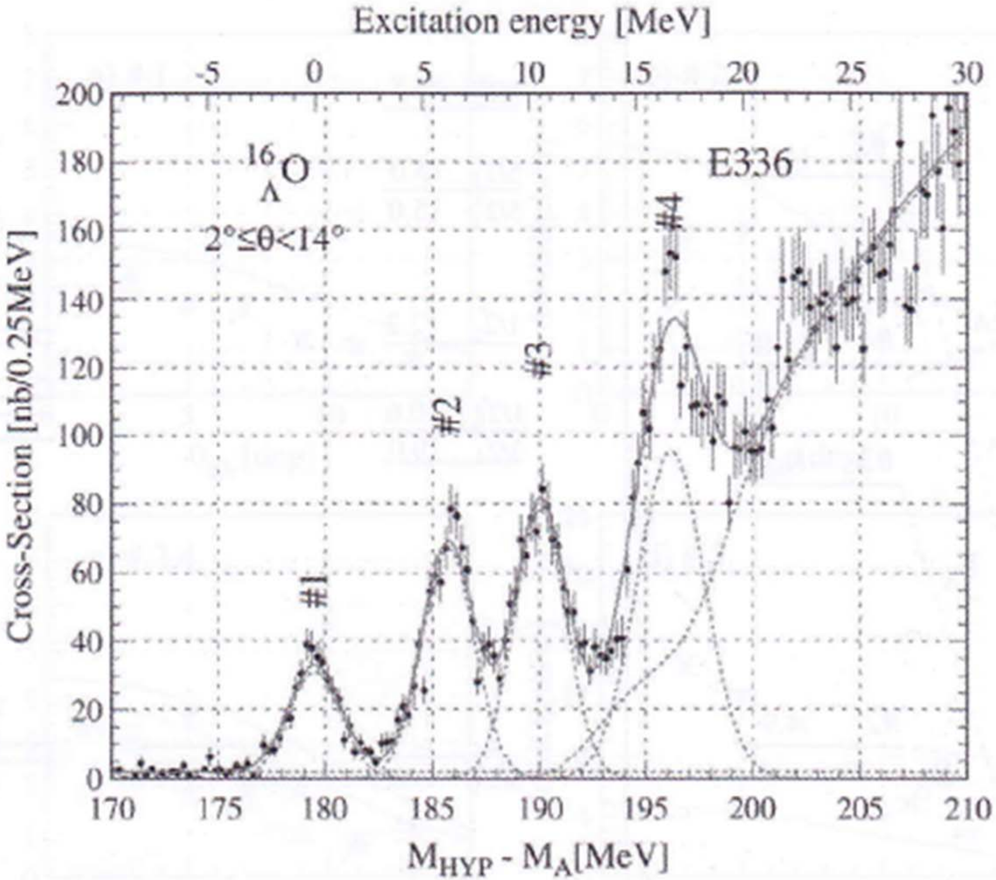
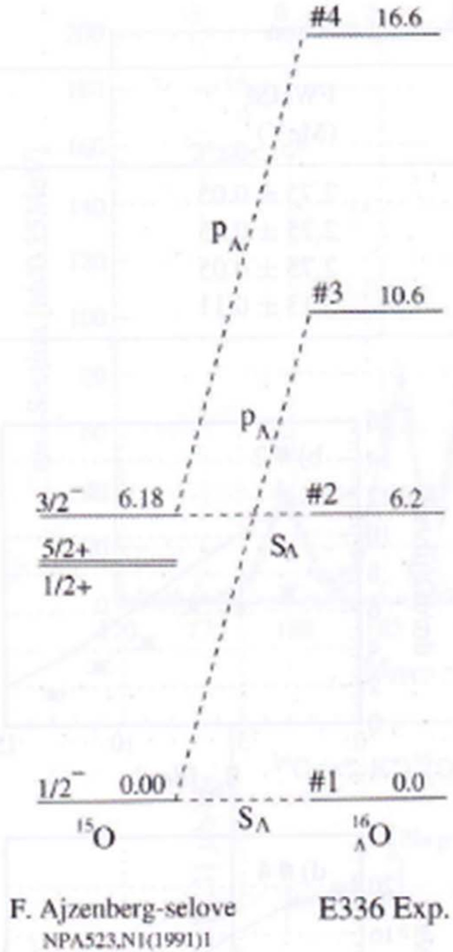


Fig. 26. Missing mass spectrum of $^{89}_{\Lambda}$Y (KEK E369) [25].

Averaged-k_F approximation



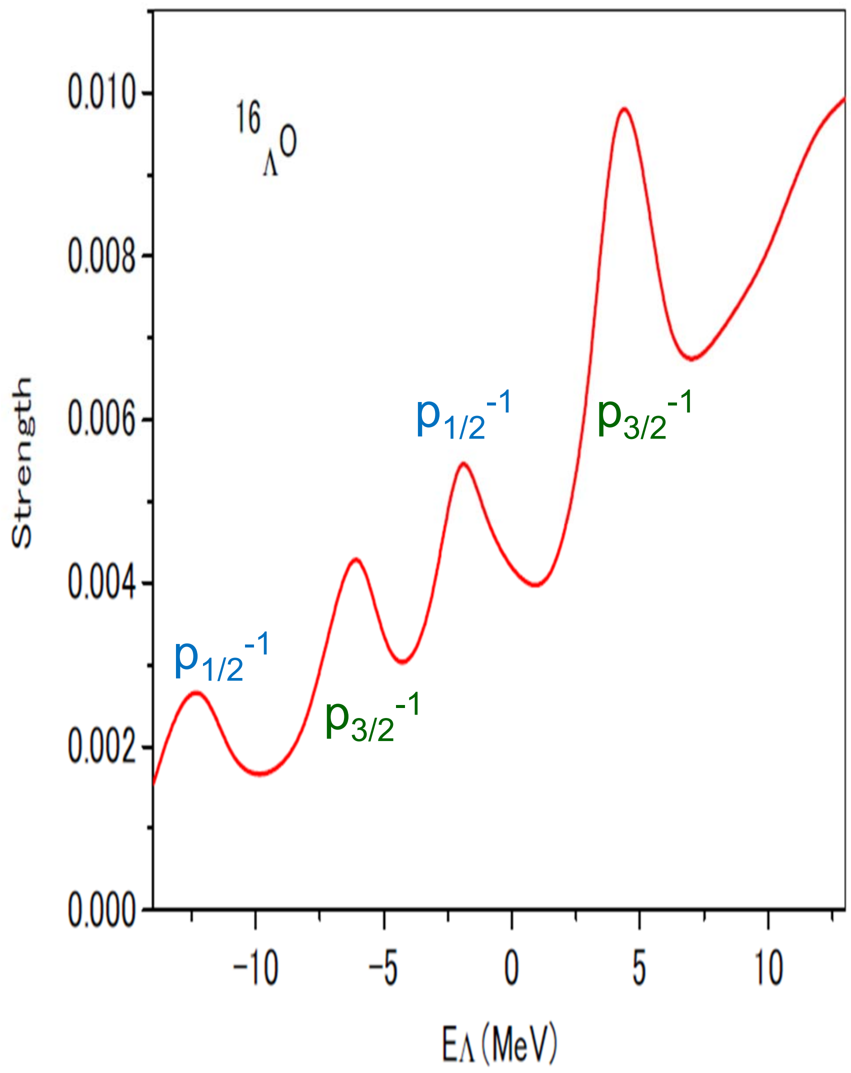


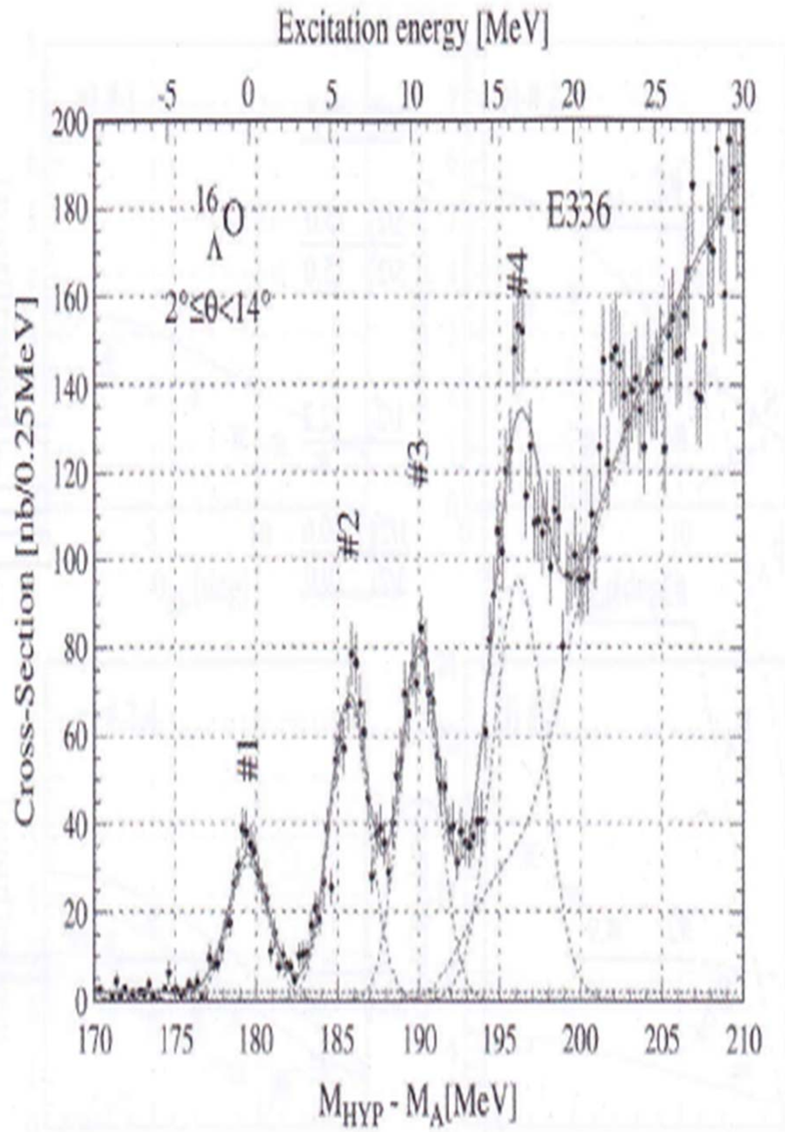
Excitation energies and cross sections of ${}^{16}_{\Lambda}O$ in the (π^+, K^+) reaction

Peaks	B_Λ or E_X (MeV)	FWHM (MeV)	Cross sections $\sigma_{2^\circ-14^\circ}$ (μb)
# 1	$B_\Lambda = 12.42 \pm 0.05$	2.75 ± 0.05	0.41 ± 0.02
# 2	$E_X = 6.23 \pm 0.06$	2.75 ± 0.05	0.91 ± 0.03
# 3	$E_X = 10.57 \pm 0.06$	2.75 ± 0.05	1.05 ± 0.03
# 4	$E_X = 16.59 \pm 0.07$	3.13 ± 0.11	1.38 ± 0.06

In ${}^{16}_{\Lambda}O$ case,
it is necessary to take $1/2^-$ and $3/2^-$ hole states
in ${}^{15}O$ core

$p_{\pi+}$=1.04 GeV/c $\theta_{\pi+}$=0°



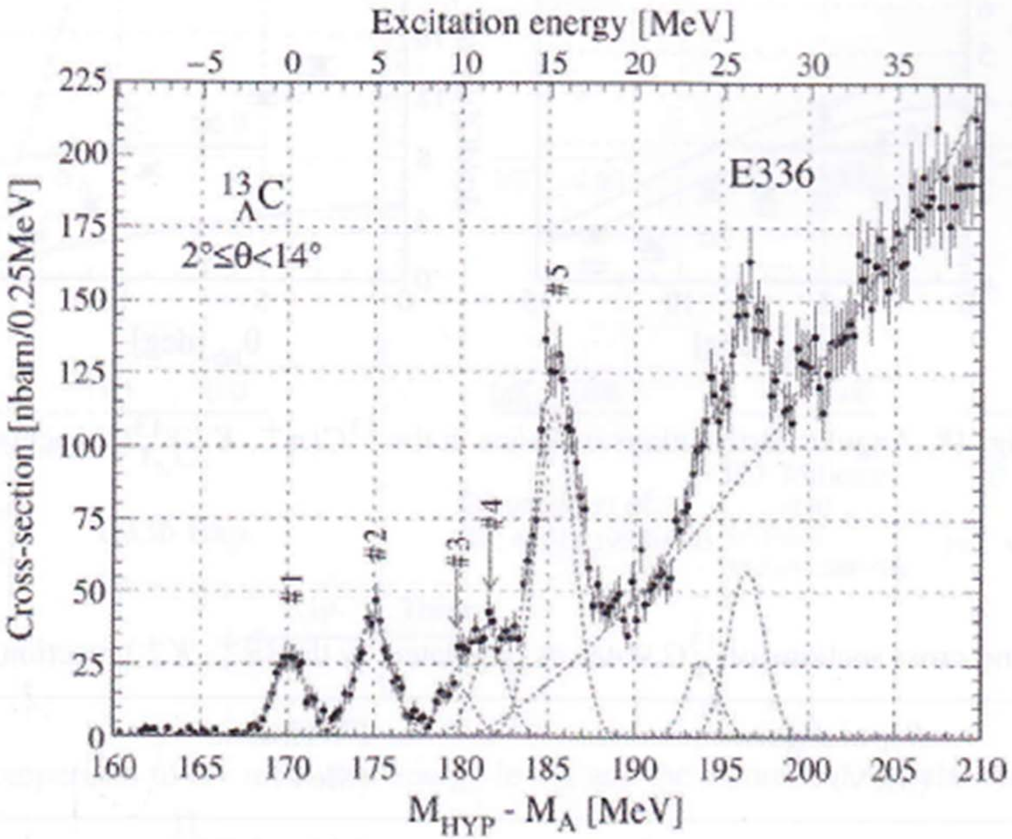


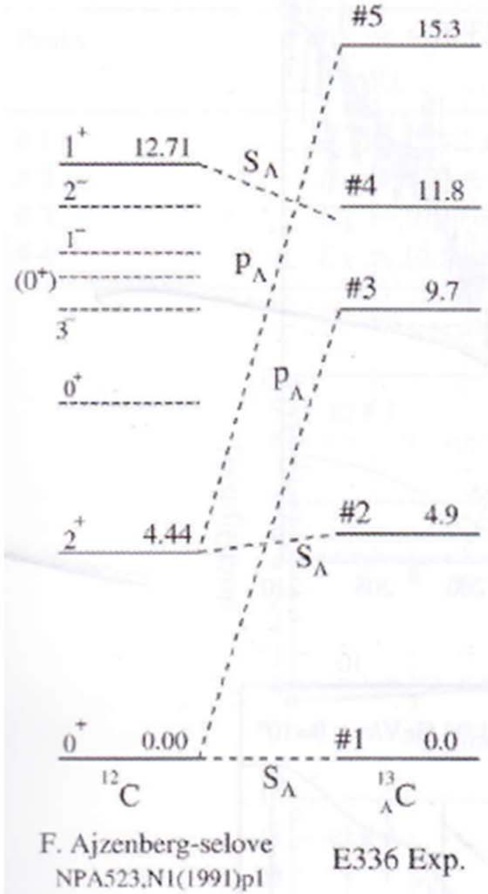
k_F is fixed for each state

GFM calculations with G-matrix folding models are quite successful to reproduce experimental (π, K) spectra

when s.p. approximation is good

When many-body calculations are needed ?





Excitation energies and cross sections of $^{13}_{\Lambda}C$ states as populated by the (π^+, K^+) reaction

Peaks	B_Λ or E_X (MeV)	FWHM (MeV)	Cross sections $\sigma_{2°-14°}$ (μb)
# 1	$B_\Lambda = 11.38 \pm 0.05$	2.23 ± 0.06	0.25 ± 0.02
# 2	$E_X = 4.85 \pm 0.07$	2.23 ± 0.06	0.42 ± 0.02
# 3	$E_X = 9.73 \pm 0.14$	2.23 ± 0.06	0.22 ± 0.02
# 4	$E_X = 11.75 \pm 0.15$	2.23 ± 0.06	0.30 ± 0.02
# 5	$E_X = 15.31 \pm 0.06$	2.46 ± 0.08	1.29 ± 0.04
# 6	$E_X = 23.68 \pm 0.16$	2.20 ± 0.29	0.33 ± 0.04
# 7	$E_X = 26.37 \pm 0.11$	2.41 ± 0.17	0.76 ± 0.06

In ${}^{13}_{\Lambda}C$ case,
it is necessary to take 0^+ and 2^+ states
in ${}^{12}C$ core nucleus

$$f_n(\mathbf{r}) \ = \ F(\mathbf{r})\,\langle n|\psi_N(\mathbf{r})|i\rangle = F(\mathbf{r})\,\langle A-1;n|\psi_N(\mathbf{r})|A\rangle$$

$|A\rangle = |{}^{13}C(1/2^-)\rangle$

$|A-1;n\rangle = |{}^{12}C(0^+)\rangle$ and $|{}^{12}C(2^+)\rangle$

$a = \langle {}^{12}C(0^+)|\psi_N|{}^{13}C(1/2^-)\rangle$

$b = \langle {}^{12}C(2^+)|\psi_N|{}^{13}C(1/2^-)\rangle$ treated as parameter

Shell model calculations are needed !!!

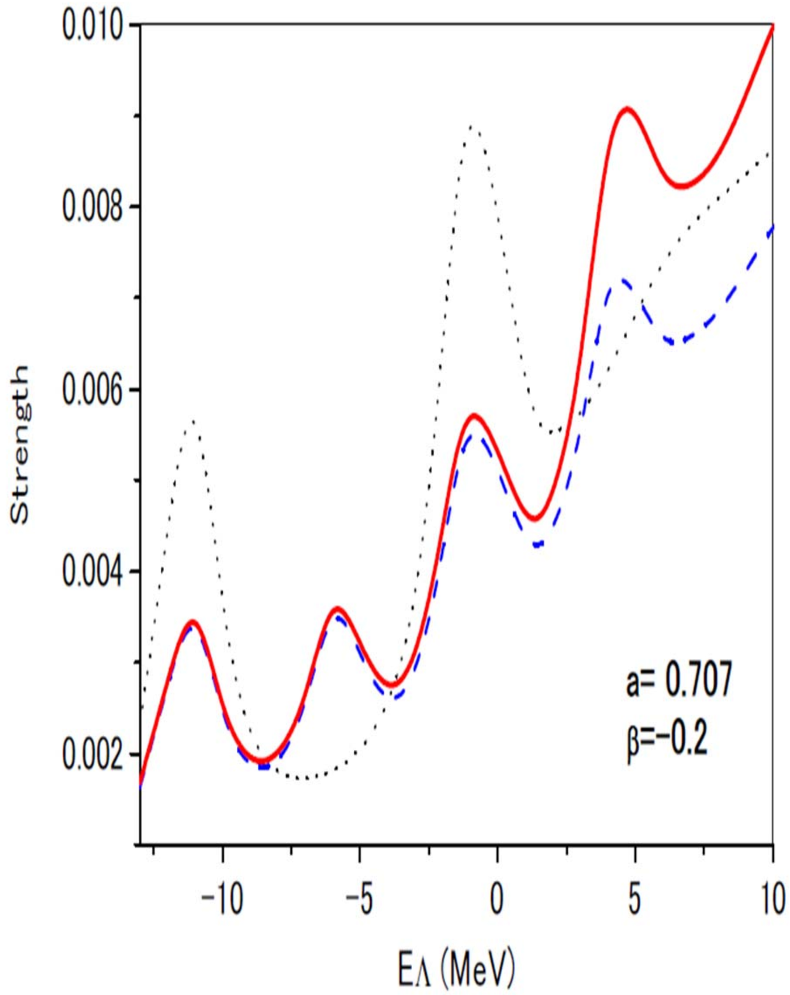
Coupled-Channel treatment

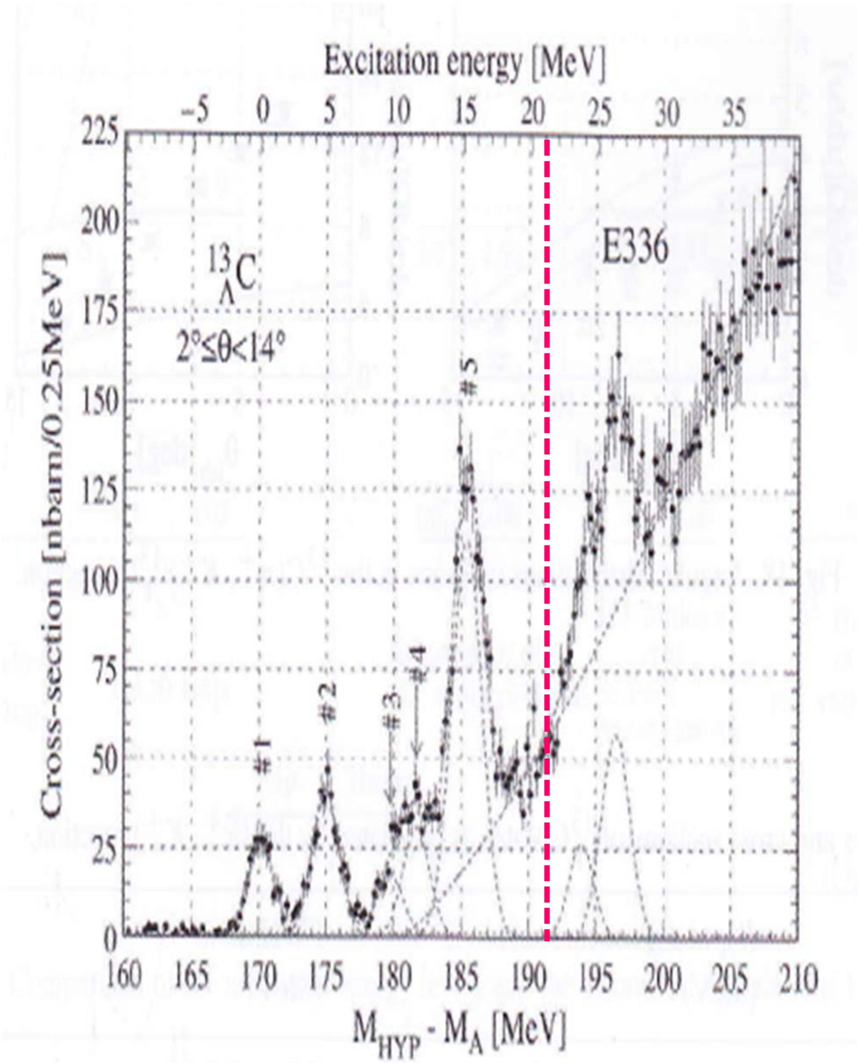
$|p^\Lambda{}_{1/2}, 0+\rangle_{J=1/2} \times |p^\Lambda{}_{3/2}, 2+\rangle_{J=1/2}$ etc

$$\begin{bmatrix} E_1 - T_1^{(l)} - U_{11}(r) & -U_{12}(r) \\ -U_{21}(r) & E_2 - T_2^{(l)} - U_{22}(r) \end{bmatrix} \begin{bmatrix} G_{11}^{(l)}(r,r') & G_{12}^{(l)}(r,r') \\ G_{21}^{(l)}(r,r') & G_{22}^{(l)}(r,r') \end{bmatrix} = \delta(r'-r)\mathbb{1}.$$

Transition densities for G-matrix folding are needed !!!

$p_{\pi+}$=1.04 GeV/c $\theta_{\pi+}$=0°





Collective model (MTT) for ^{12}C core

Ξ hypernuclei with G-matrix folding model derived from ESC08c

Experimental data suggesting attractive Ξ-nucleus interactions

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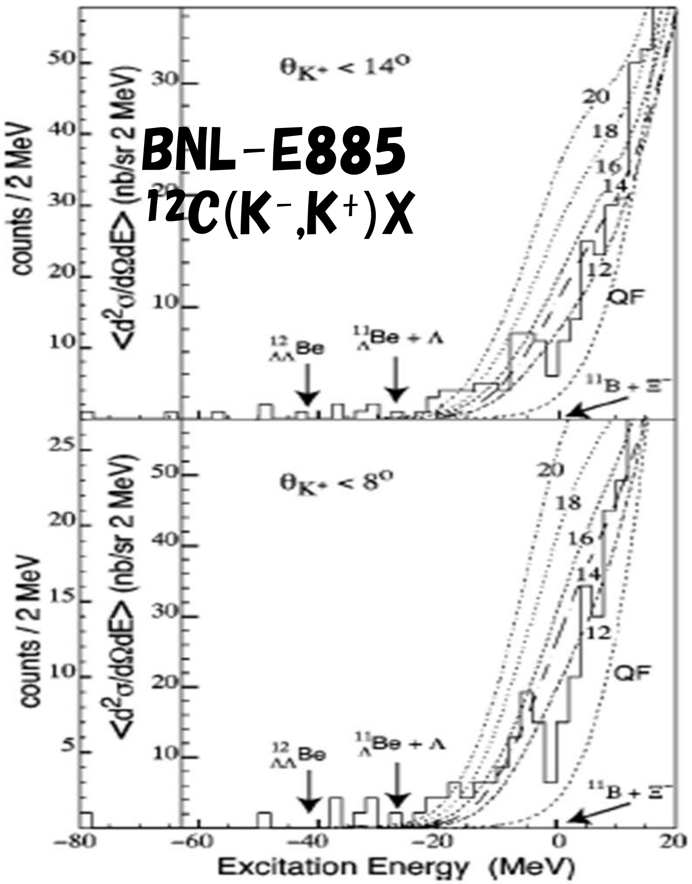
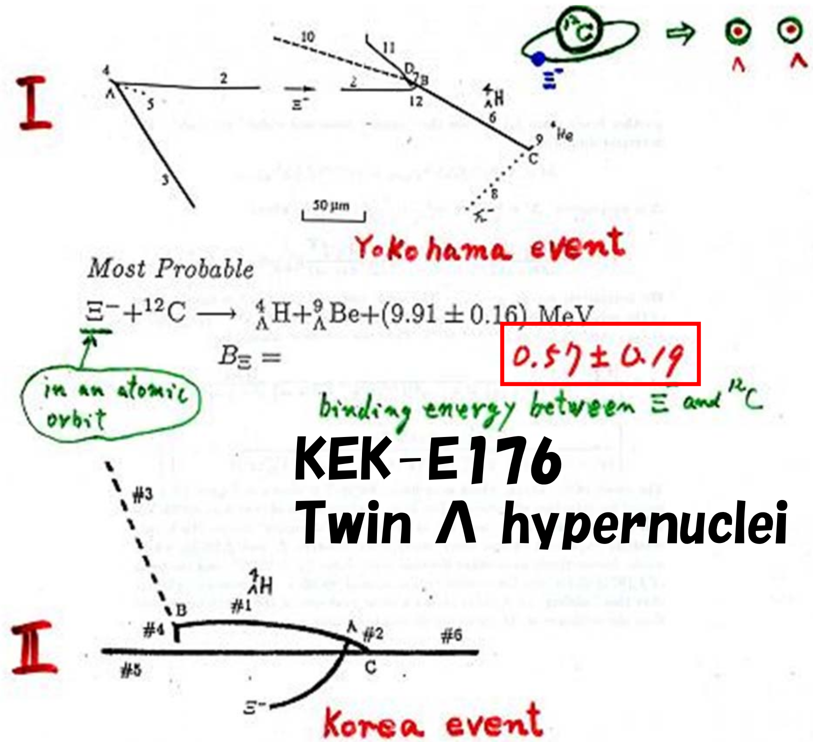


FIG. 6. Excitation-energy spectra from E885 for $^{12}C(K^-,K^+)X$



(A) $\Xi^- + ^{12}C \longrightarrow {}^4_\Lambda H + {}^9_\Lambda Be + (9.91 \pm 0.16)$ MeV

$B_\Xi = 3.70^{+0.18}_{-0.19}$ MeV

(B) $\Xi^- + ^{12}C \longrightarrow {}^4_\Lambda H + {}^9_\Lambda Be^* + (6.83 \pm 0.16)$ MeV

$B_\Xi \doteq \boxed{0.59 \pm 0.18}$

(C) $\Xi^- + ^{12}C \longrightarrow {}^4_\Lambda H^* + {}^9_\Lambda Be + (8.87 \pm 0.16)$ MeV

$B_\Xi = 2.66^{+0.18}_{-0.19}$ MeV

$U_\Xi \sim -14$ MeV

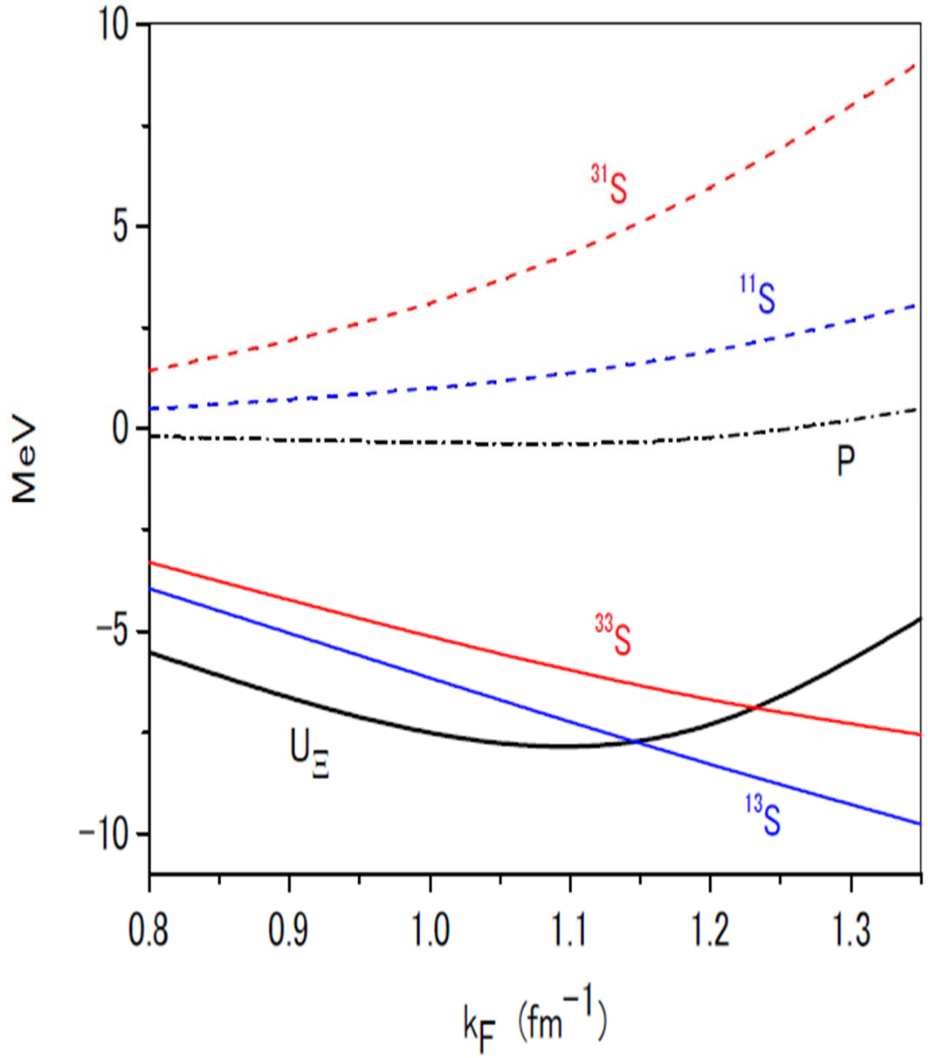
$U_\Xi \sim -16$ MeV

represented by Woods-Saxon potential

Table 1: $U_\Xi(\rho_0)$ and partial wave contributions with Continuous choice

	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	U_Ξ	Γ_Ξ
ESC08c	0	3.1	−9.8	−0.1	0.5	1.7	−1.5		
(CON)	1	9.1	−7.6	1.3	1.0	−2.4	0.0	−4.7	6.4

U_{Ξ^-} in neutron matter (T=1 components only)
repulsive in higher density region !



10
5
0
−5
−10
MeV
0.8
0.9
1.0
1.1
1.2
1.3
k_F (fm^{-1})
^{31}S
^{11}S
P
^{33}S
U_Ξ
^{13}S

Table 1: Calculated values of Ξ^- single particle energies E_{Ξ^-} and conversion widths Γ_Ξ for ${}^{12}_{\Xi^-}$Be (^{11}B+Ξ^-). ΔE_L and ΔE_C are contributions from Lane terms and Coulomb interactions, respectively. All entries are in MeV.

		E_{Ξ^-}	ΔE_L	ΔE_C	Γ_{Ξ^-}	$\sqrt{\langle r^2_\Xi \rangle}$
ESC08c	s	-4.31	$+0.27$	-2.61	2.48	3.01
	p	-0.59	$+0.06$	$*$	0.58	7.12
WS14	s	-4.89		-2.71		
	p	-0.23		$*$		

* Coulomb Assisted Bound state

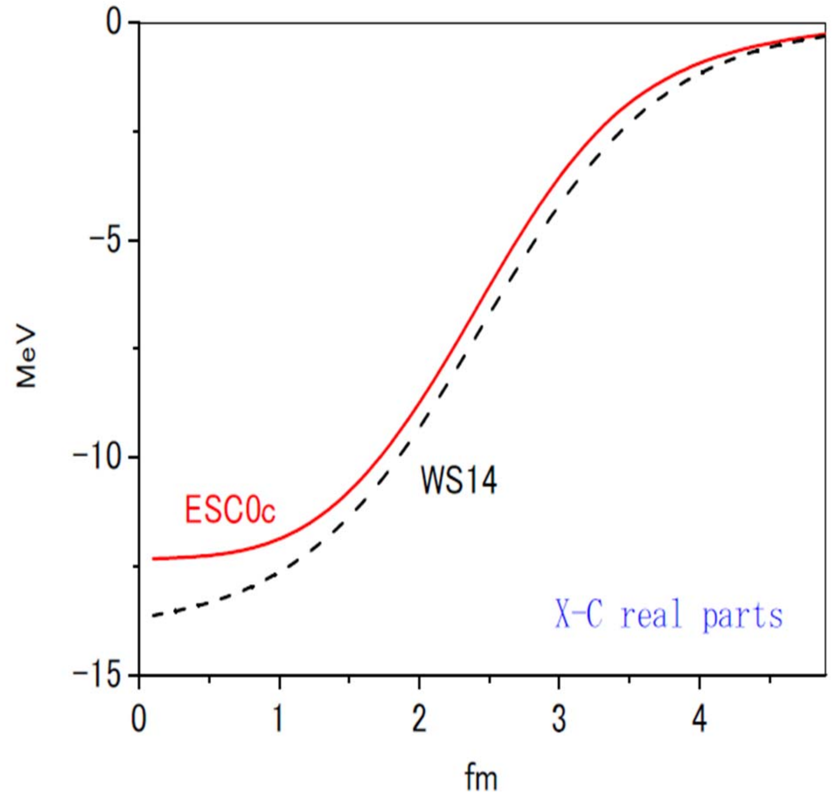
(K^-,K^+) production spectra of Ξ-hypernuclei by Green's function method in DWIA

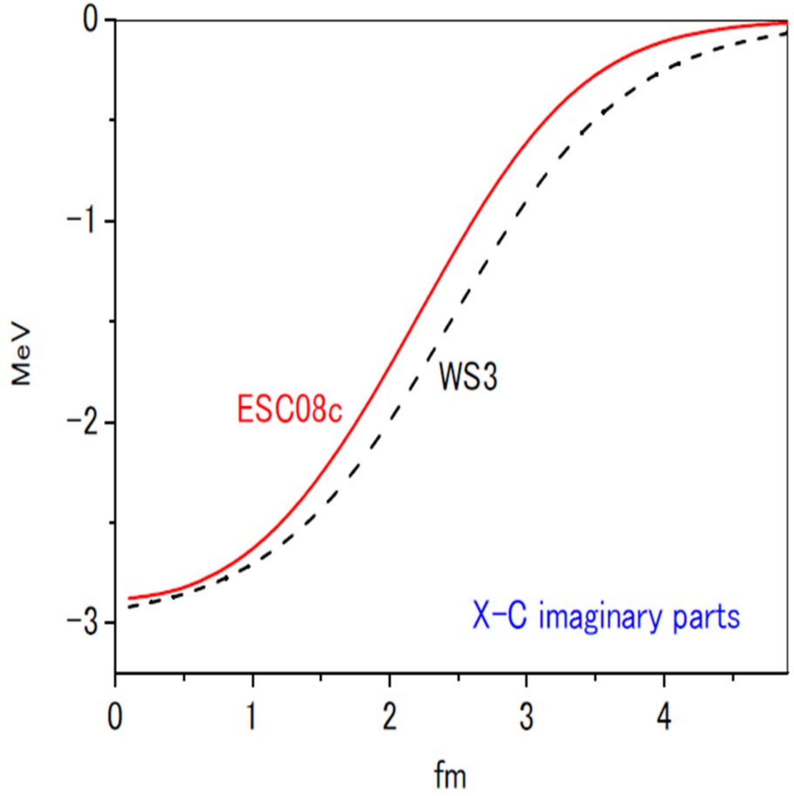
Ξ-nucleus G-matrix folding model derived from ESC08c

p_{K+}=1.65 GeV/c $\theta_{K+}=0^\circ$

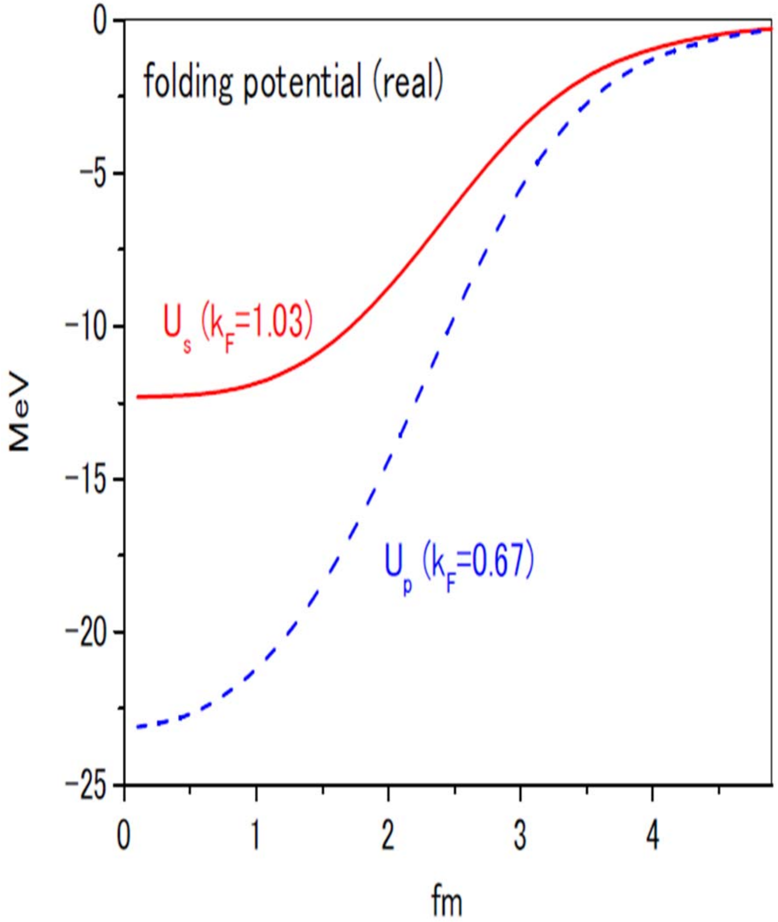
spreading width of hole-states
experimental resolution ΔE=2 MeV
are taken into account

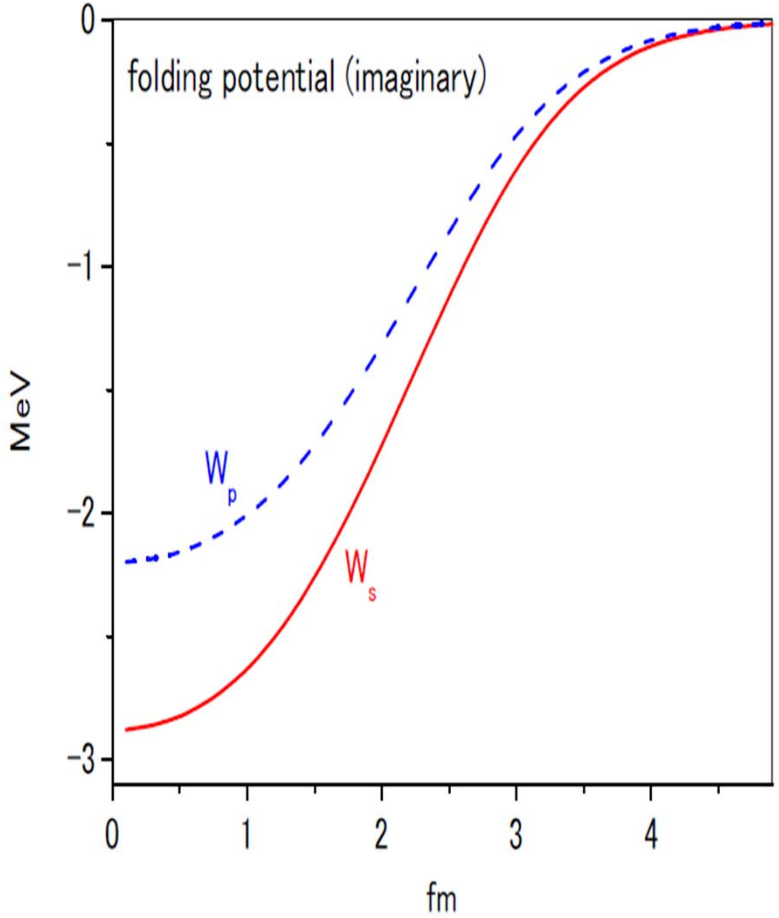
Ξ^--^{11}B potentials

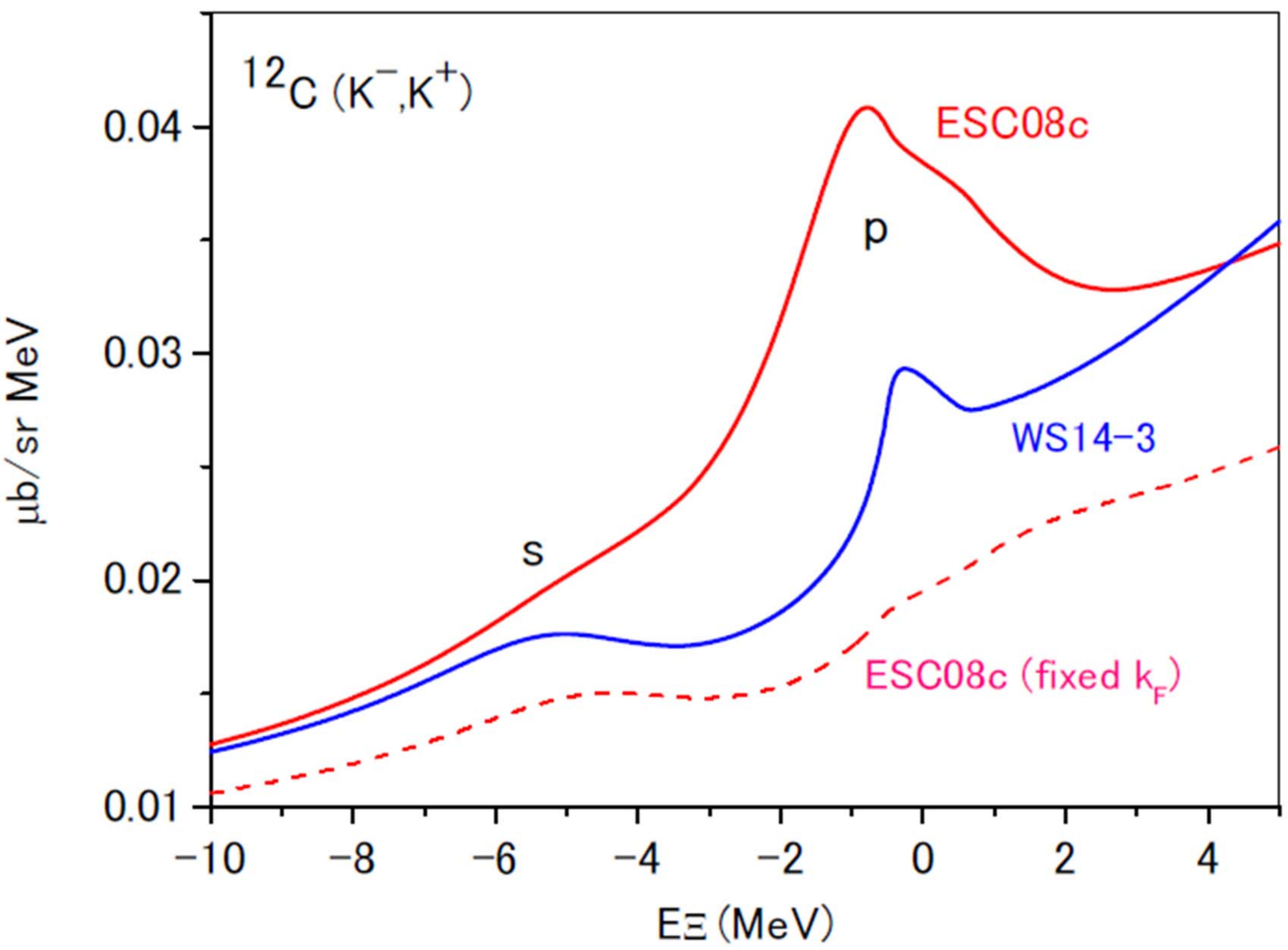




L-dependence of folding potential



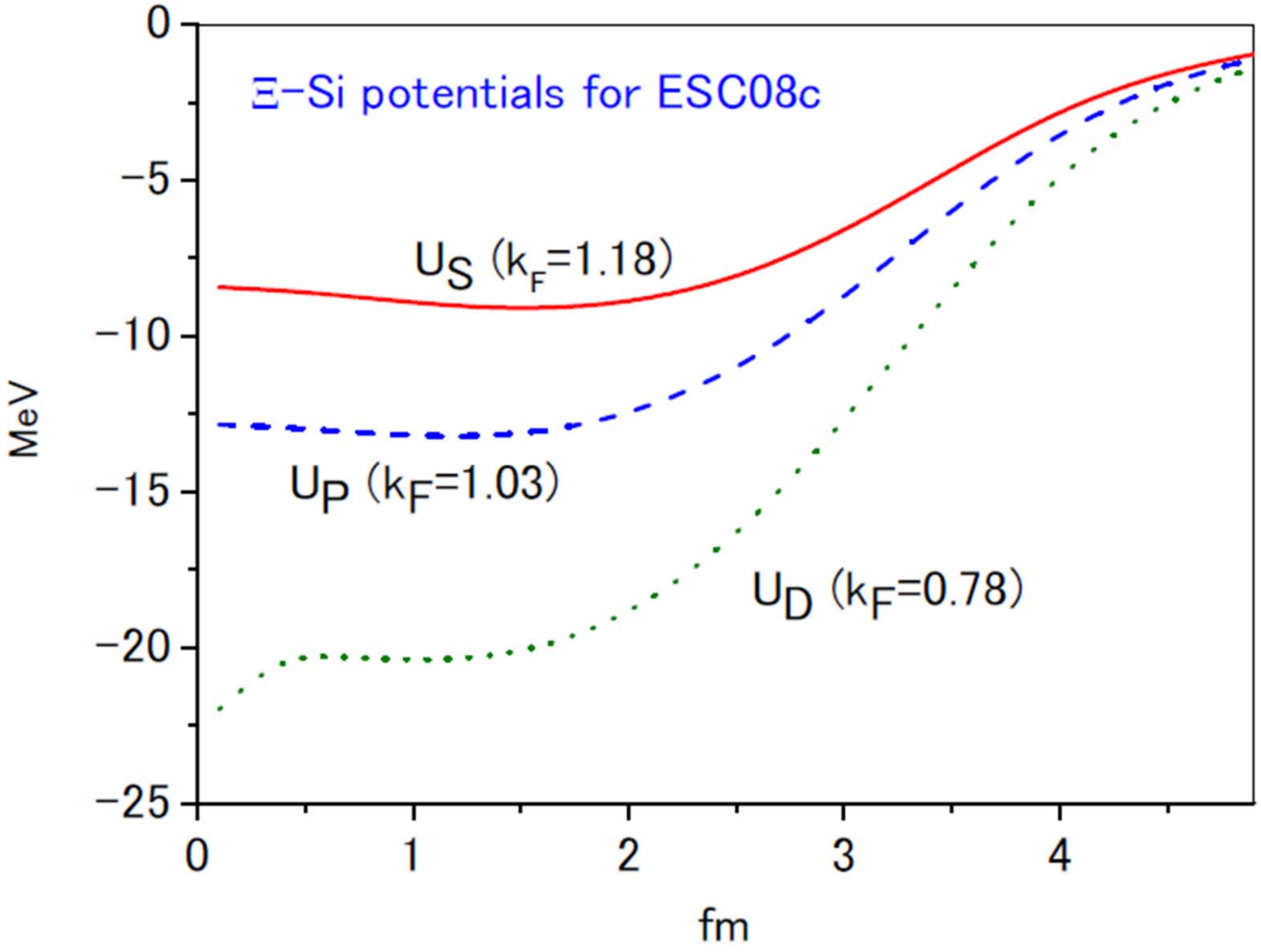




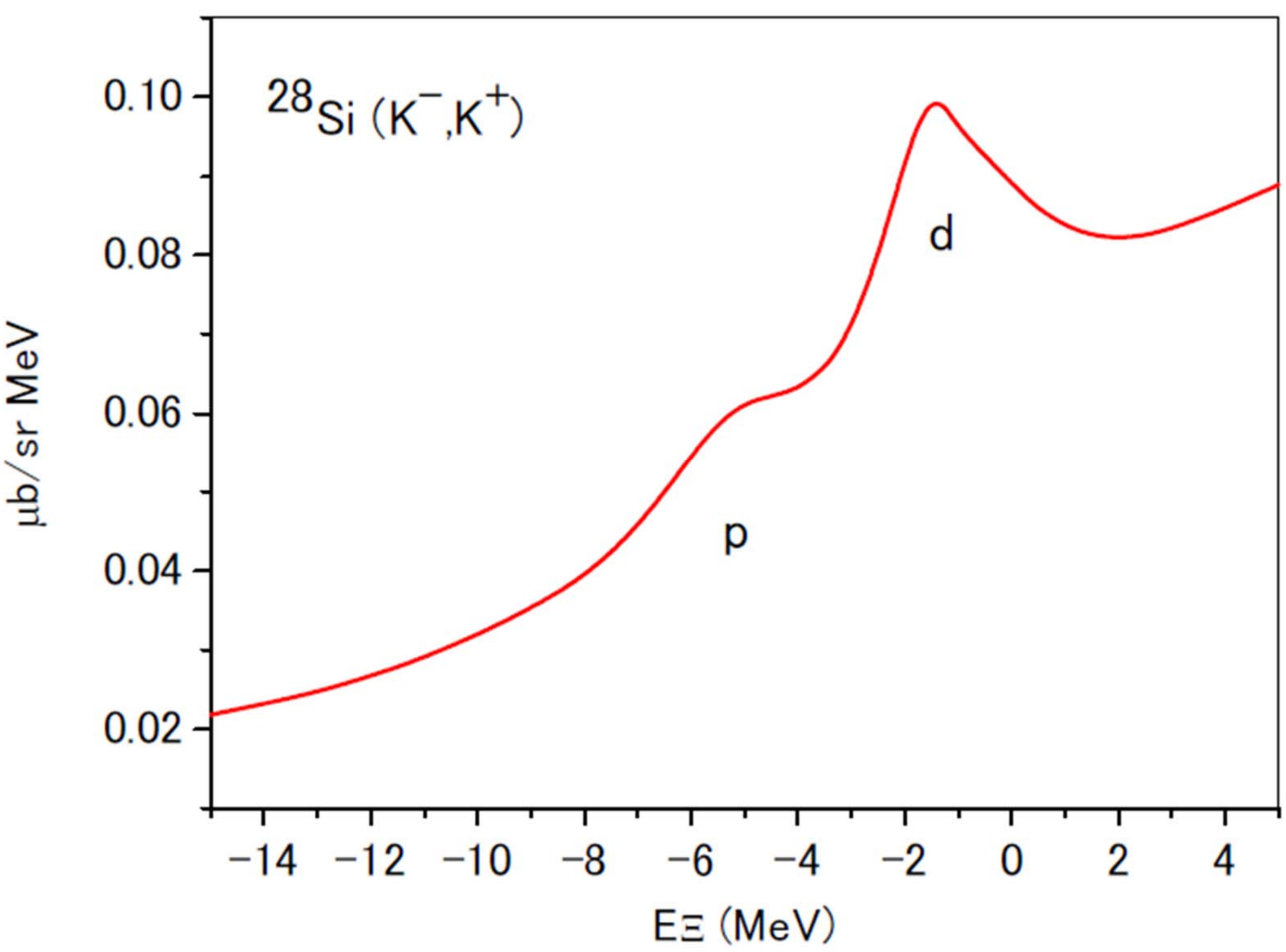
$^{12}C\ (K^-,K^+)$
ESC08c
p
s
WS14-3
ESC08c (fixed k_F)
0.01
0.02
0.03
0.04
μb/sr MeV
−10
−8
−6
−4
−2
0
2
4
EΞ (MeV)

Table 1: Calculated values of Ξ^- single particle energies E_{Ξ^-} and conversion widths Γ_Ξ for ${}^{28}_{\Xi^-}$Mg (^{27}Al+Ξ^-). ΔE_L and ΔE_C are contributions from Lane terms and Coulomb interactions, respectively. All entries are in MeV.

		E_{Ξ^-}	ΔE_L	ΔE_C	Γ_{Ξ^-}	$\sqrt{\langle r_\Xi^2 \rangle}$
ESC08c	s	-8.74	$+0.16$	-5.95	2.75	2.91
	p	-4.82	$+0.10$	$*$	1.54	3.85
	d	-1.26	$+0.03$	$*$	0.54	6.54



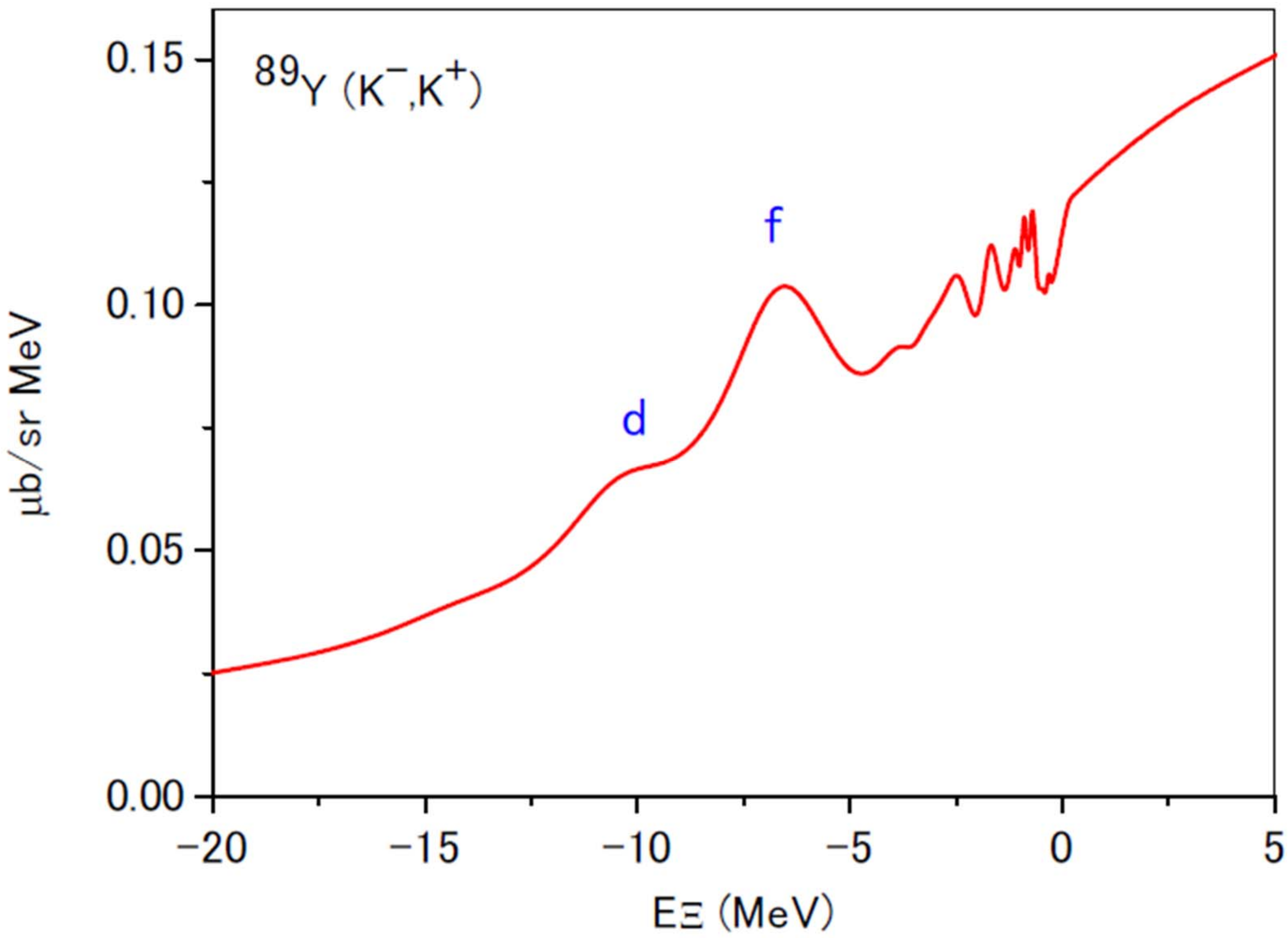
Ξ−Si potentials for ESC08c
U_S (k_F=1.18)
U_P (k_F=1.03)
U_D (k_F=0.78)
MeV
fm
0
−5
−10
−15
−20
−25
0
1
2
3
4



28Si (K−,K+)
d
p
0.10
0.08
0.06
0.04
0.02
μb/sr MeV
−14
−12
−10
−8
−6
−4
−2
0
2
4
EΞ (MeV)

Table 1: Calculated values of Ξ^- single particle energies E_{Ξ^-} and conversion widths Γ_Ξ for ${}^{89}_{\Xi^-}$Rb (^{88}Sr+Ξ^-). ΔE_L and ΔE_C are contributions from Lane terms and Coulomb interactions, respectively. All entries are in MeV.

		E_{Ξ^-}	ΔE_L	ΔE_C	Γ_{Ξ^-}	$\sqrt{\langle r_\Xi^2 \rangle}$
ESC08c	s	-17.2	$+0.68$	-13.6	2.93	3.15
	p	-13.7	$+0.56$	-12.3	1.80	4.02
	d	-10.2	$+0.43$	$*$	1.17	4.71
	f	-6.57	$+0.30$	$*$	0.76	5.43
	g	-2.16	$+0.01$	$*$	0.01	14.4



89Y (K−,K+)
f
d
0.15
0.10
0.05
0.00
μb/sr MeV
−20
−15
−10
−5
0
5
EΞ (MeV)

Σ - nucleus potential

$U_\Sigma(\rho_0)$ and partial wave contributions (Continuous Choice)

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ
ESC08a	1/2	11.4	−23.4	1.7	1.9	−5.0	0.0	−0.7	
	3/2	−12.2	44.1	−4.1	−2.3	5.1	−3.9	−0.2	12.2
ESC08b	1/2	10.4	−25.4	1.4	2.5	−5.9	0.3	−0.8	
	3/2	−11.0	52.2	−3.0	−2.8	5.6	−4.8	−0.1	18.5
ESC08c	1/2	11.5	−19.1	2.2	1.7	−5.7	−1.0	−0.7	
	3/2	−13.3	34.8	−4.6	−1.8	5.6	−1.9	−0.3	7.5
ESC04a	1/2	11.6	−26.9	2.4	2.7	−6.4	−2.0	−0.8	
	3/2	−11.3	2.6	−6.8	−2.3	5.9	−5.1	−0.2	−36.5
NSC97f	1/2	14.9	−8.3	2.1	2.5	−4.6	0.5	−0.5	
	3/2	−12.4	−4.1	−4.1	−2.1	6.0	−2.8	−0.1	−12.9

Pauli-forbidden state in QCM ➔ strong repulsion in $T=3/2$ $^3S_1$ state
taken into account by adapting Pomeron exchange in ESC approach

Optical potential

Σ-nucleus folding potential derived from complex G-matrix

$$G_{\Sigma N}(r; E, k_F)$$

In N-nucleus scattering problem
physical observables can be reproduced
with "no free parameter"

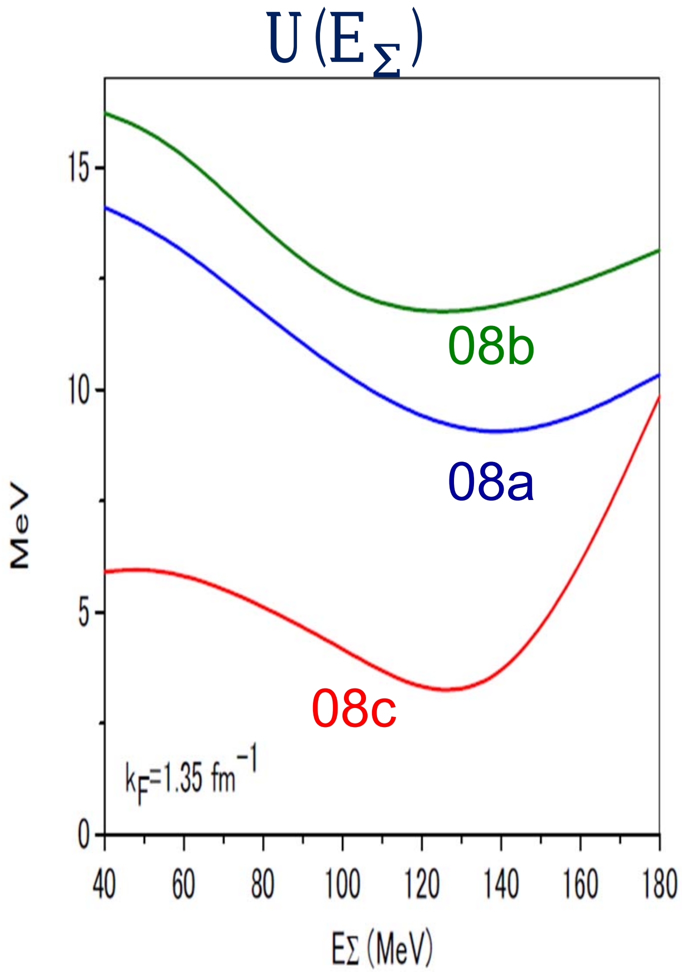
Improved LDA by JLM

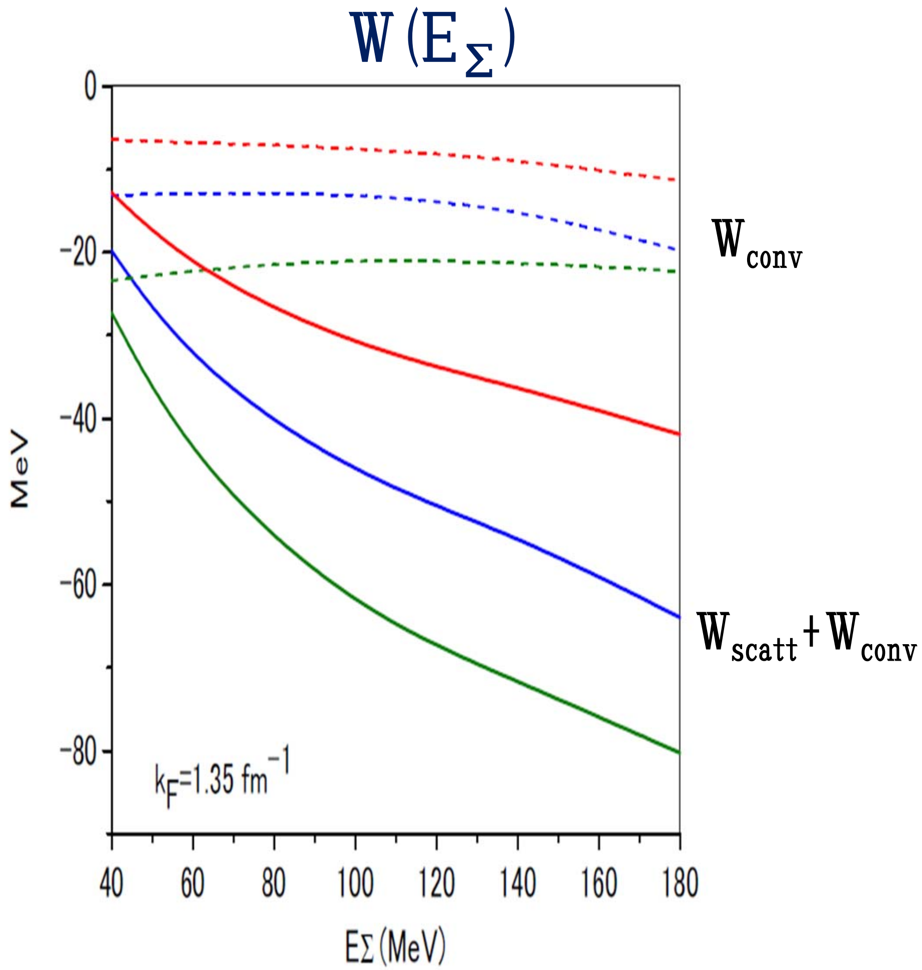
Phys. Rev. C10 (1974) 1391

$$U(\rho, E) = \sum_{ij} a_{ij}\, \rho^i\, E^{j-1}$$

$$U(r; E) = (t\sqrt{\pi})^{-3} \int U(\rho(r'), E) \exp(-|\mathbf{r} - \mathbf{r}'|^2/t^2) d\mathbf{r}'$$

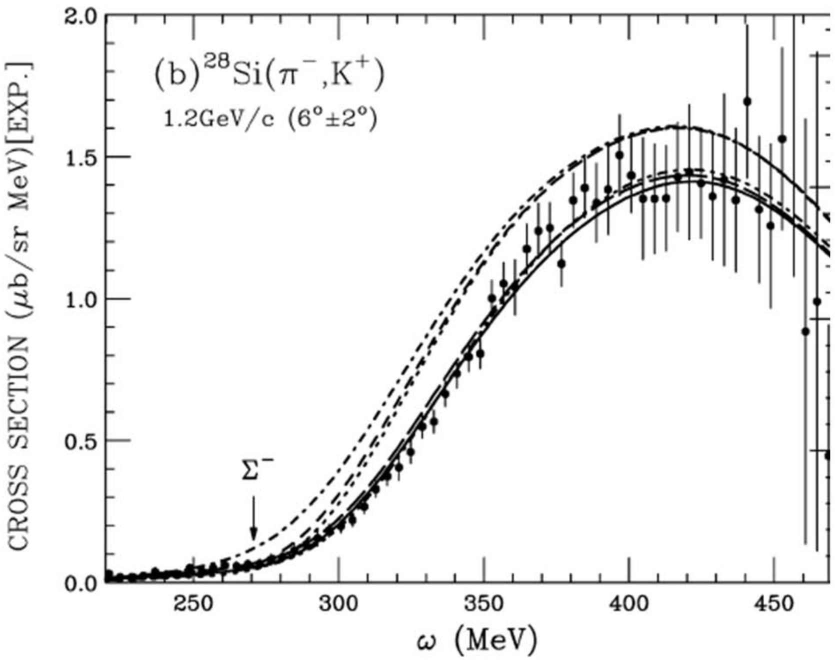
simple LDA : U($\boldsymbol{\rho}$ (r),E)



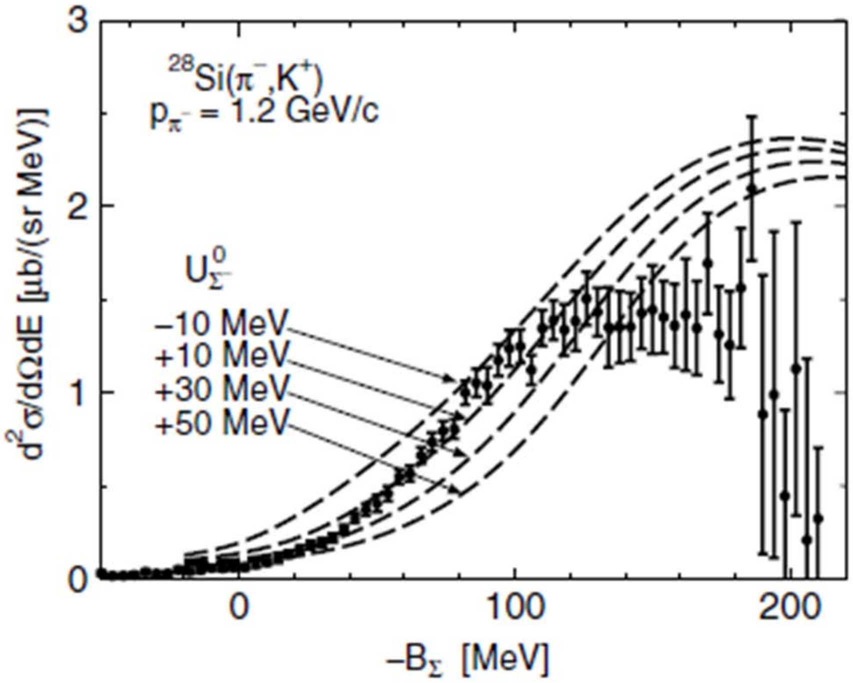


W_{scatt}が大きい理由

U_{Σ} (real)　cancelingが効く
W_{Σ}には”２乗和”で効く



Harada



Kohno

by Maekawa, at al.

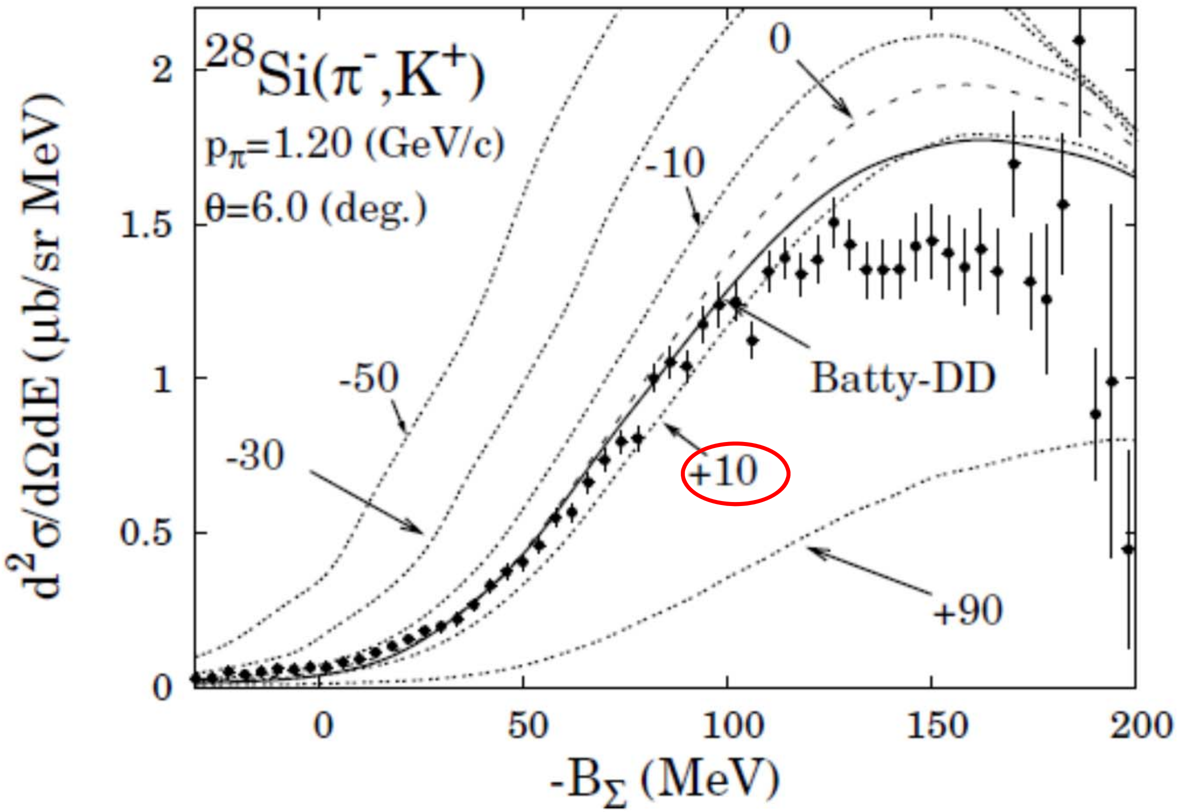
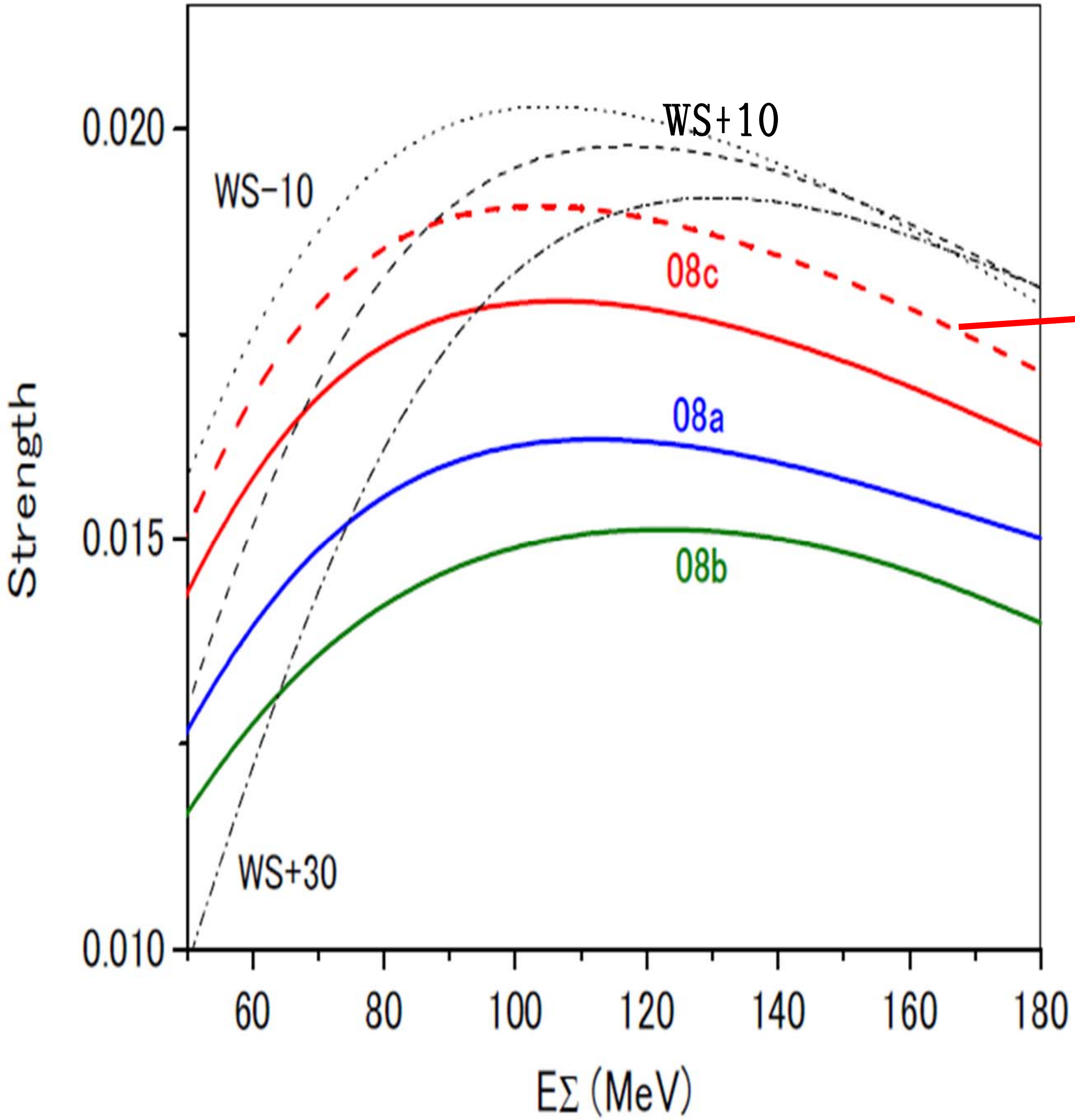


Fig. 2. Differential cross section of (π^-, K^+) reaction on ^{28}Si target at the incident momentum of p_π=1.2 GeV/c. The solid line shows result of Batty's DD potential with LOFAt + DWIA, Other line are calculated results with LOFAt + DWIA with potential depth of V_0=-50, -30, -10, 0, +10, +90 MeV(up to down), respectively. Imaginary part is fixed to be -20 MeV.

(π^-,K^+) strength function on ^{28}Si



N_W=0.65

Reliability of
Nuclear Matter
G-matrix interaction
E>50 MeV

QM Pauli-forbidden coreの強さの確定

(π,K) strength function で選別できるか？

Σ^+p scattering at JPARC

Conclusion

Properties of Λ-hypernuclei derived from ESC08 models are consistent with experimental data of energy spectra

Difference among versions (a,b,c) appear in U_Σ & U_Ξ

ESC08c (final version of ESC08) and Ξ hypernuclei

G-matrix folding model derived from ESC08c is very promising for production spectra of Ξ-hypernuclei

ESC08c folding potentials are similar to WS14

Highest-L bound states are strongly excited due to strong effects of k_F-dependence

Λ case: $U_\Lambda(\rho_0)$ = -37 MeV U_{WS} = -30 MeV

Ξ case: $U_\Xi(\rho_0)$ = -(4-5) MeV U_{WS} = -14 MeV

(ESC08c)   there appear peak structures