Structure of the constituent quark and quark distributions of hadrons*

Katsuhiko Suzuki†

Physik-Department, Technische Universität München,

D-85747 Garching, Germany

Abstract

We study the structure of constituent quarks by dressing bare quarks with the Goldstone bosons and its implications for quark distribution functions of hadrons $f_1(x)$, $g_1(x)$ and $h_1(x)$. In particular we discuss effects of the dressing on the nucleon spin structure, and find that contributions to chiral-odd $h_1(x)$ is quite different from those to $g_1(x)$, which can be measured in the semi-inclusive polarized deep inelastic scattering.

1 Introduction

Much of the success of the low energy hadron phenomenology is built on the constituent quark picture. Constituent quarks, which may be understood as quasi-particles of the non-perturbative QCD vacuum, play an important role to describe hadron properties. This success naturally leads us to study the quark distribution function observed in deep inelastic scattering within the constituent quark picture. In fact, the constituent quark models have been applied to calculate the structure function by several groups[1]. For the large Bjorken-x region valence constituent quark picture should work well as suggested by the quark counting rule. However, the valence quark description is not

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†Alexander von Humboldt fellow, e-mail: ksuzuki@physik.tu-muenchen.de

enough to explain the intermediate and small-x behavior of the structure function. In ref. [1], inclusion of the pion dressing and higher mass spectator process produces a substantial renormalization and provides a correct small-x behavior for $f_1(x)$ structure function. In this talk, we extend it to the spin dependent twist-2 structure functions $g_1(x)$ and $h_1(x)$[2].

At the scale below 1GeV, the relevant degrees of freedom are assumed to be constituent quarks (CQ) and Goldstone (GS) bosons. Once the bare quarks are dressed by GS bosons (π, K and η), we can write the constituent u-quark Fock state as

$$|U\rangle = \sqrt{Z}\,|u_0\rangle + a_\pi \left|d\pi^+\right\rangle + \frac{a_\pi}{2}\left|u\pi^0\right\rangle + a_K\left|sK^+\right\rangle + \frac{a_\eta}{6}\,|u\eta\rangle \ , \tag{1}$$

where $|u_0\rangle$ is the bare u-quark state and Z the renormalization constant for the bare state. Similar expressions can be written for other quarks. Using the SU(3) chiral quark model [5], we evaluate corrections to quark distributions illustrated in Fig.1. The GS boson fluctuation generates the renormalization of the bare state as well as depolarization effects on the CQ spin structure by emitting the GS boson in a relative P-wave state. Since we use the Infinite Momentum Frame, the factorization of the subprocess is automatic. The unpolarized u-quark distribution in the nucleon is written as

$$u(x) = Zu_0(x) + P_{u\pi/d}\otimes d_0 + V_{u/\pi}\otimes P_{\pi\, d/u}\otimes u_0 + \frac{1}{2}P_{u\,\pi/u}\otimes u_0 + \cdots \ , \tag{2}$$

where $u_0(x)$ and $d_0(x)$ are bare quark distributions in the nucleon, and $\otimes$ denotes the convolution integral. $P(y)_{j\,\alpha/i}$ is the splitting function which gives the probability to find a CQ j carrying the light-cone momentum fraction y together with a spectator GS boson ($\alpha = \pi, K, \eta$), both of which coming from a parent CQ i. $V_{i/\alpha}$ is the i-quark distribution in the GS boson α. $P(y)_{j\,\alpha/i}$ is given by

$$P(y)_{j\,\alpha/i} = \frac{1}{8\pi^2}\left(\frac{g_A\,\bar{m}}{f}\right)^2 \int dk_T^2 \frac{(m_j - m_i y)^2 + k_T^2}{y^2(1-y)\left[m_i^2 - M_{j\alpha}^2\right]^2}, \tag{3}$$

where m_i, m_j, m_α are the particle masses, $M_{j\,\alpha}^2 = (m_j^2 + k_T^2)/y + (m_\alpha^2 + k_T^2)/(1-y)$ and $\bar{m} = (m_i + m_j)/2$. The integral (3) requires a momentum cutoff to set the scale of the chiral effective theory, and we adopt the exponential cutoff, $\exp[(m_i^2 - M_{j\alpha}^2)/4\Lambda^2]$. We use $g_A = 1$, $m_u = 360$MeV and $m_s = 570$MeV in the following calculations.

We also calculate contributions to the spin structure function in terms of the spin-dependent splitting functions $\Delta P(y)_{j\,\alpha/i}$. The process (b) never contributes to $g_1(x)$, because the GS bosons carry no spin.

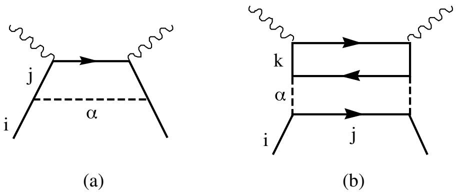

Figure 1: GS boson dressing

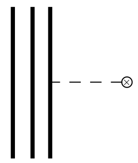

Figure 2: Pion cloud contribution

2 Gottfried sum and nucleon spin structure in CQM

Let us discuss the Gottfried sum and the nucleon spin structure in the chiral constituent quark model, as first done by Eichten *et al.*[4]. By using the dressed quark distributions, the Gottfried sum is given by

$$GSR = \int_0^1 \frac{dx}{x}\ [F_2^p(x) - F_2^n(x)] = \frac{1}{3}\left(1 - 2\langle P_\pi\rangle\right)\ . \tag{4}$$

where $\langle P_\pi\rangle = \int_0^1 dy P_\pi$, the first moment of the splitting function. We fix the cutoff to reproduce the empirical value of the Gottfried sum, and find GSR $\simeq 0.23$ ($Z = 0.67$) with $\Lambda = 1.4$GeV. This value should be understood as an upper bound of the cutoff in this model, based on the assumption that the violation of the Gottfried sum is entirely given by the GS boson dressing alone.

We next study the nucleon spin structure. The spin fraction of the constituent u-quark is modified from their bare quark values as follows:

$$\Delta u = Z\Delta u_0 + \frac{1}{2}\langle\Delta P_\pi\rangle\,\Delta u_0 + \langle\Delta P_\pi\rangle\,\Delta d_0 + \frac{1}{6}\langle\Delta P_\eta\rangle\,\Delta u_0\ , \tag{5}$$

where Δq_0 are the spin fractions of the bare quarks. Similar expressions can be written for d-quark by replacing (u, d) with (d, u). We find that the first moment of the spin-dependent splitting function is small and negative, $\langle\Delta P_\pi\rangle = -0.06$, which means depolarization caused by the P-wave coupling to the GS bosons is a significant effect.

As a first rough estimate, we start from the naive $SU(6)$ quark model values, $\Delta u_0 = \frac{4}{3}$, $\Delta d_0 = -\frac{1}{3}$. Inserting these values into eq. (5), we obtain $\Delta u = 0.86$,

$\Delta d = -0.29$, $\Delta s = -0.006$, and $\Sigma = 0.56$. Considerable part of the CQ spin is transfered to the orbital angular momentum of the GS bosons, but these values are still far from the empirical data. If we allow ourselves to vary the momentum cutoff, it would be possible to obtain a value around $\Sigma \sim 0.3$. However, the agreement with the nucleon axial-vector coupling constant $G_A = \Delta u - \Delta d$ is then lost. We therefore conclude that the nucleon spin problem cannot be solved by the GS boson dressing alone, but the depolarization effects are nevertheless significant.

Let us investigate some necessary further steps. Up to now we deal only with the $SU(3)$ octet of GS bosons to build up the CQ structure, but the constituent quark already has non-trivial spin structure due to the $U(1)_A$ axial anomaly of QCD. Also, the pion cloud directly contributes to the axial vector current (Fig.2). The relativistic effects of the quark motion in the nucleon sould be also taken into account. Including all these contributions we find in the chiral limit ($m_\pi = 0$) $\Delta u = 0.93$, $\Delta d = -0.43$, $\Delta s = -0.12$, and $\Sigma = 0.21$[2], which are now reasonably consistent with the empirical values. Although we cannot draw strong conclusion from the model dependent analysis, the present results combined with other contributions considered above go altogether into the right direction of providing a reasonable description of the nucleon spin structure.

3 Chiral-odd structure function $h_1(x)$

We now focus on the chiral-odd transversity spin structure function $h_1(x)$. The chiral-odd $h_1(x)$ distribution corresponds to a target helicity-flip amplitude in the helicity basis, and provides a correlation between the left- and right-handed quarks, which may tell us more about the chiral dynamics of QCD. In the non-relativistic limit $h_1(x) = g_1(x)$, and $h_1(x)$ is slightly larger than that of $g_1(x)$ in the simple relativistic quark model.

We use the same technique to calculate the splitting function $\delta P(x)$ for $h_1(x)$ structure function, and find that a relation

$$P(x) + \Delta P(x) = 2\delta P(x) \tag{6}$$

holds among the GS boson corrections, which implies a saturation of Soffer's inequality

[6] in the chiral quark model. Note that $\delta P(x)$ is positive in whole x region. The first moment of the transversity splitting function is $\langle \delta P_\pi \rangle = 0.05$, in contrast to the negative value of the logitudinal case, $\langle \Delta P_\pi \rangle = -0.06$.

We shall show how the $g_1(x)$ and $h_1(x)$ distributions are modified by the GS boson fluctuations. To get input distributions we use a quark-diquark model developed previously. Regarding the u-quark case, relative magnitudes of $f_1^u(x)$, $g_1^u(x)$ and $h_1^u(x)$ are not modified very much, though the renormalization reduces these distribution functions from their original ones. The axial charge and tensor charge, first moments of $g_1(x)$ and $h_1(x)$, are modified as, $(\Delta u,\, \delta u) = (1.01, 1.17) \rightarrow (0.65, 0.80)$.

However, effects of the GS boson dressing are apparent in the d-quark case. The d-quark tensor charge is much reduced ($\sim 50\%$), whereas corrections to the axial charge are small, $(\Delta d,\, \delta d) = (-0.25, -0.29) \rightarrow (-0.22, -0.15)$. Recent lattice QCD simulation indicates $\Delta u < \delta u$ and $|\Delta d| > |\delta u|$, which is consistent with our results. We show in Fig.3 $g_1^d(x)$ [a] and $h_1^d(x)$ [b] with bare results by dashed curves and dressed cases by the solid ones. It is clearly seen that $h_1^d(x)$ is reduced very much.

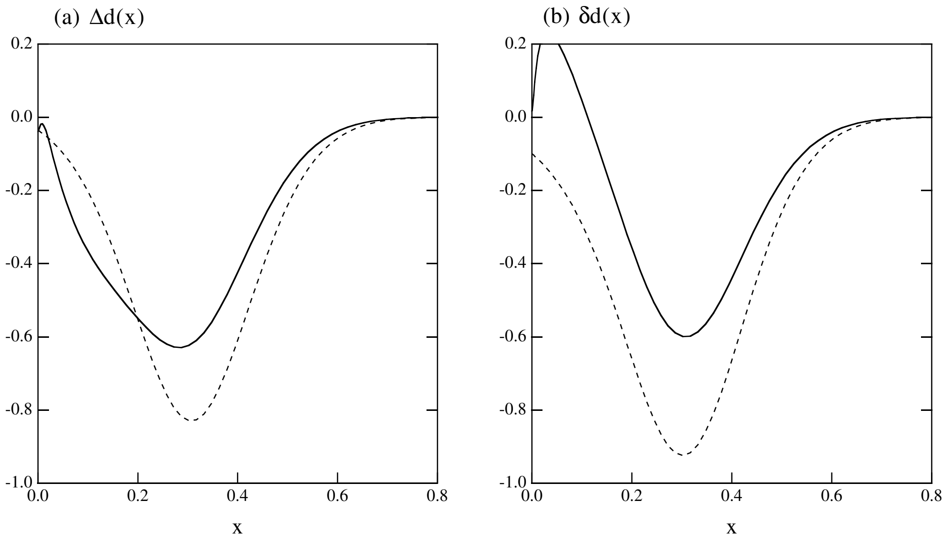

Figure 3: $g_1(x)$ and $h_1(x)$ with the GS boson dressing[2]

We want to point out that such a difference between $g_1(x)$ and $h_1(x)$ can be checked by semi-inclusive polarized deep inelastic scattering. We emphasize here the semi-inclusive DIS on the transversely polarized nucleon with pion production. We can extract $h_1^d(x)/h_1^u(x)$ from the angle weighted spin asymmetry of this process by obserbing π^+ and π^- in the final state[7, 3]. We can also obtain $g_1^d(x)/g_1^u(x)$ using the longitudinally polarized nucleon. We show $h_1^d(x)/h_1^u(x)$ (solid) and $g_1^d(x)/g_1^u(x)$ (dashed) in Fig.4[3]. Due to the existence of the GS boson dressing their shapes are quite different. Simple valence quark model gives almost the same curve for both cases.

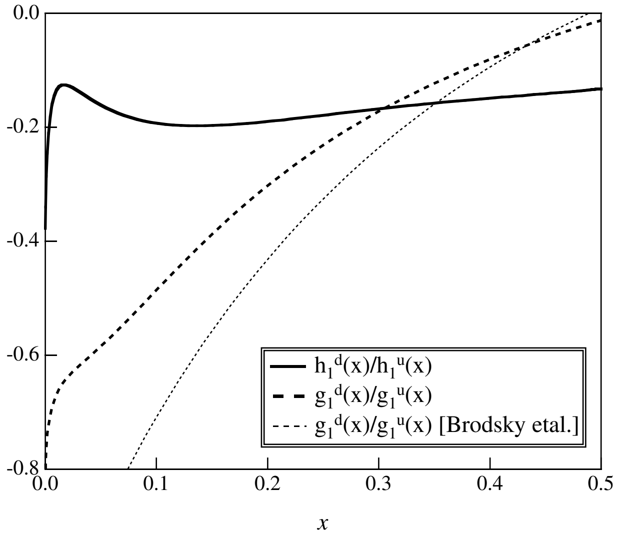

Figure 4: $h_1^d(x)/h_1^u(x)$ and $g_1^d(x)/g_1^u(x)$ with the GS boson dressing[3]

4 Conclusions

We have investigated aspects of constituent quark structure in the deep inelastic processes by introducing the Goldstone boson fluctuations. This dressing generates the renormalization of the bare CQ state and depolarization on the quark spin structure. It is also found that this approach gives a large enhancement of the sea quark distribution in the pion, whose momentum fraction is twice as large as that of the nucleon sea[2]. Experimental tests of $h_1(x)$ distribution function will provide new insights into the role of the constituent quarks as quasi-particles.

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References

[1] S.A. Kulagin, W. Melnitchouk, T. Weigl and W. Weise, Nucl. Phys. **A597** (1996) 515, and references therein

[2] K. Suzuki and W. Weise, TU-Munich preprint (1997)

[3] K. Suzuki and T. Shigetani, Nucl. Phys. **A** in press

[4] E.J. Eichten, I. Hinchliffe and C. Quigg, Phys. Rev. **D45** (1992) 2269

[5] A. Manohar and H. Georgi, Nucl. Phys. **B234** (1984) 189

[6] J. Soffer, Phys. Rev. Lett. **74** (1995) 1992

[7] A.M. Kotzinian and P.J. Mulders, hep-ph/9701330