Section VI

Probability and Statistics

Probability and Statistics

1. Definitions

Probability Density

A random variable X is a discrete random variable if there exists a function f (the probability distribution) such that for all possible values of X

$$f(x_i) \geq 0$$

$$\sum f(x_i) = 1 ,$$

and for any event E

$$P(E) = P[\, X \text{ is in } E\,] = \Sigma_E f(x) \;.$$

$\Sigma_E f(x)$ means the sum over all those values x that are in E and where $f(x) = P[X = x]$ is the probability that X is some real number x.

Similarly, X is a continuous random variable if there exists a function f (the probability density) such that for all possible values of X

$$f(x) \geq 0$$

$$\int f(x_i) = 1 ,$$

and for any event E

$$P(E) = P[X \text{ is in } E] = \int_E f(x)\, dx \;.$$

The probability that X assumes any given value of x is equal to zero. The probability that X assumes some value on the interval from x_1 to x_2 is

$$\int_{x_1}^{x_2} f(x)\, dx \,.$$

Cumulative Distribution Function

The cumulative distribution function F(x) is defined as the probability that X is less than or equal to some value x. For the continuous case

$$F(x) = P\,[\,X \leq x\,] \qquad -\infty \leq x \leq \infty$$

$$= \int_{-\infty}^{\infty} f(x)\, dx \;.$$

If the cumulative distribution is known then the probability density can be obtained from

$$f(x) = \frac{dF(x)}{dx} \; .$$

The generalization for the discrete case is obvious.

Mathematical Expectation

The expected value of a function g(x) of a random variable X having probability density f(x) is defined as

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)\, f(x)\, dx \; .$$

The expected value of X is the mean [$\mu(x)$], and the expected value of the function $(x-\mu)^2$ is the variance [$\sigma^2(x)$]. The mean is the first moment about the origin, the variance is the second moment about the mean. The quantity $\sigma(x)$ is usually referred to as the standard deviation of X.

There are several useful properties of the mean and variance for combinations of random variables:

(1) $E[aX + bY] = aE[X] + bE[Y]$

(2) $E[X \cdot Y] = E[X] \cdot E[Y]$, if X and Y are statistically independent

(3) $\sigma^2(cX) = c^2 \sigma^2(X)$

(4) $\sigma^2(X+c) = \sigma^2(X)$

(5) $\sigma^2(aX+b) = a^2 \sigma^2(X)$.

Marginal and Conditional Distributions

Cases where more than one random variable must be considered jointly arise frequently. A set of n random variables $\mathbf{X} \equiv (X_1, X_2, X_3, \ldots\ldots\ldots, X_n)$ are said to be jointly distributed if there exists a function (the joint probability density) $f(\mathbf{x}) = f(x_1, x_2, x_3, \ldots\ldots\ldots, x_n) \geq 0$ for all possible values of x_i and such that for any event E

$$P(E) = P[\mathbf{X}] \qquad \mathbf{X} \text{ is in } E$$

$$= \int_E f(\mathbf{x})\, d\mathbf{x} \; .$$

The cumulative distribution is given by

$$F(\mathbf{x}) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \int_{-\infty}^{x_3} \cdots\cdots \int_{-\infty}^{x_n} f(\mathbf{x})\, d\mathbf{x} \; .$$

Similar definitions apply for discrete random variables. Random variables are independent if and only if their joint density factorizes such that $f(x_1, x_2) = f_1(x_1)\, f_2(x_2)$.

The marginal distribution of a subset of the random variables $\mathbf{X}_p = (X_1, X_2, X_3, \ldots\ldots\ldots, X_p)$, where $p < n$ is given by integrating $f(\mathbf{x})$ over the unincluded variables

$$g(\mathbf{x}_p) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(\mathbf{x})\, dx_{p+1}\, dx_{p+2} \ldots\ldots dx_n \,.$$

The conditional distribution $h(\mathbf{x}_p \mid x_{p+1}, x_{p+2}, \ldots, x_n)$ for $\mathbf{X}_p$ is the joint distribution of this subset under the condition that the other variable have certain values

$$h(\mathbf{x}_p \mid x_{p+1}, \ldots\ldots, x_n) = \frac{f(\mathbf{x})}{g(\mathbf{x}_{p+1:n})}$$

provided that $g(\mathbf{x}_{p+1:n}) \neq 0$.

Covariance and Correlation

For random variables X_i and X_j that are jointly distributed, the covariance σ_{ij} is defined as

$$\sigma_{ij} = \rho_{ij}\,\sigma_i\,\sigma_j = E\,[\,(X_i - \mu_i)(X_j - \mu_j)\,]\ .$$

The quantity ρ_{ij} is the correlation coefficient and $\sigma_i = \sigma_{ii} = \sigma(x_i)$ and σ_j are the standard deviations of X_i and X_j respectively. Note that the covariance and correlation coefficient must be zero for independent random variables, but that they can be zero even if the random variables are not independent.

Confidence Level

The term confidence level refers to the probability content of the probability density outside some specified limit. The confidence level can be specified for a one-tailed (range divided into two intervals) or two tailed (range divided into three intervals) case. In the one-tailed case we write

$$CL = \int_{-\infty}^{x_o} f(x)\, dx\,, \text{ or}$$

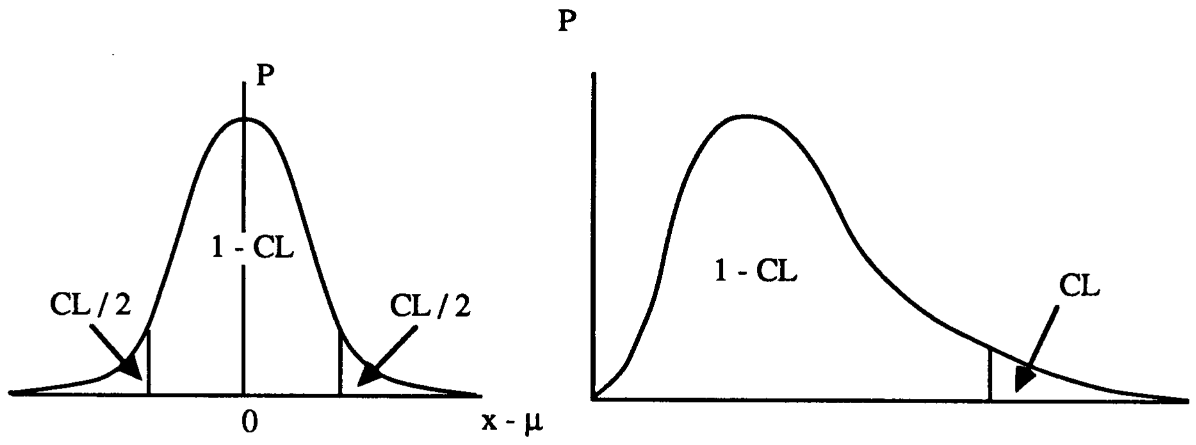
$$CL = \int_{x_o}^{\infty} f(x)\, dx\ .$$

For a two-tailed case

$$CL = \int_{x_a}^{x_b} f(x)\, dx\,.$$

In this situation it is conventional to choose the shortest possible interval (x_a, x_b). For symmetric probability distributions this always corresponds to an interval centered on the mean of the distribution.

These two cases are demonstrated in the figures below. Note that some sources use the complementary definition as the value for the confidence level (i.e. their CL corresponds to 1 - CL here).



2. Useful Probability Distributions

The properties of a number of useful probability functions are summarized in Tables VI-1 (discrete distributions) and VI-2 (continuous distributions).

Binomial Distribution

The binomial distribution applies when a physical process is characterized by a series of independent events (also called Bernoulli trials) each of which can have only one of two results (i.e. success or failure) and the probabilities for the two results are constant. For example it applies to processes such as the left-right scattering in polarization experiments or the tossing of a coin. Depending on the way the description of a binomial process is formulated, it also leads to the following distributions:

(a) Geometric distribution

This gives the probability that exactly k trails will needed before the first success is obtained. This probability is simply that of k-1 failures followed by a success.

(b) Negative binomial or Pascal distribution

This gives the probability that k failures will be observed during a series of trials for r successes. Alternately it gives the probability that exactly r+k trails will be needed for r successes.

Hypergeometric Distribution

The hypergeometric distribution is similar to the binomial distribution, but applies when the number of trials is not negligible compared to the total population of events. For example, when doing destructive testing of a finite sample. The binomial distribution does not apply because the probability of success depends on the number of preceding trails. The binomial distribution would apply for non-destructive testing with replacement of the tested item back to the sample population before the next trial. The hypergeometric distribution is often encountered in industrial quality control. Note that the variance depends on the number of trials. The binomial distribution can be safely used as an approximation when $n/N \leq 0.10$.

Some Discrete Probability Functions and Their Properties

Distribution	*Probability Function*	*Parameters and Conditions*	*Mean* $E(X)$	*Variance* $Var(X)$	*Moment-Generating Function* $\Psi_X(\theta) = E(e^{\theta x})$
Binomial	$P_X(k) = \binom{n}{k} p^k q^{n-k}$ $(k = 0, 1, \ldots, n)$	$n = 1, 2, \ldots$ $p + q = 1 \quad 0 \leq p \leq 1$	np	npq	$(pe^{\theta} + q)^n$
Poisson	$P_X(k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$ $(k = 0, 1, \ldots)$	$\lambda > 0$	λ	λ	$\exp[\lambda(e^{\theta} - 1)]$
Hypergeometric	$P_X(k) = \dfrac{\binom{Np}{k}\binom{Nq}{n-k}}{\binom{N}{n}}$ $(k = 0, 1, \ldots, n)$	$n \leq N \quad N = 1, 2, \ldots$ $p = 0, \frac{1}{N}, \frac{2}{N}, \ldots, 1$ $p + q = 1 \quad 0 \leq p \leq 1$	np	$npq\left(\dfrac{N-n}{N-1}\right)$	†
Geometric	$P_X(k) = pq^{k-1}$ $(k = 1, 2, \ldots)$	$p + q = 1 \quad 0 \leq p \leq 1$	$\dfrac{1}{p}$	$\dfrac{q}{p^2}$	$\dfrac{pe^{\theta}}{1 - qe^{\theta}}$
Negative binomial	$P_X(k) = \binom{-r}{k} p^r(-q)^k$ $(k = 0, 1, \ldots)$	$r > 0$ $p + q = 1 \quad 0 \leq p \leq 1$	$\dfrac{rq}{p}$	$\dfrac{rq}{p^2}$	$p^r(1 - qe^{\theta})^{-r}$

†See M. G. Kendall, *Advanced Theory of Statistics*, Charles Griffin, London, 1948, p. 127.

Some Important Continuous Probability Laws and Their Properties

Distribution	*Density Function*	*Parameters and Conditions*	*Mean*	*Variance*	*Moment-Generating Function* $\Psi_X^{(\theta)}$
Normal	$f_X(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right]$ $-\infty < t < \infty$	$-\infty < \mu < \infty$ $\sigma > 0$	μ	σ^2	$e^{[\theta\mu + (\theta^2\sigma^2/2)]}$
Exponential	$f_X(t) = \lambda e^{-\lambda t} \quad t \geq 0$ $= 0 \quad t < 0$	$\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - \theta}$
Uniform over Interval a to b	$f_X(t) = \frac{1}{b-a} \quad a < t < b$ $= 0$ otherwise	$-\infty < a < b < \infty$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{\theta b} - e^{\theta a}}{\theta(b-a)}$
Gamma	$f_X(t) = \frac{\lambda(\lambda t)^{k-1} e^{-\lambda t}}{\Gamma(k)} \quad t \geq 0$ $= 0$ otherwise	$\lambda > 0$ $k > 0$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$\left(1 - \frac{\theta}{\lambda}\right)^{-k}$
Chi-square	$f(\chi^2) = \frac{(\chi^2)^{[(n-2)/2]} e^{-\chi^2/2}}{2^{n/2}\Gamma(n/2)} \quad \chi^2 \geq 0$ $= 0$ otherwise	$n = 1, 2, \ldots$ (called degrees of freedom)	n	$2n$	$(1 - 2\theta)^{-(n/2)}$

Multinomial Distribution

The multinomial distribution is the generalization of the binomial distribution to the case where there are more than two possibilities for the result of a trial. The binomial formulae for the mean and variance apply if q is interpreted as the sum of probabilities for all results but one with probability p. The probability distribution is given by

$$P(n_1, n_2, \ldots\ldots, n_l) = \frac{N!}{\prod_{i=1}^{k} (n_i!)} \prod_{i=1}^{k} p_i^{n_i} ,$$

where p_i is the probability of a result of type i and k is the number of possible outcomes The covariance for results i and j is

$$\mathrm{cov}(n_i\, n_j) = -N\, p_i\, p_j \, .$$

The multinomial distribution applies to the partitioning of events into histograms.

Poisson Distribution

The Poisson distribution gives the probability of finding exactly k events during a given time (interval, etc) if the events are independent and have a constant probability. The Poisson distribution describes many natural processes (radioactive decay), and is often used in queueing theory and quality control. Along with the Normal or Gaussian distribution, it is the most frequently encountered. The Poisson distribution has only one parameter λ, and both the mean and variance equal λ. A useful property of Poisson distributions is that the sum of two Poisson random variables with means μ and $ß$ is also a Poisson random variable with mean $(\mu+ß)$.

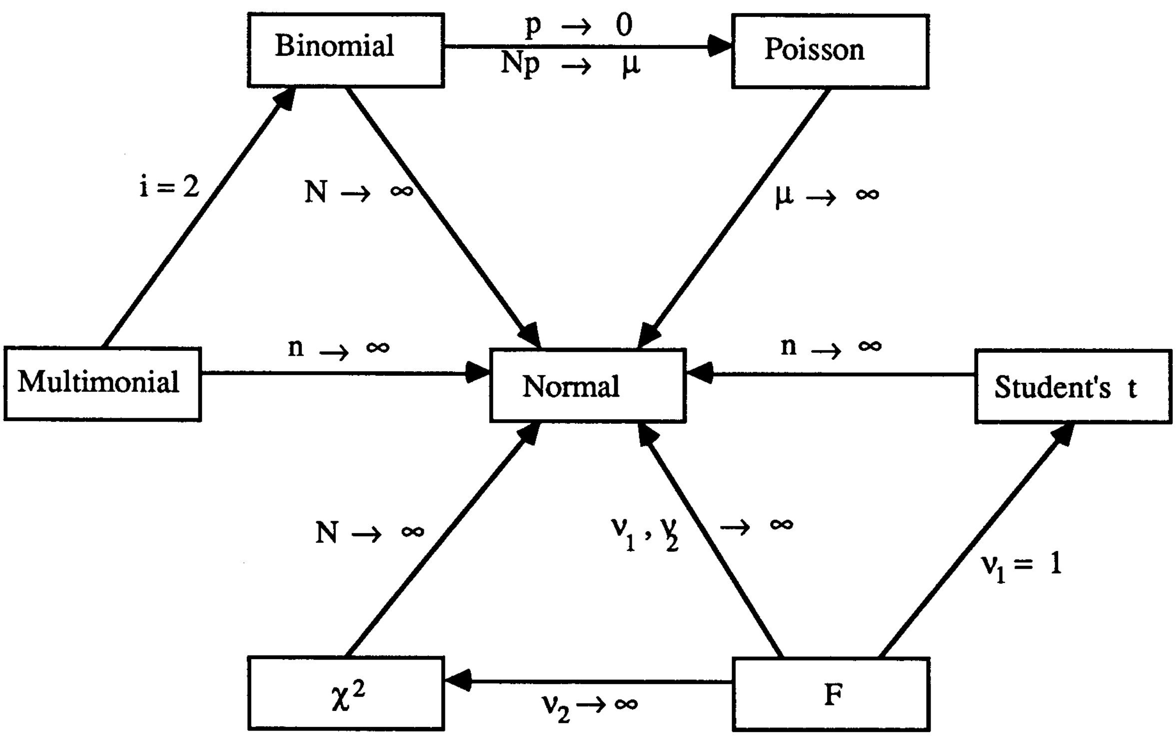
The Poisson distribution can be used as an approximation for the binomial for $p \le 0.1$, independent of the number of trials. The Poisson distribution is closely related to the exponential and χ^2 distributions.

Normal or Gaussian Distribution

The normal distribution is the most important because it is so frequently encountered. Error distributions, if they result from the combination of many small effects, are usually well described by a normal distribution. This is a result of the central limit theorem which states that the sum of a large number of random variables, independent of their original distributions provided they have finite variance, tends towards a normal distribution.

The probability that a random observation x will have $|x-\mu| < \delta$ is 1 - CL (two tailed case). If the standard deviation σ is known, commonly chosen intervals and the probability that they will contain an unknown mean μ are summarized below.

CL (%)	δ	CL (%)	δ
31.7	$1\,\sigma$	10	$1.64\,\sigma$
4.6	$2\,\sigma$	5	$1.96\,\sigma$
0.3	$3\,\sigma$	1	$2.58\,\sigma$
6×10^{-3}	$4\,\sigma$	0.1	$3.29\,\sigma$
6×10^{-5}	$5\,\sigma$	0.01	$3.89\,\sigma$



Asymptotic relationship between the Normal Distribution and other common statistical distributions

Multi-dimensional Normal Distribution

The multi-dimensional Normal distribution is a generalization of the Normal distribution to the case where there are more than one jointly distributed random variables, each of which follows a normal distribution. If the random variables are denoted by **X**, with means $\mu = (\mu_1, \mu_2, ...,\mu_n)$ and covariances σ_{ij} as described above, then the multi-dimensional normal probability distribution has the form

$$f(X) = \frac{(2\pi)^{-n/2}}{|V|^{1/2}} \exp\left\{ \frac{1}{2}(X-\mu)^T V^{-1} (X-\mu) \right\}.$$

The covariance matrix V has elements σ_{ij}.

Exponential Distribution

The exponential distribution is strongly linked to the Poisson distribution. It gives the time (or interval) between successive events if the events arrive randomly in time at a constant average rate. In this case, the number of events in a specified interval has a Poisson distribution. The exponential distribution is often used to describe the time to failure in reliability applications. This distribution also has the property that it has no memory, that is, the probability of an event occurring in the next instant is independent of the length of time one has already waited for an event to occur.

Another distribution related to the exponential is the Gamma distribution. It describes the time necessary to obtain *k* independent events if they occur at a constant rate. It is sometimes called the Erlang distribution when *k* is integer.

χ^2 Distribution

The $\chi^2(n)$ distribution describes the sum of *n* independent, normally distributed, reduced random variable. *n* is referred to as the number of degrees of freedom. A reduced random variable is one of the form $t = (x-\mu)/\sigma$. The probability density of $\chi^2(n)$ has its maximum at $\chi^2 = n-2$. The sum of two χ^2 variables with degrees of freedom N and M is also a χ^2 variable with N+M degrees of freedom.

An important relation between the χ^2 and the Poisson distributions is

$$P(k \le N_o | \mu) = 1 - P[\chi^2(2N_o+2) < 2\mu] .$$

This allows confidence intervals for one distribution to be determined from tabulations of the other. The χ^2 confidence level (one tail) for various numbers of degrees of freedom is shown in following figure.

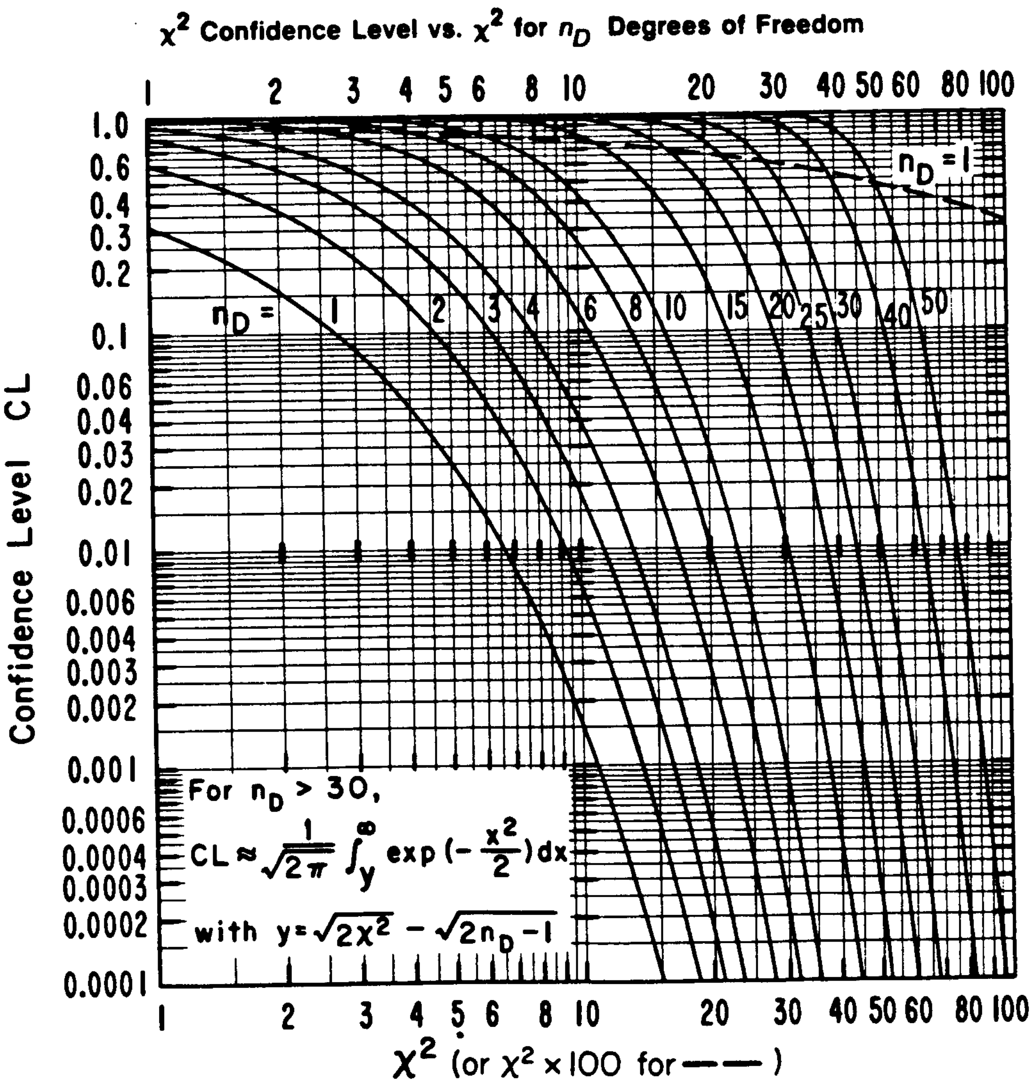
For large *n*, χ^2 becomes normally distributed with mean *n* and variance $2n$. The reduced variable is

$$y_1 = (\chi^2 - n)/\sqrt{2n} .$$

A better approximation is that χ and not χ^2 becomes normally distributed. Specifically,

$$y_2 = \sqrt{2\chi^2} - \sqrt{2n-1}$$

approaches normality with mean zero and unit standard deviation. For small CL's y_2 is much more accurate than y_1.



When an observed value n_o of n is small the usual approximation of specifying limits as $n_o \pm \sqrt{n_o}$ is poor. The following table gives some convenient CL's for n_o up to 20.

	Two tail CL's				One tail CL's		
	68.3%		95%		68%	90%	95%
n_o	lower	upper	lower	upper	limit	limit	limit
0	0	1.15	0	3.00	1.15	2.30	3.00
1	0.268	2.50	0.042	4.77	2.35	3.89	4.74
2	0.864	3.68	0.304	6.40	3.5	5.32	6.30
3	1.55	5.15	0.713	7.95	5.0	6.68	7.75
4	2.29	6.40	1.21	9.43	6.2	7.99	9.15
5	3.06	7.63	1.76	10.9	7.2	9.27	10.51
6	3.85	8.84	2.35	12.3	8.2	10.53	11.84
7	4.65	10.0	2.97	13.6	9.2	11.77	13.15
8	5.47	11.2	3.62	15.0	10.3	13.00	14.44
9	6.30	12.4	4.29	16.3	11.4	14.21	15.71
10	7.14	13.5	4.98	17.6	12.4	15.41	16.96
12	8.84	15.8	6.40	20.2			
14	10.6	18.1	7.86	22.7			
16	12.3	20.4	9.36	25.2			
18	14.1	22.6	10.9	27.7			
20	15.8	24.8	12.4	30.1			

Part of this table is taken from the RPP-86 and part from **Errors in Experiments with small Numbers of Events**, O. Helene, Nucl. Instr. Meth. **228** (1984) 120. The one tail 68% CL column was obtained by interpolation from the figure on the previous page and is less accurate. Helene obtained the limits by evaluating the integral

$$CL = \int_{a^-}^{a^+} f(a)\, da$$

where the probability distribution of a, the mean of the Poisson distribution, is

$$f(a) = \frac{e^{-a} a^N}{N!} .$$

Note that a is a continuous variable and *N* is the parameter.

3. Statistics

Samples from a common population

When one is presented with a set of N independent data, x_k, one wants to make some inference about the parameters associated with the data. Assume that the x_k are obtained from the same population with mean μ and variance σ^2. Then unbiased estimates of the mean and variance are

$$<x> = \frac{1}{N}\sum_{k=1}^{N} x_k \quad \text{and}$$

$$s^2 = \frac{1}{N-1}\sum_{k=1}^{N}(x_k - <x>)^2 = \frac{1}{N-1}\left\{\sum_{k=1}^{N} x_k^2 - N<x>^2\right\}.$$

The standard error of the mean of the sample is $s/\sqrt{N}$. This error can be made arbitrarily small by increasing the size of the sample whereas s^2 is the estimate of the variance of the population and tends asymptotically to σ^2.

Combination of several measurements

If the x_k are independent estimate of the same μ, and have known σ_k, then the weighted average of the x_k is

$$<x> = \frac{1}{w}\sum_{k=1}^{N} w_k x_k \ ,$$

where $w_k = 1/\sigma^2_k$ and $w = \Sigma\, w_k$. The choice of weight minimizes the variance of the estimate of the mean.

The case where the individual measurements provide estimate simultaneously for parameters of random variables that are not independent is also interesting. Let θ be a set of m parameters that are to be determined and the estimates are N values θ'_k with covariance matrices $\mathbf{V}_k$. The weight matrices are then $\mathbf{W} = \mathbf{V}_k^{-1}$ and the best estimate of θ is given by.

$$\theta' = \left[\sum_{i=1}^{k} W_i\, \Theta'_i\right]\left[\sum_{i=1}^{k} W_i\right]^{-1} .$$

The covariance matrix of the combined data is

$$V' = \left[\sum_{i=1}^{k} W_i\right]^{-1} .$$

In the case that the data sets θ'_k do not contain information about one or more of the parameters, the corresponding rows and columns of the weight matrices are filled with zeros.

Error Propagation

Suppose that one has a set of random variables $\mathbf{X} = (X_1, X_2, X_3, \ldots, X_n)$ and one wishes to evaluate the value and error of some function f of these variables. The mean and variance of $f(\mathbf{X})$ are approximately (to second order in $(\mathbf{x} - \mu)$)

$$\bar{f} \approx f(\langle x \rangle) + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} V_{jk}\left[\frac{\partial^2 f}{\partial x_j\,\partial x_k}\right]_{x=\langle x \rangle},$$

$$\overline{(f-\bar{f})^2} \approx \sum_{j=1}^{n}\sum_{k=1}^{k}\left[\frac{\partial f}{\partial x_j}\right]_{x=\langle x \rangle}\left[\frac{\partial f}{\partial x_k}\right]_{x=\langle x \rangle}.$$

Tests of Hypotheses

Once a set of data is obtained it is often desired to make a conclusion about the information obtained when compared to some particular hypothesis. In this case we test the null hypothesis (H_o) against an alternate hypothesis (H_1). We then decide at some confidence level α whether to accept or reject the null hypothesis. The possibilities are summarized below.

	Decision Probability	
Decision → / Actual case ↓	Choose H_o	Choose H_1
H_o true	$1 - \alpha$	α
H_1 false	β	$1 - \beta$

In this table

α is the probability of rejecting the null hypothesis when in fact it is true. This is referred to as an error of the first kind, or the loss.

β is the probability of accepting the null hypothesis when in fact it is false. This is an error of the second kind, or the contamination.

Clearly the choice of the decision procedure is to keep α to a minimum. In some cases keeping β to a minimum may be more important.

Tables VI-3 through VI-6 summarize the most common tests of hypotheses. In the tables the following definitions are used:

Z_α = refers to the cumulative distribution function (CDF) N(x) for the normal distribution. It is the value of a reduced normal variable such that the $N(Z_\alpha) = \alpha$.

$t_{\alpha\,;\,n}$ is the value of a reduced normal variable such that the CDF of the Student's *t* distribution for n degrees of freedom equals α $T(t_{\alpha\,;\,n}) = \alpha$.

$\chi^2_{\alpha\,;\,n}$ is the value of a χ^2 variable with n degrees of freedom such that the CDF evaluated at that point equals α.

$F_{\alpha\,;\,n1,\,n2}$ is the value of am F variate with n1 and n2 degrees of freedom such that the CDF evaluated at that point equals α. This distribution is used for the comparison of variances using ratios.

The tables summarizing the tests of hypotheses are followed by

(1) Table VI-7 for the cumulative Normal distribution function
(2) Table VI-8 for the cumulative Student's *t* distribution function
(3) Table VI-9 for the cumulative χ^2 distribution function
(4) Table VI-10 for the cumulative F distribution function
(5) Table VI-11 for the cumulative Poisson distribution function
(6) Table VI-12 for the cumulative Binomial distribution function

Summary of Tests of Hypotheses About the Parameters of the Normal Distribution

Hypotheses	*Unknown Parameters*	*Rejection Rule*	*Choice of Sample Size n for Given α and β*
$H_0: \mu = \mu_0$ $H_1: \mu = \mu_1 \neq \mu_0$	μ	$\lvert\bar{x} - \mu_0\rvert > Z_{1-\alpha/2}\frac{\sigma_0}{\sqrt{n}}$	$n \simeq \frac{\sigma_0^2(Z_{1-\alpha/2} + Z_{1-\beta})^2}{(\mu_1 - \mu_0)^2}$
$H_0: \mu = \mu_0$ $H_1: \mu = \mu_1 > \mu_0$	μ	$\bar{x} > \mu_0 + Z_{1-\alpha}\frac{\sigma_0}{\sqrt{n}}$	$n = \frac{\sigma_0^2(Z_{1-\alpha} + Z_{1-\beta})^2}{(\mu_1 - \mu_0)^2}$
$H_0: \mu = \mu_0$ $H_1: \mu = \mu_1 < \mu_0$	μ	$\bar{x} < \mu_0 + Z_\alpha\frac{\sigma_0}{\sqrt{n}}$	$n = \frac{\sigma_0^2(Z_{1-\alpha} + Z_{1-\beta})^2}{(\mu_0 - \mu_1)^2}$
$H_0: \mu = \mu_0$ $H_1: \mu = \mu_1 \neq \mu_0$	μ, σ	$\lvert\bar{x} - \mu_0\rvert > t_{1-\alpha/2;n-1}\frac{s}{\sqrt{n}}$	$d = \left\lvert\frac{\mu_1 - \mu_0}{\sigma_0}\right\rvert$ Enter Fig. 9.9 or 9.10.
$H_0: \mu = \mu_0$ $H_1: \mu = \mu_1 > \mu_0$	μ, σ	$\bar{x} > \mu_0 + t_{1-\alpha;n-1}\frac{s}{\sqrt{n}}$	$d = \frac{\mu_1 - \mu_0}{\sigma_0}$ Enter Fig. 9.11 or 9.12.
$H_0: \mu = \mu_0$ $H_1: \mu = \mu_1 < \mu_0$	μ, σ	$\bar{x} < \mu_0 - t_{\alpha;n-1}\frac{s}{\sqrt{n}}$	$d = \frac{\mu_0 - \mu_1}{\sigma_0}$ Enter Fig. 9.11 or 9.12.
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 = \sigma_1^2 \neq \sigma_0^2$	μ, σ	$(n-1)s^2/\sigma_0^2 < \chi^2_{\alpha/2;n-1}$ or $> \chi^2_{1-\alpha/2;n-1}$	$\frac{\chi^2_{\beta;n-1}}{\chi^2_{1-\alpha/2;n-1}} \cong \frac{\sigma_0^2}{\sigma_1^2}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 = \sigma_1^2 > \sigma_0^2$	μ, σ	$\frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{1-\alpha;n-1}$	$\frac{\chi^2_{\beta;n-1}}{\chi^2_{1-\alpha;n-1}} = \frac{\sigma_0^2}{\sigma_1^2}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 = \sigma_1^2 < \sigma_0^2$	μ, σ	$\frac{(n-1)s^2}{\sigma_0^2} < \chi^2_{\alpha;n-1}$	$\frac{\chi^2_{\beta;n-1}}{\chi^2_{\alpha;n-1}} = \frac{\sigma_0^2}{\sigma_1^2}$

Summary of Point and Interval Estimates of the Parameters of the Normal Distribution

Parameters	*Conditions*	*Point Estimates*	*Confidence Interval*
μ	known σ	$\bar{x}$	$\bar{x} - Z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}$
μ	unknown σ	$\bar{x}$	$\bar{x} - t_{1-\alpha/2;\,n-1}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{1-\alpha/2;\,n-1}\frac{s}{\sqrt{n}}$
$\mu_X - \mu_Y$	known σ_X, σ_Y	$\bar{x} - \bar{y}$	$\bar{x} - \bar{y} - Z_{1-\alpha/2}\sqrt{\frac{\sigma_X^2}{n\dagger} + \frac{\sigma_Y^2}{m\ddagger}} \le \mu_X - \mu_Y \le \bar{x} - \bar{y} + Z_{1-\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$
$\mu_X - \mu_Y$	$\sigma_X = \sigma_Y = \sigma$ σ unknown	$\bar{x} - \bar{y}$	$\bar{x} - \bar{y} - t_{1-\alpha/2;\,n+m-2}\sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}}\sqrt{\frac{1}{n} + \frac{1}{m}}$ $\le \mu_X - \mu_Y \le \bar{x} - \bar{y} + t_{1-\alpha/2;\,n+m-2}\sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}}\sqrt{\frac{1}{n} + \frac{1}{m}}$
$\mu_X - \mu_Y$	$\sigma_X \ne \sigma_Y$ both unknown	$\bar{x} - \bar{y}$	$\bar{x} - \bar{y} - t_{1-\alpha/2;\,\nu\S}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}} \le \mu_X - \mu_Y \le \bar{x} - \bar{y} + t_{1-\alpha/2;\,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$
σ^2		s^2	$\frac{(n-1)s^2}{\chi^2_{1-\alpha/2;\,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{\alpha/2;\,n-1}}$
$\frac{\sigma_X^2}{\sigma_Y^2}$		$\frac{s_X^2}{s_Y^2}$	$\frac{s_X^2}{s_Y^2}\frac{1}{F_{1-\alpha/2;\,n-1,\,m-1}} \le \frac{\sigma_X^2}{\sigma_Y^2} \le \frac{s_X^2}{s_Y^2}\frac{1}{F_{\alpha/2;\,n-1,\,m-1}}$

† n = Number of observations taken from X population.
‡ m = Number of observations taken from Y population.

$$\S\ \nu = \frac{(s_X^2/n + s_Y^2/m)^2}{\frac{(s_X^2/n)^2}{n+1} + \frac{(s_Y^2/m)^2}{m+1}} - 2$$

Summary of Tests of Hypotheses About the Parameters of Two Independent Normal Distributions

Hypothesis	*Unknown Parameters and Conditions*	*Rejection Rule*	*For Given α and β, Choice of Sample Size $n = m$*
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta \neq 0$	μ_X, μ_Y	$\lvert\bar{x} - \bar{y}\rvert > Z_{1-\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$	$n = m \cong \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2(\sigma_X^2 + \sigma_Y^2)}{\Delta^2}$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta > 0$	μ_X, μ_Y	$\bar{x} - \bar{y} > Z_{1-\alpha}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$	$n = m = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2(\sigma_X^2 + \sigma_Y^2)}{\Delta^2}$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta < 0$	μ_X, μ_Y	$\bar{x} - \bar{y} < Z_{\alpha}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$	$n = m = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2(\sigma_X^2 + \sigma_Y^2)}{\Delta^2}$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta \neq 0$	μ_X, μ_Y σ_X, σ_Y $\sigma_X = \sigma_Y$	$\lvert\bar{x} - \bar{y}\rvert > t_{1-\alpha/2;\, n+m-2}\, s_p{}^{\dagger}\sqrt{\frac{1}{n} + \frac{1}{m}}$	$d = \frac{\lvert\mu_X - \mu_Y\rvert}{2\sigma}$ Enter Fig. 9.9 or 9.10 to obtain n'. $n = m = \frac{n' + 1}{2}$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta > 0$	μ_X, μ_Y σ_X, σ_Y $\sigma_X = \sigma_Y$	$\bar{x} - \bar{y} > t_{1-\alpha;\, n+m-2}\, s_p\sqrt{\frac{1}{n} + \frac{1}{m}}$	$d = \frac{\mu_X - \mu_Y}{2\sigma}$ Enter Fig. 9.11 or 9.12 to obtain n'. $n = m = \frac{n' + 1}{2}$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta < 0$	μ_X, μ_Y σ_X, σ_Y $\sigma_X = \sigma_Y$	$\bar{x} - \bar{y} < t_{\alpha;\, n+m-2}\, s_p\sqrt{\frac{1}{n} + \frac{1}{m}}$	$d = \frac{\mu_X - \mu_Y}{2\sigma}$ Enter Fig. 9.11 or 9.12 to obtain n'. $n = m = \frac{n' + 1}{2}$

† $s_p = \sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n + m - 2}}$

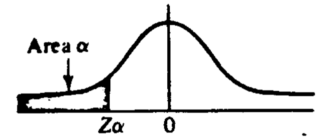
Summary of Tests of Hypotheses About the Parameters of Two Independent Normal Distributions

Hypothesis	*Unknown Parameters and Conditions*	*Rejection Rule*	*For Given α and β, Choice of Sample Size $n = m$*
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta \neq 0$	μ_X, μ_Y σ_X, σ_Y $\sigma_X \neq \sigma_Y$	$\lvert \bar{x} - \bar{y} \rvert > t_{1-\alpha/2;\, \nu}$‡$\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$	
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta > 0$	μ_X, μ_Y σ_X, σ_Y $\sigma_X \neq \sigma_Y$	$\bar{x} - \bar{y} > t_{1-\alpha;\, \nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$	
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y = \Delta < 0$	μ_X, μ_Y σ_X, σ_Y $\sigma_X \neq \sigma_Y$	$\bar{x} - \bar{y} < t_{\alpha;\, \nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$	
$H_0: \sigma_X^2 = \sigma_Y^2$ $H_1: \sigma_X^2 \neq \sigma_Y^2$	μ_X, μ_Y σ_X, σ_Y	$\frac{s_X^2}{s_Y^2} > F_{1-\alpha/2;\, n-1,\, m-1}$ $\quad \frac{s_X^2}{s_Y^2} < F_{\alpha/2;\, n-1,\, m-1}$	$\beta = \left\{ \frac{\sigma_Y^2}{\sigma_X^2} \frac{1}{F_{1-\alpha/2;\, n-1,\, m-1}} \leq F_{n-1,\, n-1} \leq \frac{\sigma_Y^2}{\sigma_X^2} F_{1-\alpha/2;\, n-1,\, m-1} \right\}$ $n = m$ is found by successive trials from F tables
$H_0: \sigma_X^2 = \sigma_Y^2$ $H_1: \sigma_X^2 > \sigma_Y^2$	μ_X, μ_Y σ_X, σ_Y	$\frac{s_X^2}{s_Y^2} > F_{1-\alpha;\, n-1,\, m-1}$	$\frac{F_{\beta;\, n-1,\, n-1}}{F_{1-\alpha;\, n-1,\, n-1}} = \frac{\sigma_Y^2}{\sigma_X^2}$ $n = m$ is found by successive trials from F tables
$H_0: \sigma_X^2 = \sigma_Y^2$ $H_1: \sigma_X^2 < \sigma_Y^2$	μ_X, μ_Y σ_X, σ_Y	$\frac{s_X^2}{s_Y^2} < F_{1-\alpha;\, n-1,\, m-1}$	$\frac{F_{1-\beta;\, n-1,\, n-1}}{F_{\alpha;\, n-1,\, n-1}} = \frac{\sigma_Y^2}{\sigma_X^2}$ $n = m$ is found by successive trials from F tables

‡ $\nu = \frac{(s_X^2/n + s_Y^2/m)^2}{[(s_X^2/n)^2/(n+1)] + [(s_Y^2/m)^2/(m+1)]} - 2$

The Cumulative Standardized Normal Distribution Function†

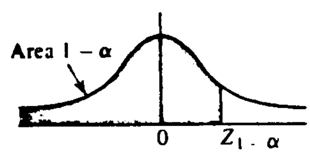
(Note: $.0^{2}1350 = .001350$)



Entry $= P\{Z < Z_\alpha\} = \alpha$

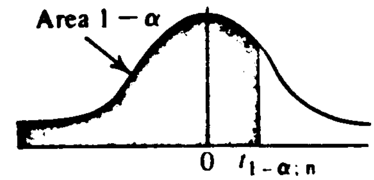
Z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
-.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-.2	.4207	.4168	.4219	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	$.0^{2}9903$	$.0^{2}9642$	$.0^{2}9387$	$.0^{2}9137$	$.0^{2}8894$	$.0^{2}8656$	$.0^{2}8424$
-2.4	$.0^{2}8198$	$.0^{2}7976$	$.0^{2}7760$	$.0^{2}7549$	$.0^{2}7344$	$.0^{2}7143$	$.0^{2}6947$	$.0^{2}6756$	$.0^{2}6569$	$.0^{2}6387$
-2.5	$.0^{2}6210$	$.0^{2}6037$	$.0^{2}5868$	$.0^{2}5703$	$.0^{2}5543$	$.0^{2}5386$	$.0^{2}5234$	$.0^{2}5085$	$.0^{2}4940$	$.0^{2}4799$
-2.6	$.0^{2}4661$	$.0^{2}4527$	$.0^{2}4396$	$.0^{2}4269$	$.0^{2}4145$	$.0^{2}4025$	$.0^{2}3907$	$.0^{2}3793$	$.0^{2}3681$	$.0^{2}3573$
-2.7	$.0^{2}3467$	$.0^{2}3364$	$.0^{2}3264$	$.0^{2}3167$	$.0^{2}3072$	$.0^{2}2980$	$.0^{2}2890$	$.0^{2}2803$	$.0^{2}2718$	$.0^{2}2635$
-2.8	$.0^{2}2555$	$.0^{2}2477$	$.0^{2}2401$	$.0^{2}2327$	$0^{2}2256$	$.0^{2}2186$	$.0^{2}2118$	$.0^{2}2052$	$.0^{2}1988$	$.0^{2}1926$
-2.9	$.0^{2}1866$	$.0^{2}1807$	$.0^{2}1750$	$.0^{2}1695$	$.0^{2}1641$	$.0^{2}1589$	$.0^{2}1538$	$.0^{2}1489$	$.0^{2}1441$	$.0^{2}1395$
-3.0	$.0^{2}1350$	$.0^{2}1306$	$.0^{2}1264$	$.0^{2}1223$	$.0^{2}1183$	$.0^{2}1144$	$.0^{2}1107$	$.0^{2}1070$	$.0^{2}1035$	$.0^{2}1001$

(Note: $.9^{2}8650 = .998650$)



Entry $= P\{Z < Z_{1-\alpha}\} = 1 - \alpha$

$Z_{1-\alpha}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8661
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.97030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	$.9^{2}0097$	$.9^{2}0358$	$.9^{2}0613$	$.9^{2}0863$	$.9^{2}1106$	$.9^{2}1344$	$.9^{2}1576$
2.4	$.9^{2}1802$	$.9^{2}2024$	$.9^{2}2240$	$.9^{2}2451$	$.9^{2}2656$	$.9^{2}2857$	$.9^{2}3053$	$.9^{2}3244$	$.9^{2}3431$	$.9^{2}3613$
2.5	$.9^{2}3790$	$.9^{2}3963$	$.9^{2}4132$	$.9^{2}4297$	$.9^{2}4457$	$.9^{2}4614$	$.9^{2}4766$	$.9^{2}4915$	$.9^{2}5060$	$.9^{2}5201$
2.6	$.9^{2}5339$	$.9^{2}5473$	$.9^{2}5604$	$.9^{2}5731$	$.9^{2}5855$	$.9^{2}5975$	$9^{2}6093$	$.9^{2}6207$	$.9^{2}6319$	$.9^{2}6427$
2.7	$.9^{2}6533$	$.9^{2}6636$	$.9^{2}6736$	$.9^{2}6833$	$.9^{2}6928$	$.9^{2}7020$	$.9^{2}7110$	$.9^{2}7197$	$.9^{2}7282$	$.9^{2}7365$
2.8	$.9^{2}7445$	$.9^{2}7523$	$.9^{2}7599$	$.9^{2}7673$	$.9^{2}7744$	$.9^{2}7814$	$.9^{2}7882$	$.9^{2}7948$	$.9^{2}8012$	$.9^{2}8074$
2.9	$.9^{2}8134$	$.9^{2}8193$	$.9^{2}8250$	$.9^{2}8305$	$.9^{2}8359$	$.9^{2}8411$	$.9^{2}8462$	$.9^{2}8511$	$.9^{2}8559$	$.9^{2}8605$
3.0	$.9^{2}8650$	$.9^{2}8684$	$.9^{2}8736$	$.9^{2}8777$	$.9^{2}8817$	$.9^{2}8856$	$.9^{2}8893$	$.9^{2}8930$	$.9^{2}8965$	$.9^{2}8999$



The t-Distribution†

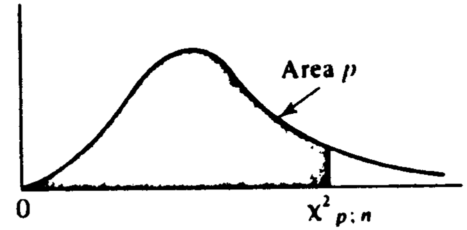
Entry $= t_{1-\alpha;n}$ where $P\{t_n < t_{1-\alpha;n}\} = 1 - \alpha$

$t_{1-\alpha;n} = -t_{\alpha;n}$

n \ $1-\alpha$	.60	.70	.80	.90	.95	.975	.990	.995	.999	.9995
1	.325	.727	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	.289	.617	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	.277	.584	.978	1.638	2.353	3.182	4.541	5.841	10.22	12.94
4	.271	.569	.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	.267	559	.920	1.476	2.015	2.571	3.365	4.032	5.893	6.859
6	.265	.553	.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	.263	.549	.896	1.415	1.895	2.365	2.998	3.499	4.785	5.405
8	.262	.546	.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	.261	.543	.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	.260	.542	.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	.260	.540	.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	.259	.539	.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	.259	.538	.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	.258	.537	.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	.258	.536	.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	.258	.535	.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	.257	.534	.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	.257	.534	.862	1.330	1.734	2.101	2.552	2.878	3.611	3.922
19	.257	.533	.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	.257	.533	.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	.257	.532	.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	.256	.532	.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	.256	.532	.858	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	.256	.531	.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	.256	.531	.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	.256	.531	.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	.256	.531	.855	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	.256	.530	.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	.256	.530	.854	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	.256	.530	.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	.255	.529	.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	.255	.528	.849	1.298	1.676	2.009	2.403	2.678	3.262	3.495
60	.254	.527	.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	.254	.527	.846	1.292	1.664	1.990	2.374	2.639	3.195	3.415
100	.254	.526	.845	1.290	1.660	1.984	2.365	2.626	3.174	3.389
200	.254	.525	.843	1.286	1.653	1.972	2.345	2.601	3.131	3.339
500	.253	.525	.842	1.283	1.648	1.965	2.334	2.586	3.106	3.310
∞	.253	.524	.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

† From W. C. Guenther, *Analysis of Variance:* Englewood Cliffs, N. J., Prentice-Hall, Inc. Used by permission.

The χ^2-Distribution†

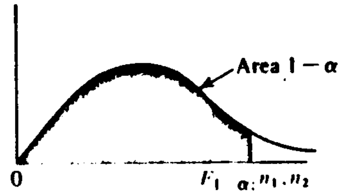


Entry = $\chi^2_{p;n}$ where $P\{\chi^2_n < \chi^2_{p;n}\} = p$

n \ p	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995	0.999
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.1015	0.4549	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	0.0100	0.0201	0.0506	0.1026	0.2107	0.5754	1.386	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	0.0717	0.1148	0.2158	0.3518	0.5844	1.213	2.366	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2070	0.2971	0.4844	0.7107	1.064	1.923	3.357	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	0.4117	0.5543	0.8312	1.145	1.610	2.675	4.351	6.626	9.236	11.07	12.83	15.09	16.75	20.52
6	0.6757	0.8721	1.2373	1.635	2.204	3.455	5.348	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	0.9893	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	1.344	1.646	2.180	2.733	3.490	5.071	7.344	10.22	13.36	15.51	17.53	20.09	21.96	26.12
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	2.603	3.053	3.816	4.575	5.578	7.584	10.34	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	3.074	3.571	4.404	5.226	6.304	8.438	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	3.565	4.107	5.009	5.892	7.041	9.299	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	4.075	4.660	5.629	6.571	7.790	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	4.601	5.229	6.262	7.261	8.547	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	5.142	5.812	6.908	7.962	9.312	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	5.697	6.408	7.564	8.672	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	6.265	7.015	8.231	9.390	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	6.844	7.633	8.907	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	7.434	8.260	9.591	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	8.034	8.897	10.28	11.59	13.24	16.34	20.34	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	8.643	9.542	10.98	12.34	14.04	17.24	21.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	9.260	10.20	11.69	13.09	14.85	18.14	22.34	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	9.886	10.86	12.40	13.85	15.66	19.04	23.34	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	16.47	19.94	24.34	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	11.16	12.20	13.84	15.38	17.29	20.84	25.34	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	11.81	12.88	14.57	16.15	18.11	21.75	26.34	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	12.46	13.56	15.31	16.93	18.94	22.66	27.34	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	13.12	14.26	16.05	17.71	19.77	23.57	28.34	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	13.79	14.95	16.79	18.49	20.60	24.48	29.34	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	29.05	33.66	39.34	45.62	51.80	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	37.69	42.94	49.33	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	46.46	52.29	59.33	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	55.33	61.70	69.33	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	51.17	53.54	57.15	60.39	64.28	71.14	79.33	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	59.20	61.75	65.65	69.13	73.29	80.62	89.33	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	67.33	70.06	74.22	77.93	82.36	90.13	99.33	109.1	118.5	124.3	129.6	135.8	140.2	149.4

†Reproduced with permission from E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, 2nd ed., Vol. 1, pp. 130–131: New York, Cambridge University Press, 1958.

The F-Distribution†



Entry = $F_{1-\alpha;\,n_1,n_2}$
where $P\{F_{n_1,n_2} < F_{1-\alpha;\,n_1,n_2}\} = 1 - \alpha$

df for denominator (n_2)	$1-\alpha$	df for numerator (n_1) 1	2	3	4	5	6	7	8	9	10	11	12
1	.75	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.95	161	200	216	225	230	234	237	239	241	242	243	244
2	.75	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.75	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.95	10.1	9.55	9.28	9.12	9.10	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.75	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.75	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.75	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.77	1.77	1.77	1.77	1.77
	.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.75	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.75	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.64	1.63	1.63	1.62
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.75	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11
10	.75	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.90	3.28	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.75	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.90	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.95	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.99	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.75	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16

† This table is abridged from Table 18 in E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, 2nd ed., Vol. 1: New York: Cambridge University Press, 1958. Used by permission.

—Cont.

df for numerator (n_1) 15	20	24	30	40	50	60	100	120	200	500	∞	$1-\alpha$	df for denominator (n_2)
9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85	.75	1
61.2	61.7	62.0	62.3	62.5	62.7	62.8	63.0	63.1	63.2	63.3	63.3	.90	
246	248	249	250	251	252	252	253	253	254	254	254	.95	
3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48	.75	2
9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49	.90	
19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	.95	
99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	.99	
2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	.75	3
5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13	.90	
8.70	8.66	8.64	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53	.95	
26.9	26.7	26.6	26.5	26.4	26.4	26.3	26.2	26.2	26.2	26.1	26.1	.99	
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	.75	4
3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76	.90	
5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63	.95	
14.2	14.0	13.9	13.8	13.7	13.7	13.7	13.6	13.6	13.5	13.5	13.5	.99	
1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	.75	5
3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	.90	
4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36	.95	
9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02	.99	
1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	.75	6
2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72	.90	
3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67	.95	
7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88	.99	
1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	.75	7
2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47	.90	
3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23	.95	
6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65	.99	
1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58	.75	8
2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29	.90	
3.22	3.15	3.12	3.08	3.04	3.02	3.01	2.97	2.97	2.95	2.94	2.93	.95	
5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86	.99	
1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53	.75	9
2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16	.90	
3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71	.95	
4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31	.99	
1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48	.75	10
2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06	.90	
2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54	.95	
4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91	.99	
1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45	.75	11
2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97	.90	
2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40	.95	
4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60	.99	
1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42	.75	12
2.10	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	.90	
2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30	.95	
4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36	.99	

F-dist'n

df for denominator (n_2)	$1-\alpha$	df for numerator (n_1) 1	2	3	4	5	6	7	8	9	10	11	12
13	.75	1.45	1.54	1.54	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.90	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.95	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.99	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.75	1.44	1.53	1.53	1.52	1.51	1.50	1.48	1.48	1.47	1.46	1.46	1.45
	.90	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.95	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.99	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.75	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.75	1.42	1.51	1.51	1.50	1.48	1.48	1.47	1.46	1.45	1.45	1.44	1.44
	.90	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.95	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.99	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.75	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.90	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.95	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.99	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.75	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.90	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.95	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.99	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.75	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.90	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.95	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.99	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.75	1.40	1.49	1.48	1.46	1.45	1.44	1.42	1.42	1.41	1.40	1.39	1.39
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23
22	.75	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.90	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.95	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.99	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.75	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.75	1.38	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.37	1.37	1.36	1.35
	.90	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.95	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.99	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.75	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.90	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.99	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.99	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90

df for numerator (n_1) 15	20	24	30	40	50	60	100	120	200	500	∞	$1-\alpha$	df for denominator (n_2)
1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.41	1.40	1.40	1.40	.75	13
2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.88	1.86	1.85	1.85	.90	
2.53	2.46	2.42	2.38	2.34	2.31	2.30	2.26	2.25	2.23	2.22	2.21	.95	
3.82	3.66	3.59	3.51	3.43	3.38	3.34	3.27	3.25	3.22	3.19	3.17	.99	
1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.39	1.38	1.38	.75	14
2.01	1.96	1.94	1.91	1.89	1.87	1.86	1.83	1.83	1.82	1.80	1.80	.90	
2.46	2.39	2.35	2.31	2.27	2.24	2.22	2.19	2.18	2.16	2.14	2.13	.95	
3.66	3.51	3.43	3.35	3.27	3.22	3.18	3.11	3.09	3.06	3.03	3.00	.99	
1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36	.75	15
1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76	.90	
2.40	2.33	2.29	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07	.95	
3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87	.99	
1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.35	1.34	1.34	.75	16
1.94	1.89	1.87	1.84	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.72	.90	
2.35	2.28	2.24	2.19	2.15	2.12	2.11	2.07	2.06	2.04	2.02	2.01	.95	
3.41	3.26	3.18	3.10	3.02	2.97	2.93	2.86	2.84	2.81	2.78	2.75	.99	
1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.34	1.34	1.33	1.33	.75	17
1.91	1.86	1.84	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.69	.90	
2.31	2.23	2.19	2.15	2.10	2.08	2.06	2.02	2.01	1.99	1.97	1.96	.95	
3.31	3.16	3.08	3.00	2.92	2.87	2.83	2.76	2.75	2.71	2.68	2.65	.99	
1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32	1.32	.75	18
1.89	1.84	1.81	1.78	1.75	1.74	1.72	1.70	1.69	1.68	1.67	1.66	.90	
2.27	2.19	2.15	2.11	2.06	2.04	2.02	1.98	1.97	1.95	1.93	1.92	.95	
3.23	3.08	3.00	2.92	2.84	2.78	2.75	2.68	2.66	2.62	2.59	2.57	.99	
1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31	1.30	.75	19
1.86	1.81	1.79	1.76	1.73	1.71	1.70	1.67	1.67	1.65	1.64	1.63	.90	
2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.94	1.93	1.91	1.89	1.88	.95	
3.15	3.00	2.92	2.84	2.76	2.71	2.67	2.60	2.58	2.55	2.51	2.49	.99	
1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29	.75	20
1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	.90	
2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84	.95	
3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42	.99	
1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28	.75	22
1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	.90	
2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78	.95	
2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.42	2.40	2.36	2.33	2.31	.99	
1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26	.75	24
1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53	.90	
2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73	.95	
2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21	.99	
1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.26	1.26	1.26	1.25	1.25	.75	26
1.76	1.71	1.68	1.65	1.61	1.59	1.58	1.55	1.54	1.53	1.51	1.50	.90	
2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.75	1.73	1.71	1.69	.95	
2.81	2.66	2.58	2.50	2.42	2.36	2.33	2.25	2.23	2.19	2.16	2.13	.99	
1.33	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25	1.25	1.24	1.24	.75	28
1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48	.90	
2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65	.95	
2.75	2.60	2.52	2.44	2.35	2.30	2.26	2.19	2.17	2.13	2.09	2.06	.99	

F-dist.

df for denominator (n_2)	$1-\alpha$	df for numerator (n_1) 1	2	3	4	5	6	7	8	9	10	11	12
30	.75	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.75	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.90	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.95	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.99	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.75	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.75	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.95	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.75	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.90	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.95	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.99	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.75	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.95	3.84	3.00	2.60	2.37	3.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

df for numerator (n_1) 15	20	24	30	40	50	60	100	120	200	500	∞	$1-\alpha$	df for denominator (n_2)
1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	.75	30
1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46	.90	
2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	.95	
2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01	.99	
1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19	.75	40
1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38	.90	
1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51	.95	
2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80	.99	
1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15	.75	60
1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29	.90	
1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39	.95	
2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60	.99	
1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10	.75	120
1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19	.90	
1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25	.95	
2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38	.99	
1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06	.75	200
1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14	.90	
1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.29	1.26	1.22	1.19	.95	
2.13	1.97	1.89	1.79	1.69	1.63	1.58	1.48	1.44	1.38	1.33	1.28	.99	
1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00	.75	∞
1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00	.90	
1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00	.95	
2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00	.99	

The Poisson Distribution Function†

$$F_X(k) = \sum_{s=0}^{k} \frac{e^{-\lambda}\lambda^s}{s!}$$

λ \ k	0	1	2	3	4	5	6	7	8	9
0.02	980	1,000								
0.04	961	999	1,000							
0.06	942	998	1,000							
0.08	923	997	1,000							
0.10	905	995	1,000							
0.15	861	990	999	1,000						
0.20	819	982	999	1,000						
0.25	779	974	998	1,000						
0.30	741	963	996	1,000						
0.35	705	951	994	1,000						
0.40	670	938	992	999	1,000					
0.45	638	925	989	999	1,000					
0.50	607	910	986	998	1,000					
0.55	577	894	982	998	1,000					
0.60	549	878	977	997	1,000					
0.65	522	861	972	996	999	1,000				
0.70	497	844	966	994	999	1,000				
0.75	472	827	959	993	999	1,000				
0.80	449	809	953	991	999	1 000				
0.85	427	791	945	989	998	1 000				
0.90	407	772	937	987	998	1,000				
0.95	387	754	929	984	997	1,000				
1.00	368	736	920	981	996	999	1,000			
1.1	333	699	900	974	995	999	1,000			
1.2	301	663	879	966	992	998	1,000			
1.3	273	627	857	957	989	998	1,000			
1.4	247	592	833	946	986	997	999	1,000		
1.5	223	558	809	934	981	996	999	1,000		
1.6	202	525	783	921	976	994	999	1,000		
1.7	183	493	757	907	970	992	998	1,000		
1.8	165	463	731	891	964	990	997	999	1,000	
1.9	150	434	704	875	956	987	997	999	1,000	
2.0	135	406	677	857	947	983	995	999	1,000	

† From *Poisson's Exponential Binomial Limit* by E. C. Molina, copyright © 1942 by Litton Educational Publishing, Inc., by permission of Van Nostrand Reinhold Company.

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λ \ k	0	1	2	3	4	5	6	7	8	9
2.2	111	355	623	819	928	975	993	998	1,000	
2.4	091	308	570	779	904	964	988	997	999	1,000
2.6	074	267	518	736	877	951	983	995	999	1,000
2.8	061	231	469	692	848	935	976	992	998	999
3.0	050	199	423	647	815	916	966	988	996	999
3.2	041	171	380	603	781	895	955	983	994	998
3.4	033	147	340	558	744	871	942	977	992	997
3.6	027	126	303	515	706	844	927	969	988	996
3.8	022	107	269	473	668	816	909	960	984	994
4.0	018	092	238	433	629	785	889	949	979	992
4.2	015	078	210	395	590	753	867	936	972	989
4.4	012	066	185	359	551	720	844	921	964	985
4.6	010	056	163	326	513	686	818	905	955	980
4.8	008	048	143	294	476	651	791	887	944	975
5.0	007	040	125	265	440	616	762	867	932	968
5.2	006	034	109	238	406	581	732	845	918	960
5.4	005	029	095	213	373	546	702	822	903	951
5.6	004	024	082	191	342	512	670	797	886	941
5.8	003	021	072	170	313	478	638	771	867	929
6.0	002	017	062	151	285	446	606	744	847	916

λ \ k	10	11	12	13	14	15	16
2.8	1,000						
3.0	1,000						
3.2	1,000						
3.4	999	1,000					
3.6	999	1,000					
3.8	998	999	1,000				
4.0	997	999	1,000				
4.2	996	999	1,000				
4.4	994	998	999	1,000			
4.6	992	997	999	1,000			
4.8	990	996	999	1,000			
5.0	986	995	998	999	1,000		
5.2	982	993	997	999	1,000		
5.4	977	990	996	999	1,000		
5.6	972	988	995	998	999	1,000	
5.8	965	984	993	997	999	1,000	
6.0	957	980	991	996	999	999	1,000

-Cont.

λ \ k	0	1	2	3	4	5	6	7	8	9
6.2	002	015	054	134	259	414	574	716	826	902
6.4	002	012	046	119	235	384	542	687	803	886
6.6	001	010	040	105	213	355	511	658	780	869
6.8	001	009	034	093	192	327	480	628	755	850
7.0	001	007	030	082	173	301	450	599	729	830
7.2	001	006	025	072	156	276	420	569	703	810
7.4	001	005	022	063	140	253	392	539	676	788
7.6	001	004	019	055	125	231	365	510	648	765
7.8	000	004	016	048	112	210	338	481	620	741
8.0	000	003	014	042	100	191	313	453	593	717
8.5	000	002	009	030	074	150	256	386	523	653
9.0	000	001	006	021	055	116	207	324	456	587
9.5	000	001	004	015	040	089	165	269	392	522
10.0	000	000	003	010	029	067	130	220	333	458
	10	11	12	13	14	15	16	17	18	19
6.2	949	975	989	995	998	999	1,000			
6.4	939	969	986	994	997	999	1,000			
6.6	927	963	982	992	997	999	999	1,000		
6.8	915	955	978	990	996	998	999	1,000		
7.0	901	947	973	987	994	998	999	1,000		
7.2	887	937	967	984	993	997	999	999	1,000	
7.4	871	926	961	980	991	996	998	999	1,000	
7.6	854	915	954	976	989	995	998	999	1,000	
7.8	835	902	945	971	986	993	997	999	1,000	
8.0	816	888	936	966	983	992	996	998	999	1,000
8.5	763	849	909	949	973	986	993	997	999	999
9.0	706	803	876	926	959	978	989	995	998	999
9.5	645	752	836	898	940	967	982	991	996	998
10.0	583	697	792	864	917	951	973	986	993	997
	20	21	22							
8.5	1,000									
9.0	1,000									
9.5	999	1,000								
10.0	998	999	1,000							

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λ \ k	0	1	2	3	4	5	6	7	8	9
10.5	000	000	002	007	021	050	102	179	279	397
11.0	000	000	001	005	015	038	079	143	232	341
11.5	000	000	001	003	011	028	060	114	191	289
12.0	000	000	001	002	008	020	046	090	155	242
12.5	000	000	000	002	005	015	035	070	125	201
13.0	000	000	000	001	004	011	026	054	100	166
13.5	000	000	000	001	003	008	019	041	079	135
14.0	000	000	000	000	002	006	014	032	062	109
14.5	000	000	000	000	001	004	010	024	048	088
15.0	000	000	000	000	001	003	008	018	037	070
	10	11	12	13	14	15	16	17	18	19
10.5	521	639	742	825	888	932	960	978	988	994
11.0	460	579	689	781	854	907	944	968	982	991
11.5	402	520	633	733	815	.878	924	954	974	986
12.0	347	462	576	682	772	844	899	937	963	979
12.5	297	406	519	628	725	806	869	916	948	969
13.0	252	353	463	573	675	764	835	890	930	957
13.5	211	304	409	518	623	718	798	861	908	942
14.0	176	260	358	464	570	669	756	827	883	923
14.5	145	220	311	413	518	619	711	790	853	901
15.0	118	185	268	363	466	568	664	749	819	875
	20	21	22	23	24	25	26	27	28	29
10.5	997	999	999	1,000						
11.0	995	998	999	1,000						
11.5	992	996	998	999	1,000					
12.0	988	994	997	999	999	1,000				
12.5	983	991	995	998	999	999	1,000			
13.0	975	986	992	996	998	999	1,000			
13.5	965	980	989	994	997	998	999	1,000		
14.0	952	971	983	991	995	997	999	999	1,000	
14.5	936	960	976	986	992	996	998	999	999	1,000
15.0	917	947	967	981	989	994	997	998	999	1,000

POISSON

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λ \ k	4	5	6	7	8	9	10	11	12	13
16	000	001	004	010	022	043	077	127	193	275
17	000	001	002	005	013	026	049	085	135	201
18	000	000	001	003	007	015	030	055	092	143
19	000	000	001	002	004	009	018	035	061	098
20	000	000	000	001	002	005	011	021	039	066
21	000	000	000	000	001	003	006	013	025	043
22	000	000	000	000	001	002	004	008	015	028
23	000	000	000	000	000	001	002	004	009	017
24	000	000	000	000	000	000	001	003	005	011
25	000	000	000	000	000	000	001	001	003	006
	14	15	16	17	18	19	20	21	22	23
16	368	467	566	659	742	812	868	911	942	963
17	281	371	468	564	655	736	805	861	905	937
18	208	287	375	469	562	651	731	799	855	899
19	150	215	292	378	469	561	647	725	793	849
20	105	157	221	297	381	470	559	644	721	787
21	072	111	163	227	302	384	471	558	640	716
22	048	077	117	169	232	306	387	472	556	637
23	031	052	082	123	175	238	310	389	472	555
24	020	034	056	087	128	180	243	314	392	473
25	012	022	038	060	092	134	185	247	318	394
	24	25	26	27	28	29	30	31	32	33
16	978	987	993	996	998	999	999	1,000		
17	959	975	985	991	995	997	999	999	1,000	
18	932	955	972	983	990	994	997	998	999	1,000
19	893	927	951	969	980	988	993	996	998	999
20	843	888	922	948	966	978	987	992	995	997
21	782	838	883	917	944	963	976	985	991	994
22	712	777	832	877	913	940	959	973	983	989
23	635	708	772	827	873	908	936	956	971	981
24	554	632	704	768	823	868	904	932	953	969
25	473	553	629	700	763	818	863	900	929	950
	34	35	36	37	38	39	40	41	42	43
19	999	1,000								
20	999	999	1,000							
21	997	998	999	999	1,000					
22	994	996	998	999	999	1,000				
23	988	993	996	997	999	999	1,000			
24	979	987	992	995	997	998	999	999	1,000	
25	966	978	985	991	994	997	998	999	999	1,000

***The Binomial Distribution Function*†**

$$F_X(x) = \sum_{k=0}^{x} \binom{n}{k} p^k (1-p)^{n-k}$$

n	x	p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7182	0.6480	0.5748	0.5000
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6562
	4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980

† Source: This table is extracted from *Tables of the Binomial Probability Distribution:* Washington, D.C., National Bureau of Standards, Applied Mathematics Series, No. 6, U.S. Department of Commerce, 1952.

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n	x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0232	0.0107
	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
	2	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327
	3	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133
	4	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744
	5	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000
	6	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256
	7	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867
	8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9941
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
	2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
	3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
	4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
	5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
	6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
	7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
13	0	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001
	1	0.8646	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0017
	2	0.9755	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0112
	3	0.9969	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0461
	4	0.9997	0.9935	0.9658	0.9009	0.7940	0.6543	0.5005	0.3530	0.2279	0.1334
	5	1.0000	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.2905
	6	1.0000	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000
	7	1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.7095
	8	1.0000	1.0000	1.0000	0.9998	0.9990	0.9960	0.9874	0.9679	0.9302	0.8666
	9	1.0000	10.000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9922	0.9797	0.9539
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9959	0.9888
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001
	1	0.8470	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009

BINOMIAL

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n	x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
14	2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065
	3	0.9958	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287
	4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898
	5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120
	6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953
	7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047
	8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9917	0.9757	0.9417	0.8811	0.7880
	9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9961	0.9886	0.9713
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9978	0.9935
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
15	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
	4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
	5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
	6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
	7	1.0000	1.0000	0.9996	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9849	0.9578	0.9050	0.8182	0.6964
	9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
	10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.10000
16	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	0.0000
	1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003
	2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021
	3	0.9930	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106
	4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384
	5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051
	6	1.0000	0.9995	0.9944	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272
	7	1.0000	0.9999	0.9989	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018
	8	1.0000	1.0000	0.9998	0.9985	0.9925	0.9743	0.9329	0.8577	0.7441	0.5982
	9	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9771	0.9417	0.8759	0.7728
	10	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9938	0.9809	0.9514	0.8949
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9851	0.9616
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9965	0.9894
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9979
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245

—*Cont.*

n	*x*	*p* = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
17	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471	0.0717
	6	1.0000	0.9992	0.9917	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1662
	7	1.0000	0.9999	0.9983	0.9891	0.9598	0.8954	0.7872	0.6405	0.4743	0.3145
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9873	0.9617	0.9081	0.8166	0.6855
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9968	0.9880	0.9652	0.9174	0.8338
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9970	0.9894	0.9699	0.9283
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9914	0.9755
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9936
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9988
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0004	0.0001	0.0000	0.0000
	1	0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0046	0.0013	0.0003	0.0001
	2	0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0236	0.0082	0.0025	0.0007
	3	0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0783	0.0328	0.0120	0.0038
	4	0.9985	0.9718	0.8794	0.7164	0.5187	0.3327	0.1886	0.0942	0.0411	0.0154
	5	0.9998	0.9936	0.9581	0.8671	0.7175	0.5344	0.3550	0.2088	0.1077	0.0481
	6	1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.5491	0.3743	0.2258	0.1189
	7	1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.7283	0.5634	0.3915	0.2403
	8	1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.8609	0.7368	0.5778	0.4073
	9	1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.9403	0.8653	0.7473	0.5927
	10	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9788	0.9424	0.8720	0.7597
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9938	0.9797	0.9463	0.8811
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9986	0.9942	0.9817	0.9519
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9846
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000	0.0000
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0004
	3	0.9868	0.8850	0.6841	0.4551	0.2630	0.1332	0.0591	0.0230	0.0077	0.0022
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4878	0.3169	0.1796
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238
	9	1.0000	1.0000	0.9999	0.9984	0.9911	0.9674	0.9125	0.8139	0.6710	0.5000
	10	1.0000	1.0000	1.0000	0.9997	0.9977	0.9895	0.9653	0.9115	0.8159	0.6762
	11	1.0000	1.0000	1.0000	1.0000	0.9995	0.9972	0.9886	0.9648	0.9129	0.8204
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9884	0.9658	0.9165
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9969	0.9891	0.9682
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9972	0.9904
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9978
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

BINOMIAL —*Cont.*

n	*x*	*p* = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
20	0	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
	1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
	2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
	3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
	4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
	5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
	6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
	7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
	8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
	9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000